Forschungszentrum Karlsruhe
in der Helmholtz-Gemeinschaft

Institut für Meteorologie und Klimaforschung

Karlsruhe Institute of Technology

Turbulence, Stability and Trend Removal in spectroscopic time series data

Peter Werle

Karlsruhe Institute of Technology, IMK-IFU

Kreuzeckbahnstrasse 19, 82467 Garmisch-Partenkirchen, Germany

Peter.Werle@kit.edu

A prerequisite (not only) for experimental work is

a measure that helps us

to judge how good we are.

Sample Variance

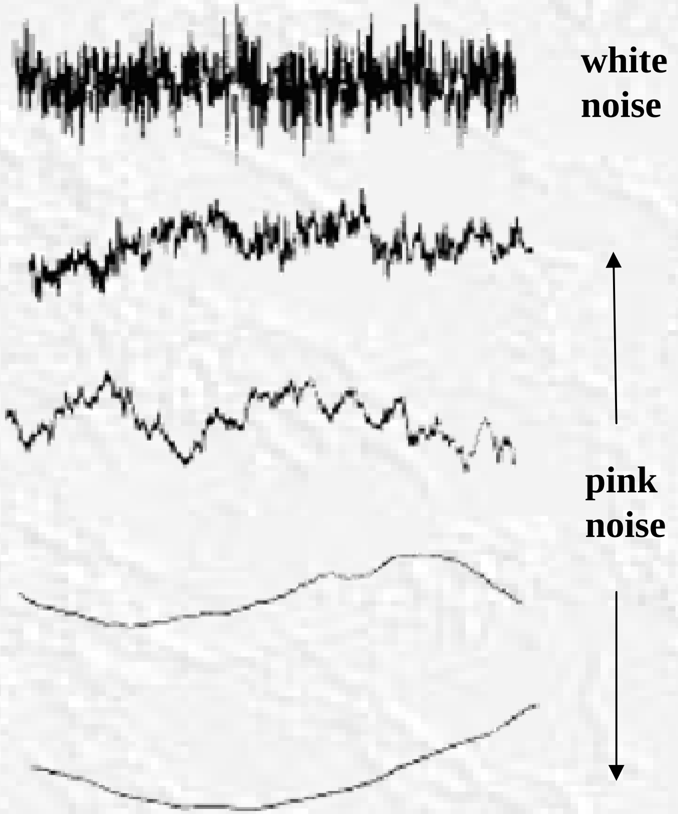
Typically, the sample variance is calculated from a data sample using the relation:

$$\sigma_{STD\ DEV\ y}(\tau) = \sqrt{\frac{1}{M-1}\sum_{i=1}^{M}\left(y_i - \bar{y}\right)^2}$$

where $\bar{y}$ is the sample mean calculated from the same data set.

What happens to the standard deviation …

… when a data set may be characterized by power law spectra which are more dispersive than classical white noise fluctuations ?



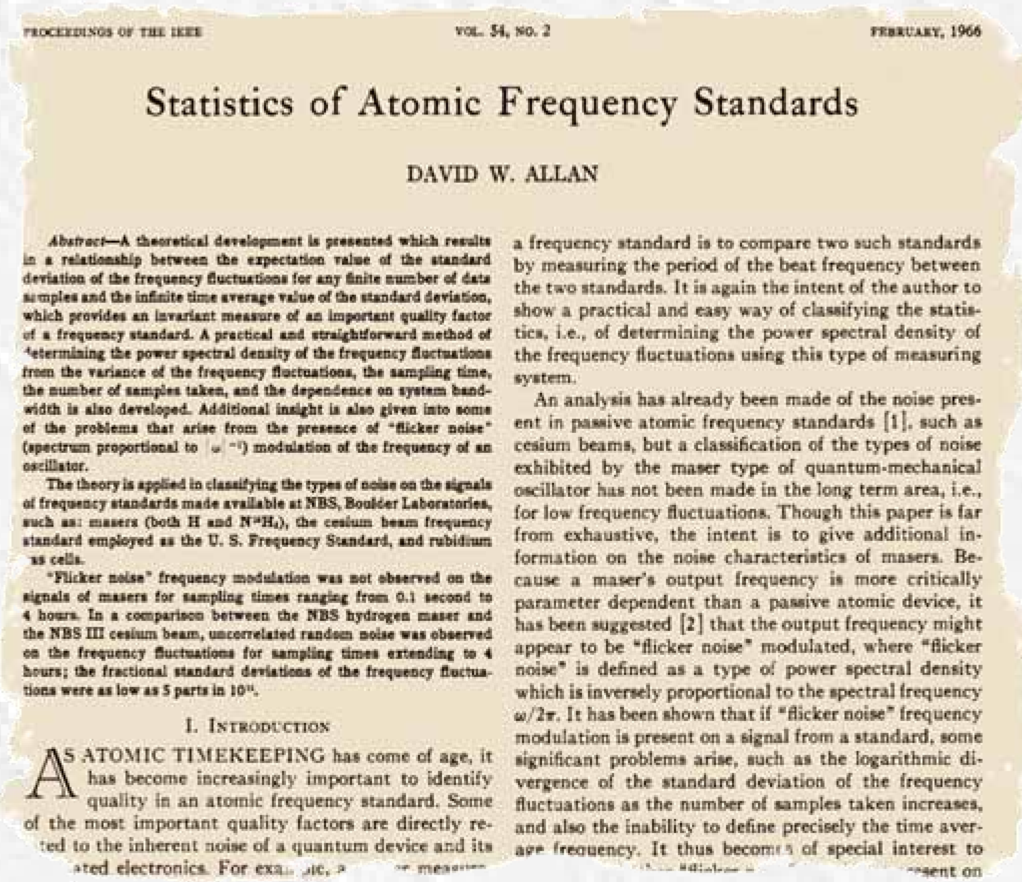
One can show that the standard deviation is a function of the number of data points in the set; it is also a function of the dead time and of the measurement system bandwidth. For example, using **flicker noise** as a model, as the number of data points increases, **the standard deviation monotonically increases without limit**.

