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Analysis of liquid phase turbulent kinetic energy balance for bubble-train flow

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Contents

- Introduction
  - Phenomenon of bubble-induced turbulence (BIT)
  - Conservation equation of liquid turbulence kinetic energy (k_L equation)

- Analysis of BIT using DNS data
  - TURBIT-VoF computer code
  - Bubble-train flow numerical experiment
  - Methodologies to evaluate balance terms in k_L equation*
  - Budget of k_L equation for bubble-train flow
  - Scrutiny and validation of closure assumptions in modelling BIT

- Conclusions and further steps

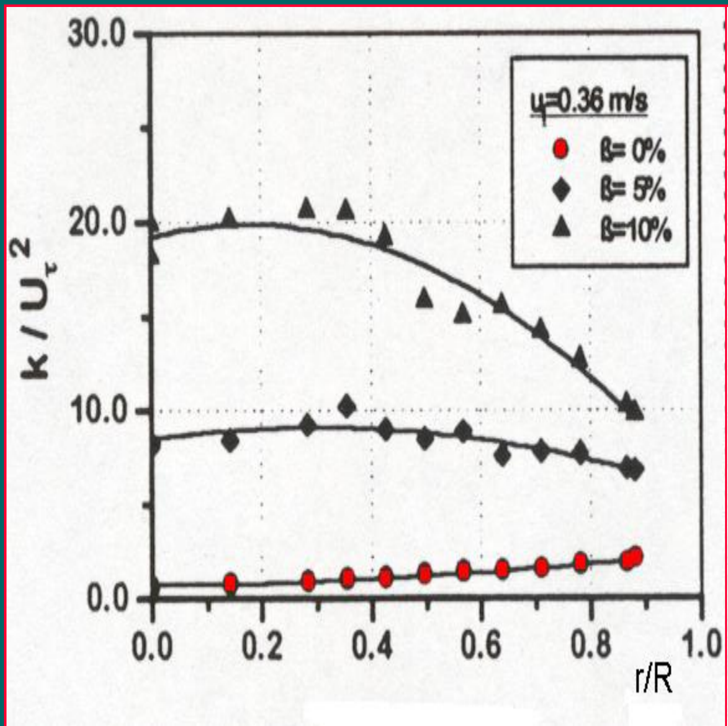
Bubble-induced turbulence

DEFINITION OF BUBBLE-INDUCED TURBULENCE:

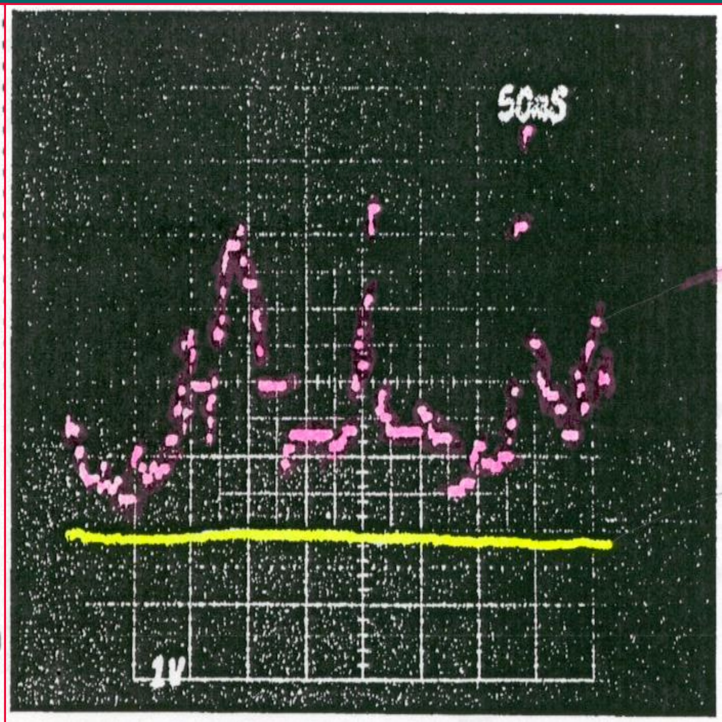
Nonlinearity of the flow; Discrete buoyancy distribution

Bubble wakes; Deformation of interfaces

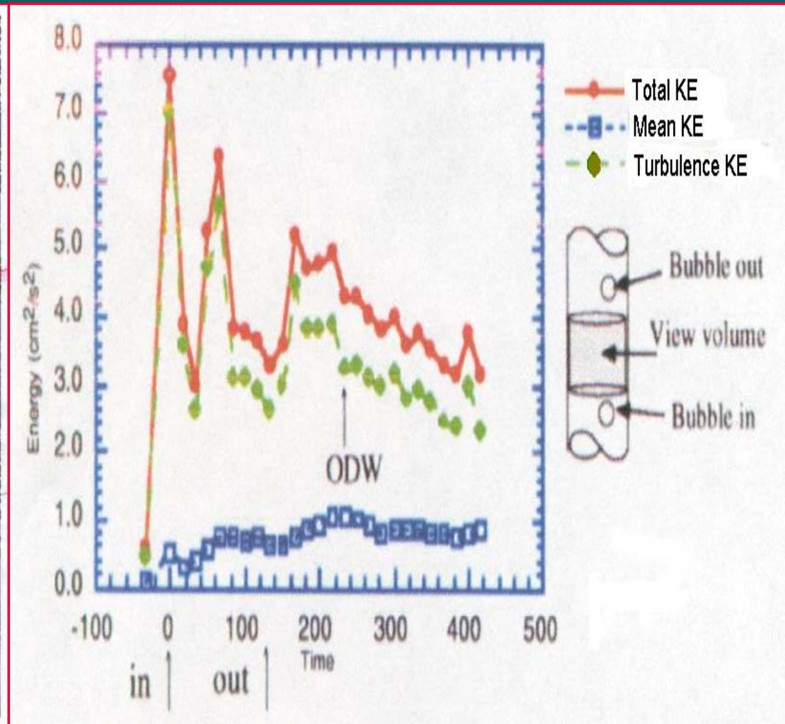
ILLUSTRATIVE EXAMPLES:



