Generalized Weibull approach to interface failure in bi-material ceramic joints

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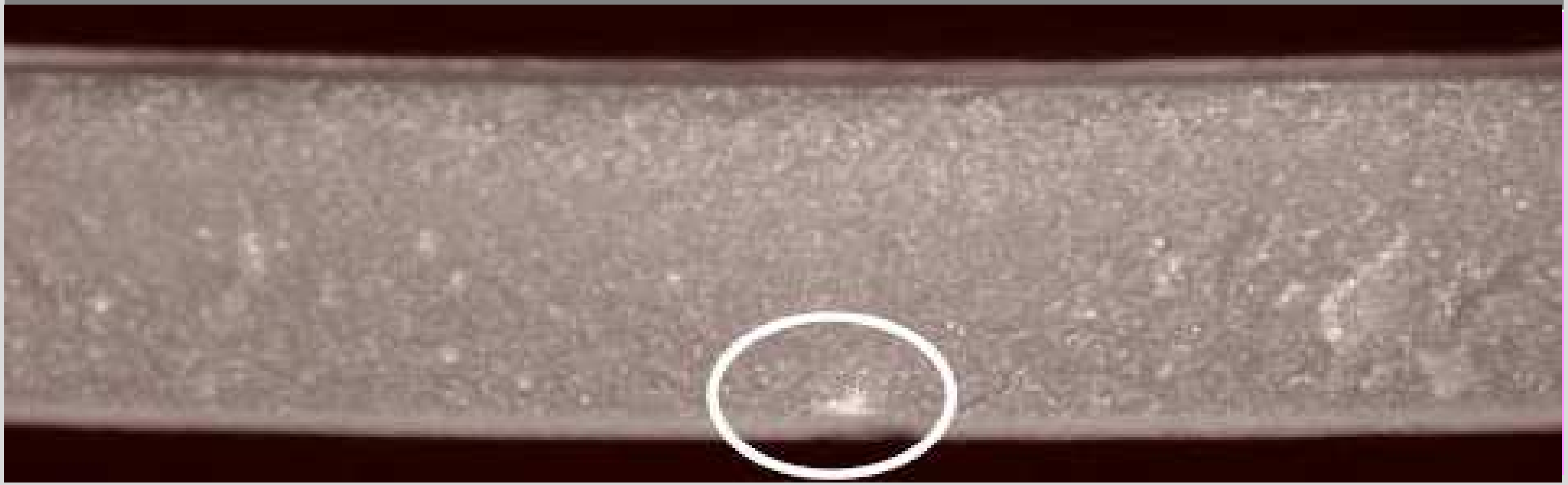

Outline

Generalization of Weibull theory
for interface failure in bi-material ceramic joints

1. Introduction / Motivation:
- what is the interface?
- where do interfaces occur?
- quality of interfaces

2. Fracture mechanics of interfaces:
- computation of interface stresses
- interface cracks: order of stress singularity
- fracture criteria

3. Weibull approach for failure probability calculation:
- generalization to the case of interface failure
- role of different parameters on interface failure:

4. Summary

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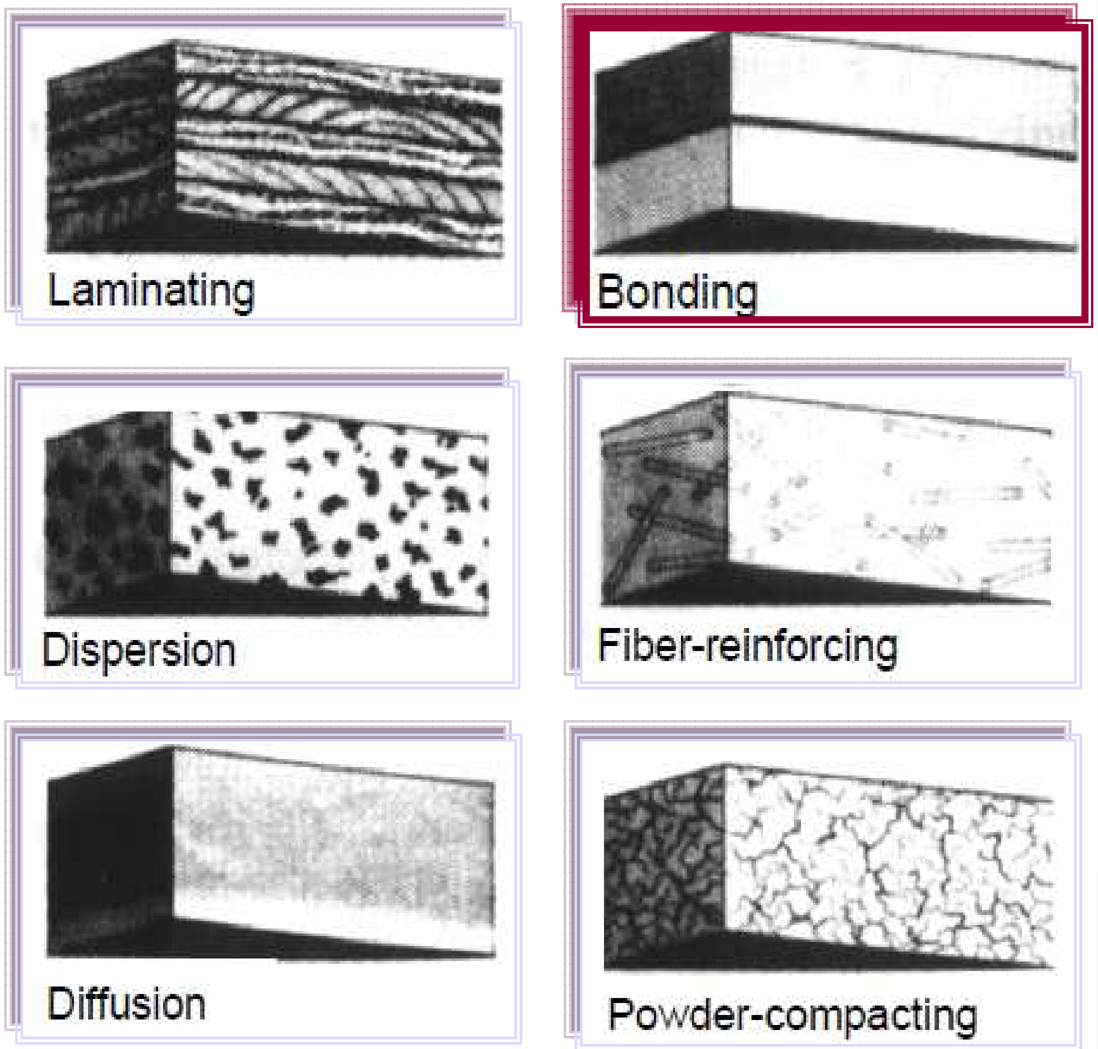
What is composite material?

Structural material made of two or more different materials

A structure or an entity made up of distinct components

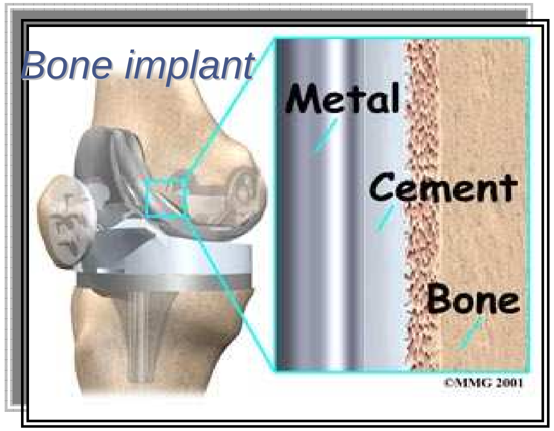
A complex material, in which two or more distinct substances (metals, ceramics, glasses, polymers) are combined to produce structural or functional properties not present in any individual component



Motivation: Where do we use material joints?

www.texarkanaortho.com

Orthopaedic surgery



www.eurodent-st.ru

Stomatology

Need in ceramic implants:

- Recovery of the functionality of damaged components

- Increase duration of implant usage

- Cosmetic reasons

- etc…

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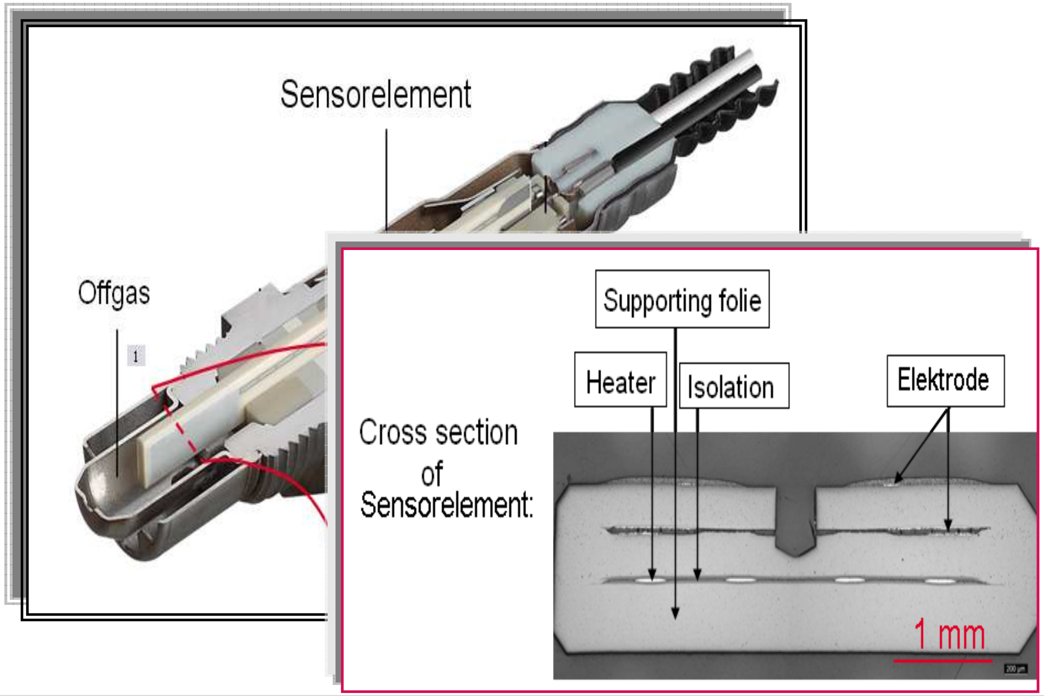
Motivation: Where do we use material joints?

- Reliability of material joints plays an important role in many engineering applications

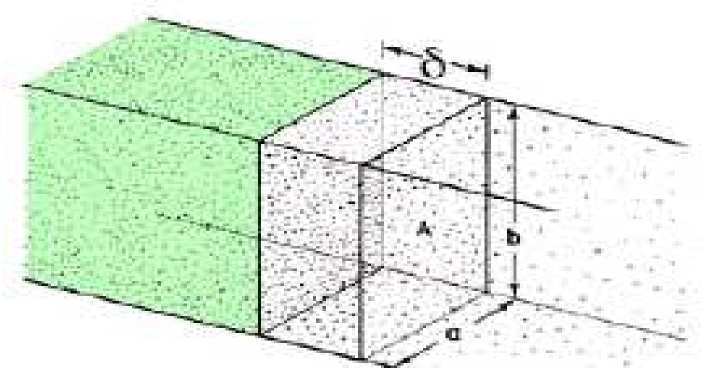
- Lambda - sensor –

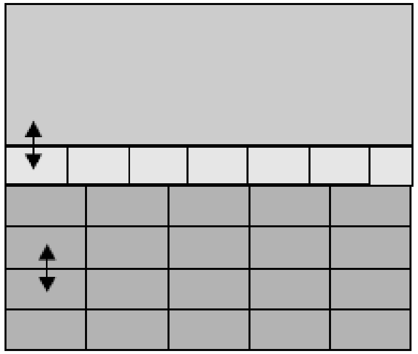
electronic device containing number of material interfaces

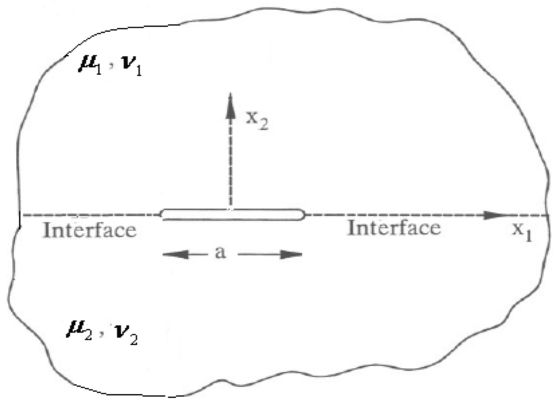


What is an interface for us?

The area of interconnection between two entities…







A thin layer or boundary between two different substances or two phases of a single substance.

Interface can be approximated with a sharp line/plane between two elastic and ideally bonded materials.

