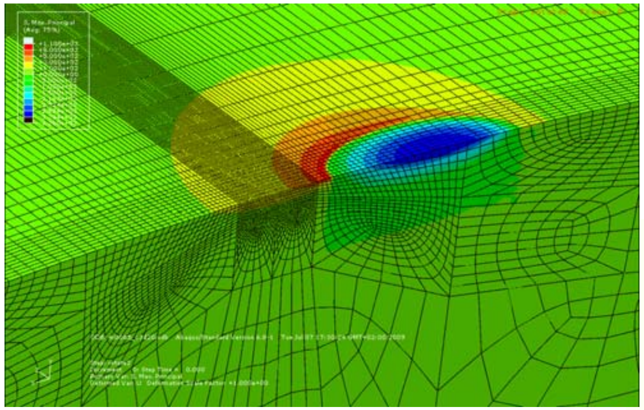
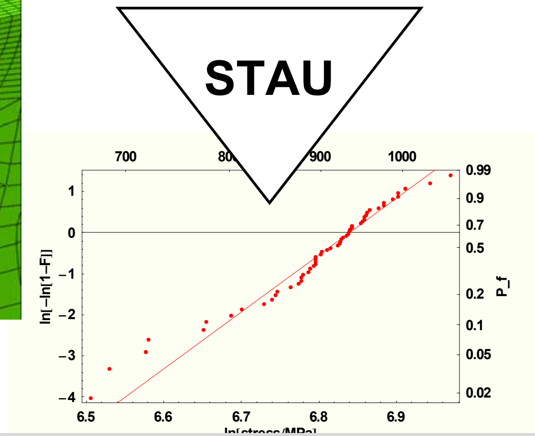
High performance sliding and friction systems based on advanced ceramics
sfb 483

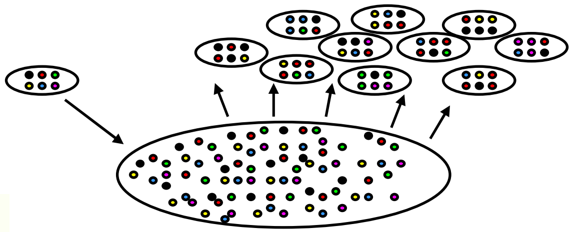
Uncertainties in Mechanics and Materials Research – Application in the field of Materials Research

H. Riesch-Oppermann, M. Härtelt, S. Scherrer-Rudiy

Karlsruhe Institute of Technology (KIT), Institute for Applied Materials (IAM)







Outline

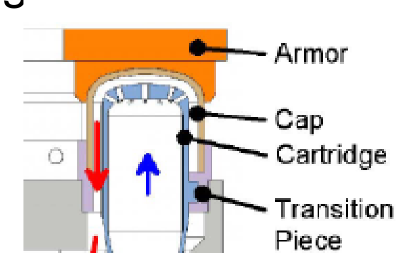
- Scope; historical account
- About ceramics…
- The Weakest Link approach and Fracture Mechanics
- STAU – the interface to Finite Elements
- Uncertainty analysis: Bootstrap & Bayes‘ methods
- Reducing uncertainties by pooling
- (uncertainty: predictions/boundary conditions)
- Summary

Scope I

- Reliability of brittle materials

ceramics

tungsten



glass

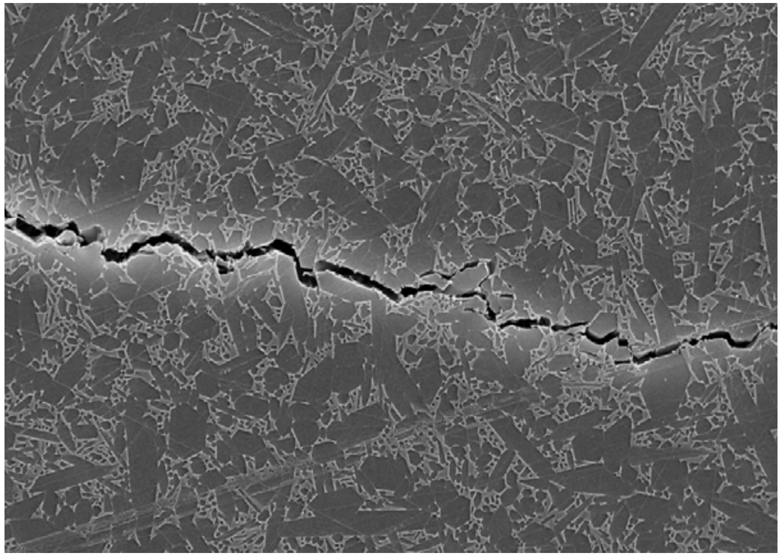
steel (ferrite)