High Re liquid flow



Low Re liquid flow



Originally stagnant liquid

Liquid phase turbulence kinetic energy

1. Definition: $k_L = \overline{u'^2_{L\alpha}} / 2$

2. Conservation equation (Kataoka and Serizawa, 1989):

$$\frac{D}{Dt}\left(\overline{\Phi}_L k_L\right) = \underbrace{-\frac{\partial}{\partial x_\alpha}\left(\overline{\Phi}_L \overline{\overline{p'_L u'_{L\alpha}}}\right) - \frac{\partial}{\partial x_\beta}\left(\frac{1}{2}\overline{\Phi}_L \overline{\overline{u'^2_{L\alpha} u'_{L\beta}}}\right) + \frac{1}{Re_{ref}}\frac{\partial}{\partial x_\beta}\left(\overline{\Phi}_L \frac{\partial k_L}{\partial x_\beta}\right)}_{DIFFUSION}$$

$$\underbrace{-\overline{\Phi}_L \overline{\overline{u'_{L\alpha} u'_{L\beta}}} \frac{\partial \overline{\overline{u}}_{L\alpha}}{\partial x_\beta}}_{PRODUCTION} \quad \underbrace{-\frac{1}{Re_{ref}} \overline{\Phi}_L \overline{\overline{\frac{\partial u'_{L\alpha}}{\partial x_\beta}\frac{\partial u'_{L\alpha}}{\partial x_\beta}}}}_{DISSIPATION}$$

$$\underbrace{-\overline{p'_{Lin} u'_{Lin\alpha} n_{Lin\alpha} a_{in}} + \frac{1}{Re_{ref}} \overline{u'_{Lin\alpha} \frac{\partial u'_{Lin\alpha}}{\partial x_\beta} n_{Lin\beta} a_{in}}}_{INTERFACIAL\ TERMS}$$

$\overline{A_L}$ averaging

$\overline{\overline{A_L}} = \overline{A_L \Phi_L} / \overline{\Phi_L}$ phase averaging

$A'_L = A_L - \overline{\overline{A_L}}$ $\quad A'_{Lin} = A_{Lin} - \overline{\overline{A_L}}$ fluctuation

$\overline{\Phi_L}$ liquid volumetric fraction

a_{in} interfacial area concentration

Data on local liquid flow field and interface topology evaluated by DNS !!!

DNS of bubbly flows

TURBIT-VoF computer code developed at IRS, FZK

(1) $$\frac{\partial u_\alpha}{\partial x_\alpha} = 0$$

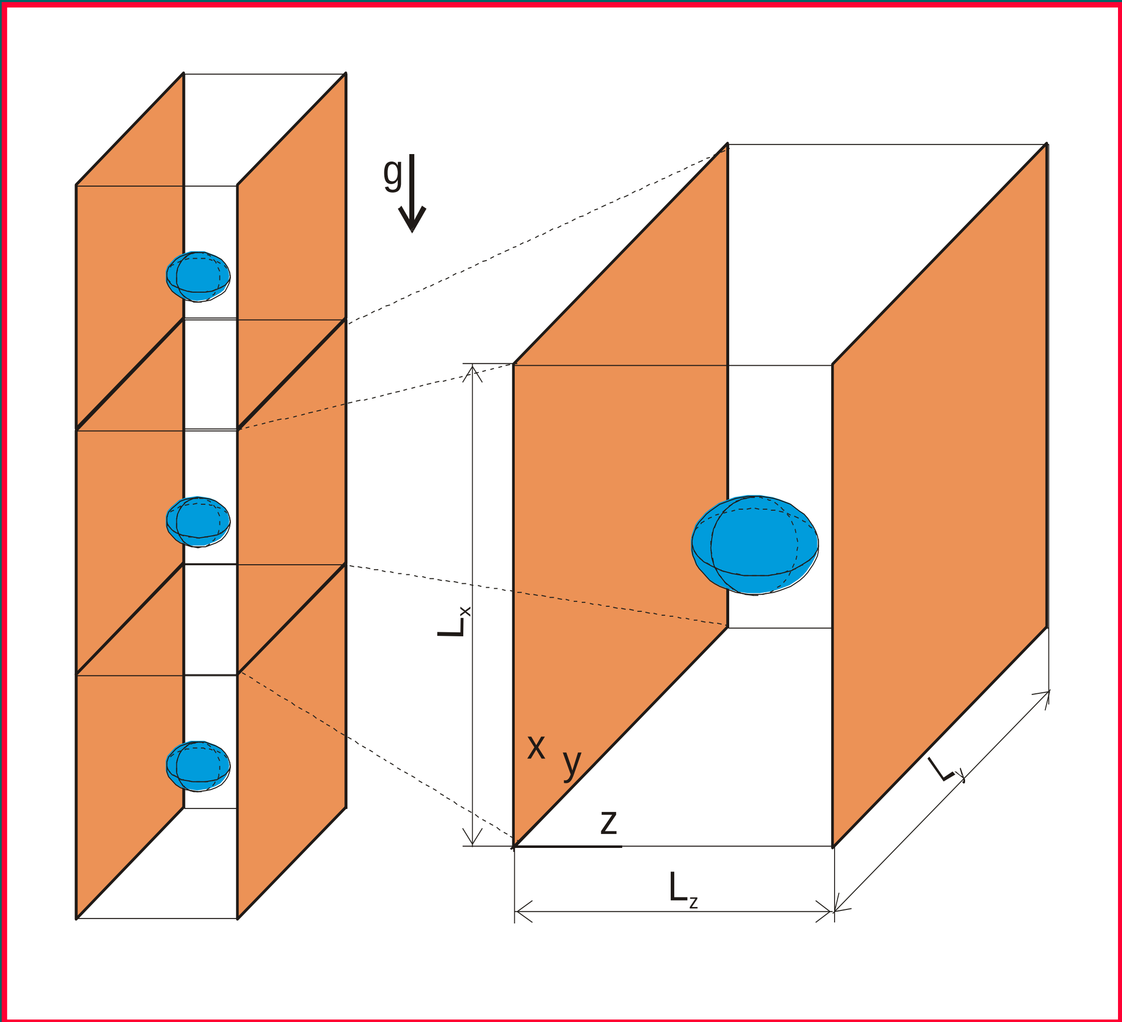
(2) $$\frac{\partial(\rho u_\alpha)}{\partial t} + \frac{\partial(\rho u_\alpha u_\beta)}{\partial x_\beta} = -\frac{\partial p}{\partial x_\alpha} + \frac{1}{Re_{ref}} \frac{\partial \tau_{\alpha\beta}}{\partial x_\beta} - \frac{(1-f)E\ddot{o}_{ref}}{We_{ref}} \frac{g_\alpha^*}{g^*} + \frac{\kappa n_\alpha a_{in}}{We_{ref}}$$

