Assessment of closure assumptions in turbulence kinetic energy equation for bubble-driven liquid flow

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Multiphase Flows: Simulation, Experiment and Application
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Content

- Introduction and motivation
- Direct numerical simulation
  - Verification for single bubbles
  - Results for bubble swarms
- Analysis of k_L-equation
  - Profiles of balance terms (budget of k_L)
  - Assessment of closure assumptions
- Conclusions and outlook

Introduction

- Background:
  - Computations of turbulent bubbly flows with the two-fluid-model
- Status:
  - See keynote lecture of Prof. Lopez de Bertodano on Friday
- Problem:
  - No established and for wide range of parameters validated turbulence model available for bubbly flows
- Often made simplification:
  - Application of k-ε model for liquid phase with standard set of coefficients derived for single-phase flow

Motivation

- Liquid phase turbulence kinetic energy: $k_{\mathrm{L}} \equiv \overline{\overline{\mathbf{u}_{\mathrm{L}}^{'} \cdot \mathbf{u}_{\mathrm{L}}^{'}}} / 2$
- Analytical transport equation* for k_{L}:

$$\frac{\partial}{\partial t}\left(\alpha_{\mathrm{L}} k_{\mathrm{L}}\right) + \nabla \cdot \left(\alpha_{\mathrm{L}} k_{\mathrm{L}} \overline{\overline{\mathbf{u}}}_{\mathrm{L}}\right) = \underbrace{\frac{1}{Re_{\mathrm{ref}}} \nabla \cdot \left(\alpha_{\mathrm{L}} \overline{\overline{\mathbb{T}_{\mathrm{L}}^{'} \cdot \mathbf{u}_{\mathrm{L}}^{'}}}\right) - \nabla \cdot \left[\alpha_{\mathrm{L}} \left(\overline{\overline{p_{\mathrm{L}}^{'} \mathbf{u}_{\mathrm{L}}^{'}}} + \frac{1}{2} \overline{\overline{(\mathbf{u}_{\mathrm{L}}^{'} \cdot \mathbf{u}_{\mathrm{L}}^{'}) \mathbf{u}_{\mathrm{L}}^{'}}}\right)\right]}_{DIFFUSION}$$

$$\underbrace{-\alpha_{\mathrm{L}} \overline{\overline{\mathbf{u}_{\mathrm{L}}^{'} \mathbf{u}_{\mathrm{L}}^{'}}} : \nabla \overline{\overline{\mathbf{u}}}_{\mathrm{L}}}_{PRODUCTION} \underbrace{- \frac{1}{Re_{\mathrm{ref}}} \alpha_{\mathrm{L}} \overline{\overline{\mathbb{T}_{\mathrm{L}}^{'} : \nabla \mathbf{u}_{\mathrm{L}}^{'}}}}_{DISSIPATION} + \underbrace{\overline{\left[\frac{1}{Re_{\mathrm{ref}}} \mathbb{T}_{\mathrm{L;in}}^{'} - p_{\mathrm{L;in}}^{'} \mathbb{I}\right] \cdot \mathbf{u}_{\mathrm{L;in}}^{'} \cdot \mathbf{n}_{\mathrm{L;in}} a_{\mathrm{in}}}}_{INTERFACIAL\ \ TERM}$$

- Terms on right hand side of equation must be modeled
- Experimental data for individual closure terms is missing

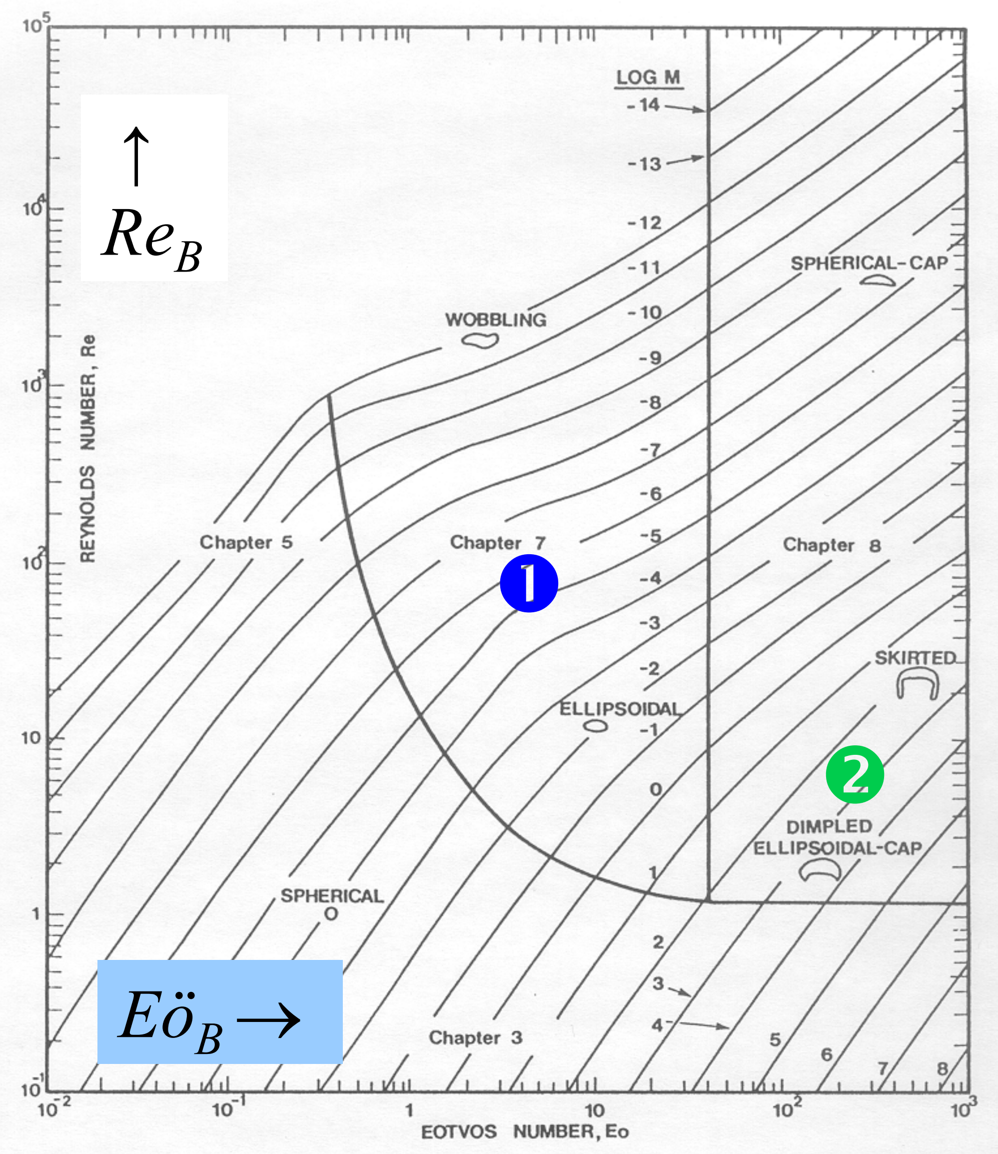
  $\Rightarrow$ **Direct numerical simulation of bubble swarm flow**

* Kataoka & Serizawa, Int. J. Multiphase Flow, 15 (1989) 843

In-house code TURBIT-VOF

- Volume-of fluid method for interface tracking
  - Interface is locally approximated by plane (PLIC method)
- Governing equations for two incompressible fluids
  - Single field momentum equation with surface tension term
  - Zero divergence condition for center-of-mass velocity
  - Advection equation for liquid volumetric fraction f
- Solution strategy
  - Projection method resulting in pressure Poisson equation
  - Explicit third order Runge-Kutta time integration scheme
- Discretization in space
  - Finite volume formulation for regular staggered grid
  - Second order central difference approximations

