PROBABILISTIC ANALYSIS OF A ROLLING CONTACT FATIGUE TEST FOR SILICON NITRIDE

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Motivation

High strength ceramics (Si$_3$N$_4$) are used in rolling applications
- Time-dependent failure due to slow crack propagation
- Design of such components requires probabilistic methods
- Rolling contact fatigue test as model system for rolling applications
Theory: Crack propagation mechanisms

- Crack loading

\[ \Delta K = K_{\text{max}} - K_{\text{min}} \]

\[ R = \frac{K_{\text{min}}}{K_{\text{max}}} \]

- Subcritical crack propagation
  - Stress corrosion/ chemical reaction
  - Quasi-static effect

- Cyclic crack propagation (fatigue)
  - Depends on load sequence
  - Degradation of strengthening effects (grain bridging)

\[ \frac{da}{dt} = A_S \cdot \left( \frac{K_1}{K_{lc}} \right)^{n_S} \]

\[ \frac{da}{dN} = \frac{A}{(1 - R)^{n-p}} \cdot \left( \frac{\Delta K_1}{K_{lc}} \right)^n \]

\[ A_S, n_S: \text{material properties} \]

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Theory: Failure probability

Weibull Theory

- Integration over component surface/volume and flaw orientation
- Equivalent stress $\sigma_{eq}$: local failure criterion

Subcritical crack propagation

$$P_f = 1 - \exp \left[ -\frac{1}{A_0} \int_A \frac{1}{2\pi} \int_{\alpha} \left( \frac{\sigma_{eq}(t)}{\sigma_0} \right)^{n_S-2} \max_{t \in [0,T]} \left( \frac{\sigma_{eq}(t)}{\sigma_0} \right) \left( \frac{\sigma_{eq}(t')}{\sigma_0} \right)^{n_S} \int_0^\alpha \left( \frac{\sigma_{eq}(t')}{\sigma_0} \right)^{n_S} dt' \right] d\alpha dA$$

Cyclic crack propagation

$$P_f = 1 - \exp \left[ -\frac{1}{A_0} \int_A \frac{1}{2\pi} \int_{\alpha} \left( \frac{\sigma_{eq,max}(\eta)}{\sigma_0} \right)^{n_S-2} \max_{\eta \in [N_1,N_k]} \left( \frac{\sigma_{eq,max}(\eta)}{\sigma_0} \right) \left( \frac{\sigma_{eq,max}(N')}{\sigma_0} \right)^{n} \int_{N_1}^\alpha \left( \frac{\sigma_{eq,max}(N')}{\sigma_0} \right)^{n} (1 - R(N'))^p dN' \right] d\alpha dA$$

Time/Cycle dependent load history
Theory: Numerical evaluation

**STAU**$^1$: Finite-Element Postprocessor for reliability assessment of ceramics

Reliability analysis for rolls $\Rightarrow$ Complex load history must be considered

Results: Material parameters Si$_3$N$_4$-SL200

- **Strength**
  \[ \sigma_{4PB} = 1044 \text{ MPa} \]
  \[ m = 11.5 \]

- **Subcritical crack growth**
  \[ n = 42, \quad A_S = 10^{-6} \text{ m/s} \]

- **Cyclic fatigue parameters (air)**

  - **Crack growth exponent** \( n \) depends on load ratio \( R \) (n=20,31)
  - **Curves must be represented by common exponent** \( n \):
    \[ \frac{da}{dN} = \frac{A}{(1 - R)^{n-p}} \left( \frac{\Delta K}{K_{lc}} \right)^n \]
    \[ n = 24, \quad p = 2.2, \quad A = 3 \cdot 10^{-8} \text{ m/cycle} \]

Results: Material parameters $\text{Si}_3\text{N}_4$

- Cyclic fatigue parameters (water)

- Water enhances fatigue effect!

- Measurements for $R=0.5$: $n=29.9$

- Parameters are evaluated assuming $p=2.2$ (air):

$$\frac{da}{dN} = \frac{A}{(1-R)^{n-p}} \left( \frac{\Delta K_1}{K_{lc}} \right)^n$$

$n=29.9$, $p=2.2$, $A=5 \cdot 10^{-5}$ m/cycle
Results: Rolling contact fatigue (RCF) test

- Steel-disk
- Si₃N₄-disk
- lubricant

**Stress distribution**

- Damage after $10^5$ rotations

- Lubricant: friction coefficient $\mu = 0.085$
- $F = 1700\text{N}$
- Relative slip: $\sim 22\%$
- Max. principal stress: $\sim 1100\ \text{MPa}$

RCF tests: Iyas Khader, Fraunhofer Institute IWM, Freiburg
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Results: STAU analysis

- Contact damage: initiation of macroscopic flaws
- Predicting of a certain flaw density on the surface

- STAU: probability of the initiation of one flaw in the considered subarea
- Size of subarea ↔ crack density
- Results refer to crack density of 1 crack per 250 μm along the circumference
Results: Failure probability

- Probability to initiate one macroscopic crack every 250 μm

\[ \ln\left(\ln\left(\frac{1}{1 - P_f(Z)}\right)\right) \]

Weibull function

\[ P_f = 1 - \exp\left[-\left(\frac{Z}{N_0}\right)^m\right] \]

- Weibull function

- Highest failure probability obtained for fatigue parameters in water.

\begin{itemize}
  \item Weibull function
  \item \[ P_f = 1 - \exp\left[-\left(\frac{Z}{N_0}\right)^m\right] \]
  \item \( m \) – slope of the curve
  \item \( N_0 \) - characteristic lifetime (63%-quantile)
  \item \( Z \) – no. of rotations
\end{itemize}
Results: Failure probability

- Relation with experimental crack density after 10 h

- Initiation probability low for parameters in (air)
Results: Parametric study

- Influence of crack growth exponent $n$

- Crack growth exponent has a strong influence on the results:
  For $n<25$, crack initiation probability is $>70%$;
  Characteristic lifetimes are below 10 h
Discussion

- Agreement with experimental results for lower \( n \)-values if water parameters are used

- Stress history: \( R=0 \) dominates
  - Using a lower \( n \) value (\( n \approx 20 \)) in the case of water.

- Limitations of the analysis scheme:
  - Stress gradients in the range of natural flaws
  - Interaction of macroscopic cracks
  - Wear
Summary

- STAU as general tool for reliability assessment under complex loading
- Prediction of evolving crack patterns on roll surface is possible
- Sensitivity to crack growth parameters is challenging
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