(3) $$\frac{\partial f}{\partial t} + \frac{\partial(u_\alpha f)}{\partial x_\alpha} = 0$$

Homogeneous mixture model in interfacial cells ($0<f<1$): u_R=0, $p_L=p_G$

Interface evolution tracked using Volume-of-Fluid procedure

TURBIT-VoF numerical experiment



Definition of bubble-train flow

Computational domain: 1x1x1
Grid: 64x64x64

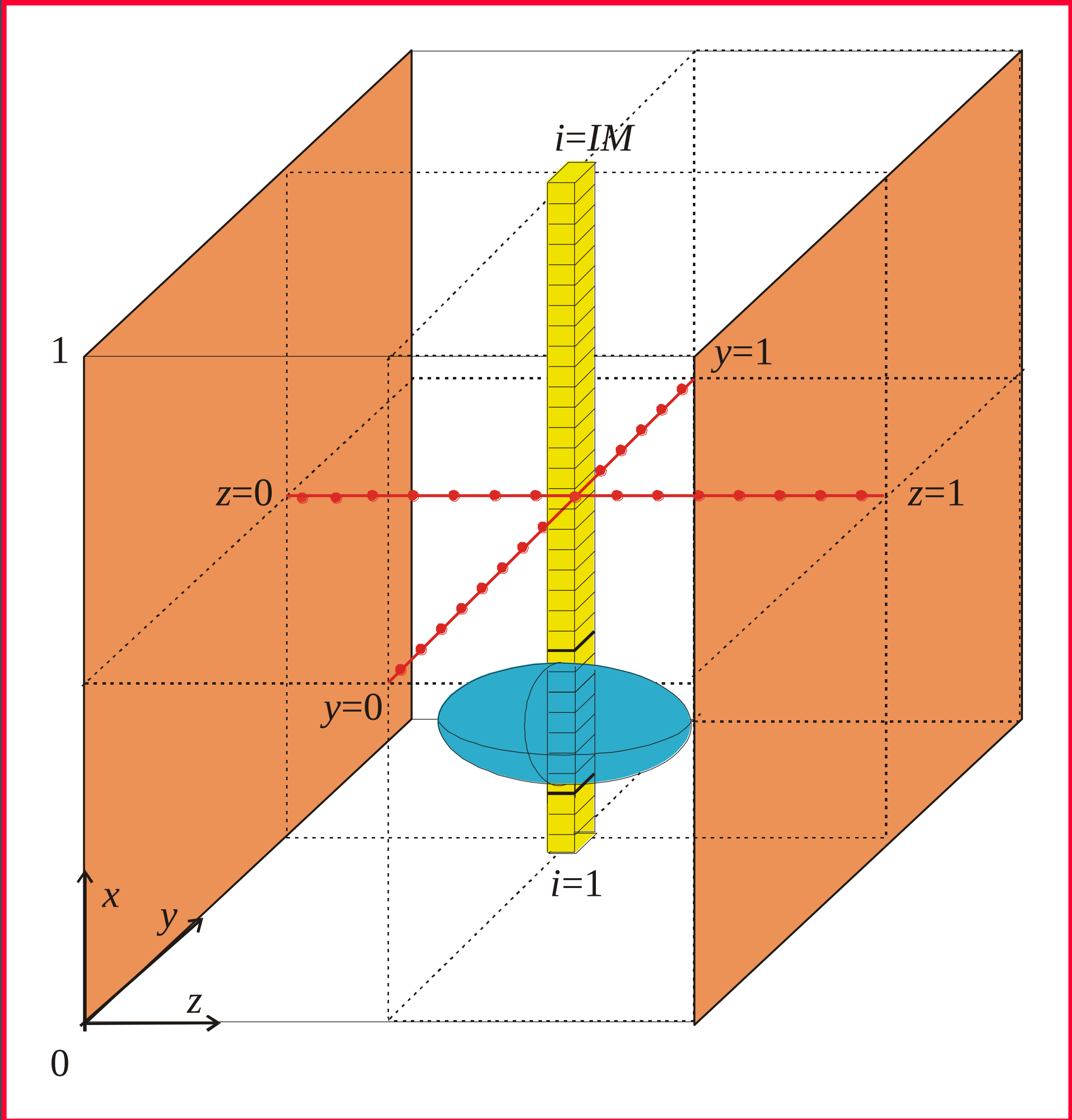
Bubble diameter: 0.25
Gas volumetric fraction: 0.818%

Phase density ratio: 0.5
Bubble Eötvös number: 3.065
Morton number: $3.06 \cdot 10^{-6}$

Time step: $1 \cdot 10^{-4}$
Number of time steps: 65 000
Bubble-path: approx. rectilinear
Bubble Reynolds number: ~125
Quasi-steady flow for time > 5.5

Data on instantaneous 3D flow field and interface topology are available !!!