Code verification



❶ Medium Morton number ellipsoidal bubble

DNS for $\Gamma_\mu = \mu_d / \mu_c = 1$,

$Mo = 3.09 \cdot 10^{-6}$, $E\ddot{o}_B = 3.06$

$\Gamma_\rho = \rho_d / \rho_c = 0.5; 0.2; 0.1; 0.02$

❷ High Morton number ellipsoidal cap bubble

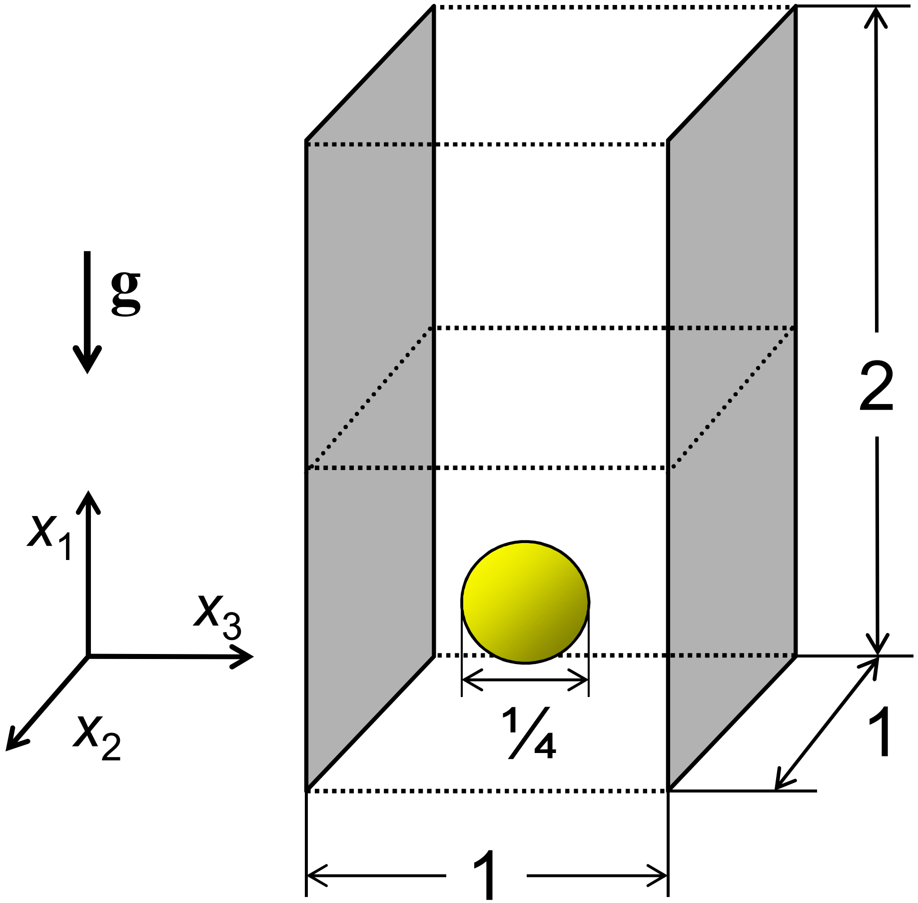
DNS for $\Gamma_\mu = \mu_d / \mu_c = 1$,

$Mo = 266$, $E\ddot{o}_B = 243$

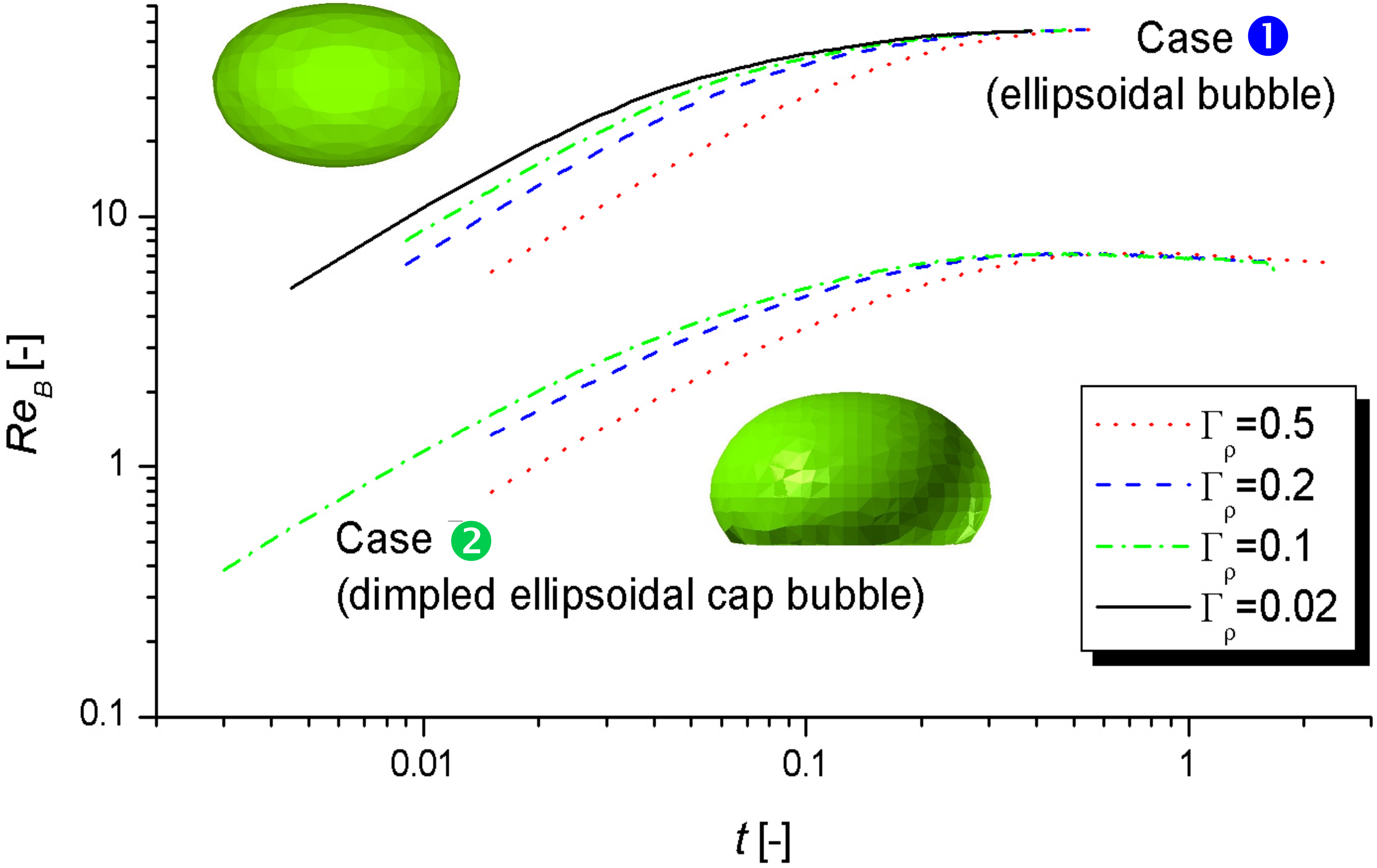
$\Gamma_\rho = \rho_d / \rho_c = 0.5; 0.2; 0.1$

Computational set-up

- Domain: 2 x 1 x 1
- Grid: 128 x 64 x 64
- Bubble diameter: 0.25
  (= 16 mesh cells)
- Gas holdup: ≈ 0.4%
- Boundary conditions
  - walls at $z = 0$ and $z = 1$
  - periodic in x and y
- Liquid & gas initially at rest



Bubble Reynolds number over time



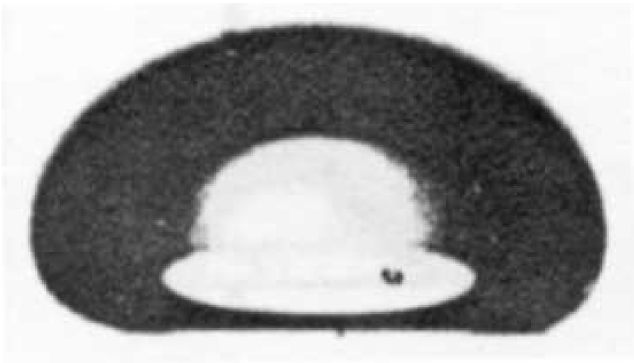
Comparison of bubble shape (case ❷)

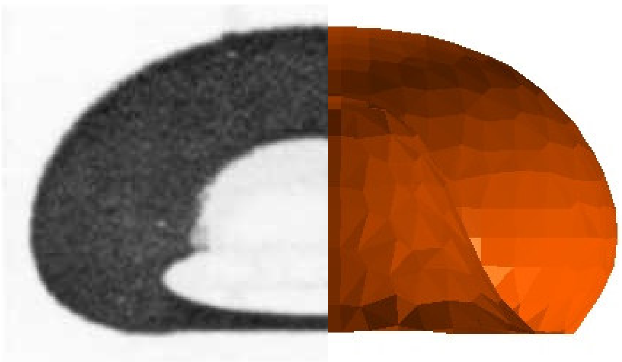
Experiment Bhaga & Weber

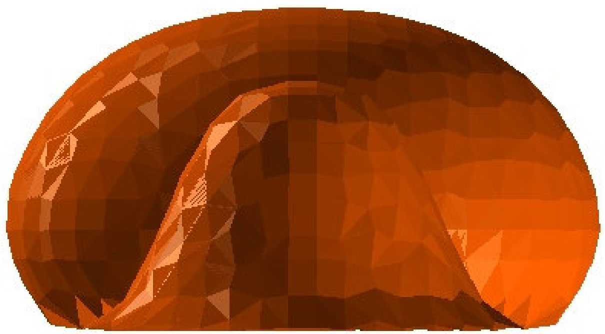
($\Gamma_\rho \approx 0{,}0008$; $\Gamma_\mu \approx 10^{-5}$)