- high strength, low wear, heat resistance, chemical inertness

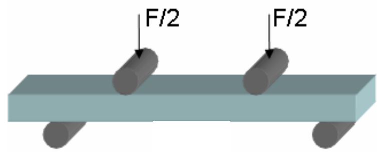
Scope II

- fields of interest

microstructure (silicon nitride Si_3N_4)



mechanical properties

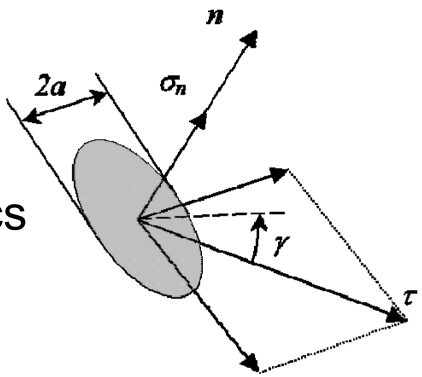


failure behaviour

Scope III

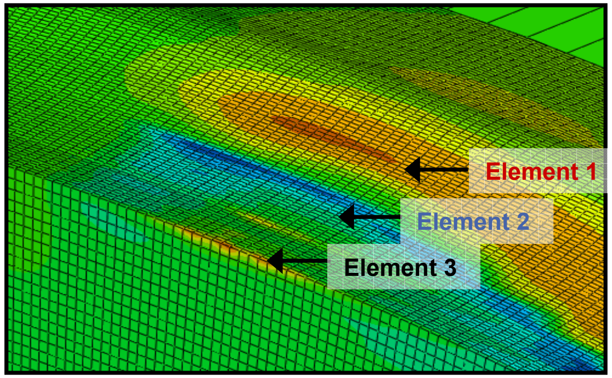
- methods

fracture mechanics



probability theory

engineering



From then to now (history)

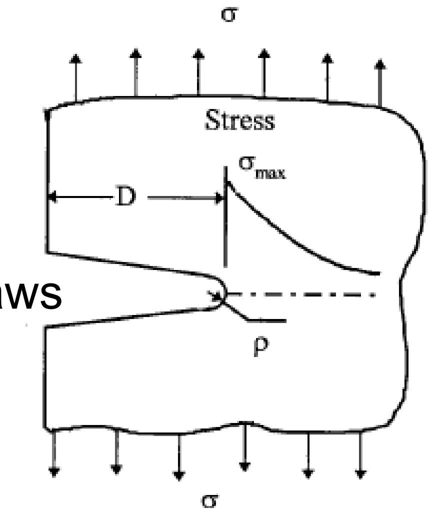
- fracture of materials (mechanics)

  Griffith (~1920)

  Irwin (~1960)

  the importance of material flaws

  Hutchinson, Rice (~1970)



- fracture of materials (statistics)

  Weibull (~1930)

  Freudenthal (~1950)

  Batdorf, Evans, Matsuo (~1980-90)

  fracture mechanics based weakest link approach

- material flaws as stress raisers

From then to now II

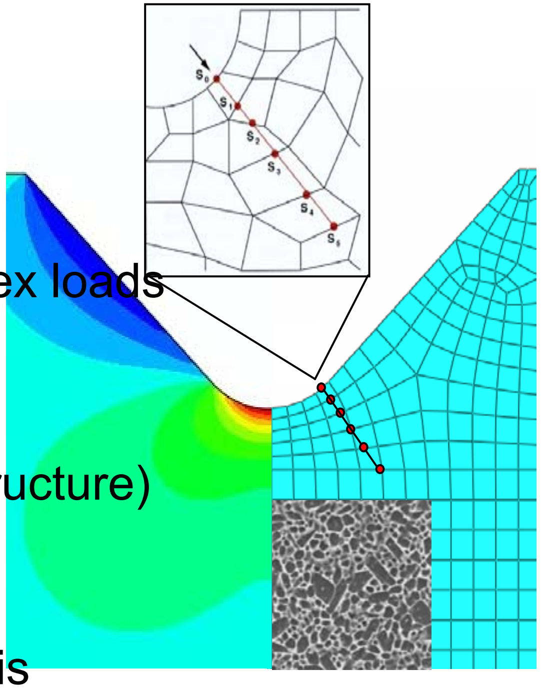
- computers (engineering)