Averaging procedure



Ergodic theorem valid in *x* direction

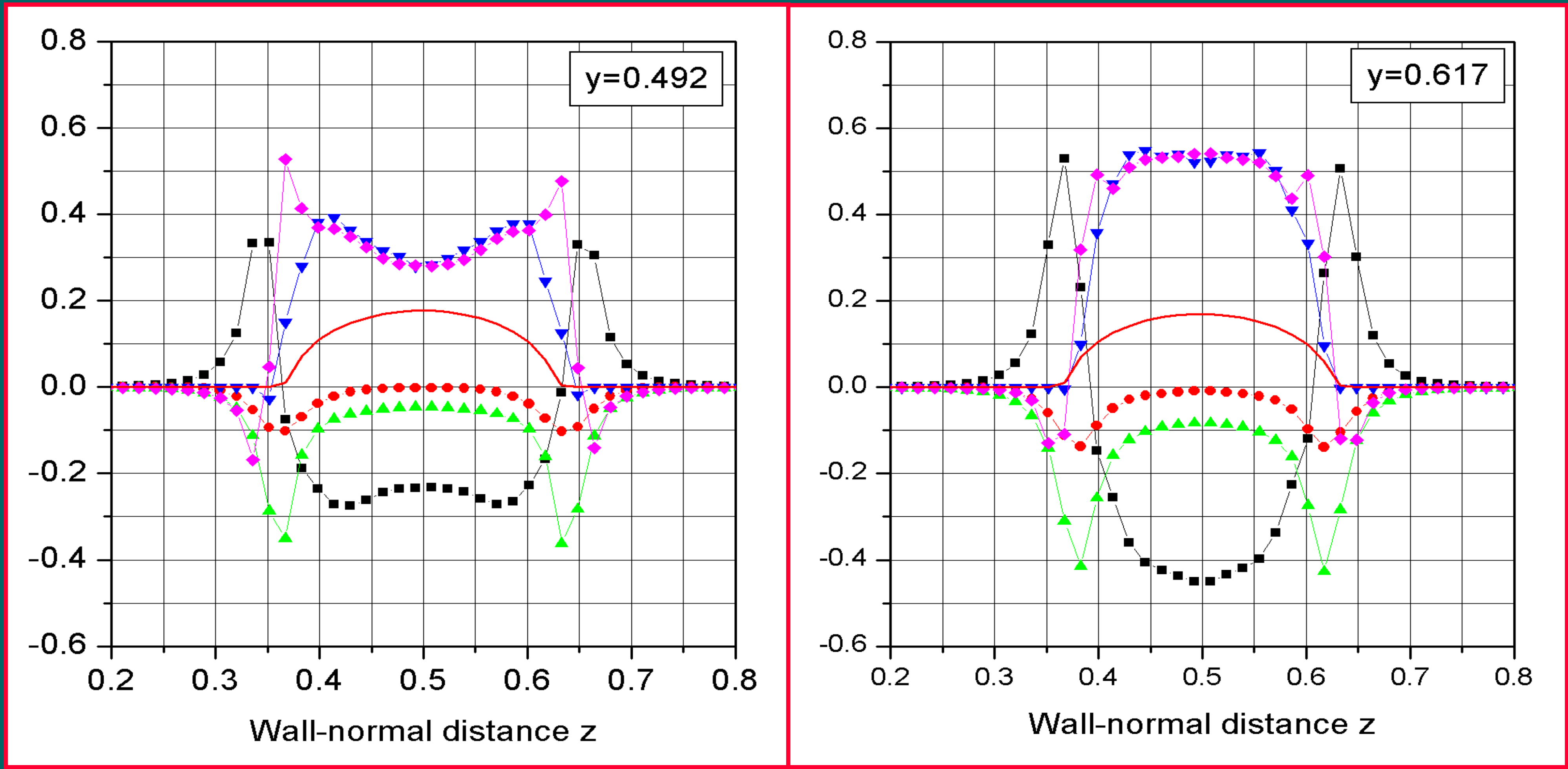
→ Time averaging replaced by spatial-line averaging:

$$\overline{A}_L = \overline{A}^{i}_{L(j,k)} = \frac{\sum_{i=1}^{IM} A_{(i,j,k)}}{IM} \qquad (1)$$

$$\overline{\overline{A}}_L = \overline{\overline{A}}^{i}_{L(j,k)} = \frac{\sum_{i=1}^{IM} f_{(i,j,k)} A_{(i,j,k)}}{\sum_{i=1}^{IM} f_{(i,j,k)}} \qquad (2)$$

Balance terms in exact k_L equation

DIFFUSION (D) PRODUCTION (P) DISSIPATION (ε) INTERFACIAL TERMS $IFT^B=-(D+P+\varepsilon)$



Turbulence in bubbly flow is gained by **interfacial terms**

and lost through **dissipation** and **production** term !!!

Modelling of turbulence in bubbly flows

Use of exact balance terms: validation of closure terms in modelled k_L equation

Models:

1. Laminar
2. Algebraic
3. One-equation (k-l)
4. Two-equation (k-ε)
5. Reynolds-stress (τ-ε)

Modelled k_L equation:

$$\frac{D(\alpha_L k_L)}{Dt} = \underbrace{\frac{\partial}{\partial x_\beta}\left[\alpha_L \nu_L^{eff} \frac{\partial k_L}{\partial x_\beta}\right]}_{DIFFUSION} + \underbrace{\alpha_L \tau_{L\alpha\beta}^{t} \frac{\partial \overline{\overline{u}}_{L\alpha}}{\partial x_\beta}}_{PRODUCTION} - \underset{DISSIPATION}{\alpha_L \varepsilon_L} + \underbrace{IFT^M}_{INTERFACIAL\ TERMS}$$

Modelled versus exact production term

Modelling approaches:

- Pfleger and Becker, 2001
  Grienberger and Hofmann, 1992
  Svedsen et al., 1992

$$P = \alpha_L \left[2\left(\frac{1}{Re_{ref}} + \nu_L^t \right) \overline{S}_{L\alpha\beta} \right] \frac{\partial \overline{u}_{L\alpha}}{\partial x_\beta}$$

