Forschungszentrum Karlsruhe in der Helmholtz-Gemeinschaft

Institut für Meteorologie und Klimaforschung





Winter College on Optics in Environmental Science

February 2-13, 2009 - Trieste, Italy

Atmospheric Monitoring with Tunable Diode Lasers

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H₂O - Fingerprints



Spectral signatures in the near- and midinfrared allow a unique quantitative identifikation of trace gases



Semiconductor Lasers

Relevant Characteristics for Gas Sensing



Optical Gas Analysis



Laser Spectroscopy



Lambert-Beer's Law

$$I(v) = I_0(v) e^{-\varepsilon(v) c l}$$

in the limit of low extinction : $\varepsilon(v) c l \ll 1$



Apply sensitive measurement technique with sufficient detection limits (e.g. 10⁻⁵ with modulation schemes)

P. Werle, "A review of recent advances in semiconductor laser based gas monitors", Spectrochimica Acta A - Review 54, 197-236, (1998) (for paper click <u>here</u>)

Select strong absorption line (e.g. from HITRAN Database) and check interferences !

Use long optical pathlength (White/Herriott multi-pass cells, e.g. 100m)

A Cryogenically Operated Laser Diode (COLD)

Reg Harris

S. Viciani, F. D'Amato, P. Mazzinghi, F. Castagnoli, G. Toci, P. Werle, Appl. Phys. B90, 581-592 (2008) View-PDF

Laser sources:

single mode FP lead salt TDL $\lambda = 5.8 \ \mu m$ for HNO₃ and H₂O $\lambda = 4.6 \ \mu m$ for N₂O and CO

Multipass cell:

astigmatic Herriott cell absorption path 36 m low volume (0.3 *l*)





A "Field" Diode Laser Spectrometer for Methane



P. Werle and R. Kormann, "A fast chemical sensor for eddy correlation measurements of Methane emissions from rice paddy fields", Appl. Opt. 40, 846-858 (2001). <u>View-PDF</u>



Dynamic Spectrometer Calibration is an issue



P. Werle, P. Mazzinghi, F. D'Amato, M. De Rosa, K. Maurer, F. Slemr, "Signal Processing and Calibration Procedures for in-situ Diode-Laser Absorption Spectroscopy", Spectrochimica Acta A - Review 60, 1685-1705 (2004) (for paper click <u>here</u>)

Signal Processing is an issue



P. Werle, P. Mazzingh, F. D'Amato, M. De Rosa, K. Maurer, F. Slemr, "Signal Processing and Calibration Procedures for in-situ Diode-Laser Absorption Spectroscopy", Spectrochimica Acta A - Review 60, 1685-1705 (2004) (for paper click here)

Background Drift



What are the Limits of Signal Averaging ?

What is the result of digital averaging of 10⁴ spectra ?

Which curve does the detection limit follow ?



Sample Variance

Typically, the sample variance is calculated from a data sample using the relation:

$$\sigma_{STD DEV y}(\tau) = \sqrt{\frac{1}{M-1} \sum_{i=1}^{M} (y_i - \overline{y})^2}$$

Where it is implicitly assumed that the y_i ,'s are random and uncorrelated (i.e., white) and where \overline{y} is the sample mean calculated from the same data set.

What happens to the standard deviation ...

... when a data set may be characterized by power law spectra which are more dispersive than classical white noise fluctuations ?



One can show that the standard deviation is a function of the number of data points in the set; it is also a function of the dead time and of the measurement system bandwidth. For example, using **flicker noise** as a model, as the number of data points increases, **the standard deviation monotonically increases without limit**.

Some statistical measures have been developed which do not depend upon the data length and which are readily usable for characterizing random fluctuations in precision oscillators.

IEEE subcommittee on frequency stability ...