  Finite elements

  stress analysis under complex loads

  stress analysis for complex geometries

  (microstructure)



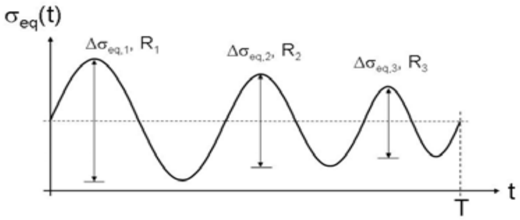
- FE postprocessing (CARES, STAU)

  link between stress analysis and reliability analysis

  spontaneous fracture

  delayed (time-dependant) fracture

  cyclic loading



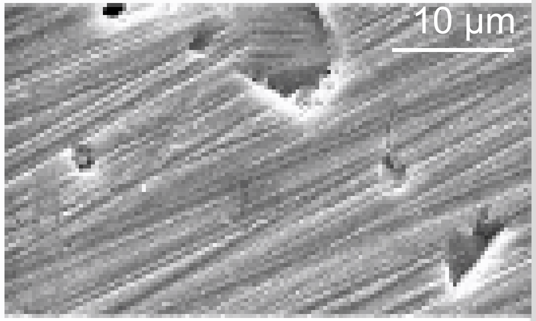
- the importance of microstructure

The importance of microstructure

- the importance of microstructure (high strength ceramics)

sintering: pores, inclusions, grain boundaries

phases, fibres, ...



machining: surface flaws

(surface treatment; hardening)

- FE postprocessing & microstructure

crack propagation

critical crack size; fracture toughness

crack resistance

contact loading; thermal shock

Aspects of uncertainty

- material behaviour (inherent scatter)

  strength: characteristic strength, scatter

  crack propagation: power law parameters

  lifetime: strength & crack propagation parameters

  (surface treatment; hardening)

- material characterization (data uncertainties)

  limited amount of data

  data from different sources

- modelling; analysis

  unknown boundary conditions

  various models for strength, lifetime, crack propagation

Reliability uncertainty

- Failure probability

  obtained by numerical analysis of mechanical stress field

  including routines for crack propagation

  uncertainty in input parameters transforms to results

- lifetime prediction

  dito

  data from different sources

- size effect

  mechanisms of fracture (microcrack failure)

Data base uncertainties → **prediction uncertainties**

Strategies for uncertainty assessment

- material behaviour (inherent scatter)

  strength: use appropriate pdf

  crack propagation: assess parameter uncertainty

  lifetime: use stress/lifetime approach via suitable pdf

- material characterization (data uncertainties)

  pooling: transferring of data to reference conditions

  use data from different sources

- modelling; analysis

  parametric studies...

  both, probabilistic and non-probabilistic approaches !!

About ceramics

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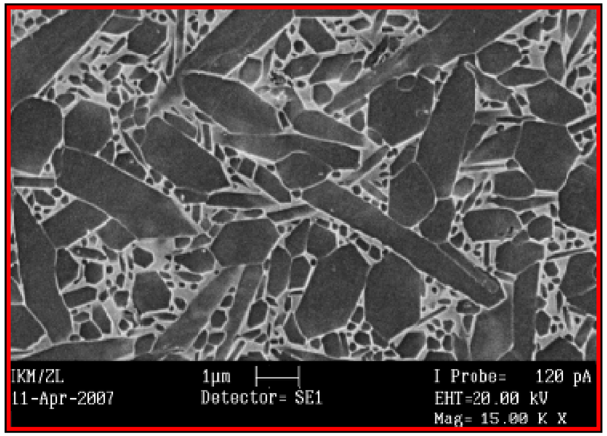