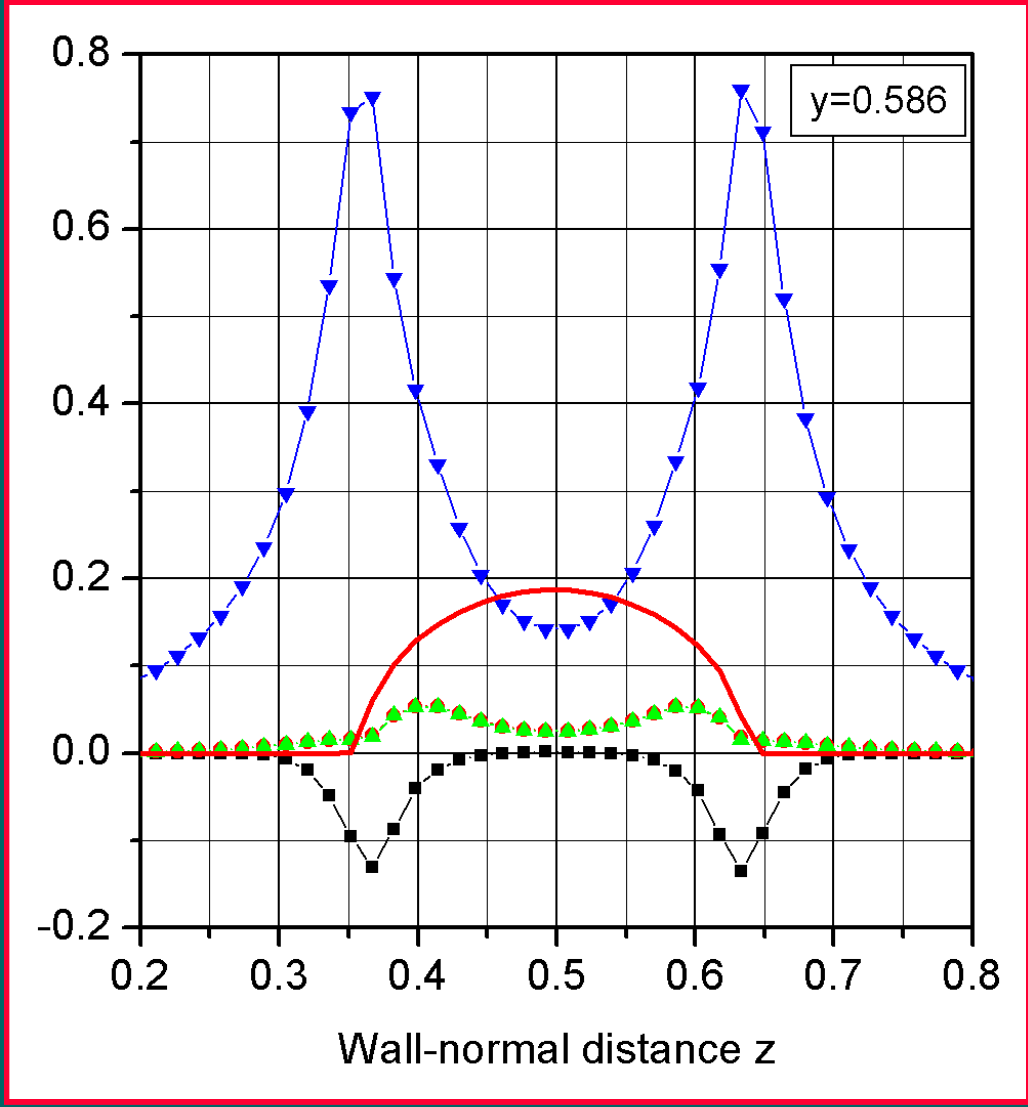
- De Bertodano et al., 1994
  Boisson and Malin, 1996; Lain et al., 2001

$$P = \alpha_L \left[2\nu_L^t \overline{S}_{L\alpha\beta} \right] \frac{\partial \overline{u}_{L\alpha}}{\partial x_\beta}$$

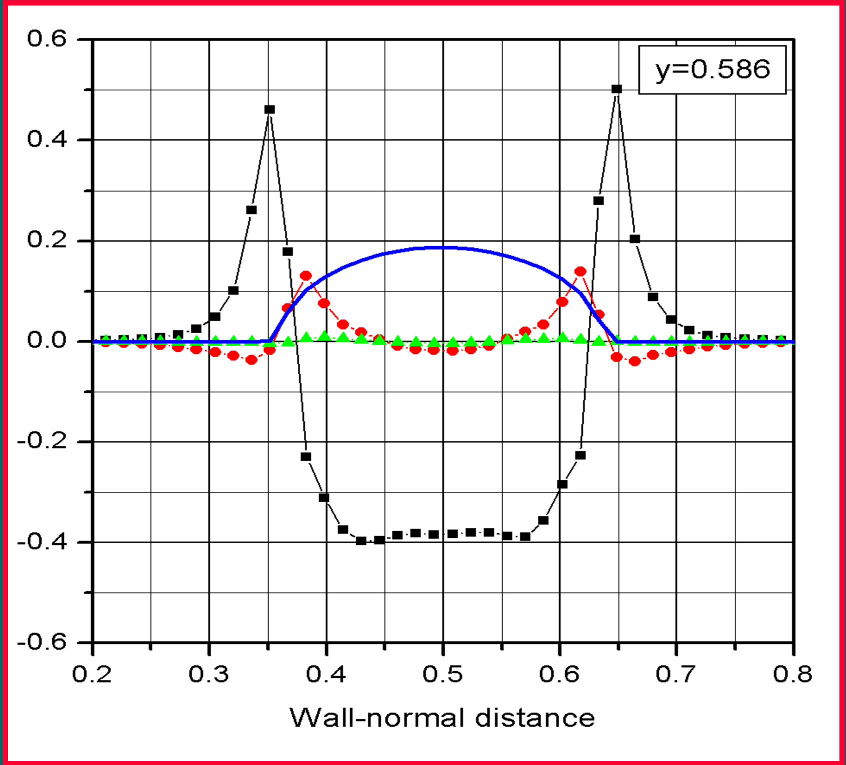
- Troshko and Hassan, 2001
  Morel, 1997; Hill et al., 1995