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Advantages and disadvantages of interfaces

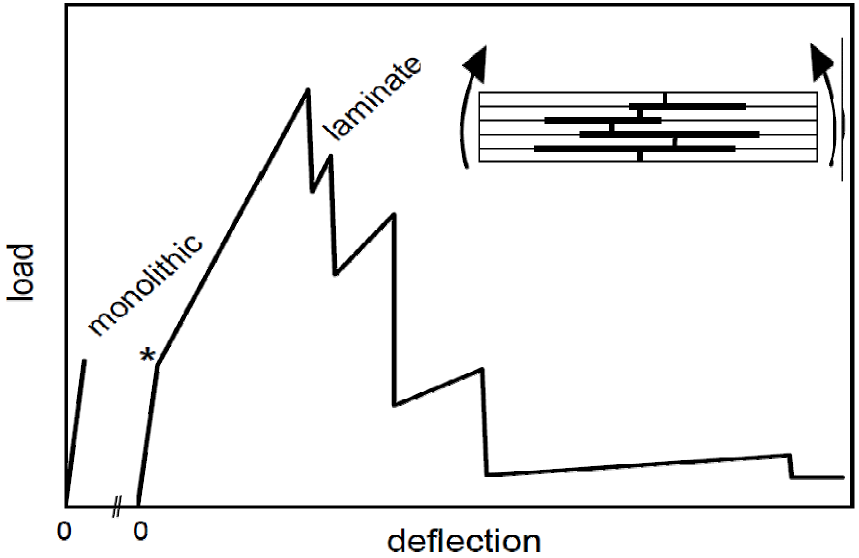
Advantages of ceramic materials:

✓ low electrical / thermal conductivity
✓ high strength at high temperatures
✓ corrosion resistance

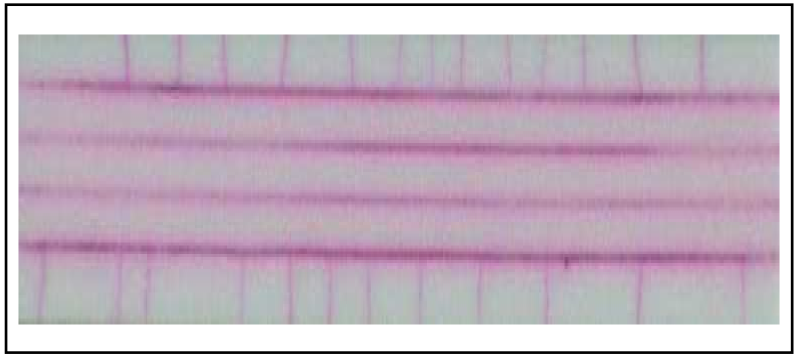
Disadvantages of ceramic materials:

✓brittleness
✓subcritical crack extension

Weak interfaces



Strong interfaces



T. Lube (2007)

Advantages and disadvantages of interfaces

1. Improvement of **functional behaviour**

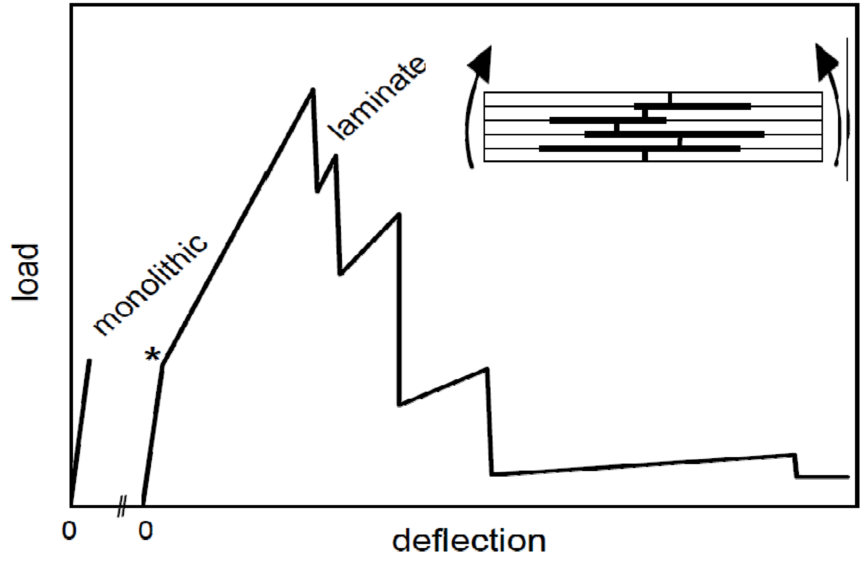
- Layered system with weak interfaces presents stronger and tougher ceramic component.

- Cracks arresting at the material joint with strong interfaces

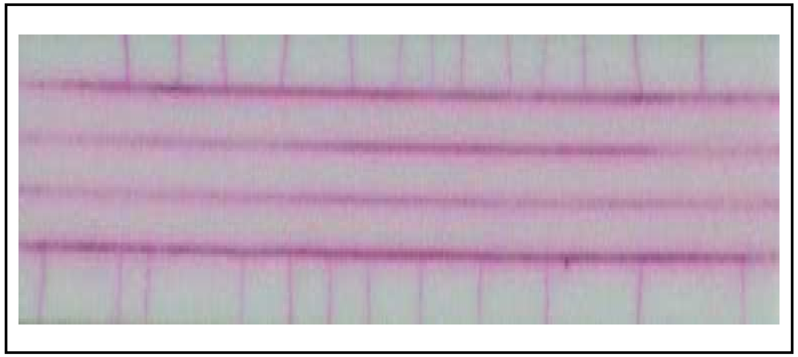
2. **Strength reliability** under transient service conditions

- Delamination fracture of the material joint with weak interfaces

Weak interfaces



Strong interfaces



T. Lube (2007)

Bi-material mechanical properties

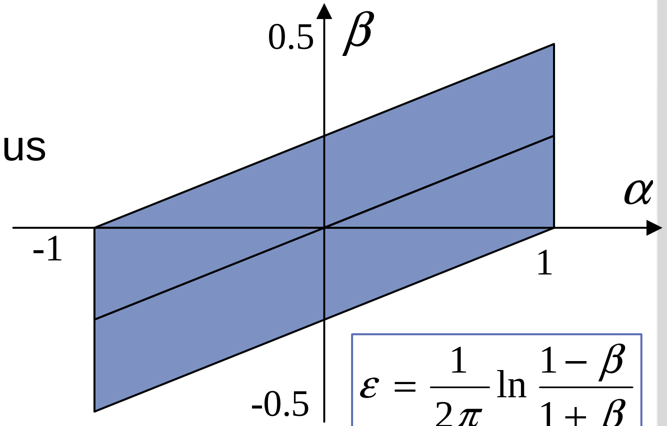
- *DunDurs' parameters*

$$\alpha = \frac{\mu_2(1-\nu_1)-\mu_1(1-\nu_2)}{\mu_2(1-\nu_1)+\mu_1(1-\nu_2)}$$

Mismatch in tensile modulus across the interface

$$\beta = \frac{1}{2}\frac{\mu_2(1-2\nu_1)-\mu_1(1-2\nu_2)}{\mu_2(1-\nu_1)+\mu_1(1-\nu_2)}$$

Mismatch in bulk modulus across the interface



$$\boxed{\varepsilon = \frac{1}{2\pi}\ln\frac{1-\beta}{1+\beta}}$$

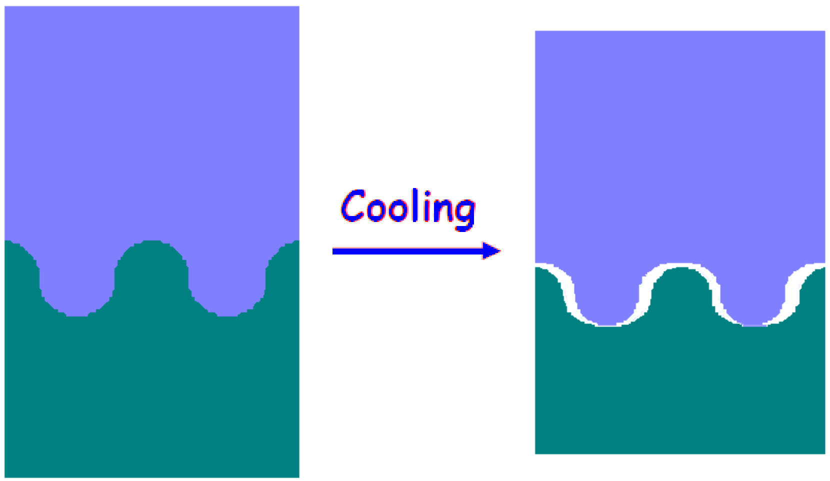
Dundurs (1967)

- *Coefficient for thermally induced stress intensity factor*

$$K_T = \frac{2\Delta\alpha_T\mu_1\mu_2}{\sqrt{\left(\mu_1+\mu_2(3-4\nu_1)\right)\left(\mu_2+\mu_1(3-4\nu_2)\right)}}$$

$\Delta\alpha_T$ - thermal expansion coefficients

Suga et al. (1984)



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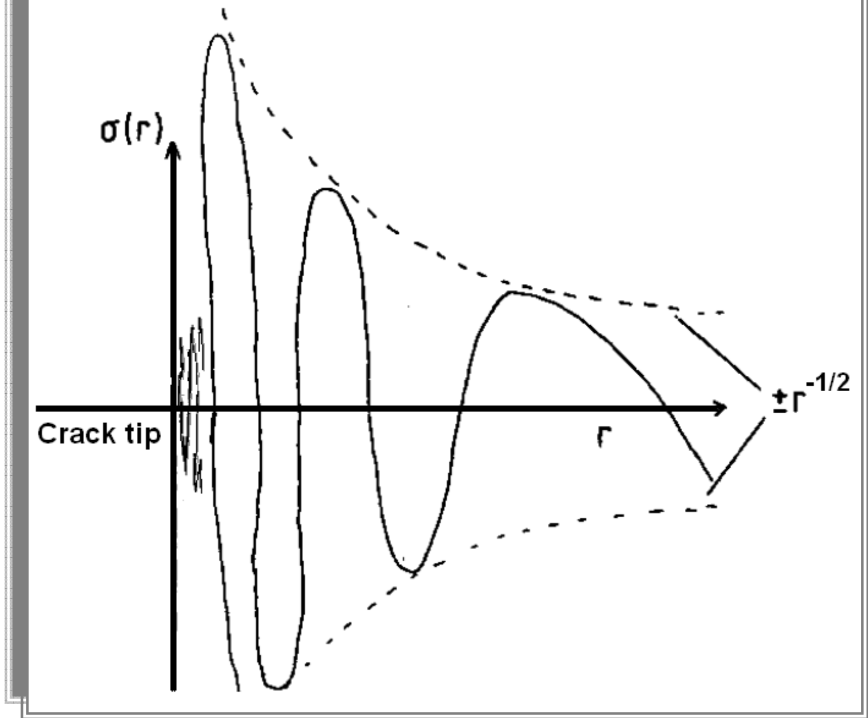
Computation of interface stresses

- *Nature of interface stresses*

- Crack in homogeneous material $r^{-1/2}$
- Crack at the bi-material interface $r^{-1/2+i\varepsilon}$

→ $\sigma \to r^{-1/2}(\sin[\varepsilon \ln r], \cos[\varepsilon \ln r])$

→ $U \to r^{1/2}(\sin[\varepsilon \ln r], \cos[\varepsilon \ln r])$



G. Loebel (1984)

Oscillations of stress and displacement fields
at tip of interface crack

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Stress intensity factors (SIFs)

The difference in elastic properties of materials causes high stress gradients on the crack faces

- *Crack in homogeneous material*

$$\boxed{K \to (Force)\cdot(length)^{1/2}}$$

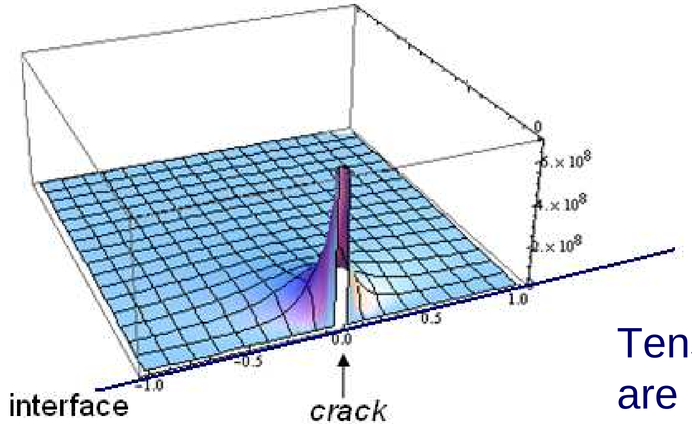
$$K_I = \sigma_{22}\sqrt{2\pi r}$$

$$K_{II} = \sigma_{12}\sqrt{2\pi r}$$

- *Crack at the bi-material interface*

$$\boxed{K \to (Force)\cdot(length)^{1/2}\cdot(length)^{-i\varepsilon}}$$

Stress σ_{yy} in the upper material



$K = K_1 + iK_2$ - Complex Stress Intensity Factor

$$\mathrm{Re}[\,Kr^{\,i\varepsilon}\,] = \sigma_{22}\sqrt{2\pi r}$$

$$\mathrm{Im}[\,Kr^{\,i\varepsilon}\,] = \sigma_{12}\sqrt{2\pi r}$$

Tensile and shear effects near the tip of the crack are inseparable into analogues of classical mode I and mode II

Stress intensity factors (SIFs)

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- *Crack in homogeneous material*

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- *Crack at the bi-material interface*

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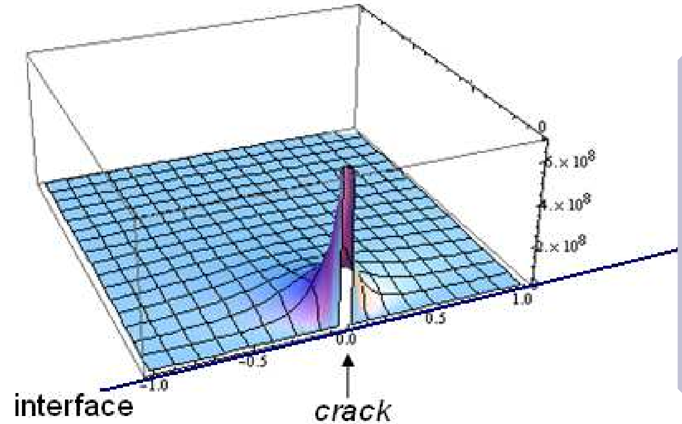
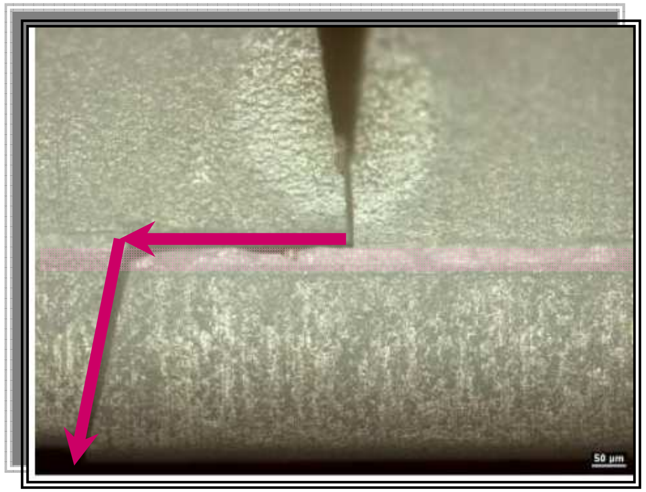
Stress σ_{yy} in the upper material