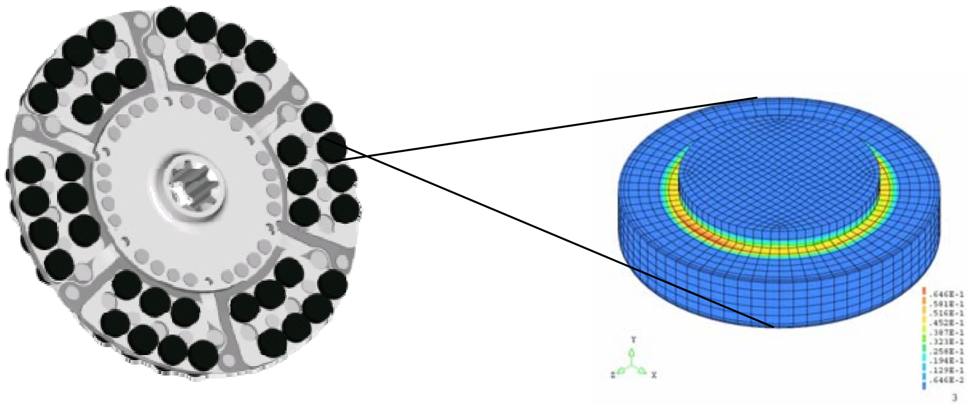

About ceramics II

from powder to product: typical steps in ceramic processing

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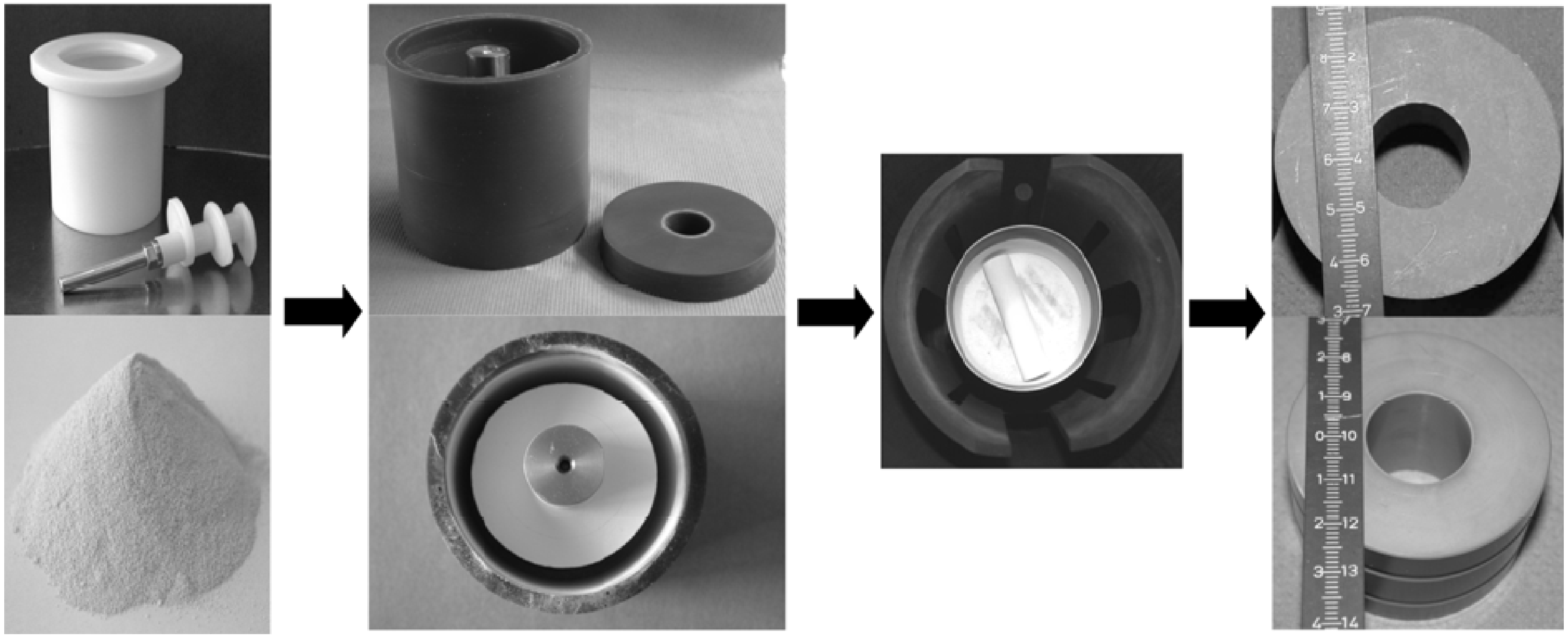
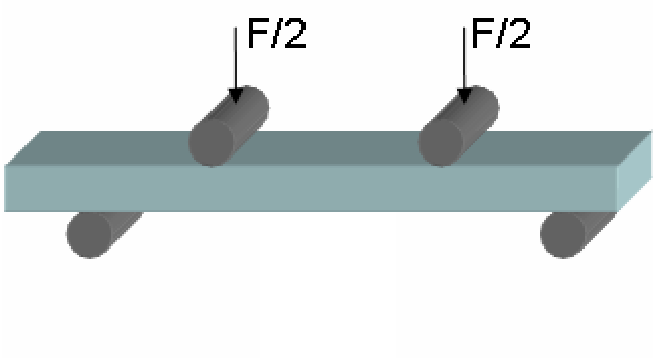