TURBIT-VOF

($\Gamma_\rho = 0{,}5$; $\Gamma_\mu = 1$)

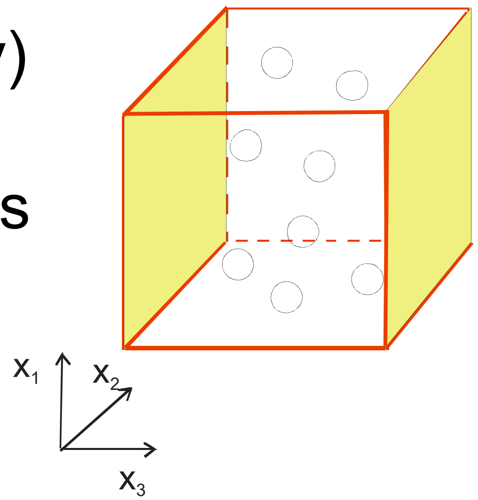


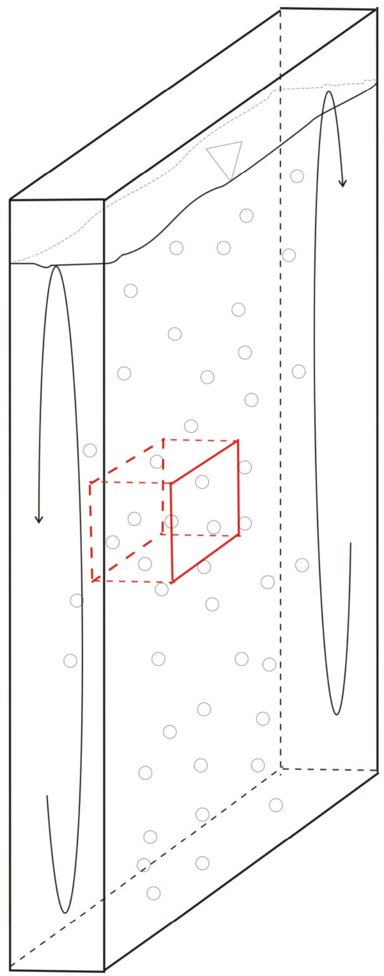




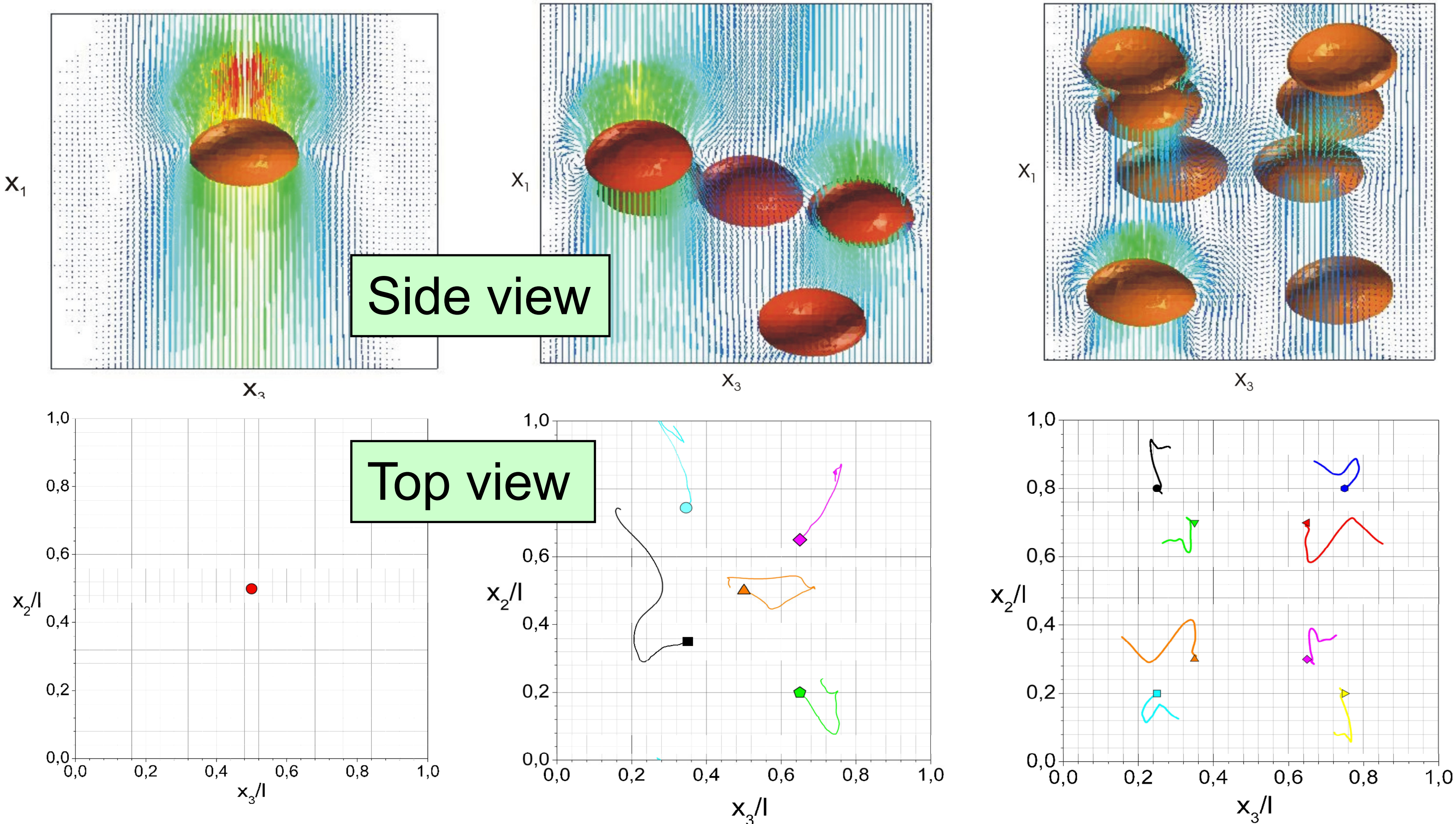
Bubble swarm simulations

- Simulation mimics part of a flat bubble column
  - two lateral walls
  - periodic b.c. in vertical and lateral direction
- Parameters fixed
  - $Eö_B = 3{,}06$; $\Gamma_\rho = 0{,}5$; $\Gamma_\mu = 1$
- Parameters varied
  - Morton number (liquid viscosity) $Mo = 3\times10^{-6}$; 3×10^{-4} ; 3×10^{-2}
  - Gas content: 1, 5 and 8 bubbles with $Mo = 3\times10^{-6}$
- Cubic computational domain
  - Grid: 64 × 64 × 64 mesh cells





Bubble shape and path (case ❶)