$K = K_1 + iK_2$ - Complex Stress Intensity Factor

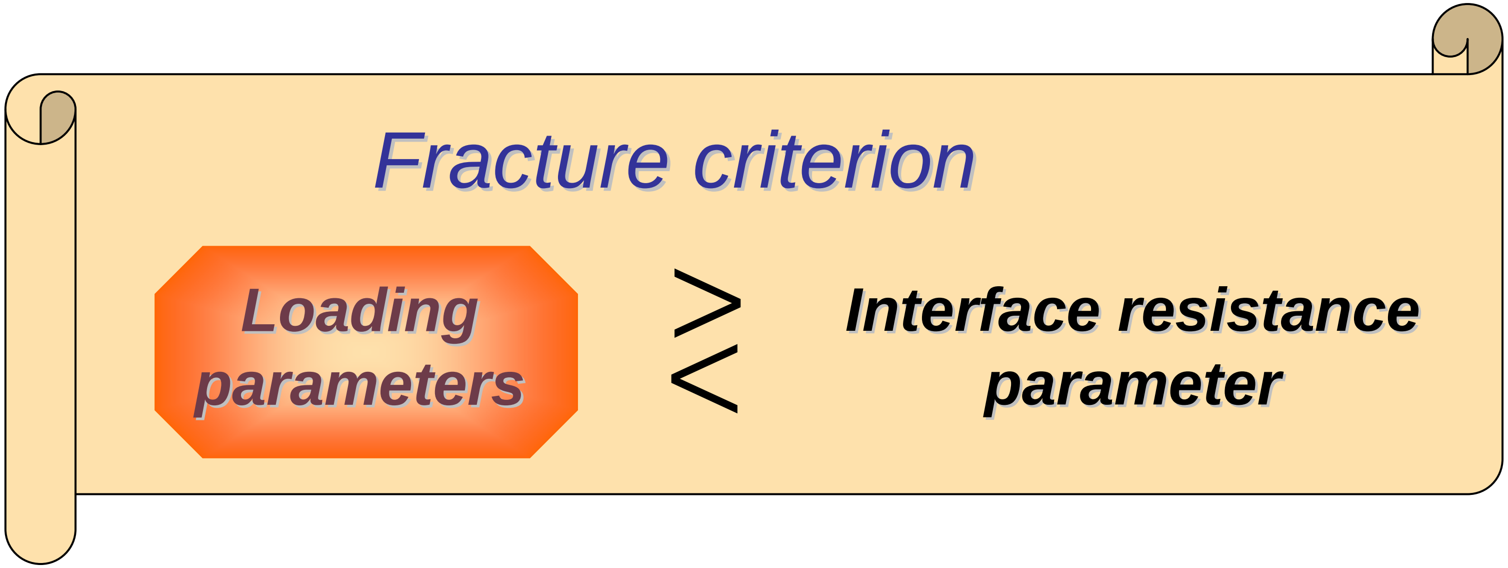
- ➢ Stress intensity magnitude $\quad |K| = \sqrt{K_1^2 + K_2^2}$
- ➢ Phase angle $\quad \psi = \arctan\left[\dfrac{\mathrm{Im}(KL^{i\varepsilon})}{\mathrm{Re}(KL^{i\varepsilon})}\right] = \arctan\left[\dfrac{\sigma_{12}}{\sigma_{22}}\right]\Bigg|_{\theta=0, r=L}$
- ➢ Arbitrary characteristic length L $\quad$ L = 100 μm

Interface fracture criteria



Crack propagation path:

- No crack extension
- Crack propagates along the interface
- Crack kinks into one of the materials



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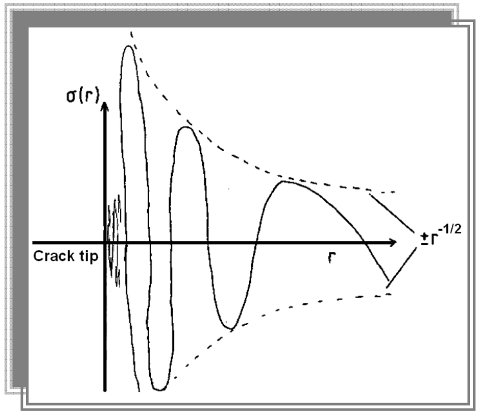
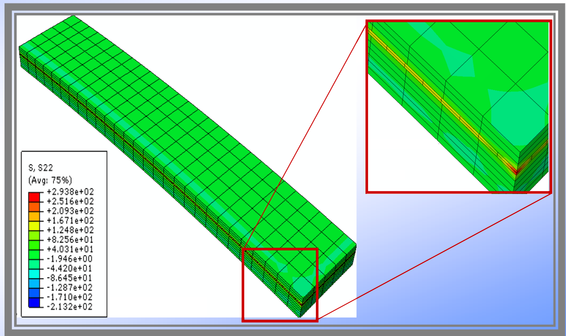
Computation of interface stresses



Determination of interface stresses

- FE modeling



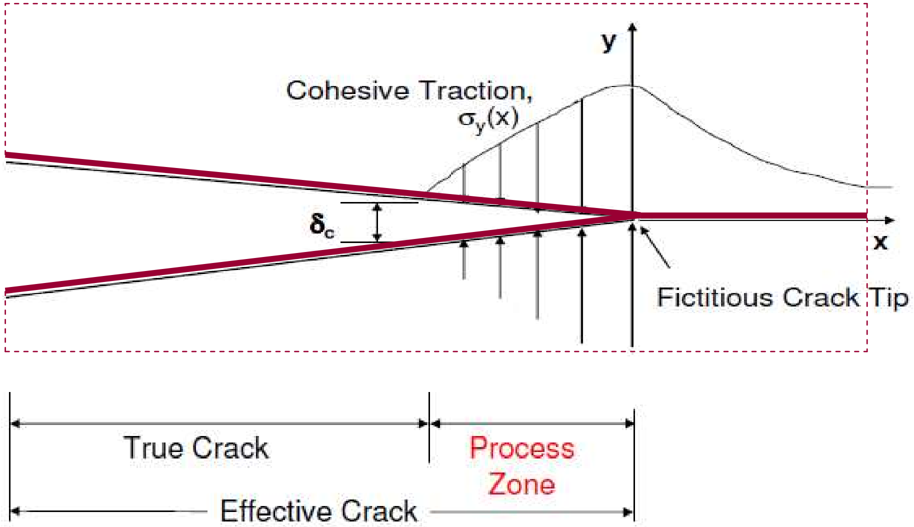
- Analytical stress computation

$$\sigma_{ij} = \frac{\mathrm{Re}[Kr^{i\varepsilon}]}{\sqrt{2\pi r}} \Sigma_{ij}^{(1)}(\theta,\varepsilon) + \frac{\mathrm{Im}[Kr^{i\varepsilon}]}{\sqrt{2\pi r}} \Sigma_{ij}^{(2)}(\theta,\varepsilon)$$

FE modelling: Numerical computation of interface stresses

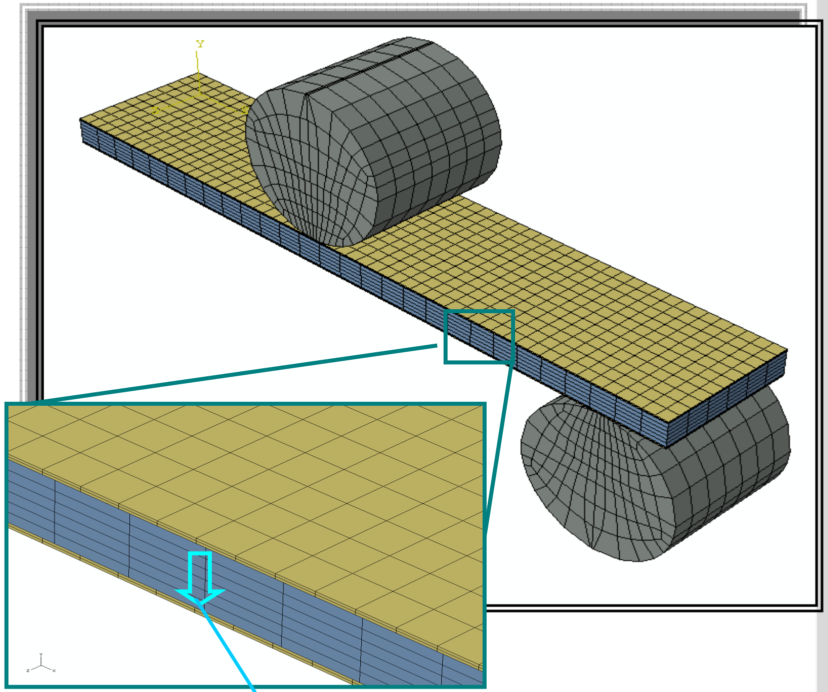
Crack tip models for FE analyses

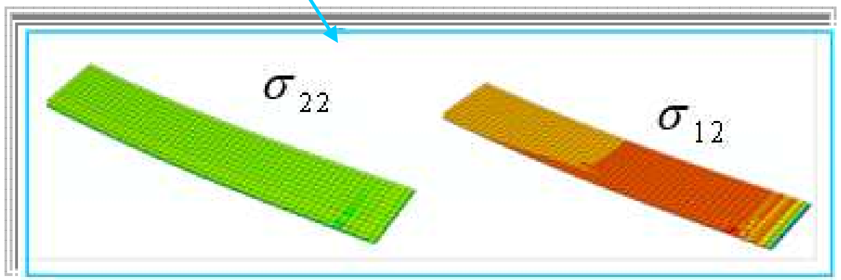
- Cohesive zone model
- Bridged zone model



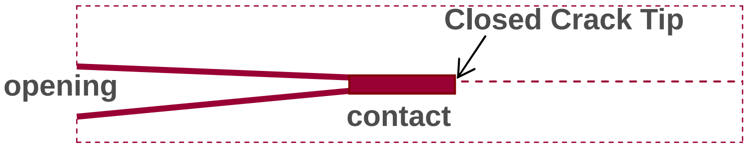
Perfect brittle materials Barenblatt (1959,1962)

Perfect plastic materials Dugdale (1962)





- Contact zone model



Comninou (1990)

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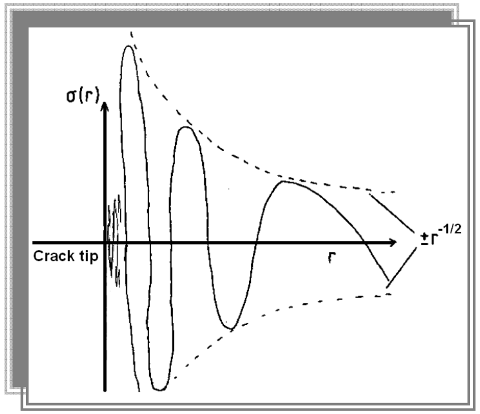
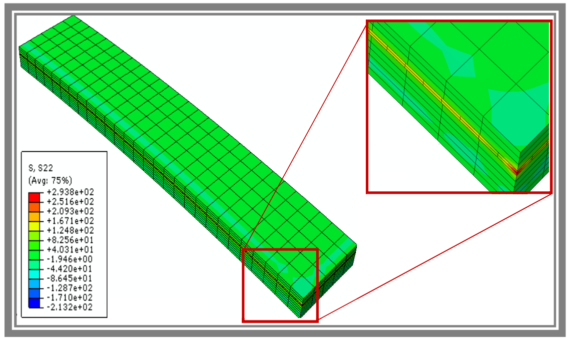
Computation of interface stresses

Determination of interface stresses



➢ FE modeling



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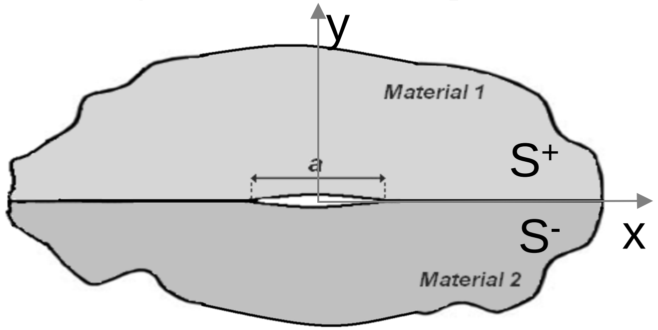
Analytical computation of interface stresses



S^+: material 1 - μ_1, k_1

S^-: material 2 - μ_2, k_2

$|x| \leq a \ (y=0)$: flat line crack is opened by normal pressures

Complex variable formulation

Williams(1959), Erdogan(1965)
England(1965)

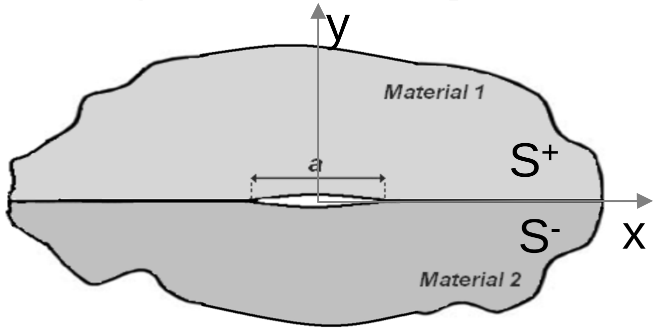
$$z = x + iy$$

$$\sigma = \sigma_{yy} - i\sigma_{xy}$$

$$U = U_x + iU_y$$

- *Mathematical statement of the problem*
  - *Boundary conditions*
  - *Governing equations*
- *Solution*

Analytical computation of interface stresses



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- *Boundary conditions*

Stress conditions on the crack faces:

$$\sigma_{yy}^{(1)} - i\sigma_{xy}^{(1)} = -p(x) \quad -iq(x)$$
$$\sigma_{yy}^{(2)} - i\sigma_{xy}^{(2)} = -p(x) \quad -iq(x)$$

Stress conditions on the uncracked interface:

$$\sigma_{yy}^{(1)} - i\sigma_{xy}^{(1)} = \sigma_{yy}^{(2)} - i\sigma_{xy}^{(2)}$$
$$U_x^{(1)} + iU_y^{(1)} = U_x^{(2)} + iU_y^{(2)}$$

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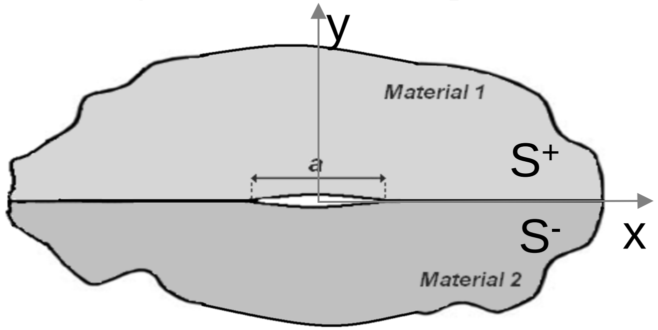
Analytical computation of interface stresses