Bild 7:Schema der Keramikherstellung am IKM am Beispiel Keramikwalzen

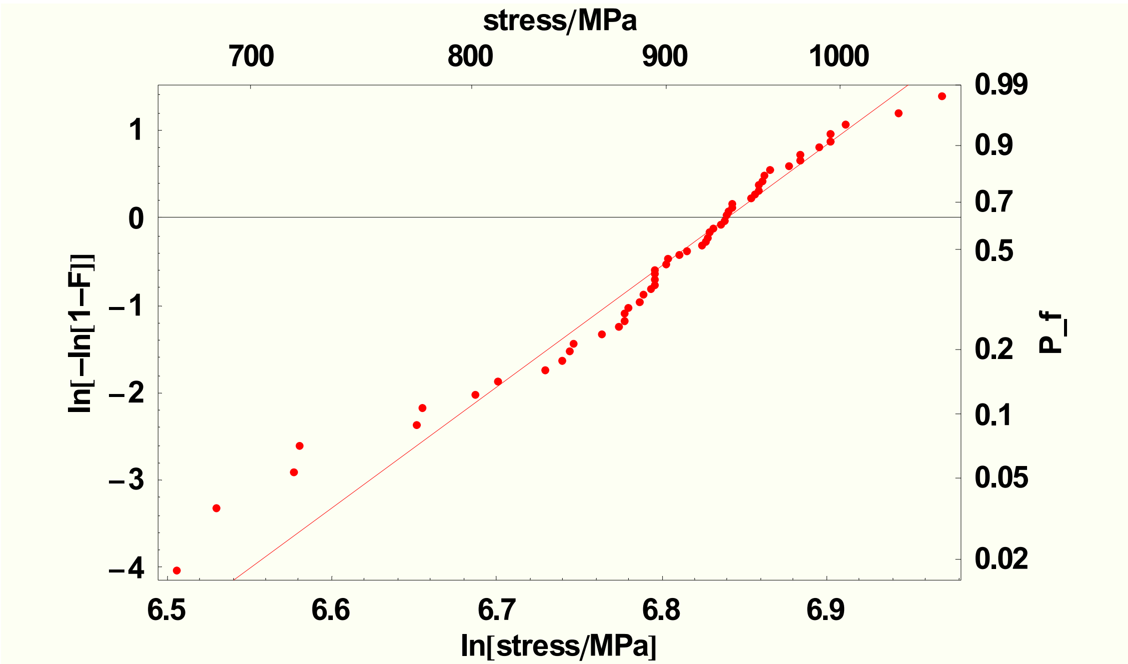
About ceramics III

after sintering: grinding to final geometry ➔ surface flaws

material characterization: four-point bend test
(inert strength, lifetime under static & cyclic loads)

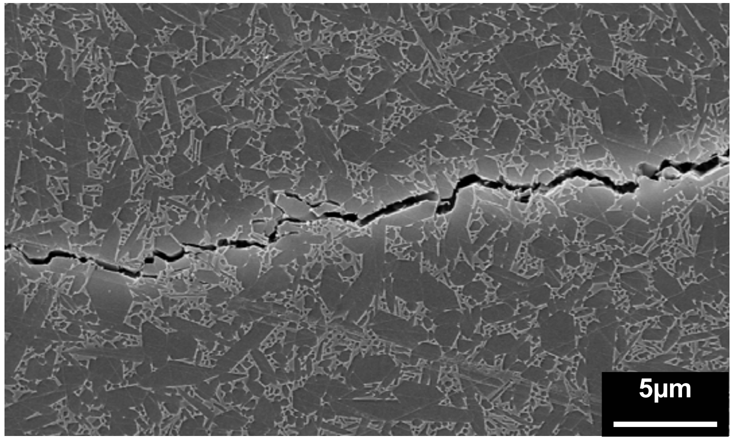


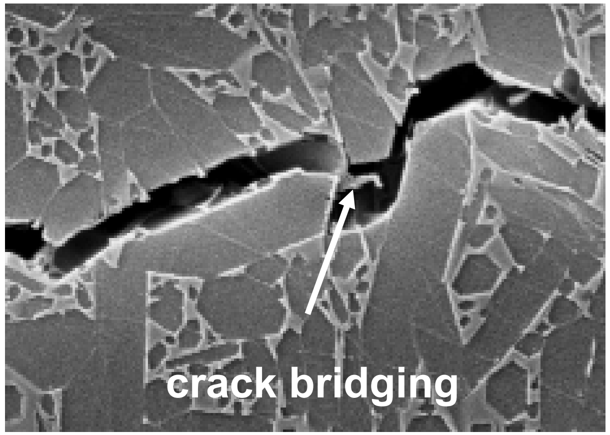
typical Weibull diagram for 4PB fracture strength of Si_3N_4 ceramic; F=i/(n+1):