Some statistical measures have been developed which do not depend upon the data length and which are readily usable for characterizing random fluctuations in precision oscillators.

IEEE subcommittee on frequency stability …

… has recommended von Neumann´s two sample variance and an experimental estimation of the square root of the "Allan variance" is

$$\sigma_y(\tau) = \sqrt{\frac{1}{2(M-1)}\sum_{i=1}^{M-1}\left(y_{i+1}-y_i\right)^2}$$


PROCEEDINGS OF THE IEEE VOL. 54, NO. 2 FEBRUARY, 1966

Statistics of Atomic Frequency Standards

DAVID W. ALLAN

Abstract—A theoretical development is presented which results in a relationship between the expectation value of the standard deviation of the frequency fluctuations for any finite number of data samples and the infinite time average value of the standard deviation, which provides an invariant measure of an important quality factor of a frequency standard. A practical and straightforward method of determining the power spectral density of the frequency fluctuations from the variance of the frequency fluctuations, the sampling time, the number of samples taken, and the dependence on system bandwidth is also developed. Additional insight is also given into some of the problems that arise from the presence of "flicker noise" (spectrum proportional to $|\omega|^{-1}$) modulation of the frequency of an oscillator.

The theory is applied in classifying the types of noise on the signals of frequency standards made available at NBS, Boulder Laboratories, such as: masers (both H and $N^{15}H_3$), the cesium beam frequency standard employed as the U. S. Frequency Standard, and rubidium gas cells.

"Flicker noise" frequency modulation was not observed on the signals of masers for sampling times ranging from 0.1 second to 4 hours. In a comparison between the NBS hydrogen maser and the NBS III cesium beam, uncorrelated random noise was observed on the frequency fluctuations for sampling times extending to 4 hours; the fractional standard deviations of the frequency fluctuations were as low as 5 parts in 10^{11}.

I. Introduction

As ATOMIC TIMEKEEPING has come of age, it has become increasingly important to identify quality in an atomic frequency standard. Some of the most important quality factors are directly related to the inherent noise of a quantum device and its associated electronics. For example, a … measure… a frequency standard is to compare two such standards by measuring the period of the beat frequency between the two standards. It is again the intent of the author to show a practical and easy way of classifying the statistics, i.e., of determining the power spectral density of the frequency fluctuations using this type of measuring system.