PROCEEDINGS OF THE IEEE

... has recommended what has come to be known as the "Allan variance" taken from the set of useful variances developed, and an experimental estimation of the square root of the Allan variance is

$$\sigma_{y}(\tau) = \sqrt{\frac{1}{2(M-1)} \sum_{i=1}^{M-1} (y_{i+1} - y_{i})^{2}}$$

This equation is very easy to implement experimentally as one simply need add up the squares of the differences between adjacent values of y_i divide by the number of them and by two, and take the square root.

vol. 54, no. 2

FEBRUARY, 1966

Statistics of Atomic Frequency Standards

DAVID W. ALLAN

Abstract—A theoretical development is presented which results in a relationship between the expectation value of the standard deviation of the frequency fluctuations for any finite number of data samples and the infinite time average value of the standard deviation, which provides an invariant measure of an important quality factor of a frequency standard. A practical and straightforward method of "tetermining the power spectral density of the frequency fluctuations from the variance of the frequency fluctuations, the sampling time, the number of samples taken, and the dependence on system bandwidth is also developed. Additional insight is also given into some of the problems that arise from the presence of "flicker noise" (spectrum proportional to $|\omega|^{-1}$) modulation of the frequency of an oscillator.

The theory is applied in classifying the types of noise on the signals of frequency standards made available at NBS, Boulder Laboratories, such as: masers (both H and N³H₄), the cesium beam frequency standard employed as the U. S. Frequency Standard, and rubidium ias cells.

"Flicker noise" frequency modulation was not observed on the signals of masters for sampling times ranging from 0.1 second to 4 hours. In a comparison between the NBS hydrogen master and the NBS III cesium beam, uncorrelated random noise was observed on the frequency fluctuations for sampling times extending to 4 hours; the fractional standard deviations of the frequency fluctuations were as low as 5 parts in 10°.

I. INTRODUCTION

AS ATOMIC TIMEKEEPING has come of age, it has become increasingly important to identify quality in an atomic frequency standard. Some of the most important quality factors are directly reited to the inherent noise of a quantum device and its iated electronics. For example, a cor measure

a frequency standard is to compare two such standards by measuring the period of the beat frequency between the two standards. It is again the intent of the author to show a practical and easy way of classifying the statistics, i.e., of determining the power spectral density of the frequency fluctuations using this type of measuring system.

An analysis has already been made of the noise present in passive atomic frequency standards [1], such as cesium beams, but a classification of the types of noise exhibited by the maser type of quantum-mechanical oscillator has not been made in the long term area, i.e., for low frequency fluctuations. Though this paper is far from exhaustive, the intent is to give additional information on the noise characteristics of masers. Because a maser's output frequency is more critically parameter dependent than a passive atomic device, it has been suggested [2] that the output frequency might appear to be "flicker noise" modulated, where "flicker noise" is defined as a type of power spectral density which is inversely proportional to the spectral frequency $\omega/2\pi$. It has been shown that if "flicker noise" frequency modulation is present on a signal from a standard, some significant problems arise, such as the logarithmic divergence of the standard deviation of the frequency fluctuations as the number of samples taken increases, and also the inability to define precisely the time average frequency. It thus becomes of special interest to Lar #Ricker m resent on

D. W. Allan, "Statistics of atomic frequency standards," Proc. IEEE, vol. 54, pp. 221–230, Feb. 1966.

Sample Variance and Allan Variance



Time series x(t) with indicated sample time τ over which each y is measured. Equations are for standard deviation and for estimate of $\sigma_y(t)$ for a finite data set of M measurements.

Allan Plot



"Allan-Plot" for two different drift components. The rightmost trace corresponds to the "white noise and drift" data shown before.

Minimum of Allan Plot defines maximum integration time (= detection limit !) for signal averaging

First application in trace gas analysis

The concept of the Allan Variance has been proposed and applied to characterize spectroscopic instrumentation

P. Werle, R. Muecke and F. Slemr, "The limits of signal averaging in atmospheric trace gas monitoring by tunable diode-laser absorption spectroscopy", Appl. Phys. B 57, 131-139 (1993). <u>View-PDF</u>

and has become a well established tool for researchers and instrument developers to describe the performance of laseroptical trace gas sensors.