About ceramics IV

high strength ceramics: strengthening mechanisms





crack bridging stresses affects critical crack size: „R-curve effect“

About ceramics V

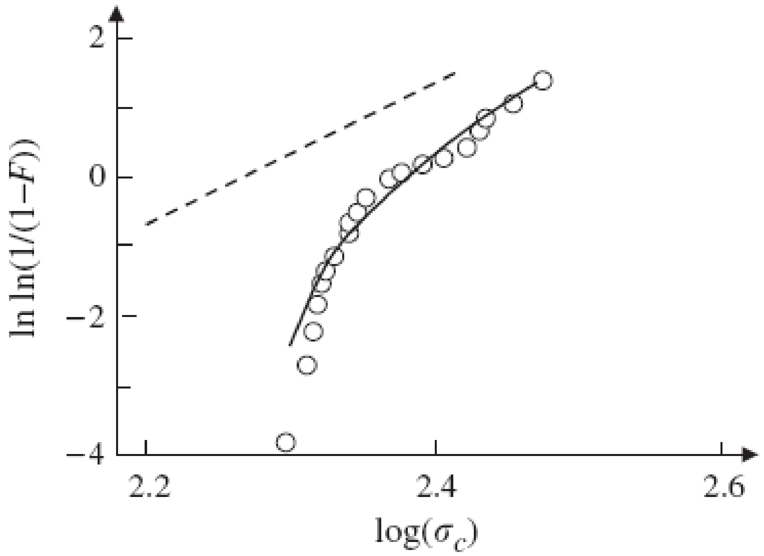


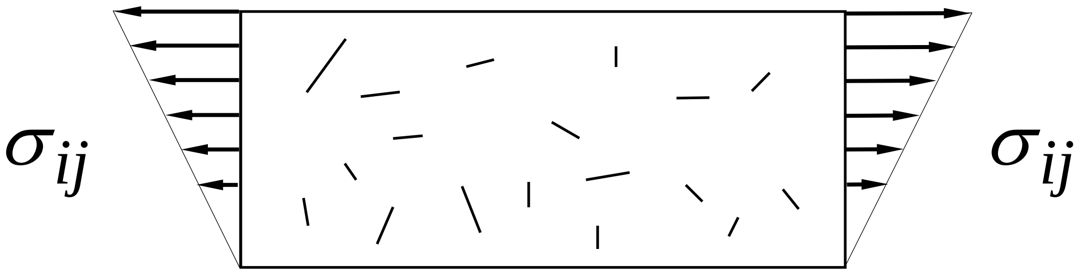
Fig. 23. Weibull plot of the strength for an Al_2O_3 (full line: calculated from bridging relation; broken line: corresponding distribution for a flat *R* curve, Fett and Munz[143]).

crack bridging stresses affects critical crack size: „R-curve effect“

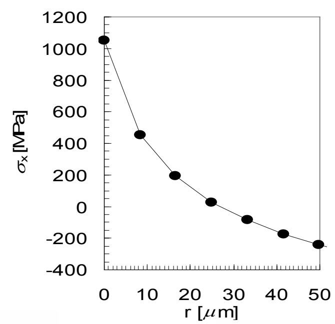
About ceramics VI

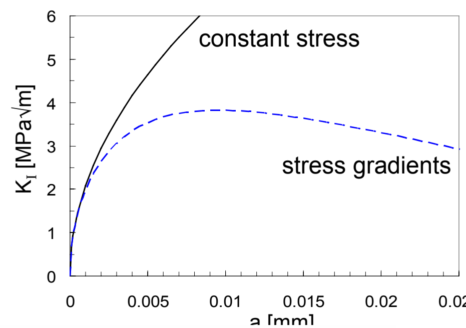
critical crack size with respect to local stress field

inert strength; K-concept: $K_{I,eq} = \sigma_{eq} Y_I \sqrt{a}$ ➡ $\sigma_{eq} = \dfrac{K_{I,eq}}{Y_I \sqrt{a}}$



inert strength; strongly varying local stresses (contact; thermal shock):





$$K_I(a) = \int_0^a \left(h_I^{(1)}(x,a) \cdot \sigma(x) + h_I^{(2)}(x,a) \cdot \tau(x) \right) \mathrm{d}x$$