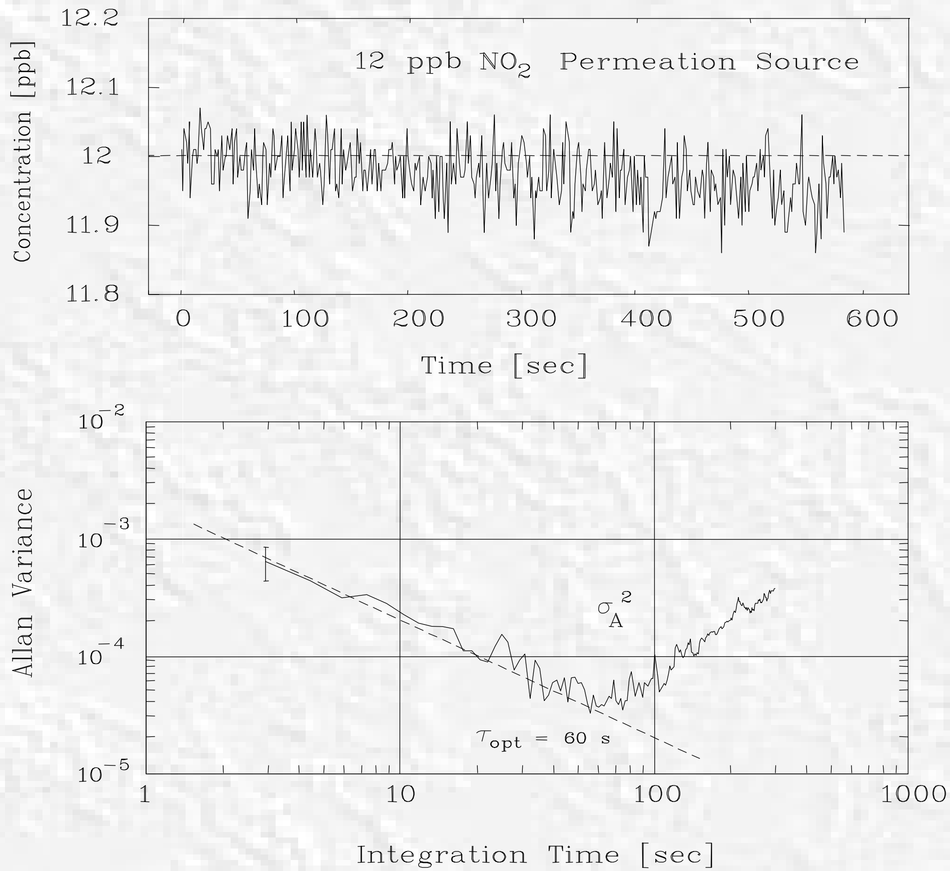
An analysis has already been made of the noise present in passive atomic frequency standards [1], such as cesium beams, but a classification of the types of noise exhibited by the maser type of quantum-mechanical oscillator has not been made in the long term area, i.e., for low frequency fluctuations. Though this paper is far from exhaustive, the intent is to give additional information on the noise characteristics of masers. Because a maser's output frequency is more critically parameter dependent than a passive atomic device, it has been suggested [2] that the output frequency might appear to be "flicker noise" modulated, where "flicker noise" is defined as a type of power spectral density which is inversely proportional to the spectral frequency $\omega/2\pi$. It has been shown that if "flicker noise" frequency modulation is present on a signal from a standard, some significant problems arise, such as the logarithmic divergence of the standard deviation of the frequency fluctuations as the number of samples taken increases, and also the inability to define precisely the time average frequency. It thus becomes of special interest to [illegible] present on

D. W. Allan, "Statistics of atomic frequency standards," *Proc. IEEE*, vol. 54, pp. 221–230, Feb. 1966.

The two sample variance approach is also useful in trace gas analysis

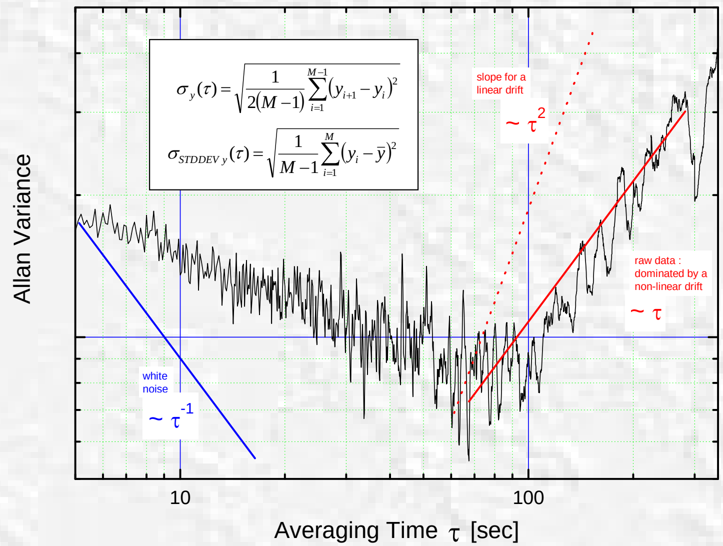
The concept of the Allan Variance has been applied to characterize spectroscopic instrumentation

and has become a well established tool for researchers and instrument developers to describe the performance of laseroptical trace gas sensors.