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$$z = x + iy$$

$$\sigma = \sigma_{yy} - i\sigma_{xy}$$

$$U = U_x + iU_y$$

- *Governing equations*

$$\begin{cases} \sigma_{xx}^{(1)} + \sigma_{yy}^{(1)} = 2\left[\phi_1'(z) + \overline{\phi_1'(z)} \right] \\ \sigma_{yy}^{(1)} - \sigma_{xx}^{(1)} + 2i\sigma_{xy}^{(1)} = 2\left[\bar{z}\phi_1''(z) + \Psi_1'(z) \right] \\ 2\mu_1 (U_x^{(1)} + iU_y^{(1)}) = k_1\phi_1(z) - z\overline{\phi_1'(z)} - \Psi_1(z) \end{cases}$$

Equilibrium conditions

Airy stress function

Complex potentials

21.03.2011
Werkstoffkolloquium
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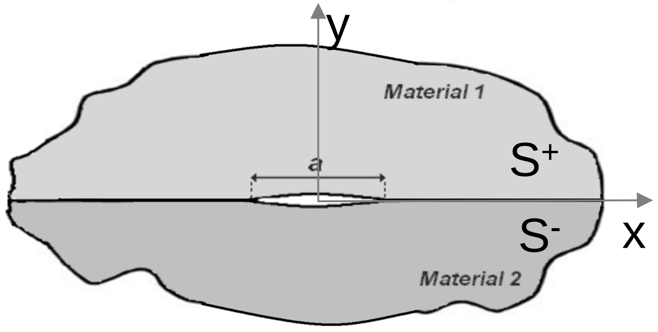
Analytical computation of interface stresses



S^+: material 1 - μ_1, k_1

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Stress conditions on the interface

$$\sigma^{(1)} = \sigma^{(2)}; \quad U^{(1)} = U^{(2)};$$

Complex variable formulation

Williams(1959), Erdogan(1965)
England(1965)

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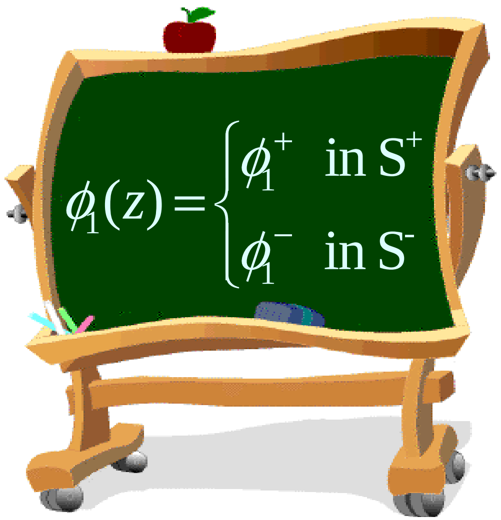
$$U = U_x + iU_y$$

- *Mathematical statement*

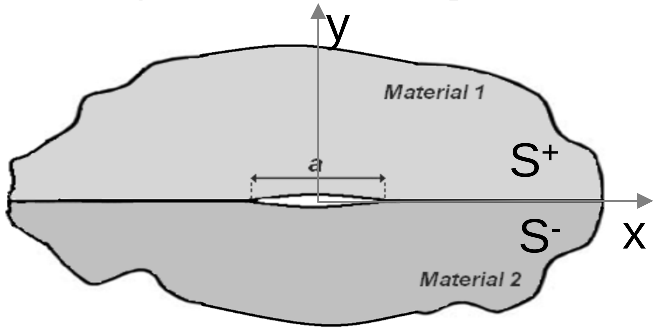
$$\phi_1^+(x) + \eta\phi_1^-(x) = -\gamma p(x)$$

on crack faces

Non-homogeneous Hilbert Problem



Analytical computation of interface stresses



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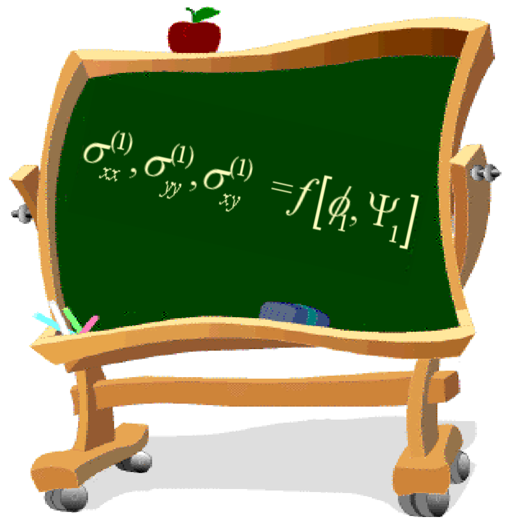

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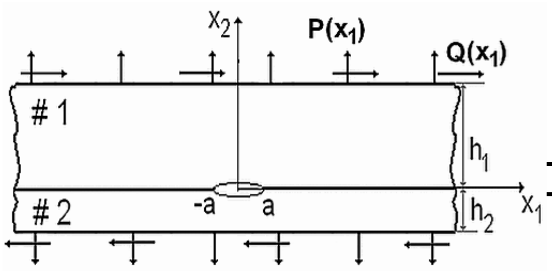
How to use analytical solution for infinite bi-material?

Stress state at the interface of bi-material strip with internal crack

- Lamé equation (equilibrium equation)
- Hooke's law
- Boundary conditions
- Continuity conditions at interface

Superposition method:



Bi-material strip with internal crack

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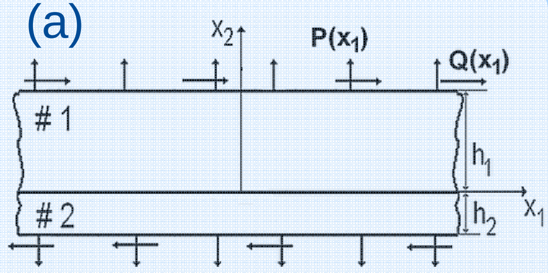
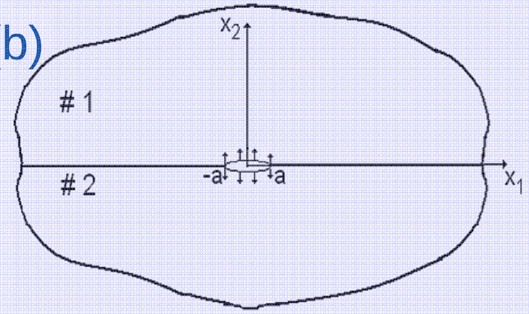
(a)



Uncracked bi-material strip

+

(b)



Interface crack in infinite bi-material

$$\boxed{\sigma^{com} = \sigma^{(a)} + \sigma^{(b)}}$$

Problem (a):

$$\int_0^\infty Q(x,\tau)\cdot\Sigma(\tau)d\tau = P(x)$$

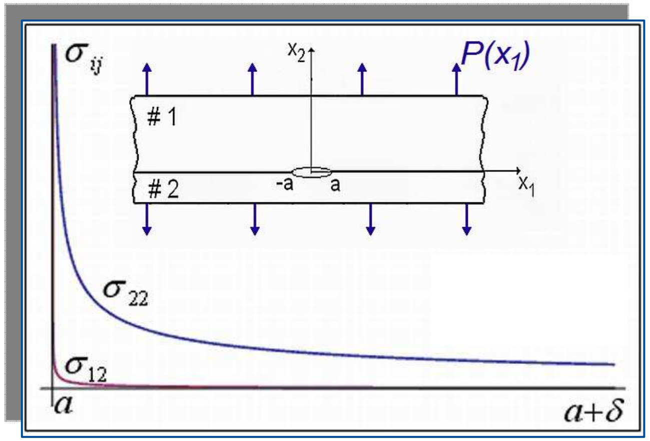
System of singular integral equations

Problem (b):

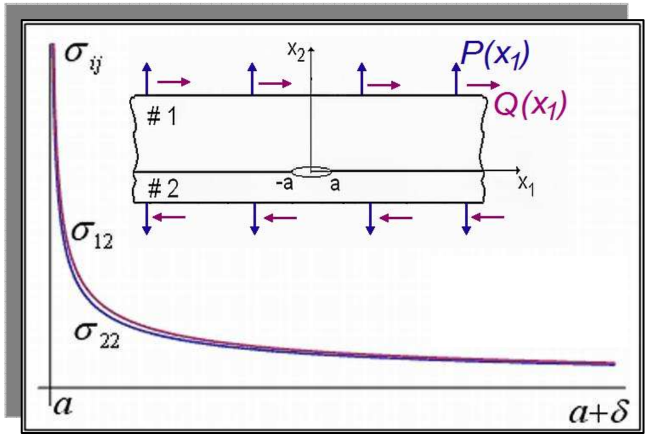
$$\phi'^{+}(x) + \eta\phi'^{-}(x) = \gamma[\Sigma_1(x) - i\Sigma_2(x)]$$

Hilbert problem

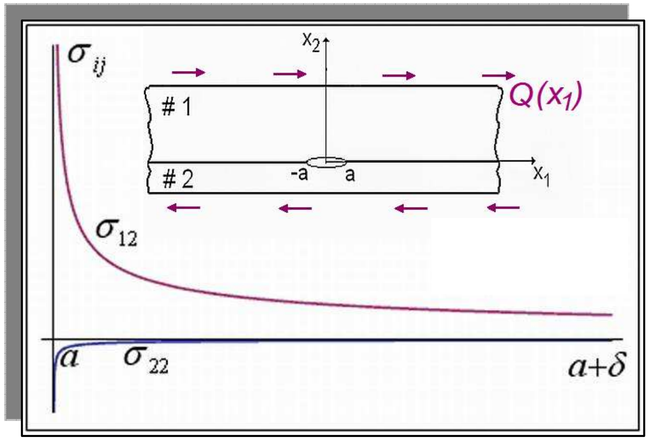
Stress state at the interface of bi-material (Al_2O_3/ ZrO_2) strip



Bi-material strip under pure tensile loading



Equal bi-action of tensile and shear stresses



Bi-material strip under pure shear stresses

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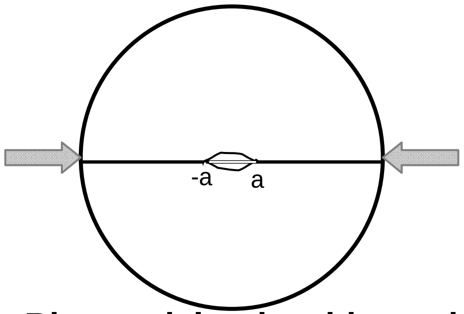
How to use analytical solution for infinite bi-material?

Stress state at the interface of bi-material Brazilian disk with internal crack

- Lamé equation (equilibrium equation)
- Hooke's law
- Boundary conditions
- Continuity conditions at interface

Superposition method:



Bi-material strip with crack

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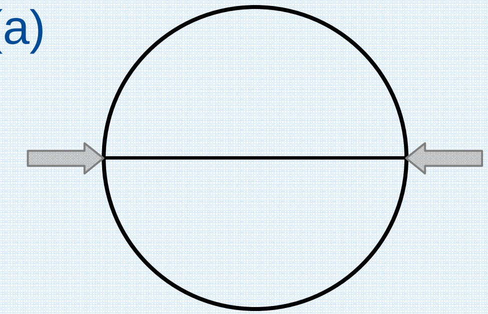
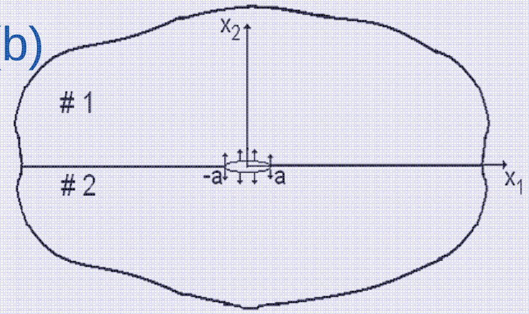
(a)



<u>Uncracked</u> Brazilian disk

+

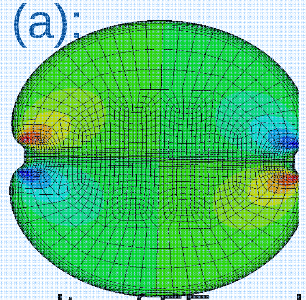
(b)



Interface crack in <u>infinite</u> bi-material

$$\boxed{\sigma^{com}=\sigma^{(a)}+\sigma^{(b)}}$$

Problem (a):



Results of FE analysis

Problem (b):

$$\phi'^{+}(x)+\eta\phi'^{-}(x)=\gamma\left[\Sigma_1(x)-i\Sigma_2(x)\right]$$

Hilbert problem

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Interface fracture criteria

Crack propagation path:

- No crack extension
- Crack propagates along the interface
- Crack kinks into one of the materials

Fracture criterion

Loading parameters $\gtrless$ ***Interface resistance parameter***

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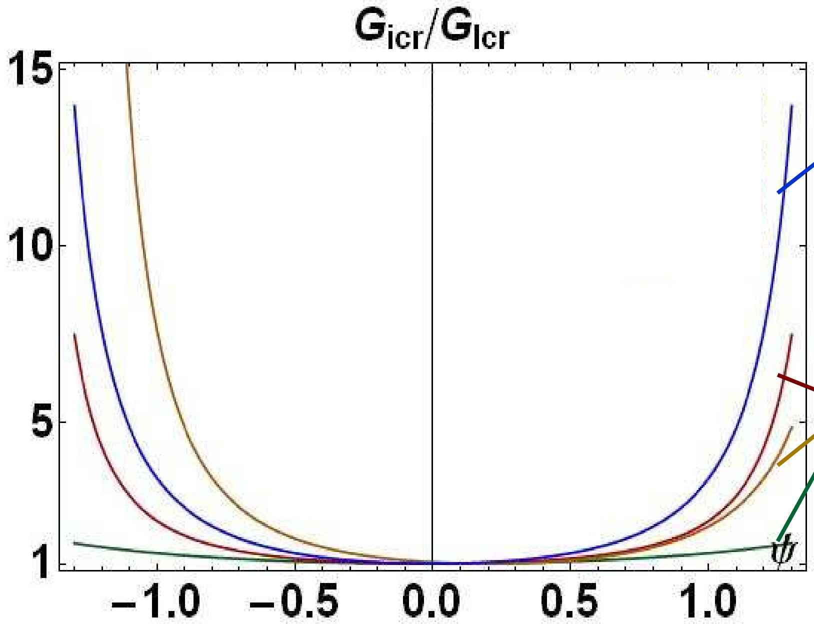
Interface toughness