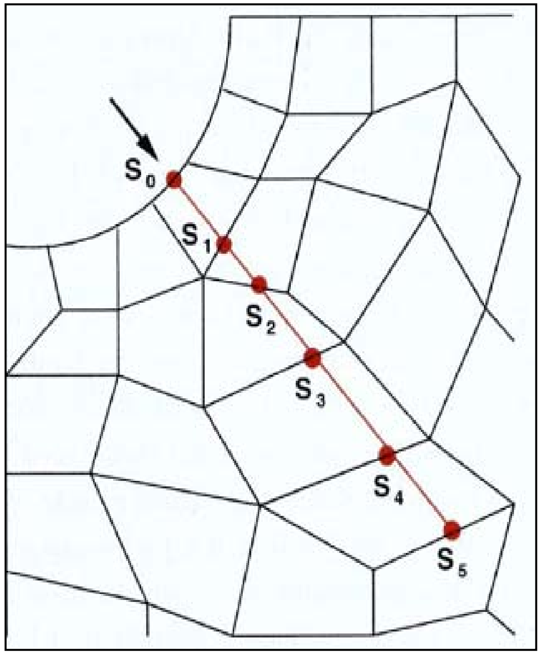
$$h_I^{(i)}(x,a) = \sqrt{\frac{2}{\pi a}} \sum_{m=0}^{\infty} D_{I,m}^{(i)} \cdot \left(1 - \frac{x}{a}\right)^{m-\frac{1}{2}}$$

About ceramics VII

inert strength; R-curve behaviour:

$$K_{tip}(a) = K_{appl}(a) - K_{br}(a) \quad = \int_0^a h(x,a)\,\sigma_{appl}(x)\,dx - \int_0^a h(x,a)\,\sigma_{br}(x)\,dx$$

iterative procedure for critical crack size necessary: $K_{tip}(a_c) = K_{Ic}$



Basic ideas of the weakest link approach

Basic idea (components & system): system failure, if weakest link fails

probabilistic model

- isotropy; i.e. flaws are uniformly distributed
- size and orientation are random
- flaws are independent (no interaction)

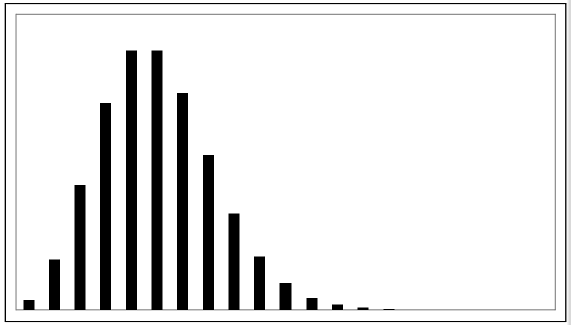
failure model

- fracture mechanics: flaws as planar cracks
- most unfavourable combination of stress, flaw size and orientation determines failure of a component

The weakest link approach in brief

$$1 - Q_n = (1 - Q_1)^n$$

$$p_n = \frac{M^n}{n!} e^{-M}$$



$$P_f = 1 - \exp[-M\, Q_1]$$

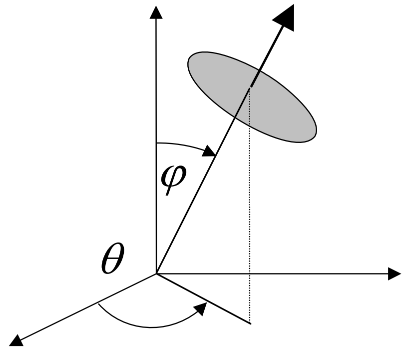
failure of individual flaws: Q_1

flaw population: p_n

The weakest link approach in detail (enter fracture mechanics)

1st step (individual flaw) - *k*=1

e.g. flaw at location (*x, y, z*) with given orientation but random size *a*



failure, if $a \geq a_c(\vec{x}, \theta, \varphi)$ **since *a* random variate with distribution F(a)**

i.e. with probability $$P\left(a \geq a_c(\vec{x}, \theta, \varphi)\right) = \int_{a_c}^{\infty} f(a)da = 1 - F\left(a_c(\vec{x}, \theta, \varphi)\right)$$

If location and orientation are also random: $$Q_1 = \frac{1}{V}\int_V \frac{1}{4\pi}\int_\Omega \left(1 - F(a_c)\right) d\Omega\, dV$$

Q_1 is the failure probability for a component with <u>exactly one</u> flaw of <u>random</u> size and orientation at a <u>random</u> location of the component !!