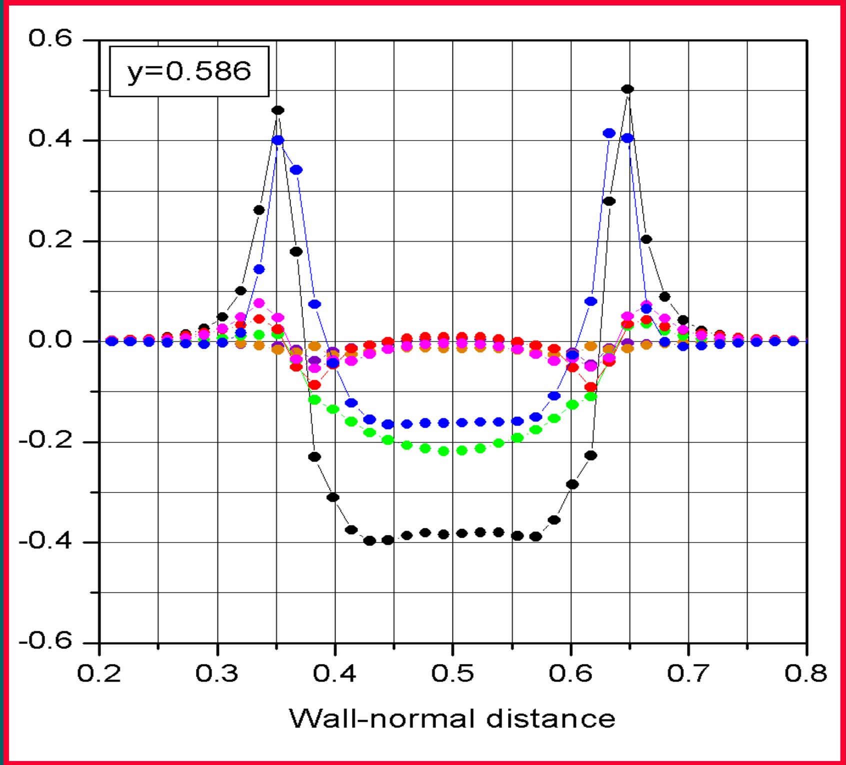
$$P = \alpha_L \left[2\nu_L^t \overline{S}_{L\alpha\beta} k_L - \frac{2}{3}\left(k_L + \nu_L^t \frac{\partial \overline{u}_{L\alpha}}{\partial x_\beta} \right) I \right] \frac{\partial \overline{u}_{L\alpha}}{\partial x_\beta}$$

- Exact production term



Modelled versus exact diffusion term





● Troshko and Hassan, 2001
De Bertodano et al., 1994
Boisson and Malin, 1996;
Morel, 1997;

$$\frac{\partial}{\partial x_\beta}\left[\alpha_L\left(\frac{1}{Re_{ref}}+\frac{\nu_L^t}{\sigma_k}\right)\frac{\partial k_L}{\partial x_\beta}\right]$$

▲ Pfleger and Becker, 2001
Grienberger and Hofmann, 1992
Svedsen et al., 1992; Hill et al., 1995

$$\frac{\partial}{\partial x_\beta}\left(\alpha_L\frac{\nu_L^t}{\sigma_k}\frac{\partial k_L}{\partial x_\beta}\right)$$

■ Exact diffusion term

Pressure corelation sub-terms ● ●

dominant in exact diffusion term ●

→ Closure for pressure correlation

$$\overline{\overline{p_L^{'} u_{L\beta}^{'}}} \propto -\nu_L^t \frac{\partial k_L}{\partial x_\beta}$$