Fracture criteria based on Energy Release Rate (ERR)

$$G_i = \frac{1}{H}(K_1^2 + K_2^2) \le G_{icr}$$

$$G_{cr} = \begin{cases} = const & \text{for homogeneous materials} \\ \neq const & \text{for interfaces} \end{cases}$$



$$G_{icr} = G_1\left(1 + tg^2\psi\right)$$

$$G_{icr} = G_1\left(1 + \lambda tg^2\psi\right)$$

$$\psi = \arctan\left[\frac{\mathrm{Im}(KL^{i\varepsilon})}{\mathrm{Re}(KL^{i\varepsilon})}\right]$$

G_1 - pure mode-I toughness

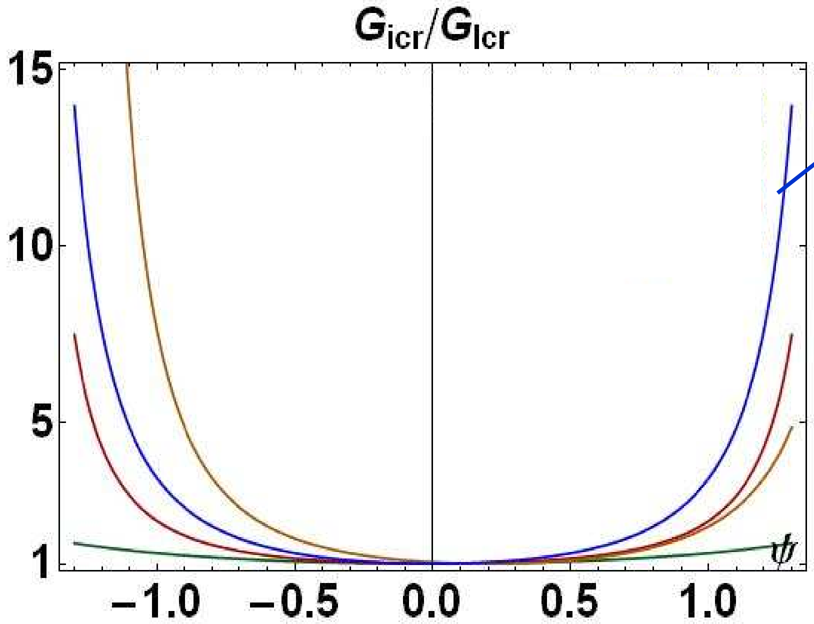
λ - calibration parameter

Y. Freed; L. Bank-Sills (2008)

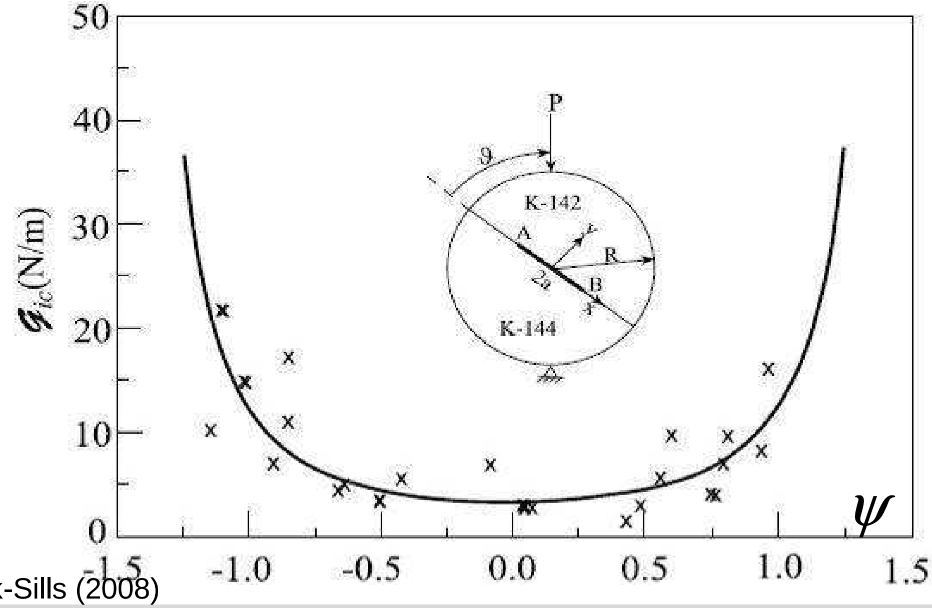
Interface toughness

Fracture criteria based on Energy Release Rate (ERR)

$$G_i = \frac{1}{H}\left(K_1^2 + K_2^2\right) \le G_{icr} \qquad G_{cr} = \begin{cases} = const & \text{for homogeneous materials} \\ \neq const & \text{for interfaces} \end{cases}$$



$$G_{icr} = G_1\left(1 + tg^2\psi\right)$$



G_1 - pure mode-I toughness

λ - calibration parameter

Y. Freed; L. Bank-Sills (2008)

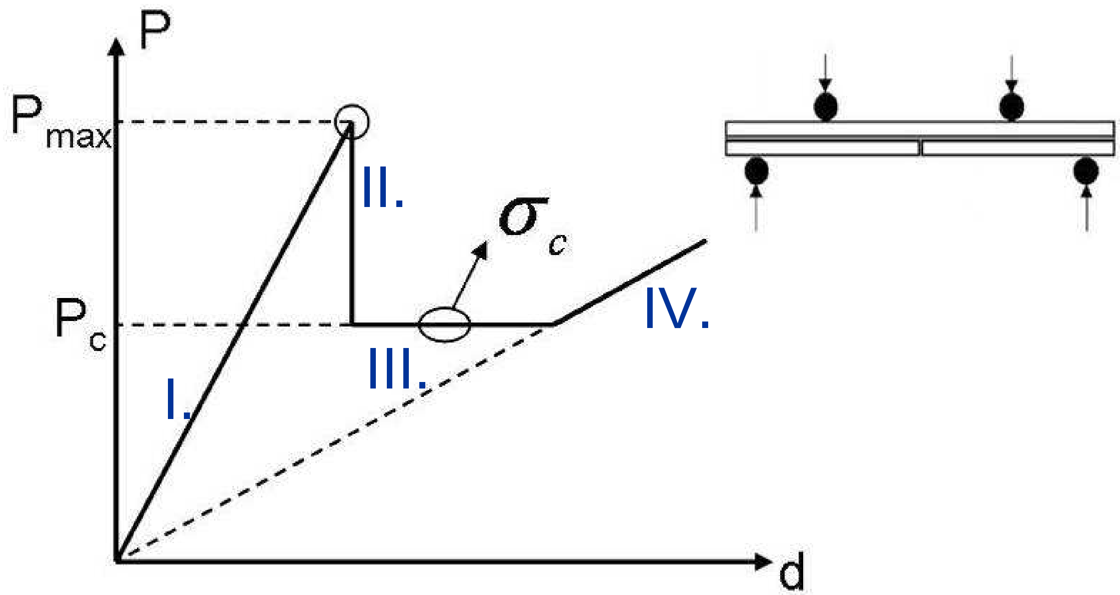
Experimental characterization of interfaces

I. Straight line segment - the linear-elastic behaviour (Hooke's law)

II. Load drop - instantaneous crack propagation towards to interface

III. Plateau - crack growth along the interface

IV. Straight line segment - crack propagation into the substrate material



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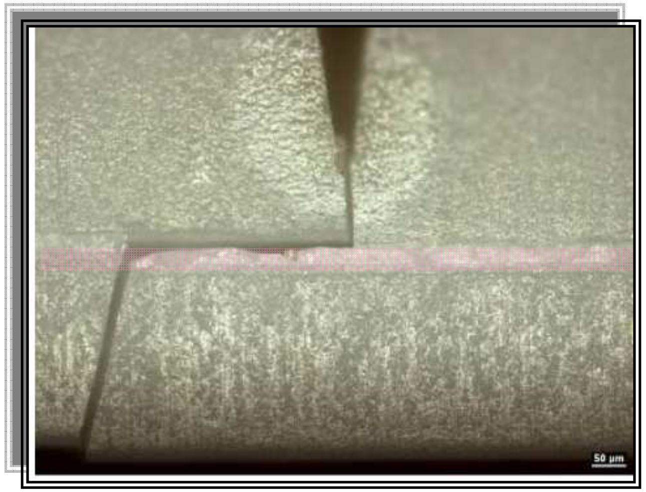
Interface fracture criteria



Crack propagation path:

- No crack extension
- Crack propagates along the interface
- Crack kinks into one of the materials

Fracture criterion

Loading parameters $>$ / $<$ ***Interface resistance parameter***

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Generalization of Weibull theory for interface failure in bi-material ceramic joints

Main assumptions to statistical model

- ✓ *Material contains a* ***number of defects*** *of different size, which can be described as cracks with fracture mechanics methods;*
- ✓ *The* ***size of the defect is a random*** *variable, which is described by a certain distribution function;*
- ✓ *There are* ***no interactions between the natural flaws****, i.e. failure of a crack is not affected by the presence of other cracks;*
- ✓ *Failure of the* ***worst*** *natural* ***flaw*** *(i.e. where the combination between high stress and large size is most unfavourable)* ***causes the failure*** *of the whole component;*
- ✓ ***Location and orientation*** *of the natural flaws* ***are random****.*

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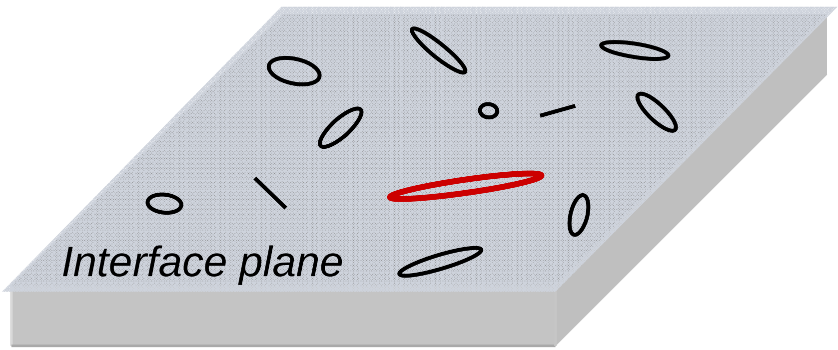
The combination of fracture mechanics and probabilistic methods



- Material parameters
- Geometry of the component
- Loading conditions

"Weakest-link" approach

Failure probability

Most <u>dangerous flaw</u> has unfavourable combination of:

- Size
- Location in the stress field
- Orientation of the stress field

The combination of fracture mechanics and probabilistic methods

Interface energy release rate fracture criteria:

$$G_i \geq G_{i\,\mathrm{cr}}(\psi)$$

- Material parameters
- Geometry of the component
- Loading conditions

Fracture mechanics

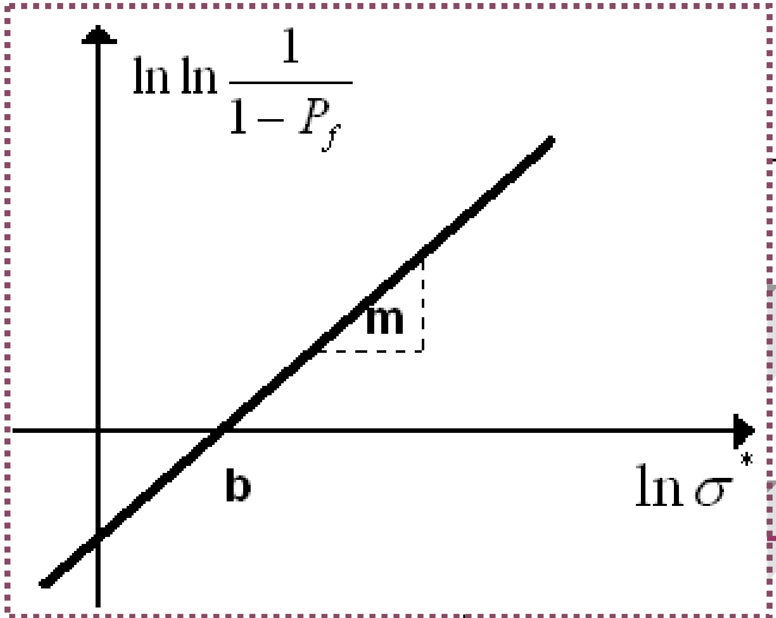
a_{cr} – critical crack length

“Weakest-link” approach

$$P_f = 1 - \exp\left(- \left(\frac{\sigma^*}{b} \right)^m \right)$$

Failure probability

The combination of fracture mechanics and probabilistic methods



b, m – Weibull parameters

Experimental data

- Material parameters
- Geometry of the component
- Loading conditions

$$P_f = 1 - \exp\left(-\left(\frac{\sigma^*}{b}\right)^m\right)$$

Failure probability

Natural flaw size distribution
Average number of natural flaws
Interface toughness, geometry

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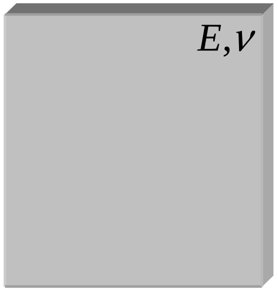
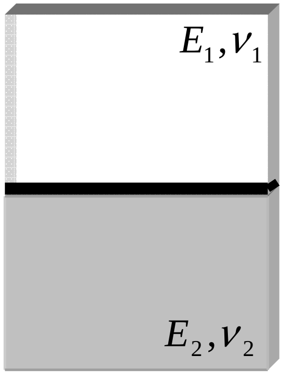
Failure probability: comparison of results

- Failure probability of homogeneous material



$$P_f = 1 - \exp\left(-\frac{1}{4\pi V_0} \int_V \int_\Omega \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0} \right)^m dV d\Omega \right)$$