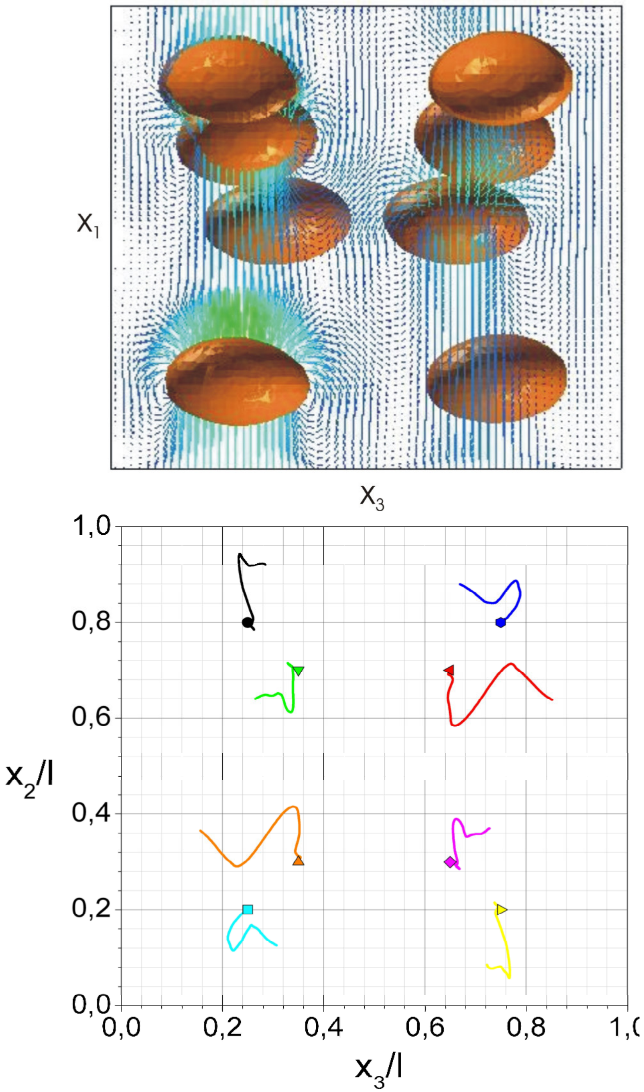


1BM6: $Mo=3\times10^{-6}$

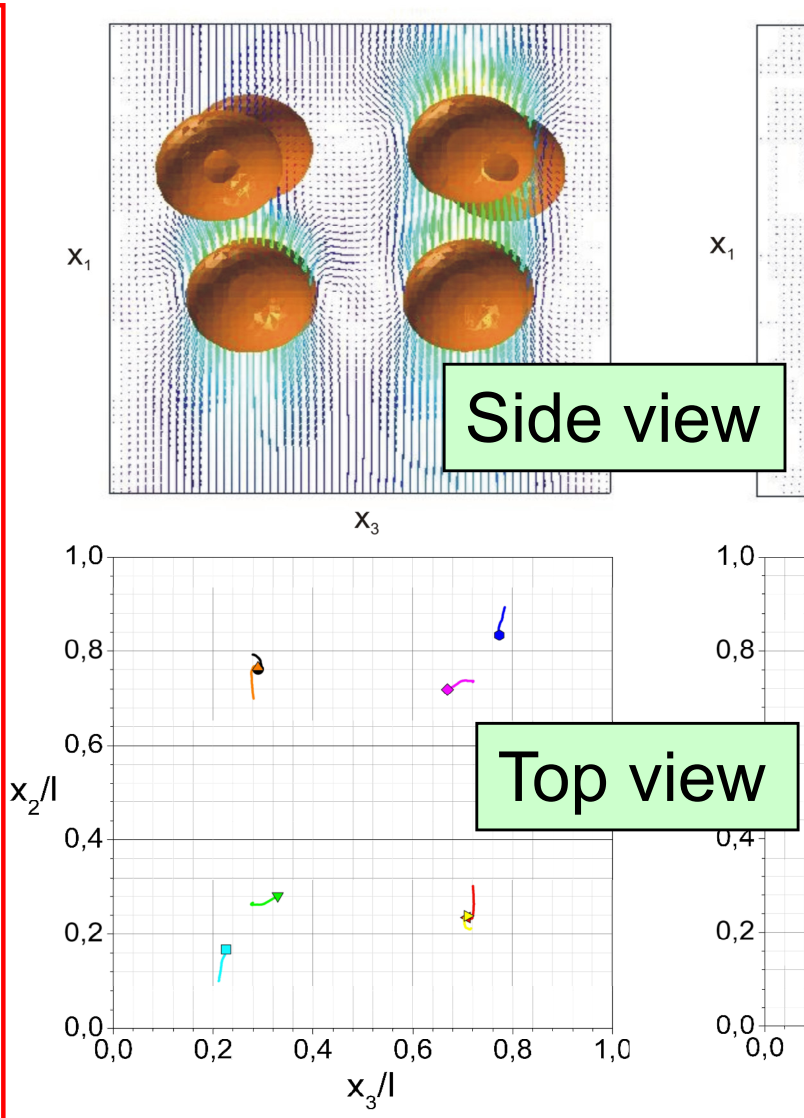
5BM6: $Mo=3\times10^{-6}$

8BM6: $Mo=3\times10^{-6}$

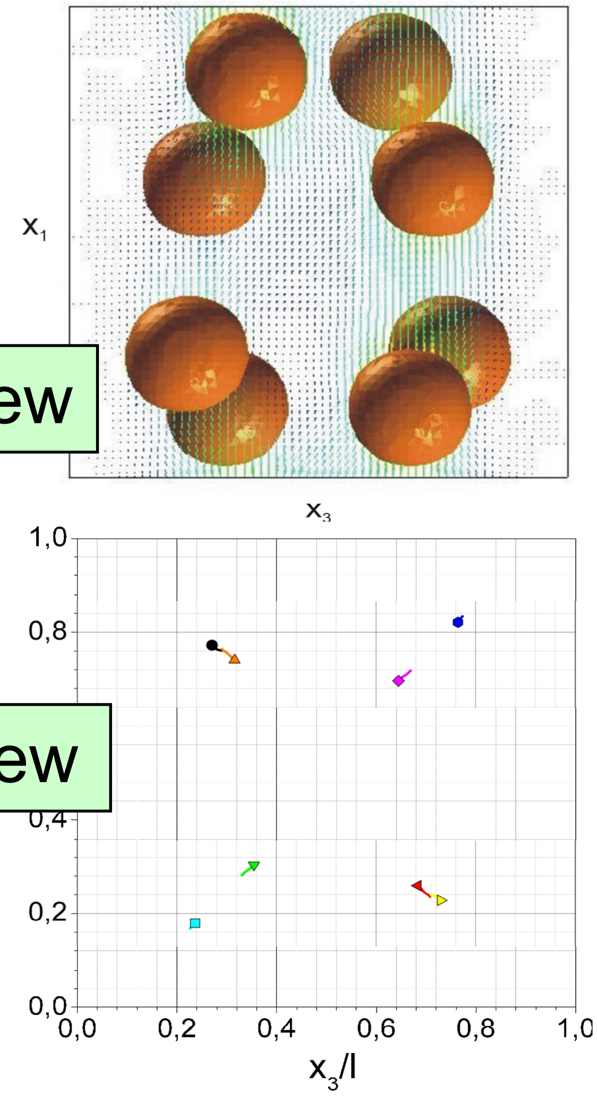
Bubble shape and path



8BM6: $Mo=3\times10^{-6}$

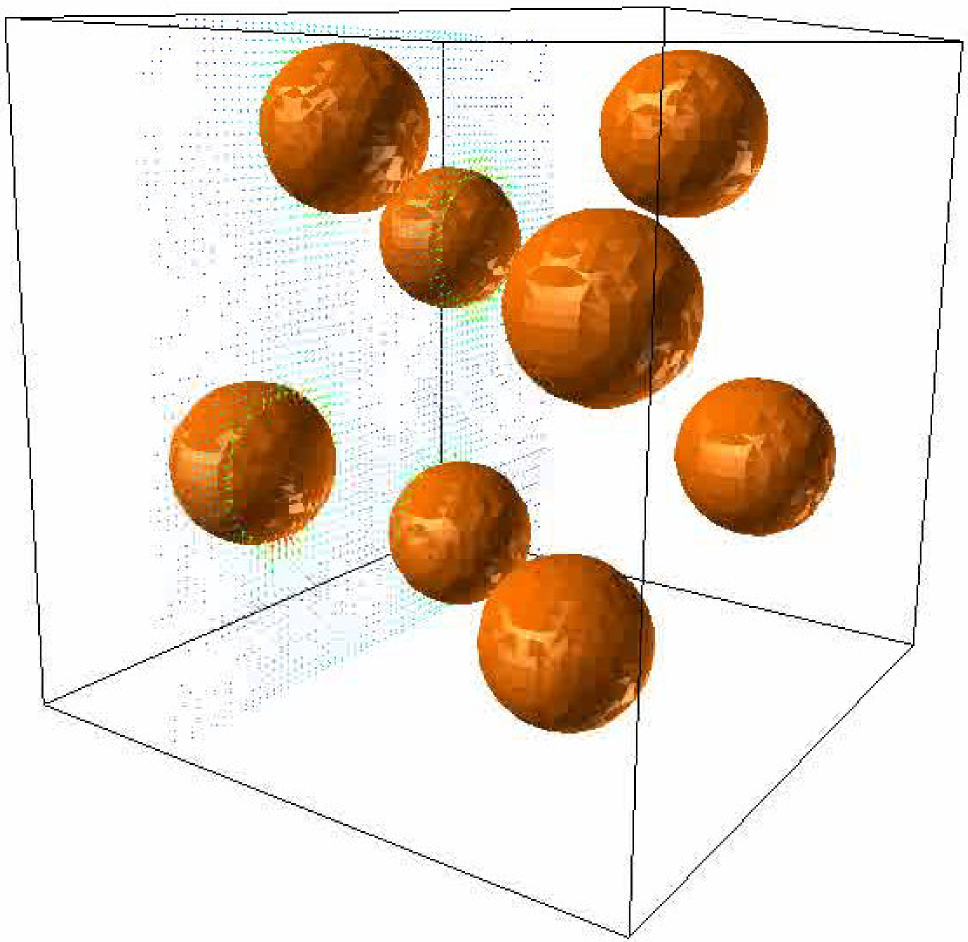


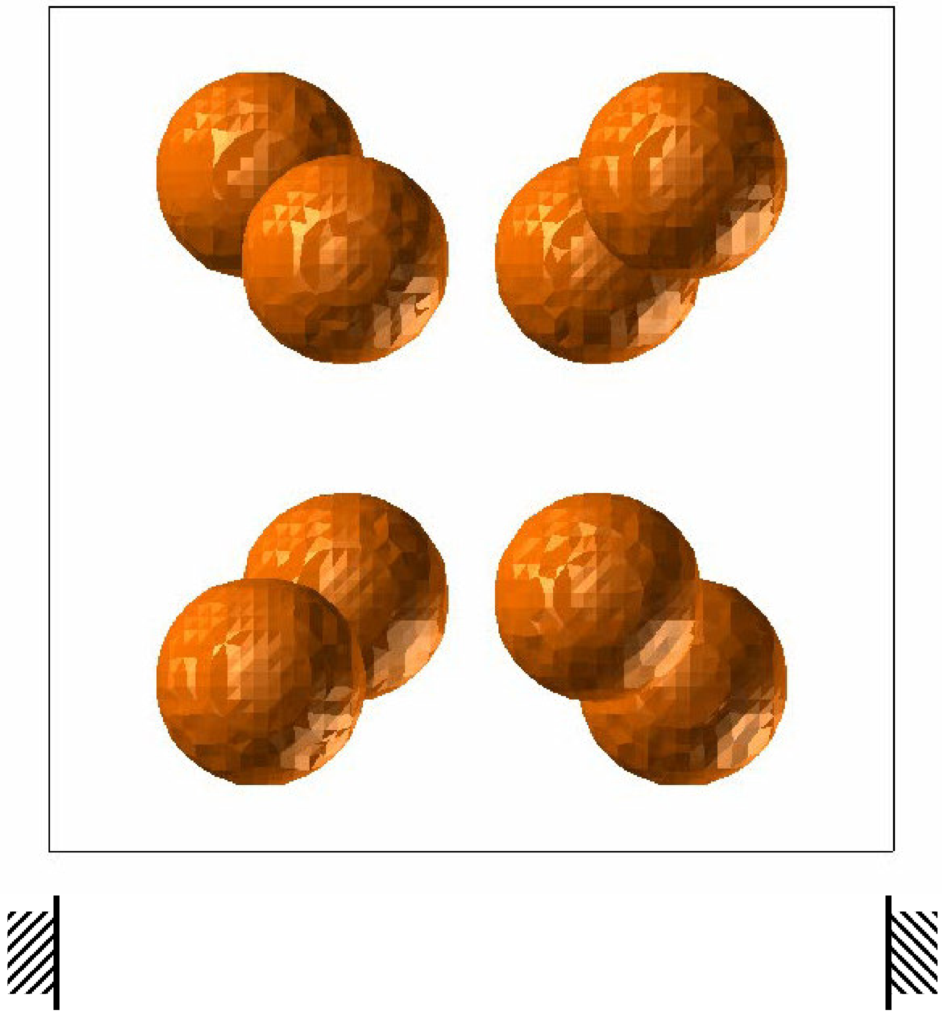
8BM4: $Mo=3\times10^{-4}$



8BM2: $Mo=3\times10^{-2}$

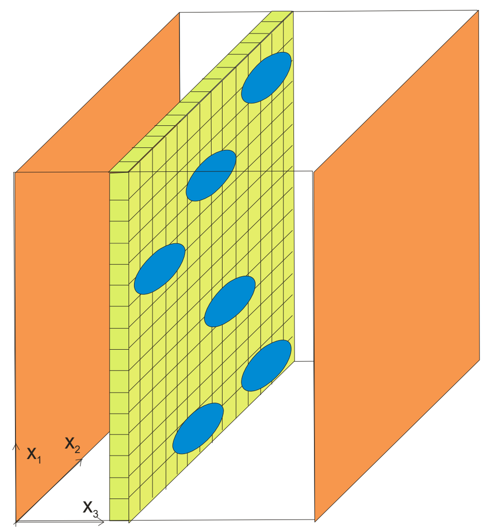
Visualization of bubble motion

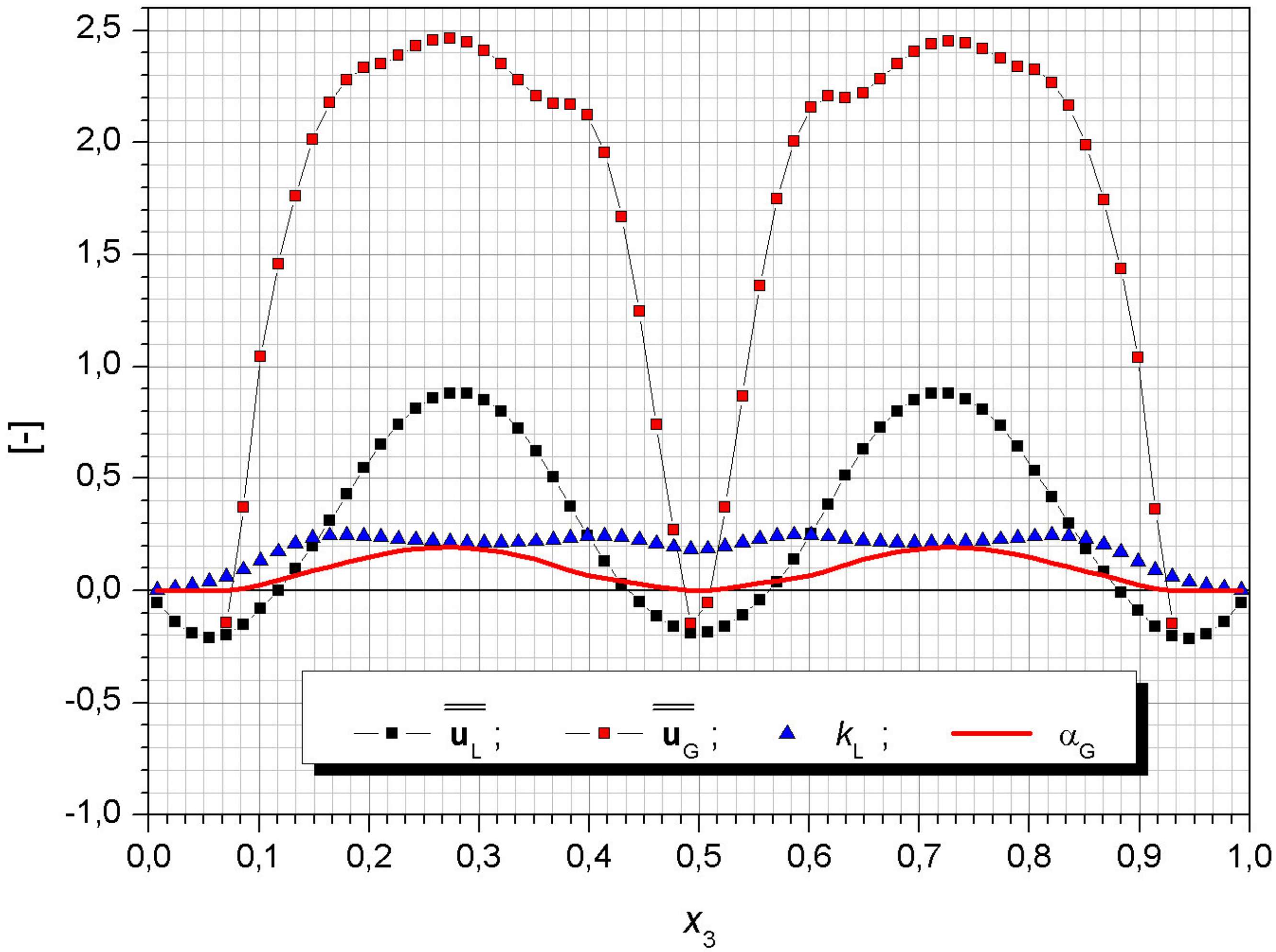




Averaging of simulation results

Averaging over planes parallel to the side walls





Terms in exact k_L-equation

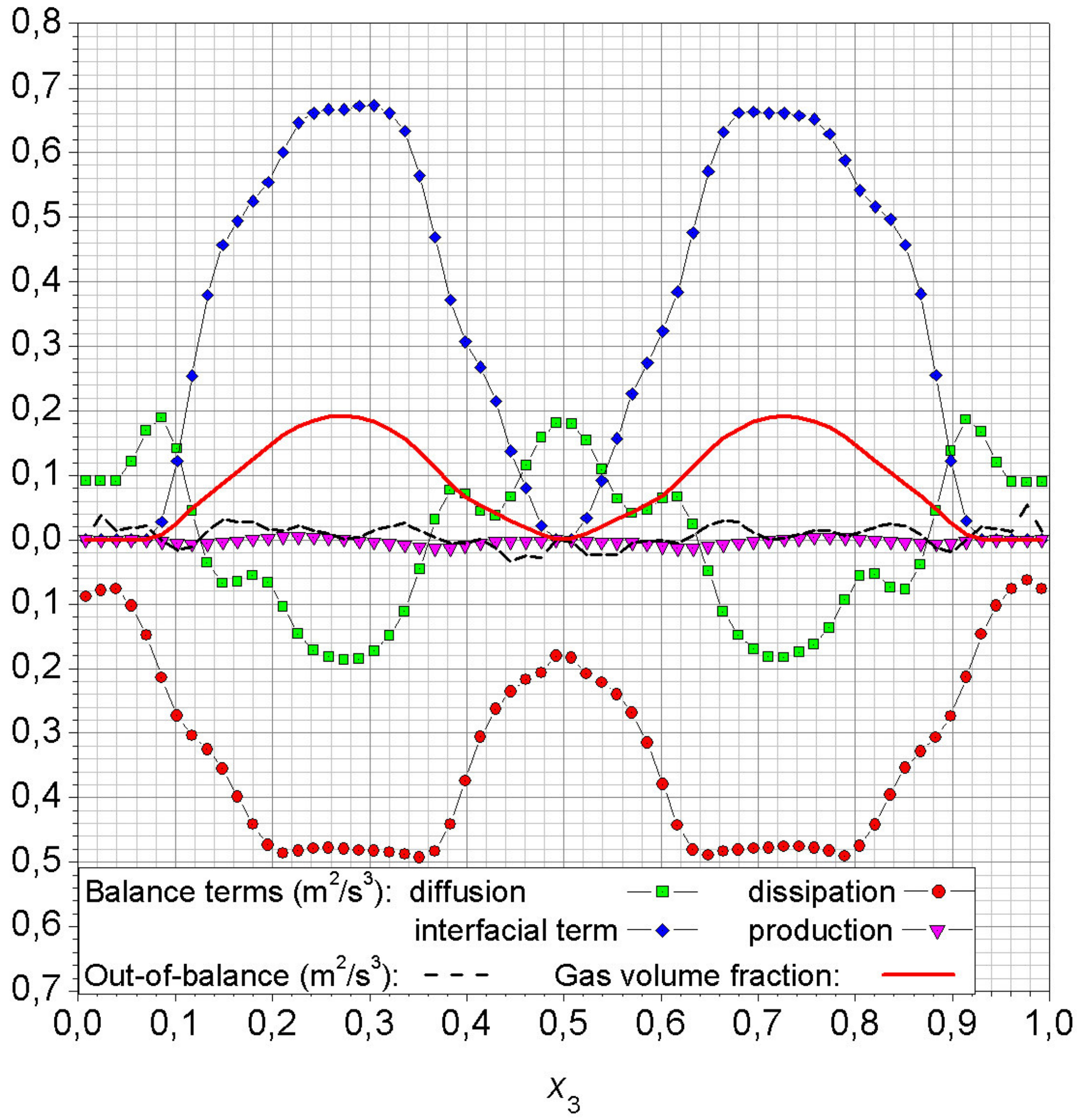
$$\frac{\partial}{\partial t}\left(\alpha_L k_L\right) + \nabla\cdot\left(\alpha_L k_L \overline{\overline{\mathbf{u}_L}}\right) =$$