Detection Limits

for atmospheric constituents for a sampling pressure of 30 mbar and T=296 K based upon a minimum detectable absorbance of 10^{-5} and 100 m optical path-length

Species	λ μm	ν cm ⁻¹	S 10 ⁻¹⁹ cm/molec	$\frac{\gamma_{v}}{10^{-3}}$ cm ⁻¹	γ_v MHz	σ 10 ⁻¹⁷ cm²/molec	DL pptv
N ₂ O	7.69	1301	1.56	2.77	83.1	1.92	70
СО	4.65	2151	1.89	3.85	115.5	1.85	73
CH ₄	7.56	1322	0.53	3.25	97.5	0.60	225
NO	5.33	1876	0.34	3.26	97.8	0.39	350
SO ₂	7.29	1372	0.49	3.86	115.8	0.41	330
NO ₂	6.25	1600	2.18	2.90	87.0	2.70	50
NH ₃	10.74	931	5.20	2.92	87.6	6.18	23
HNO ₃	5.81	1722	0.72	3.75	112.5	0.25	218
HF	1.24	7856	0.76	12.09	362.7	0.27	500
HCI	3.40	2945	5.03	4.30	129.0	4.51	30
HBr	3.78	2649	0.45	2.90	87.0	0.57	240
CIO	11.6	860	0.08	2.73	81.9	0.09	1500
OCS	4.87	2053	10.30	3.55	106.5	10.00	13
НСНО	3.56	2781	1.19	5.17	155.1	0.84	160
CH ₃ CI	3.29	3040	0.02	4.12	123.6	0.02	7800
H ₂ O ₂	7.79	1284	0.45	3.52	105.6	0.43	320
H ₂ S	7.33	1365	0.01	4.89	146.7	0.007	20000
НСООН	8.98	1113	0.44	3.32	99.6	0.44	310
HO ₂	7.08	1411	0.12	3.63	108.9	0.11	1200

Current Limitations



background is **time dependent** moves due to small temperature/pressure drifts

repeated background measurement necessary duty cycle ~ 50% - 60%



Concept : Sample Modulation - Stark Effekt

a periodic modulation of sample energy levels ...



Background Suppression

... influences signal from sample only - background is not affected



A Laser – Sample Double Modulation Setup



Signal comparison

no sample modulation

sample modulation





a promising approach ...

no sample modulation

sample modulation



... the next step

a multi-pass cell with small volume of 0.5 Liter for a fast gas exchange and a laminar gas flow to avoid turbulence



Stark Multipass Cell Design

ring mirror 11 m radius 61 spots 122 passes 46 cm baselength 0,9 I Volume





Let's move from the laboratory to ...

Industrial Process Control & Emission Monitoring High Purity Gases (Semiconductor Industry) Plasma Diagnostics, Combustion Diagnostics, Lifescience & Agriculture & Medicine

Environmental Research

from the upper Troposphere



and lower Stratosphere

to Biosphere - Atmosphere





exchange over complex terrain









FLAIR - Industry Session

http://www.inoa.it/flair/industry.htm

... an assessment of the present status

Industrial Analytical Systems based on Tunable Diode-Lasers



DFB Diode Lasers

Nanosystems and Technologies GmbH Nanoplus









- Longitudinal single mode
- Circular and low divergence output beam
- High fiber coupling efficiency
- Small power consumption
- One and two dimensional arrays
- Simplified manufacturing, testing & packaging
- Low-cost potential

www.vertilas.com

Laser type	Threshold current	Output power	Current tuning rate	Current tuning range	Temperature tuning rate	Maximum modulation
VCSEL	0.5–2 mA	0.4-3 mW	0.7 nm/mA	3-5 nm	~0.11 nm/°C	> 1 MHz
DFB	20 mA	5 mW	<0.01 nm/mA	< 1 nm	~0.11 nm/°C	< 100 kHz

Lead Salt Diode Lasers







Lead-salt diode-laser

- Spectral Positions: 400 3250 cm⁻¹
- Spectral Resolution: 10⁻⁴ cm⁻¹
- min. power 0.1 mW up to typ. 1 mW
- CW and pulsed to MHz