P. Werle et al. „The limits of signal averaging in atmospheric trace gas monitoring“, Appl. Phys. B 57, 131-139 (1993).

Detrending : What is the optimum time constant ?

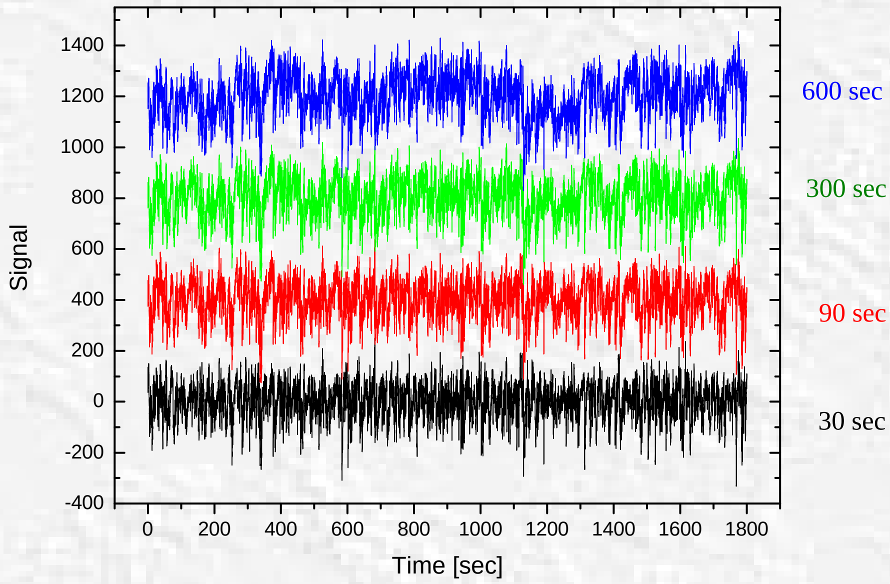


Plot of the two sample variance as a function of the subensemble averaging time t. The line following t^{-1} indicates the expected behaviour for a “white noise” dominated system.

Detrending of time series data

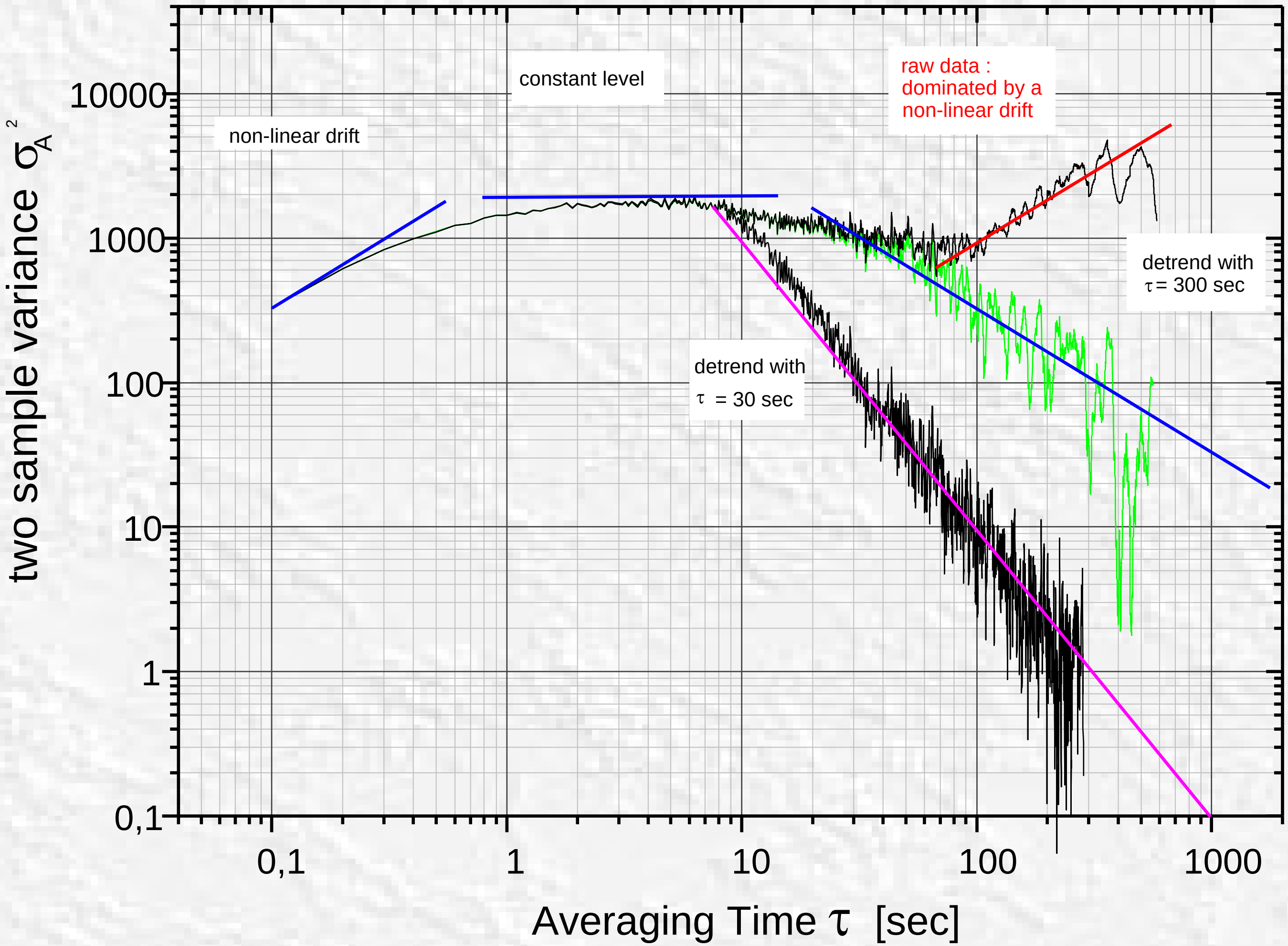
Smoothing effect of the moving average filter with different time constants