$$\underbrace{\frac{1}{Re_{ref}}\nabla\cdot\left(\alpha_L \overline{\overline{\mathbb{T}_L^{'}\cdot\mathbf{u}_L^{'}}}\right) - \nabla\cdot\left[\alpha_L\left(\overline{\overline{p_L^{'}\mathbf{u}_L^{'}}} + \frac{1}{2}\overline{\overline{(\mathbf{u}_L^{'}\cdot\mathbf{u}_L^{'})\mathbf{u}_L^{'}}}\right)\right]}_{DIFFUSION}$$

$$\underbrace{-\alpha_L \overline{\overline{\mathbf{u}_L^{'}\mathbf{u}_L^{'}}} : \nabla\overline{\overline{\mathbf{u}_L}}}_{PRODUCTION}$$

$$\underbrace{-\frac{1}{Re_{ref}}\alpha_L \overline{\overline{\mathbb{T}_L^{'} : \nabla\mathbf{u}_L^{'}}}}_{DISSIPATION}$$

$$\underbrace{+\overline{\left[\frac{1}{Re_{ref}}\mathbb{T}_{L;in}^{'} - p_{L;in}^{'}\mathbb{I}\right]\cdot\mathbf{u}_{L;in}^{'}\cdot\mathbf{n}_{L;in}a_{in}}}_{INTERFACIAL\ TERM}$$



Models for production term

- Exact term: $\mathrm{Prod}(k_{\mathrm{L}}) = -\alpha_{\mathrm{L}} \overline{\overline{\mathbf{u}'_{\mathrm{L}} \mathbf{u}'_{\mathrm{L}}}} : \nabla \overline{\overline{\mathbf{u}}}_{\mathrm{L}}$
- Common ansatz: $\mathrm{Prod}(k_{\mathrm{L}}) \approx \alpha_{\mathrm{L}} \nu_{\mathrm{L}}^{\mathrm{eff}} \left[\nabla \overline{\overline{\mathbf{u}}}_{\mathrm{L}} + \nabla \overline{\overline{\mathbf{u}}}_{\mathrm{L}}^{T} \right] : \nabla \overline{\overline{\mathbf{u}}}_{\mathrm{L}}$
- One-equation model:

$$\nu_{\mathrm{L}}^{\mathrm{eff}} = \beta_1 l_{\mathrm{TP}} \sqrt{k_{\mathrm{L}}} \quad \text{with} \quad \beta_1 = 0,56 \quad \text{and} \quad l_{\mathrm{TP}} = \alpha_{\mathrm{G}} d_{\mathrm{B}} / 3$$