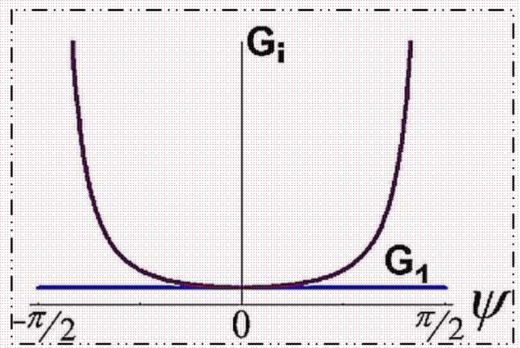
- Interface failure probability of bi-material



$$P_f = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i} \int_\Omega \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2\psi}} \right)^m dA d\Omega \right)$$

$$\psi = \arctan\left[\frac{\mathrm{Im}(KL^{i\varepsilon})}{\mathrm{Re}(KL^{i\varepsilon})} \right] = \arctan\left[\frac{\sigma_{12}}{\sigma_{22}} \right]\Bigg|_{\theta=0,r=L}$$

parameter measures the mode mixity at the tip of the crack



21.03.2011
Werkstoffkolloquium

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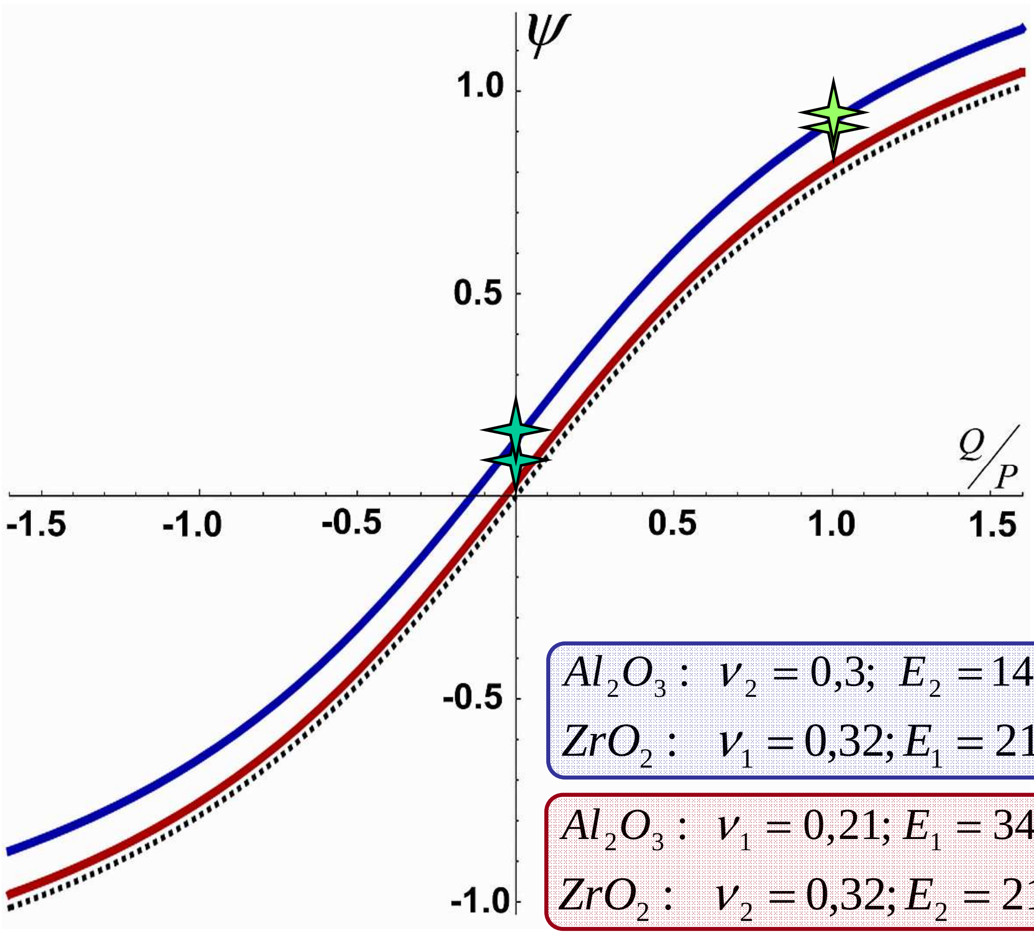
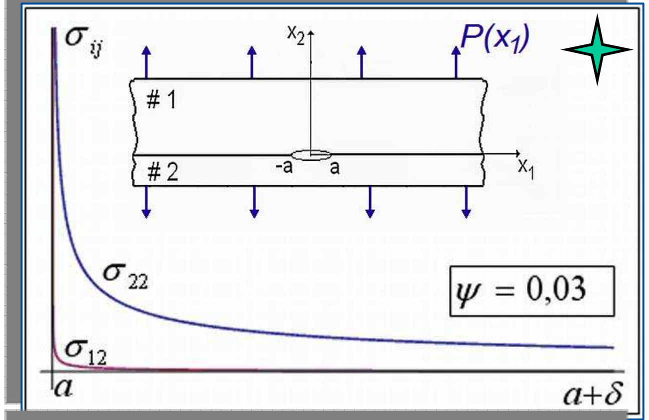
Mode – mixity parameter

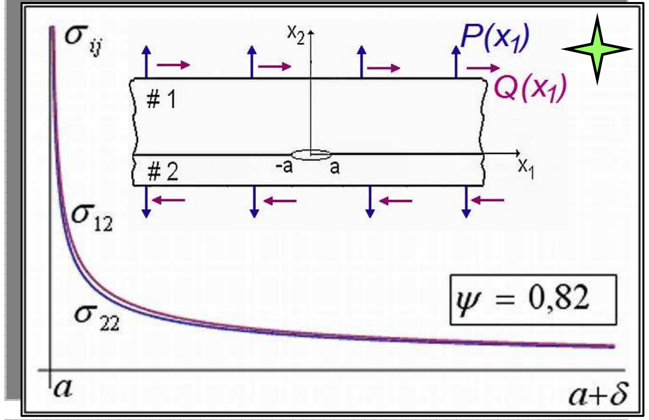
$$\psi = ArcTan\left(\sigma_{12} / \sigma_{22} \right) \quad \text{at the tip of the crack}$$

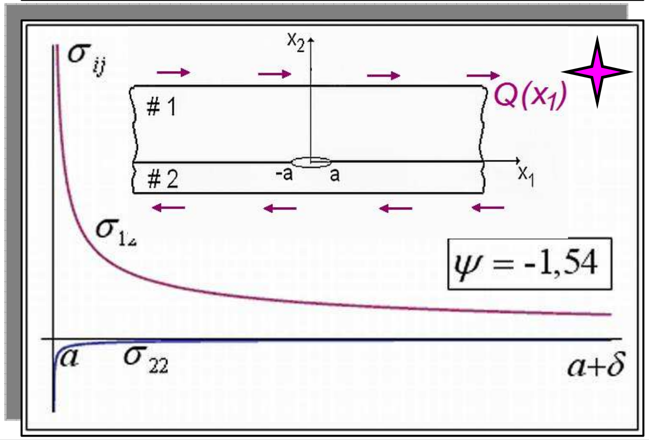


$Al_2O_3: \ \nu_2 = 0{,}3; \ E_2 = 147\,GPa$
$ZrO_2: \ \nu_1 = 0{,}32; E_1 = 213GPa$

$Al_2O_3: \ \nu_1 = 0{,}21; E_1 = 347\,GPa$
$ZrO_2: \ \nu_2 = 0{,}32; E_2 = 213GPa$







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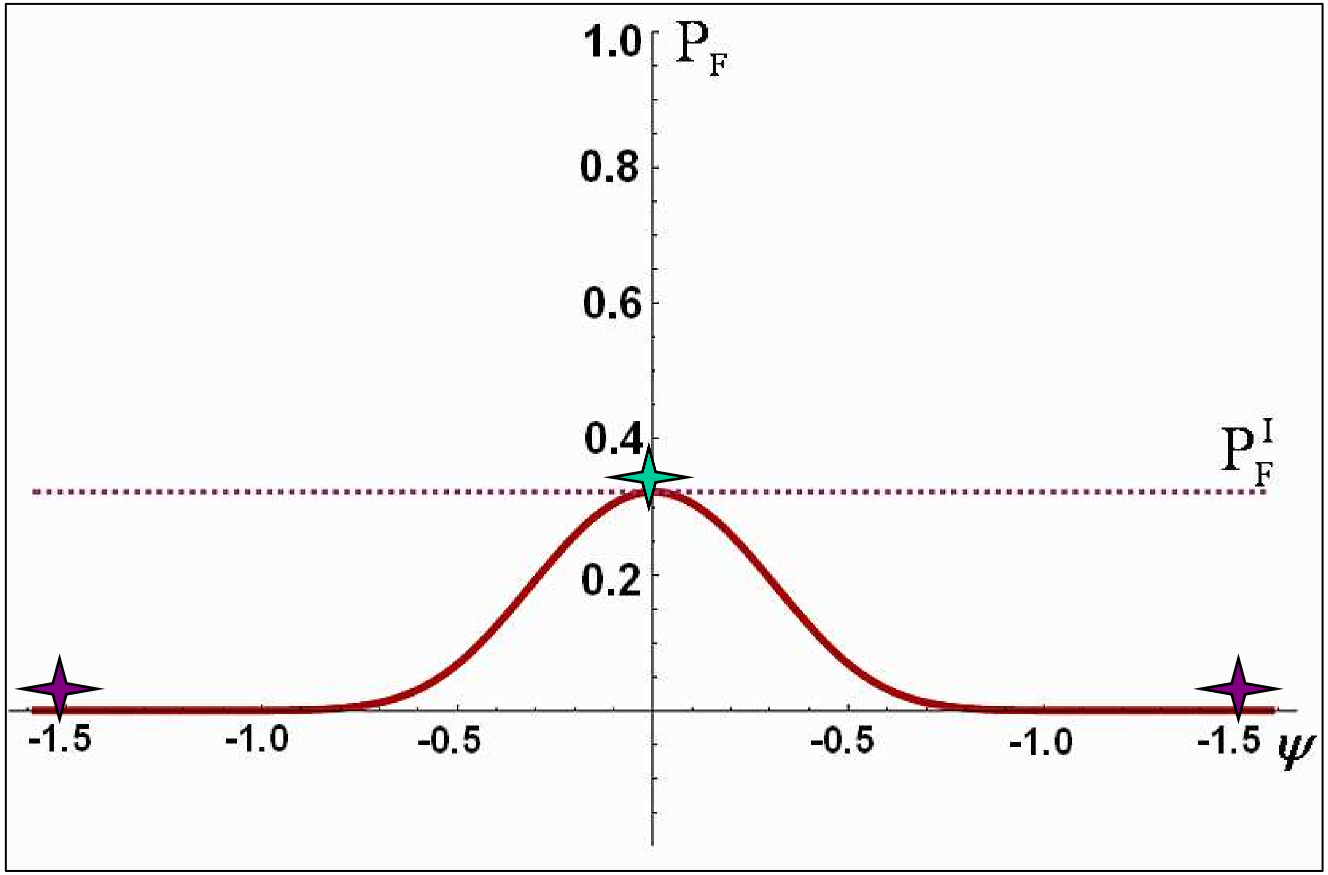
Influence of the mode-parameter ψ on P_F

$$P_F = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i} \int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2 \psi}} \right)^m dA\, d\Omega \right)$$



$\psi = 0$

max probability of failure

$\psi = \pi/2$

min probability of failure (kinking is probable)

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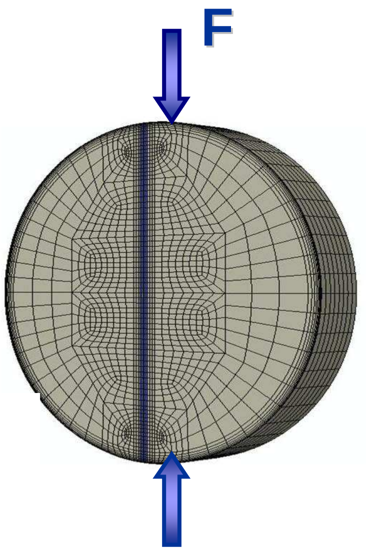
Influence of the Weibull parameters on P_F

$$P_F = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i} \int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2\psi}} \right)^m dA\, d\Omega \right)$$

3D Brazilian disk under diametrical compression



$Y - ZrO_2$

Al_2O_3

R = 22,5 mm – radius of the disk
w = 5 mm – thickness of the disk
F = 50 kN – applied force
t = 1.0 mm – thickness of Al-layer

Interface failure probability $P_F = P_F(m, \sigma_0)$

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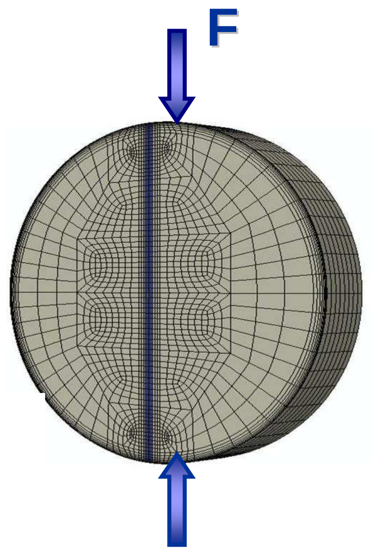
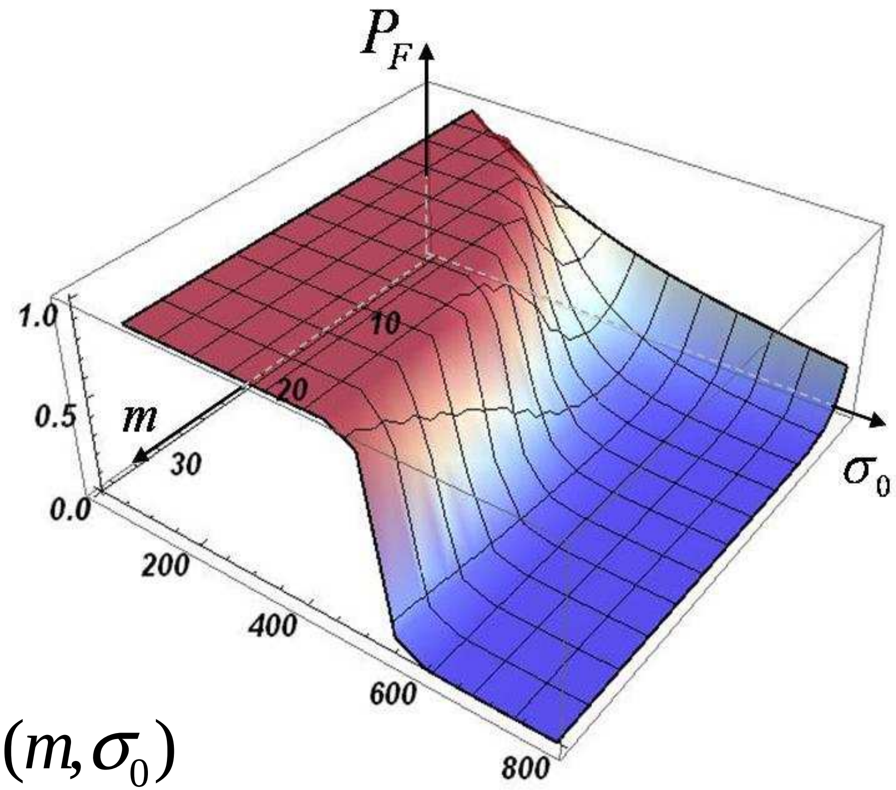
Influence of the Weibull parameters on P_F

$$P_F = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i}\int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2\psi}} \right)^m dA\, d\Omega \right)$$