- Current: 50 to 2,000 mA
- Temperature: 25 to 120 K
- Delivery time 6-8 weeks

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Quantum Cascade Laser

LN₂ and RT continuous-wave DFB

Available wavelengths off-the-shelf: 4.2 to 10.4 μm

Characteristics:

Power typical 2 mW (max up to **100 mW**) Operating current: 0.3 - 2 A, typical 0.8 A Operating voltage: 8-10 V Tuning range: > **0.4%** Spectral linewidth ~ **3 MHz**

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QCL Spectrometers



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AERODYNE RESEARCH

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TDL & Cavity ring down instruments for Field Application

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- Thin-Film Absorption Measurements
- Substrate-Absorption Measurements
- Process Control and Validation
- •Quality Control
- •Testing of Optical Coatings
- •Total Insertion-Loss Measurements
- Material Research
- Atmospheric Monitoring
- •Industrial-Process Monitoring and Control
- Homeland Security
- Trace-Gas Sensing
- Optimization of Singlet-Oxygen Generators
- •Chemical Oxygen-Iodine Laser Studies
- •Fundamental Physical Sciences

- •Plasma Decontamination
- Carbon Sequestration in Ocean Waters

G

- Eddy-Correlation Flux Measurements
- •Chamber-Flux Measurements
- Combustion Diagnostics
- Trace-Gas Monitoring
- •Leak Detection from Natural-Gas Pipelines
- Hydrological Applications
- •Methane-Hydrate Studies
- •Hydrotherrmal-Vent-Effluent Analysis
- Analytical Applications
- Biological-Science Applications
- •Atmospheric Sciences
- Methanogenic-Bacteria Studies



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Rec .

Field Laser Applications in Research

Airborne in-situ measurements ...

are a useful tool for calibration and validation of remote sensing instruments for atmospheric research

the fast response of TDLS allows detection of fine structures also when operating at the cruise speed of jet aircrafts

achieved scientific results indicates further area to be investigated by the next generation of airborne or satellite remote sensors

- study of ultra-thin clouds (UTTC, PSC)
- small scale effects leading to mixing phenomena very important in homogeneous and heterogeneous chemistry (tropical convection, polar vortex filaments)

Stratospheric Research Aircraft





Aircraft	WB-57	ER-2	M-55	Proteus
Make	EEC	Lockheed	Myasishchev	Scaled Composites
Operated by	NASA	NASA	Geophysica EEIG	Angel Technol. Corp.
Birth	1944 (B57)	1955 (U2)	1988	1998
Flight				
parameters	Sec. 12		1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
Altitude (m)	18000	21300	21830	18600 (22000 design)
Range (km)	4600	4100	4500	5500
Cruise Speed (m/s)	210	210	210	120
Payload (kg)	2700	1000	1500	3200
Duration (h)	6 h 30'	6 h 30'	5 h 40'	20



INOA instruments :

Mid-Infrared TDL for HNO₃/H₂O or N₂O/CO

Near-Infrared TDL for CH₄ and CO₂

The TDLS on the Geophysica are part of a complete payload system including *in situ* and remote sensing instruments, particle analysis etc...

 \Rightarrow This is necessary to achieve valuable scientific results



A Cryogenically Operated Laser Diode (COLD)

S. Viciani, F. D'Amato, P. Mazzinghi, F. Castagnoli, G. Toci, P. Werle, Appl. Phys. B90, 581-592 (2008) View-PDF

Laser sources:

single mode FP lead salt TDL $\lambda = 5.8 \ \mu m$ for HNO₃ and H₂O $\lambda = 4.6 \ \mu m$ for N₂O and CO

Multipass cell:

astigmatic Herriott cell absorption path 36 m low volume (0.3 *l*)





COLD – Data Processing

Acquisition

- direct absorption with fast sweep integration
- sample rate 5 MS/s @ 12 bit resolution
- 1000 points spectra acquired @ 2 kHz
- up to 64,000 averaged scans
- equivalent resolution up to 28 bit (2.6 10⁸)
- interleaved acquisition of reference spectra