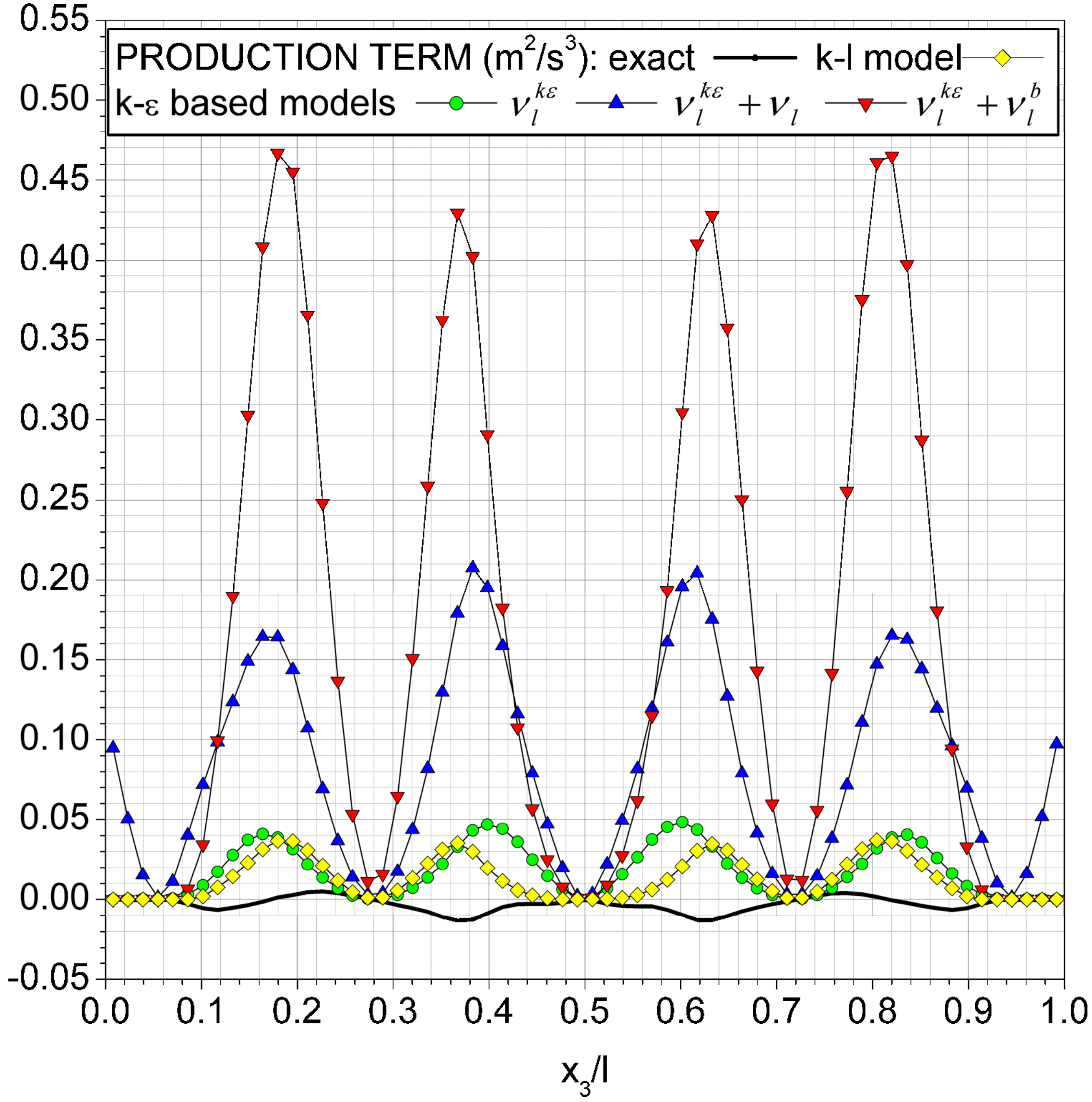
- Two-equation model:

$$\nu_{\mathrm{L}}^{\mathrm{eff}} = \nu_{\mathrm{L}}^{k-\varepsilon} = C_{\mu} k_{\mathrm{L}}^{2} / \varepsilon_{\mathrm{L}}$$

$$\nu_{\mathrm{L}}^{\mathrm{eff}} = \nu_{\mathrm{L}}^{k-\varepsilon} + \nu_{\mathrm{L}}$$

$$\nu_{\mathrm{L}}^{\mathrm{eff}} = \nu_{\mathrm{L}}^{k-\varepsilon} + 0,6 \alpha_{\mathrm{G}} d_{\mathrm{B}} \left| \overline{\overline{\mathbf{u}}}_{r} \right| = \nu_{\mathrm{L}}^{k-\varepsilon} + \nu_{\mathrm{L}}^{\mathrm{B}}$$

Evaluation of production term models



Models for diffusion term

- Exact term:

$$\mathrm{Diff}(k_\mathrm{L}) = \nu_\mathrm{L}\nabla\cdot\left(\alpha_\mathrm{L}\overline{\overline{\mathbb{T}'_\mathrm{L}\cdot\mathbf{u}'_\mathrm{L}}}\right) - \nabla\cdot\left[\alpha_\mathrm{L}\left(\overline{\overline{p'_\mathrm{L}\mathbf{u}'_\mathrm{L}}} + \frac{1}{2}\overline{\overline{(\mathbf{u}'_\mathrm{L}\cdot\mathbf{u}'_\mathrm{L})\mathbf{u}'_\mathrm{L}}}\right)\right]$$

- Common ansatz: $\mathrm{Diff}(k_\mathrm{L}) \approx \nabla\cdot\left(\alpha_\mathrm{L}\nu_\mathrm{L}^\mathrm{Diff}\nabla k_\mathrm{L}\right)$

- One-equation model:

$$\nu_\mathrm{L}^\mathrm{Diff} = 0{,}5\nu_\mathrm{L} + \beta_2 l_\mathrm{TP}\sqrt{k_\mathrm{L}} \quad \text{with} \quad \beta_2 = 0{,}38 \quad \text{and} \quad l_\mathrm{TP} = \alpha_\mathrm{G} d_\mathrm{B}/3$$

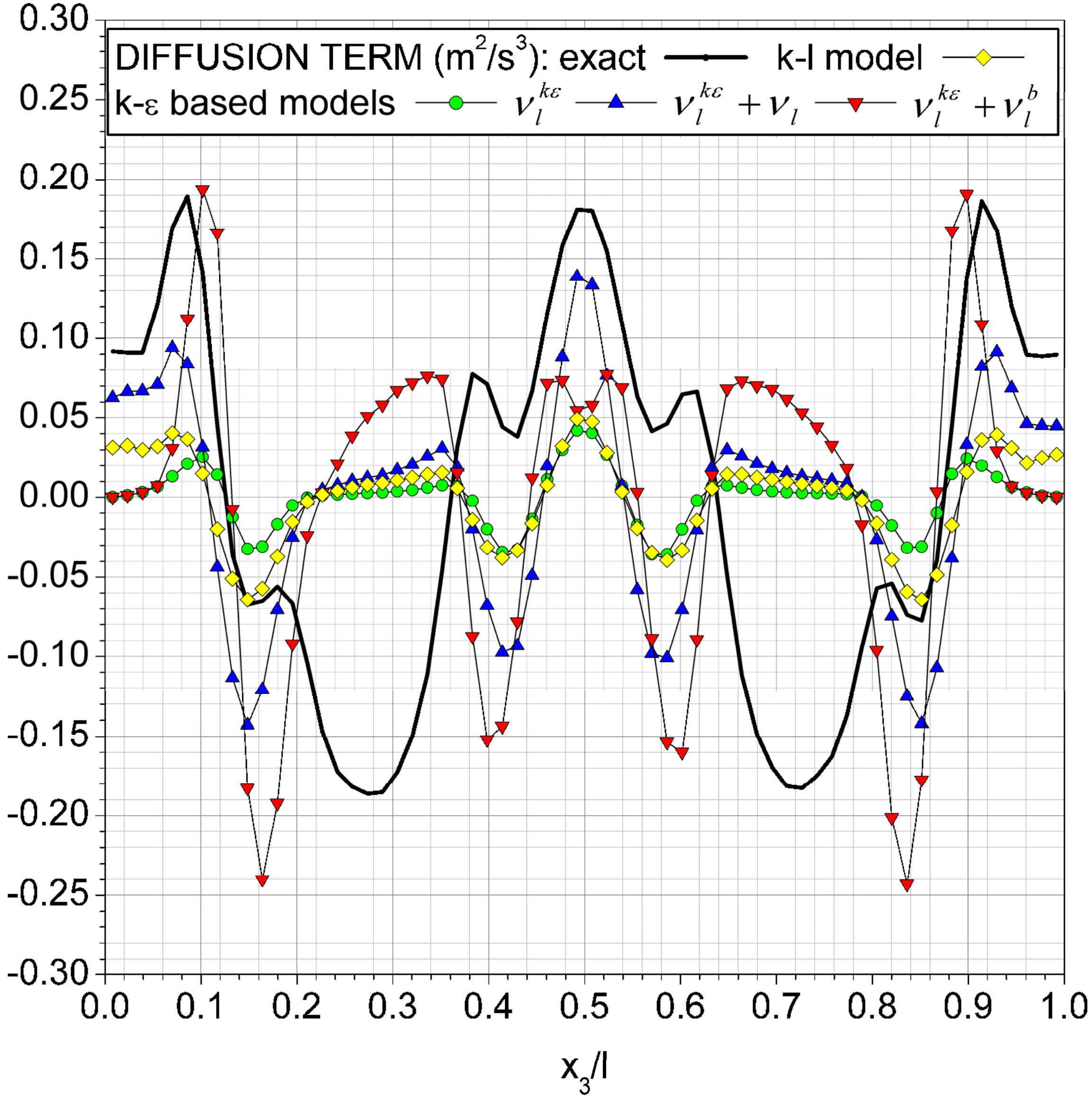
- Two-equation model:

$$\nu_\mathrm{L}^\mathrm{Diff} = \nu_\mathrm{L}^{k-\varepsilon} = C_\mu k_\mathrm{L}^2/\varepsilon_\mathrm{L}$$

$$\nu_\mathrm{L}^\mathrm{Diff} = \nu_\mathrm{L}^{k-\varepsilon} + \nu_\mathrm{L}$$

$$\nu_\mathrm{L}^\mathrm{Diff} = \nu_\mathrm{L}^{k-\varepsilon} + 0{,}6\alpha_\mathrm{G} d_\mathrm{B}\left|\overline{\overline{\mathbf{u}_r}}\right| = \nu_\mathrm{L}^{k-\varepsilon} + \nu_\mathrm{L}^\mathrm{B}$$