High pass filtered



$$y(t) = [\delta(t) - h(t)] * x(t)$$

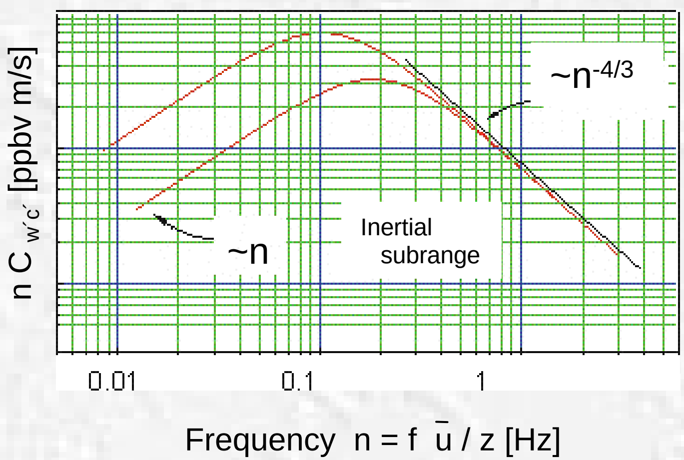
Stability Analysis vs Filter Time Constant



Frequency domain and time domain

Frequency Domain

Cospectra and Covariances

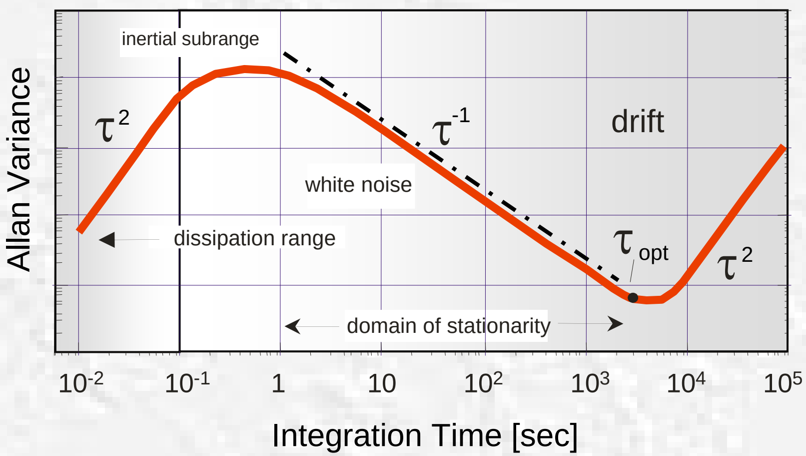


J.C. Kaimal et al., “Spectral characteristics of surface-layer turbulence“, Quart. J. R. Met. Soc. 98, 563-589 (1972)

„It is quite apparent that there is considerable similarity in the shapes of the logarithmic spectra and cospectra. On the high frequency side they fall off according to -2/3, **-4/3**, or *-3/2* depending on the parameter; on the low-frequency side the slope is very nearly +1 “

Time Domain

Allan Variance Analysis



P. Werle and R. Kormann, "Fast chemical sensor for eddy-correlation measurements of methane emissions from rice paddy fields “, Applied Optics 40, 851 (2001).