Data processing

- FFT filtering and/or fringe subtraction
- lineshape fitting of the averaged spectra
- number density retrieved from HITRAN
- concentration calculated as f(T, P)

Achieved performance

Absorption sensitivity

- < 10⁻⁵ in laboratory tests
- $\approx 10^{-4}$ during flights

Ultimate sensitivity is due to fringes, not to detection technique

COLD after several campaigns



Learned from campaigns

Liquid nitrogen is not a problem Actually sometimes it is an advantage

in case of power failure \Rightarrow the laser remains cool

in case of high humidity on ground \Rightarrow keeps the whole optics dry \Rightarrow used for flushing the cell

Reliability was found quite high



 \Rightarrow No laser failures during campaigns, just replacements for different targets \Rightarrow The same set of mirrors still in use after 6 years and 5 campaigns! \Rightarrow Alignment found very stable (no realignment required in campaigns).

Main problems were:

multimode emission from some lasers communication failure with the laser controller

Future developments



- Increasingly-sophisticated scientific questions addressed by *in situ* payloads (aircraft, balloon) has increased demand for higher precision, higher accuracy measurements of tracers and water
- Aircraft platforms need duplication with differing techniques for continuous intercomparison (TDL, QCL, CRD, DFG,)

- fast systems for airborne eddy correlation measurements
- in-situ measurements of reactive species in open multipass cells
- airborne measurements of water isotopes ...

Biogeochemical cycles - Turbulent Fluxes



The fast diode-laser trace gas sensor



TO DO :

- detrending
- frequency spectra
- covariance and cospectrum
- time lag



Detrending of time series data

Smoothing effect of the moving average filter with different time constants



Low pass filtered

High pass filtered

Detrending : What is the optimum time constant ?



Signal

10 Hz time series data together with selected plots for subensemble averages with bin-sizes of 30s, 90s and 300s.



Plot of the two sample variance as a function of the subensemble averaging time t. The line following t-1 indicates the expected behaviour for a "white noise" dominated system.

Stability Analysis vs Filter Time Constant



Two sample variance characterization of a trace gas concentration time series in the time domain. For averaging intervals below 1 s atmospheric turbulence is well resolved and appears as a 'non-linear drift', while in the range between 1s to 10s 'flicker noise' dominates. The upper right part is the drift dominated behaviour of the raw data. The middle trace can be considered as ideal and corresponds to the application of a 300 s moving average filter for detrending. The lower trace belongs to the 30 sec time constant.

Detrended time series data to get the fluctuations



"running means" or "high-pass filtering" : Allan Variance and be used to determine the high pass constant

Turbulent flux after time lag correction



Covariance & Cospectrum

$$Cov[w,c](\Delta t) \propto \int w'(t+\Delta t) \cdot c'(t) \cdot dt \qquad C_{w'c'}(n) \propto \operatorname{Re}\left[W(n) \cdot C^*(n) \cdot e^{2\pi i n \tau_d}\right]$$

Turbulent Flux $F' = Cov [w, c](\tau_d)$





Mean daytime flux vs wind direction





Mean daytime flux : 6,35 ppbv m/s

Footprint estimation





Schuepp et. al., Bound.-Lay. Met 50, 355 (1990)

Results



The closed chamber measurements reported to about 70% higher CH₄ fluxes than the eddy corrrelation instruments (3 independent systems).

S.

Closed chamber measurements may overestimate methane fluxes from rice paddy fields.

Werle, P., Kormann, R., 2001. A fast chemical sensor for eddy correlation measurements of Methane emissions from rice paddy fields. Appl. Opt. 40, 846-858. Denmead, O.T., 2008. Approaches to measuring fluxes of methane and nitrous oxide between landscapes and the atmosphere. Plant Soil 309, 5–24.

For more Field Laser Applications in Industry and Research

FLAIR 2009 – Sept. 6-11, 2009 Garmisch-Partenkirchen, Germany



http://imk-ifu.kit.edu/flair/

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http://www.inoa.it/home/pwwerle/papers.html