not appropriate for bubble-driven flows

Interfacial terms modelling

Interfacial terms $\propto$ work of interfacial forces

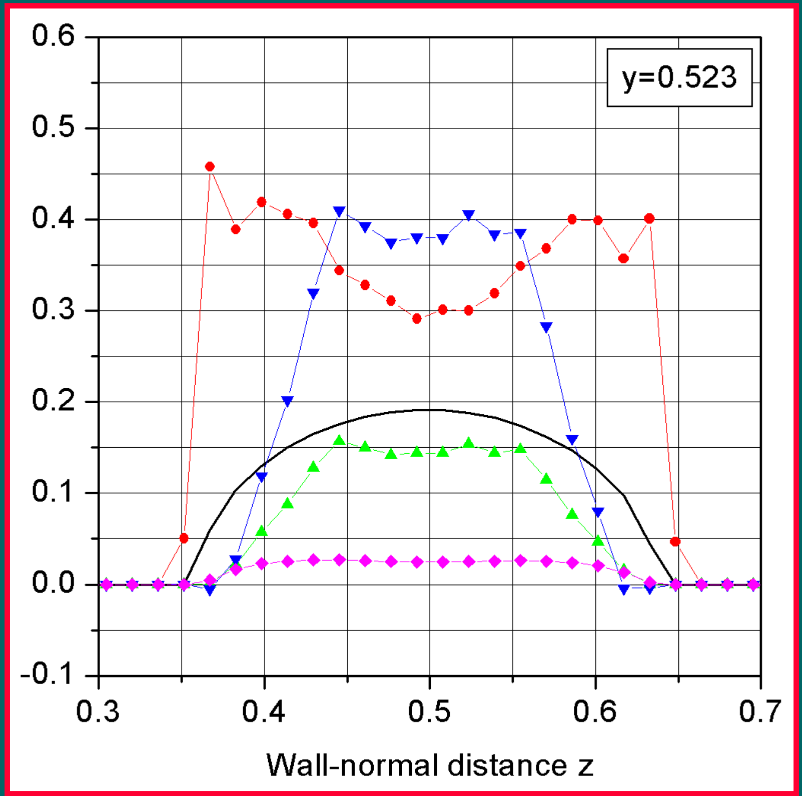
$$IFT = W^{D} + W^{ND}$$

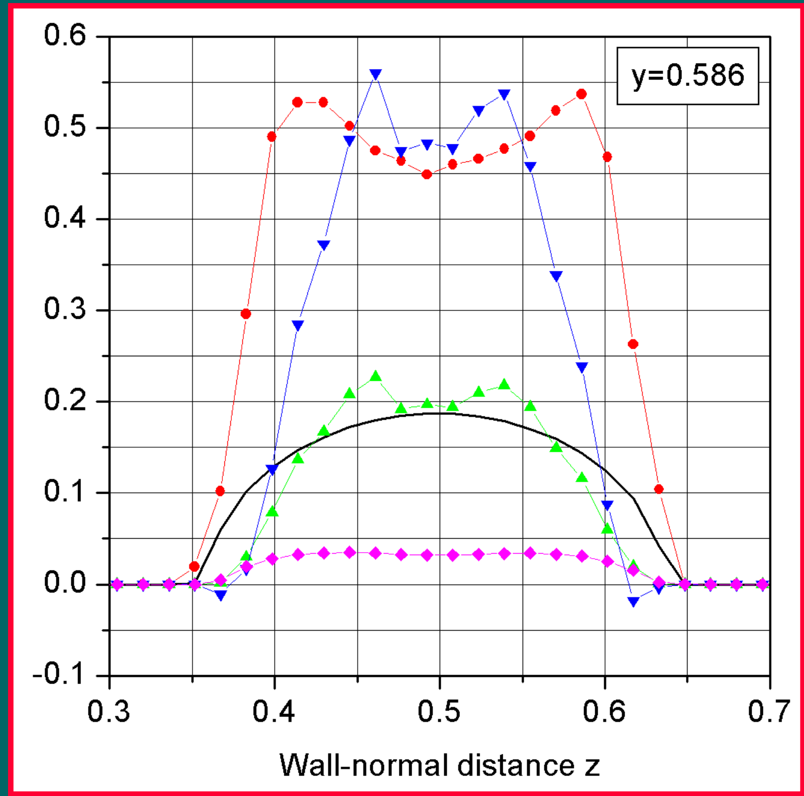
Reference	Work of drag force: W^{D}	$M^{D}_{L\alpha}$	C_D
Morel, 1997	$M^{D}_{L\alpha}\bar{u}_{R\alpha}$	$\frac{3}{4}\alpha_G\frac{C_D}{D_b}\bar{U}_R\bar{u}_{R\alpha}$	$\frac{2}{3}\sqrt{E\ddot{o}_B}\cdot f(\alpha_L)$
de Bertodano et al., 1994	$M^{D}_{L\alpha}\bar{u}_{R\alpha}$	$\frac{3}{4}\alpha_G\frac{C_D}{D_b}\bar{U}_R\bar{u}_{R\alpha}$	not given
Pfleger and Becker, 2001	$1.44\alpha_L M^{D}_{L\alpha}\bar{u}_{R\alpha}$	$\frac{3}{4}\alpha_G\frac{C_D}{D_b}\bar{U}_R\bar{u}_{R\alpha}$	0.44
Svendsen et al.,1995	$0.75 M^{D}_{L\alpha}\bar{u}_{R\alpha}$	not given	
Hill et al. 1995	$\frac{3}{4}\frac{\alpha_G C_D}{D_b}\bar{U}_R\left(\frac{\bar{u}_{R\alpha}\partial\alpha_G/\partial x_\alpha}{0.3Re_{ref}\alpha_L\alpha_G}+2k_L(C_t-1)\right)$	not contained explicitly in W^D	$\frac{2}{3}\sqrt{E\ddot{o}_B}\cdot f(\alpha_L)$

Work of added-mass force:
(Morel, 1997)

$$W^{AM} = \frac{1}{2}\left(\bar{u}_{G\alpha} - \bar{u}_{L\alpha}\right)\frac{1+2\alpha_G}{\alpha_L}\alpha_G\left(\frac{D_G\bar{u}_{G\alpha}}{Dt} - \frac{D_L\bar{u}_{L\alpha}}{Dt}\right)$$

Modelled versus exact interfacial terms





Modelled IFT:

- ♦ Morel, 1997
- Pflerger&Becker,2001
- Hill et al. 1995

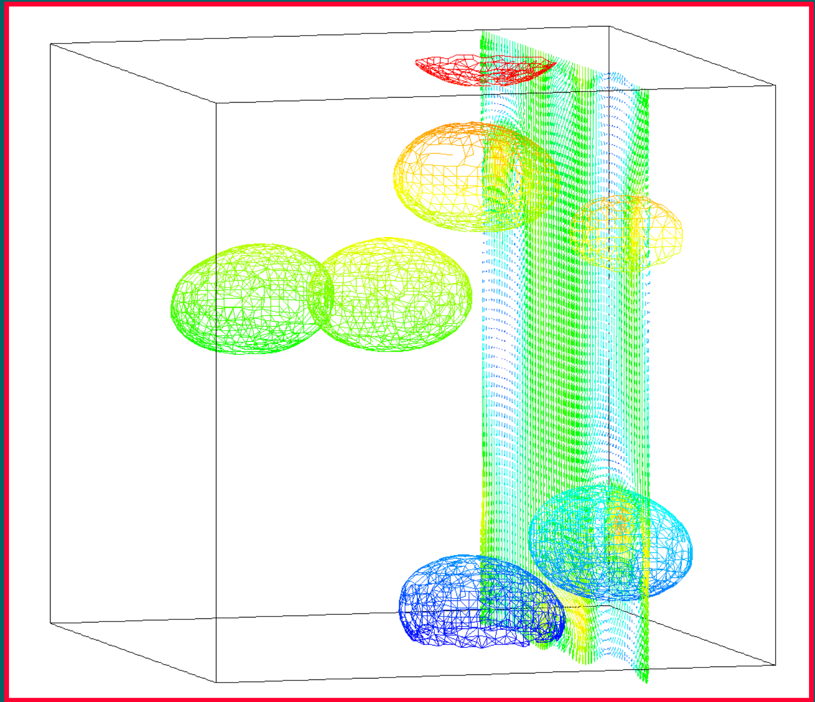
● Exact IFT

♦ Morel	$M_{L\alpha}^{D} = \frac{3}{4}\alpha_G \frac{C_D}{D_b}\overline{U}_R \overline{u}_{R\alpha}$	$W^D = M_{L\alpha}^{D}\overline{u}_{R\alpha}$	$C_D = \frac{2}{3}\sqrt{E\ddot{o}_B} \cdot f(\alpha_L)$
Pfleger&Becker		$W^D = 1.44\alpha_L M_{L\alpha}^{D}\overline{u}_{R\alpha}$	C_D=0.44

Importance of including local flow details in model assumptions for IFT !!!

Conclusions and future steps

- **Use of bubble-train flow DNS data to study BIT**
  - Production term is negative
  - Interfacial terms are only source terms
- **Closure assumptions**
  - Models for production and diffusion perform poor
  - Morels' model for interfacial terms performs well
- **Future steps**
  - DNS data for bubble swarm
  - Improvement of BIT models



Evaluation of interfacial terms

Definition: $IFT^{E} = -\overline{p'_{Lin} u'_{Lin\alpha} n_{Lin\alpha} a_{in}} + \frac{1}{Re_{ref}} \overline{u'_{Lin\alpha} \frac{\partial u'_{Lin\alpha}}{\partial x_{\beta}} n_{Lin\beta} a_{in}}$

Fluctuation of interfacial quantity: $A'_{Lin} = A_{Lin} - \overline{\overline{A_L}}$

Homogeneous mixture model in interfacial cells: $u_G = u_L = u$ and $p_G = p_L = p$

Liquid phase interfacial pressure $p_{Lin} = ?$

Assumption: $p_{Lin} \cong p_{(i,j,k)}$ where (i,j,k) is a neighbouring cell with $f_{(i,j,k)} = 1$

Liquid phase interfacial velocity $\vec{u}_{Lin} = ?$

No assumptions. Methodology for evaluation of $\vec{u}_{Lin}$ is developed.