Evaluation of diffusion term models



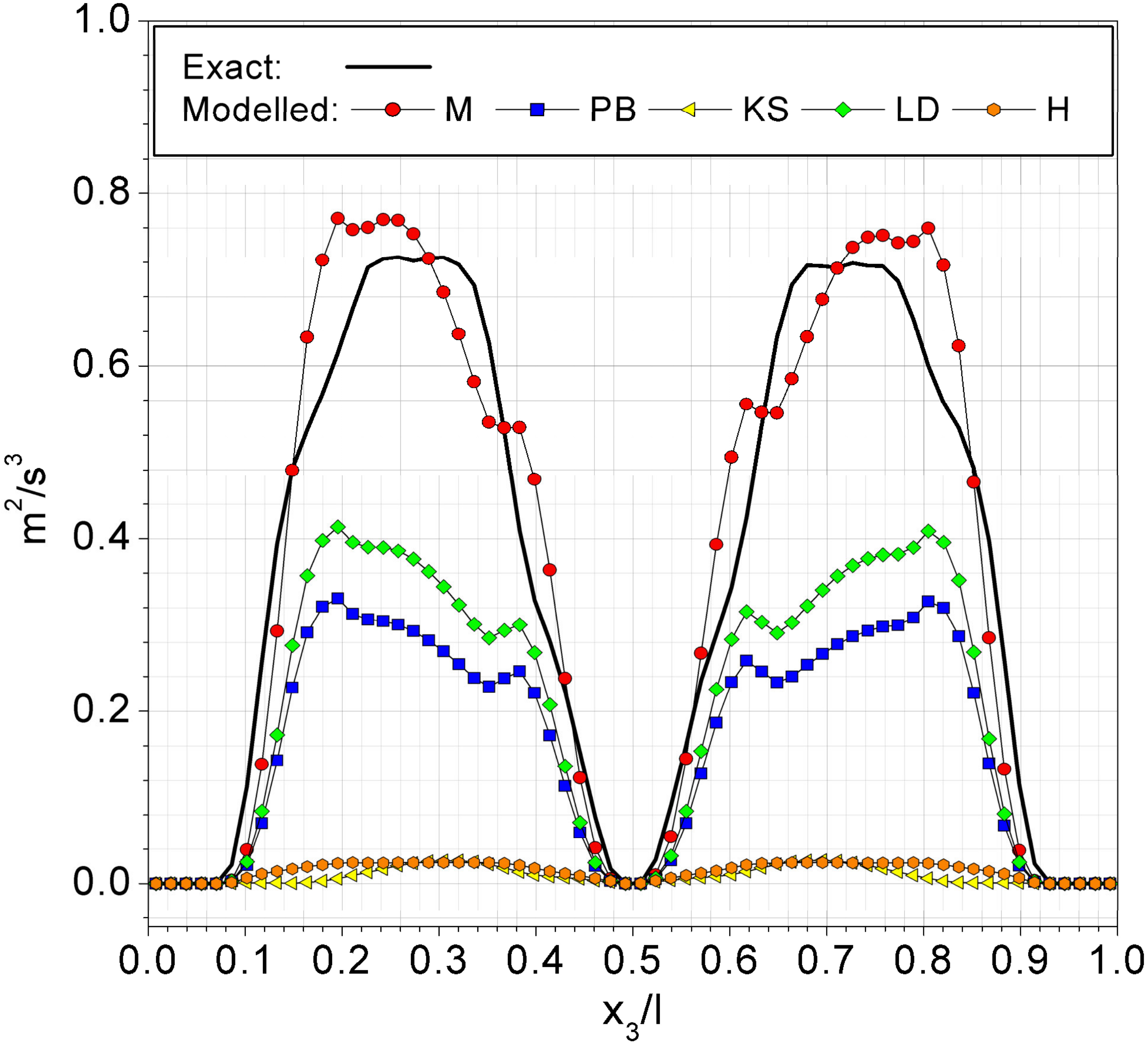
Models for interfacial term

Exact term:

$$\mathrm{IFT}(k_\mathrm{L}) = \overline{\left[\frac{1}{Re_\mathrm{ref}}\mathbb{T}'_\mathrm{L;in} - p'_\mathrm{L;in}\mathbb{I}\right]\cdot \mathbf{u}'_\mathrm{L;in}\cdot \mathbf{n}_\mathrm{L;in} a_\mathrm{in}}$$

Reference	Work of drag force, W_D^*	Other contributions, W_ND^*
Kataoka & Serizawa (1997) Model 1, KS	$0.075 f_\mathrm{w}\left[\frac{3}{4}\alpha_\mathrm{G}\frac{C_\mathrm{D}}{d_\mathrm{B}^*}U_\mathrm{T}^{*3}\right]$	$-\alpha_\mathrm{G}\frac{k_\mathrm{L}^{*3/2}}{d_\mathrm{B}^*}$
Hill *et al.* (1995) Model 2, H	$\frac{3}{4}\frac{\alpha_\mathrm{G} C_\mathrm{D}}{d_\mathrm{B}^*}\left\lvert\overline{\overline{\mathbf{u}}}_\mathrm{R}^*\right\rvert\left\{\frac{\mu_\mathrm{L}^*\overline{\overline{\mathbf{u}}}_\mathrm{R}^*\cdot\nabla^*\alpha_\mathrm{G}}{0.3\rho_\mathrm{L}^*\alpha_\mathrm{L}\alpha_\mathrm{G}} + 2k_\mathrm{L}^*(C_\mathrm{t}-1)\right\}$	None
Lahey & Drew (2000) Model 3, LD	$\frac{1}{4}\alpha_\mathrm{L}\left(1+C_\mathrm{D}^{4/3}\right)\alpha_\mathrm{G}\frac{\left\lvert\overline{\overline{\mathbf{u}}}_\mathrm{R}^*\right\rvert^3}{d_\mathrm{B}^*}$	None
Morel (1997) Model 4, M	$\frac{3}{4}\alpha_\mathrm{G}\frac{C_\mathrm{D}}{d_\mathrm{B}^*}\left\lvert\overline{\overline{\mathbf{u}}}_\mathrm{R}^*\right\rvert^3$	$\frac{1+2\alpha_\mathrm{G}}{2\alpha_\mathrm{L}}\alpha_\mathrm{G}\left\{\frac{\mathrm{D}_\mathrm{G}\overline{\overline{\mathbf{u}}}_\mathrm{G}^*}{\mathrm{D}t^*} - \frac{\mathrm{D}_\mathrm{L}\overline{\overline{\mathbf{u}}}_\mathrm{L}^*}{\mathrm{D}t^*}\right\}\cdot\overline{\overline{\mathbf{u}}}_\mathrm{R}^*$
Pfleger & Becker (2001) Model 5, PB	$1.44\alpha_\mathrm{L}\left[\frac{3}{4}\alpha_\mathrm{G}\frac{C_\mathrm{D}}{d_\mathrm{B}^*}\left\lvert\overline{\overline{\mathbf{u}}}_\mathrm{R}^*\right\rvert^3\right]$	None

Evaluation of models for interfacial term



Conclusions and outlook

- Detailed analysis of transport equation for liquid turbulence kinetic energy in bubbly flow
  - Production by shear stresses is negligible
  - Importance of interfacial term and diffusion term
- Evaluation of model assumptions
  - Production term, diffusion term
  - Interfacial term
- Outlook
  - Development of improved models
  - Implementation of improved models in CFX code and recalculation of experiments for bubble columns