3D Brazilian disk under diametrical compression





Interface failure probability $P_F = P_F(m, \sigma_0)$

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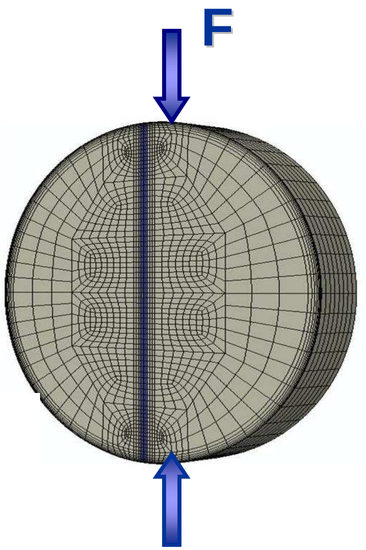
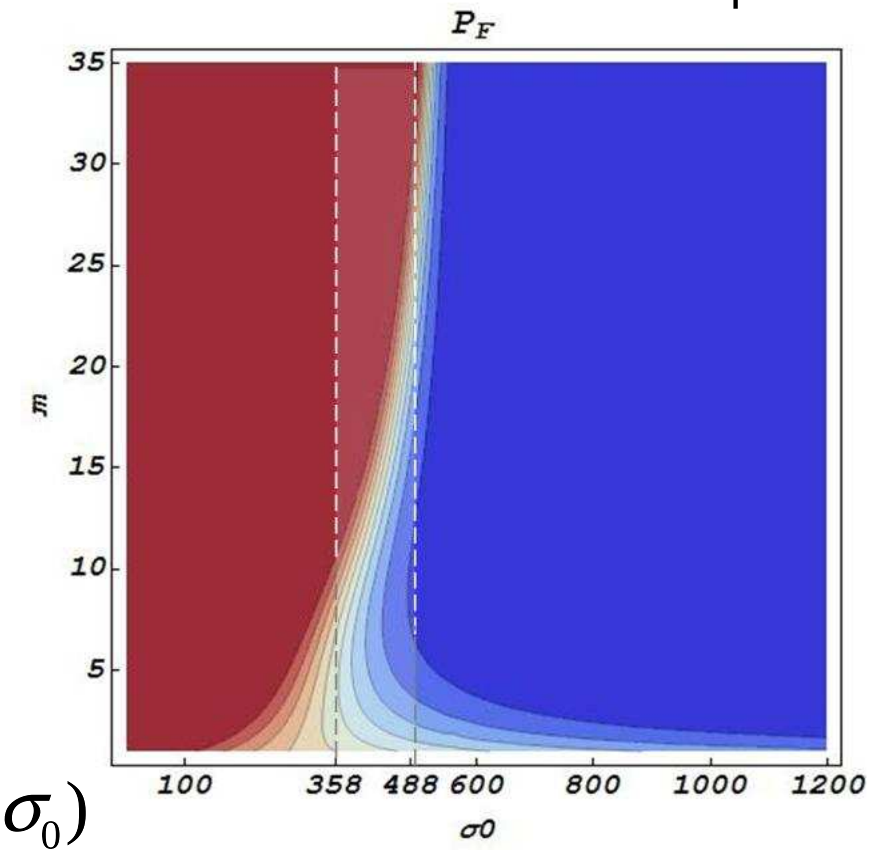
Influence of the Weibull parameters on P_F

Karlsruhe Institute of Technology

$$P_F = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i} \int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2\psi}} \right)^m dA\, d\Omega \right)$$

3D Brazilian disk under diametrical compression





Interface failure probability $P_F = P_F(m, \sigma_0)$

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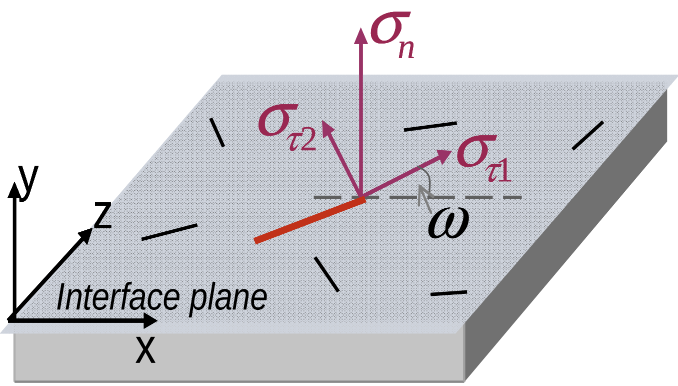
Influence of the crack model on P_F

$$P_F = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i}\int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2\psi}} \right)^m dA\, d\Omega \right)$$

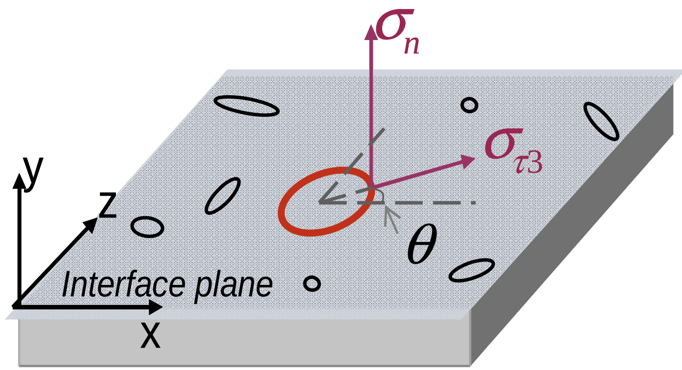
- ✓ *"Through – wall" crack model*

$$\sigma_{eq} = \sqrt{\sigma_n^2 + \sigma_{\tau 1}^2 + \frac{H_1}{(1+4\varepsilon^2)H_2}\sigma_{\tau 2}^2}$$



- ✓ *"Penny - shaped" crack model*

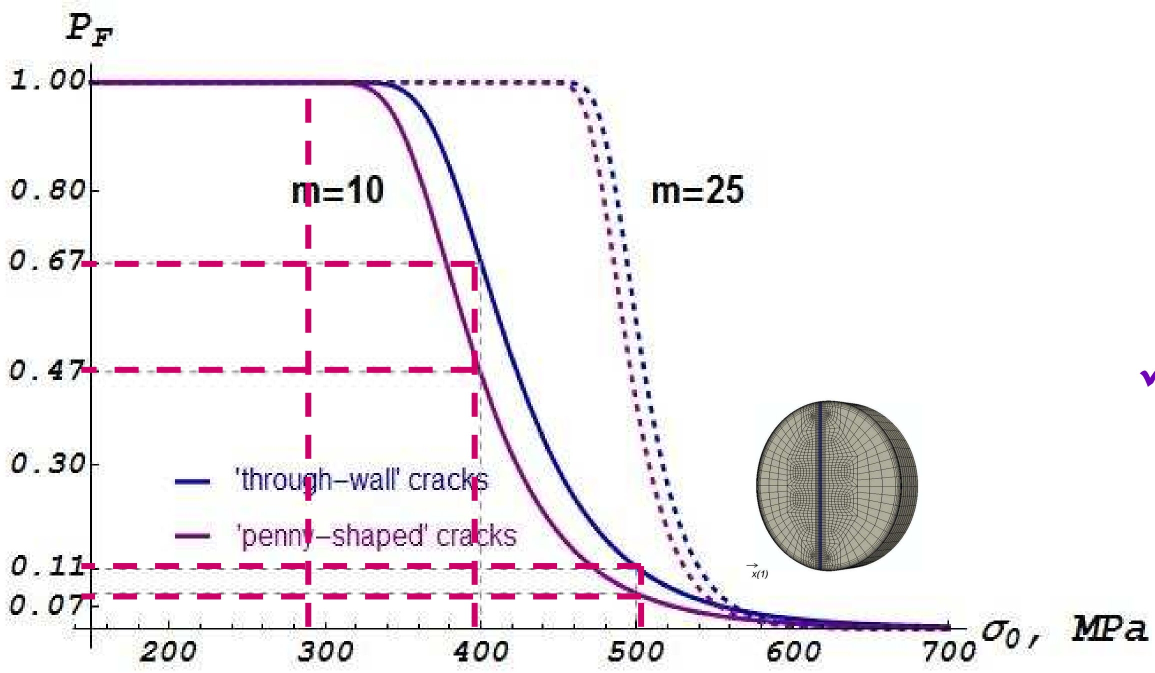
$$\sigma_{eq} = \sqrt{\sigma_n^2 + Y_{II}^2\sigma_{\tau 3}^2\cos^2\theta + \frac{H_1}{\left|\Gamma(2+i\varepsilon)/\Gamma(0.5+i\varepsilon)\right|^2 H_2} Y_{III}^2\sigma_{\tau 3}^2\sin^2\theta}$$



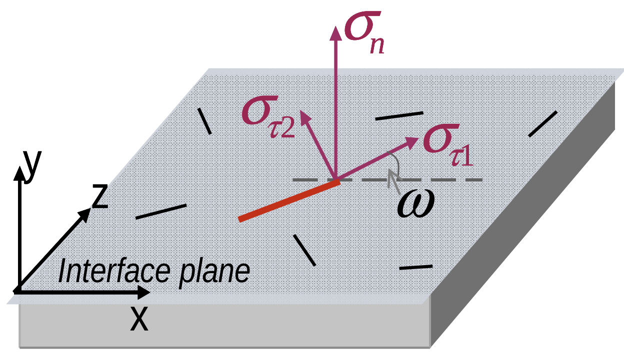
Influence of the crack model on P_F

$$P_F = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i} \int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2\psi}} \right)^m dA\, d\Omega \right)$$

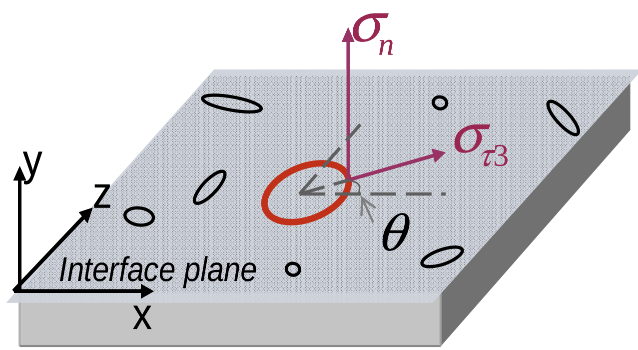
Failure probability as a function of interface strength



✓ *"Through – wall" crack model*



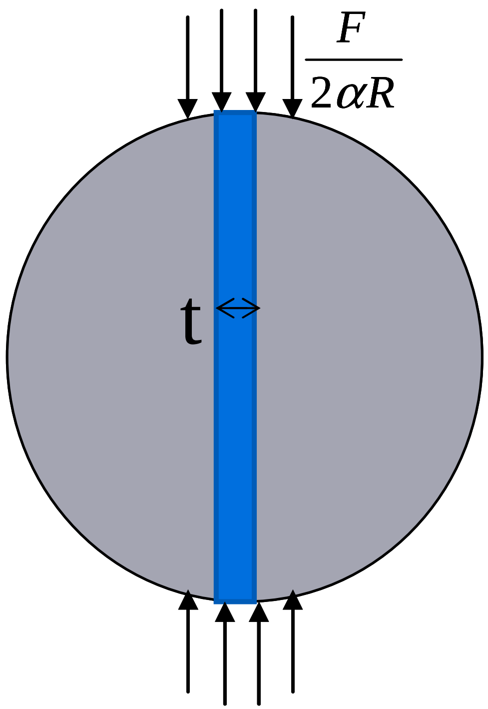
✓ *"Penny - shaped" crack model*



Influence of interface geometry on P_F

$$P_F = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i}\int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2\psi}} \right)^m dA\, d\Omega \right)$$

Brazilian disk under diametrical compression



ZrO_2: $\nu_1 = 0{,}32; \quad E_1 = 213 GPa$
Al_2O_3: $\nu_2 = 0{,}3; \quad E_2 = 147 GPa$

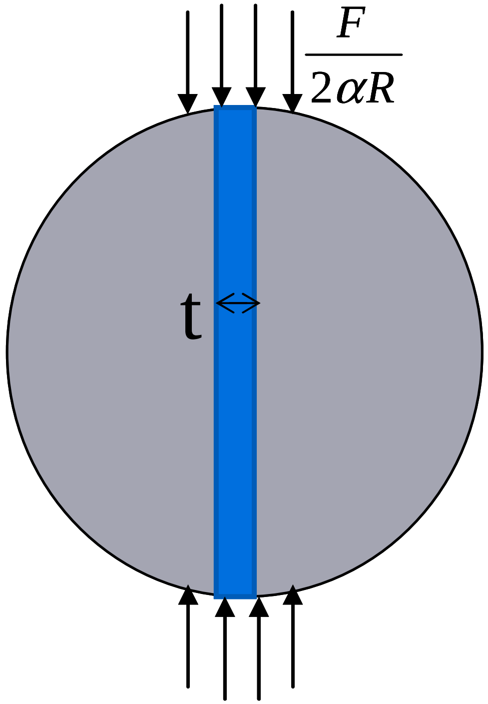
R = 22,5mm – radius of the disk
F = 50 kN – applied force

Thickness of inter-layer:
- t = 0.1 mm
- t = 1.0 mm
- t = 3.0 mm
- Analytical solution (t = 0.0 mm)

Influence of interface geometry on P_F

$$P_F = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i} \int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2 \psi}} \right)^m dA\, d\Omega \right)$$