Evaluation of interfacial velocity

No phase change: $\vec{u}_{Lin} = \vec{u}_{Gin} = \vec{u}_{in}$

$\vec{u}_{in} = \vec{u}_{int} + \vec{u}_{inn}$

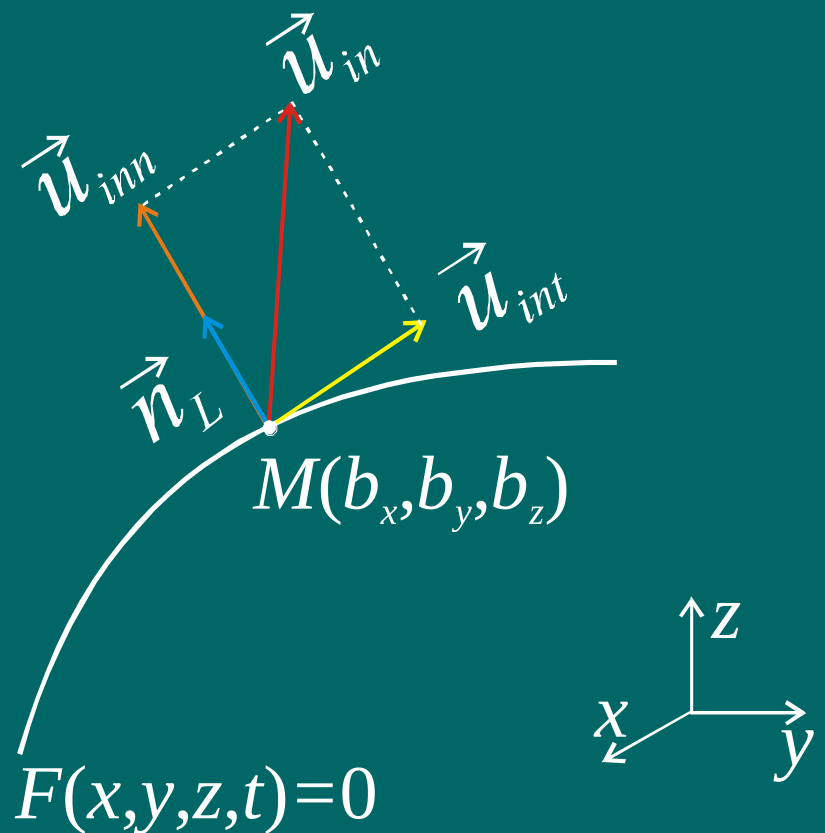
Tangential component (Ishii 1975):

$$\vec{u}_{int} = \vec{u}_t = \vec{u} - \left(\vec{u} \cdot \vec{n}_L\right) \cdot \vec{n}_L$$

Normal component (Kataoka et al., 1986):

$$\vec{u}_{inn} = \left(\vec{u}_{in} \cdot \vec{n}_L\right) \cdot \vec{n}_L$$

$$\vec{u}_{in} \cdot \vec{n}_L = \frac{\partial F / \partial t}{\sqrt{\left(\partial F/\partial x\right)^2 + \left(\partial F/\partial y\right)^2 + \left(\partial F/\partial z\right)^2}}$$



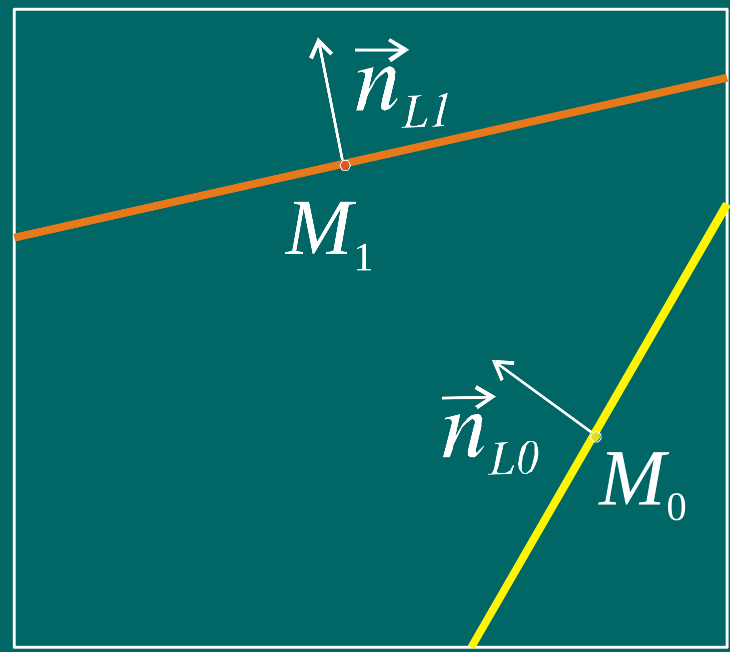
TURBIT-VoF definition: $F(x,y,z,t) = \left(b_x - x\right)\cdot n_{Lx} + \left(b_y - y\right)\cdot n_{Ly} + \left(b_z - z\right)\cdot n_{Lz} = 0$

$F(x,y,z,t)$ is not explicit function of $t \rightarrow \quad \dfrac{\partial F}{\partial t} = ?$

$$\frac{\partial F}{\partial t} = ?$$

(I) $F\left(b_{x0}, b_{y0}, b_{z0}, t_0\right) = 0$

(II) $F\left(b_{x0} + \delta x, b_{y0} + \delta y, b_{z0} + \delta z, t_0 + \Delta t\right) = 0$



1. Expand (I) in Taylor series
2. Neglect HOT
3. Sustract (II) from expanded (I)
4. Rearrange the difference

$$\rightarrow \quad \frac{\partial F}{\partial t} = -\frac{1}{\Delta t}\left(n_{Lx0}\delta x + n_{Ly0}\delta y + n_{Lz0}\delta z\right)$$

Idea comes from experimental determination of interfacial velocity. There M_0 and M_1 are fixed (sensor positions) and Δt is variable (Kataoka et al. 1986).