Brazilian disk under diametrical compression

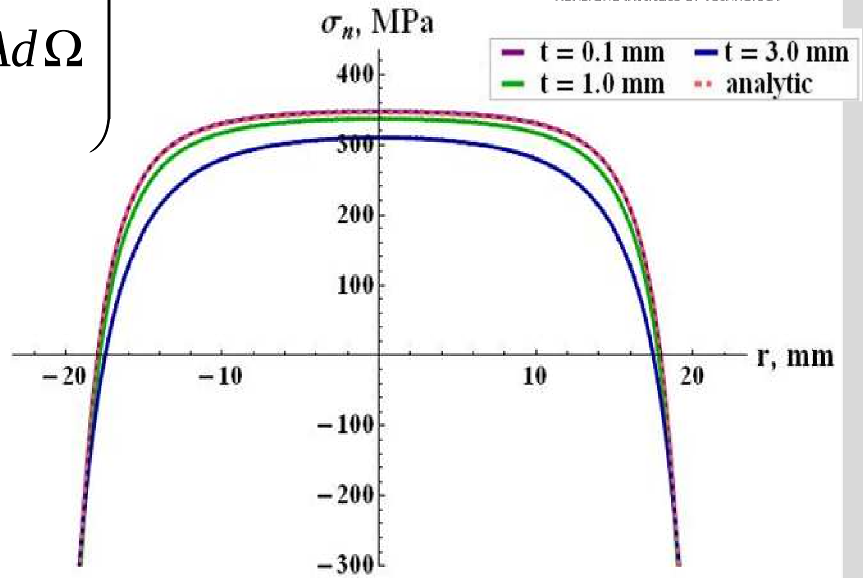


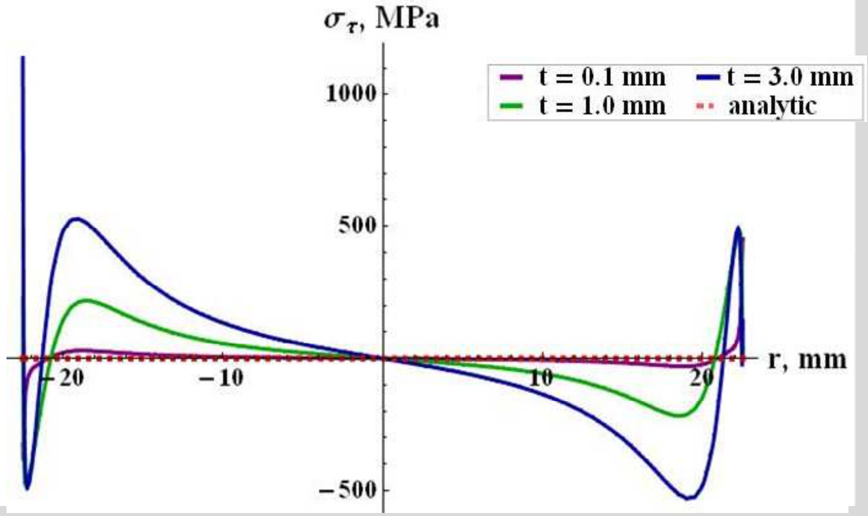
t = 0.1 mm

t = 1.0 mm

t = 3.0 mm

Analytical





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Influence of interface geometry on P_F

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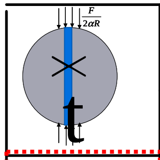
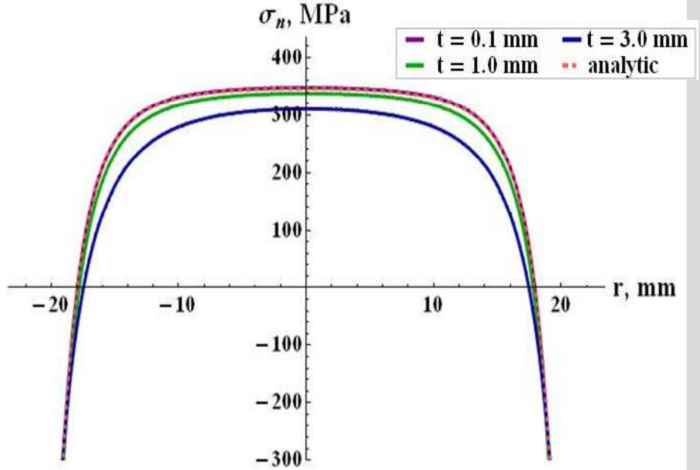
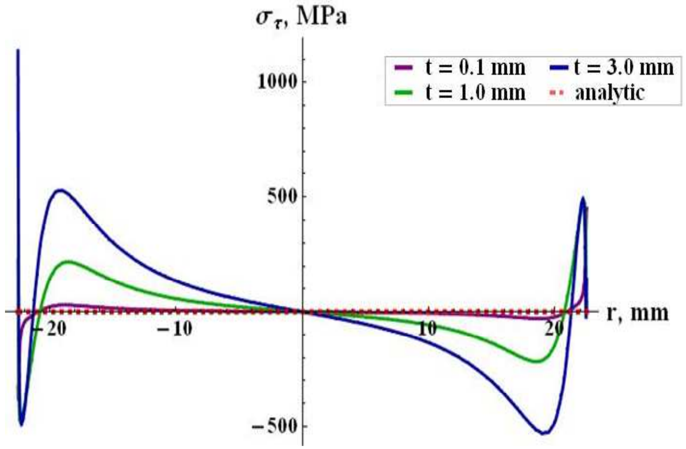
$$P_F = 1 - \exp\left(- \frac{1}{2\pi A_0} \int_{A_i}\int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2\psi}} \right)^m dA\, d\Omega \right)$$

Failure probability values for some material combinations

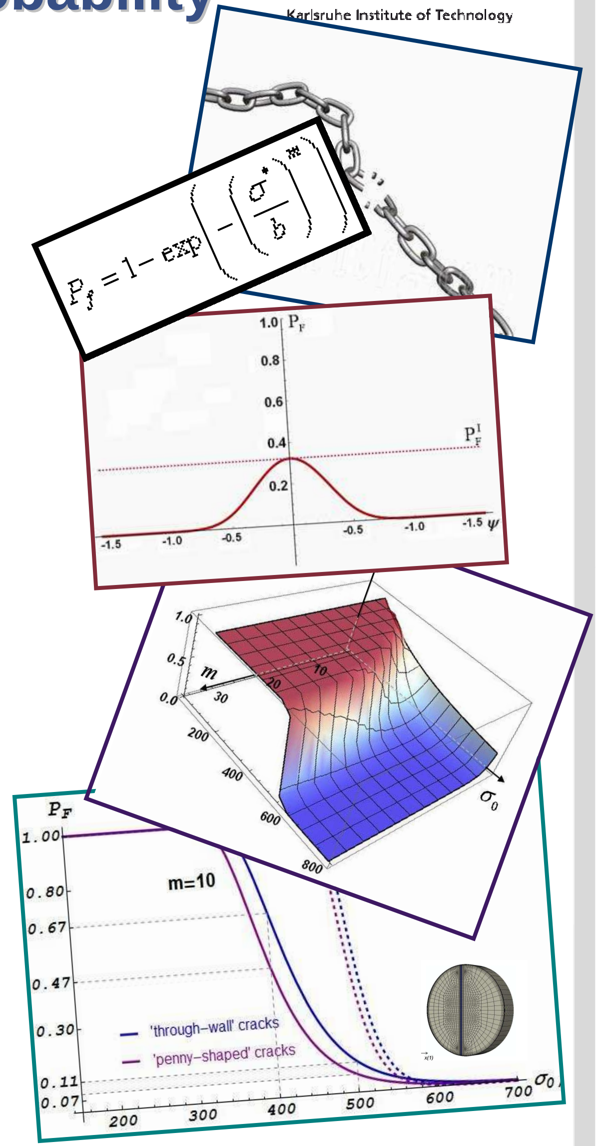
	$m=10$, $\sigma_0 = 320MPa$		$m=10$, $\sigma_0 = 430MPa$		$m=18$, $\sigma_0 = 380MPa$	
	exact	conserv.	exact	conserv.	exact	conserv.
Analyt.	63.2%	63.5%	5.1%	5.1%	7.0%	7.01%
$t=0.1mm$	64.7%	65.0%	5.3%	5.3%	7.5%	7.6%
$t=1.0mm$	69.3%	71.8%	6.0%	6.4%	12.6%	13.2%
$t=3.0mm$	61.9%	100.0%	4.9%	69.1%	10.5%	100%

- ✓ P_F grows (↑) till some certain value of ***t***, then it starts to decrease (↓)
- ✓ For some geometries conservative approach leads to overestimation





Generalized Weibull approach for interface failure probability



⟶ Generalization of the Weibull theory for the case of interface failure is introduced:

$$P_F = 1 - \exp\left(-\frac{1}{2\pi A_0} \int_{A_i}\int_{\Omega} \left(\frac{\sigma_{eq}(x,\omega)}{\sigma_0 \sqrt{1+\tan^2\psi}} \right)^m dA\, d\Omega \right)$$

⟶ Equivalent stress are calculated with respect to considered crack model.

⟶ The failure probability of the interface peaks for mode-I loading (the sharpness of the peak depends on material parameters).

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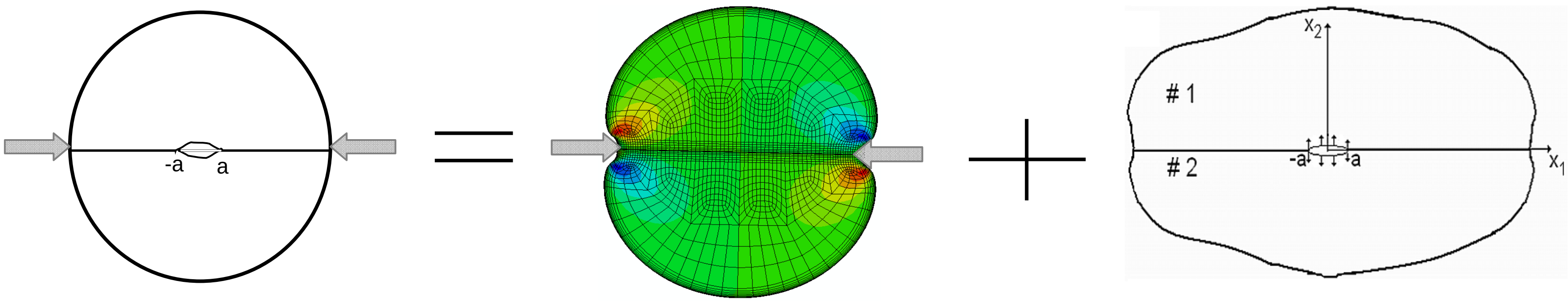
Summary

→ An algorithm for failure probability calculation is available and ready to be implemented in STAU

→ A superposition approach leads to a semi-analytic solution for the stress field and thus the failure probability



→ The approach is promising for layered materials in functional applications

→ The sequential issue is to obtain relevant statistical parameters

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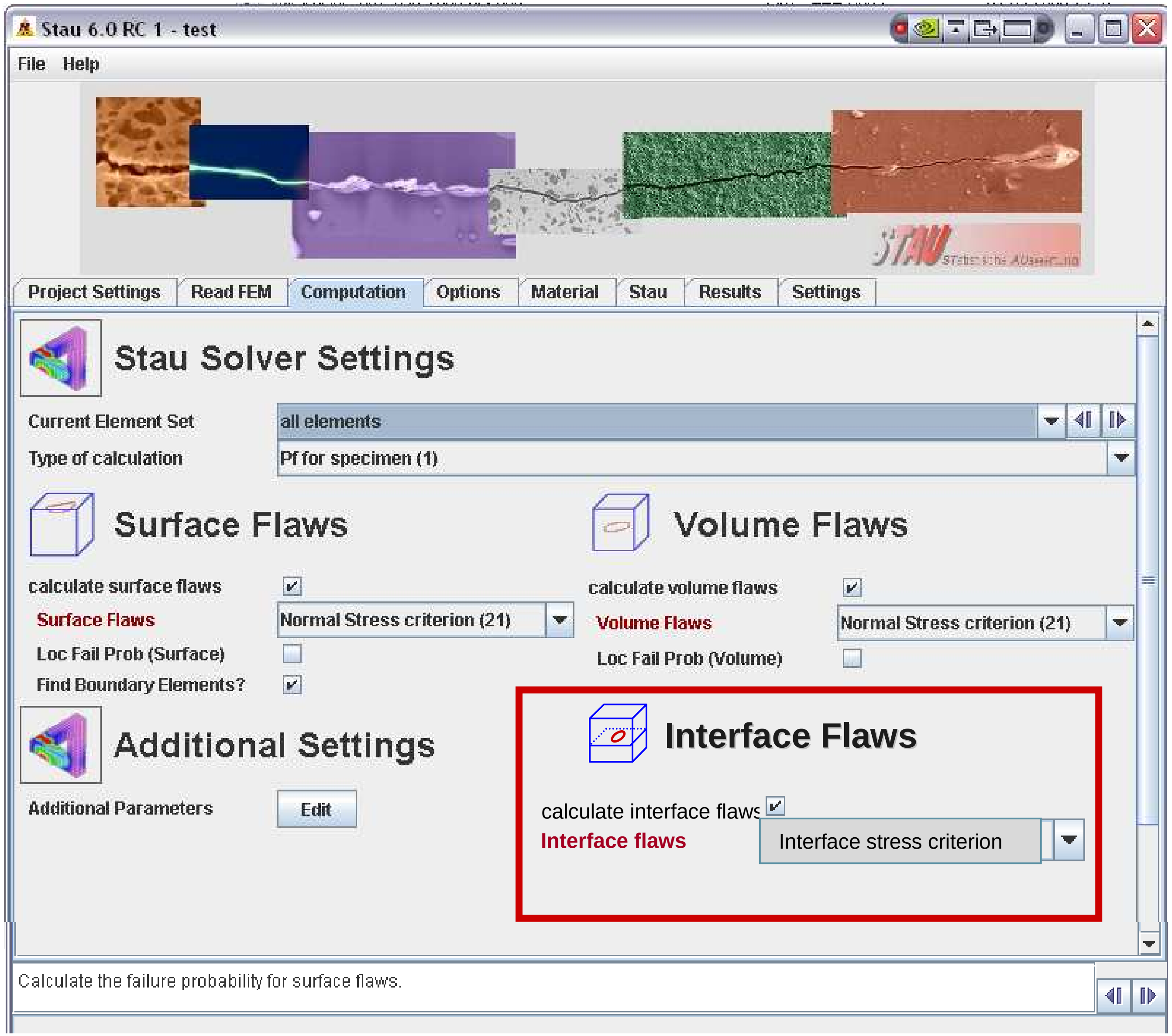
Changes to STAU (Version 6)



Thank you for your attention!