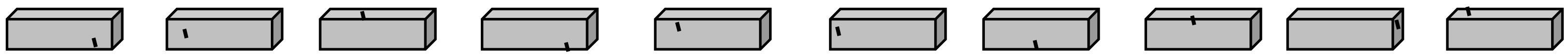
The weakest link approach in detail (enter fracture mechanics)

2nd step (fixed number of flaws / dislocations / extrusions / ...) - *k* = *n*

flaws are independent (no interaction)

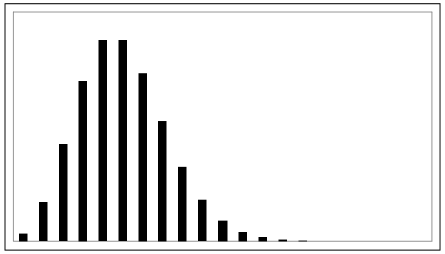
➡ **1 component with *n* flaws == *n* components, each with 1 flaw**

consider n components each with 1 flaw: each components fails with probability Q_1



➔ **survival probability *1-Q_n* for *n* components:** $$1 - Q_n = (1 - Q_1)^n$$

number of flaws follows a Poisson distribution: $$p_n = \frac{M^n}{n!} e^{-M}$$



***M* - average number of flaws per component**

➔ **probability that a component has exactly *n* flaws and fails:** $$p_n\, Q_n$$

The weakest link approach in detail (enter fracture mechanics)

3rd step (summing it all up ...)

component with random number of flaws has failure probability:

$$\overline{\underline{P_f}} = \sum_{n=0}^{\infty} p_n Q_n = 1 - \sum_{n=0}^{\infty} \frac{M^n}{n!} e^{-M} \left(1 - Q_1\right)^n = 1 - e^{-M} e^{M(1-Q_1)} \overline{\underline{= 1 - \exp\left(-MQ_1\right)}}$$

***MQ_1* is just the <u>average number of critical flaws</u> in the component**

Now we need some information about the flaw size distribution F(a): $F(a) \propto a^{-r} \propto \sigma_{eq}^{2r}$

⇧ **fracture mechanics**

$$P_f = 1 - \exp\left[-\frac{1}{V_0} \int_V \frac{1}{4\pi} \int_\Omega \left(\frac{\sigma_{eq}}{\sigma_0} \right)^m d\Omega \, dV \right]$$

and finally we obtain a size effect relation:

$$P_f = 1 - \exp\left[-\frac{1}{V_0} \int_V \frac{1}{4\pi} \int_\Omega \left(\frac{\sigma_{eq}}{\sigma^*} \right)^m d\Omega \, dV \left(\frac{\sigma^*}{\sigma_0} \right)^m \right] = 1 - \exp\left[-\frac{V_{eff}}{V_0} \left(\frac{\sigma^*}{\sigma_0} \right)^m \right] = 1 - \exp\left[-\left(\frac{\sigma^*}{b} \right)^m \right]$$

The size effect in the weakest link approach

a look at geometry…

what we learn from the final result:

$$P_f = 1 - \exp\left[-\frac{1}{V_0} \int_V \frac{1}{4\pi} \int_\Omega \left(\frac{\sigma_{eq}}{\sigma^*} \right)^m d\Omega \, dV \left(\frac{\sigma^*}{\sigma_0} \right)^m \right] = 1 - \exp\left[-\frac{V_{eff}}{V_0} \left(\frac{\sigma^*}{\sigma_0} \right)^m \right] = 1 - \exp\left[-\left(\frac{\sigma^*}{b} \right)^m \right]$$

the component strength distribution is a Weibull distribution with parameters *m* and *b*

the component size leads to a geometry dependence of the distribution parameter *b*

it is possible to define a geometry-independent distribution parameter σ_0

the size effect is described by the volume integral V_{eff} using a reference stress value σ^*

The weakest link model – ready for FE use

Numerical integration of stress field:

$$P_f(t) = 1 - \exp\left[-\frac{1}{V_0} \int_V \frac{1}{4\pi} \int_\Omega \left(\max_{\tau\in[0,t]} \left\{ \left[\left(\frac{\sigma_{eq}}{\sigma_0} \right)^{n-2} + \Psi \right]^{\frac{1}{n-2}} \right\} \right)^m d\Omega \, dV \right]$$

spontaneous failure

Sub-critical crack growth

Fracture at time *t* is governed by the <u>maximum</u> load in [0, t]!

$$\Psi = \frac{\sigma_0}{B} \int_0^\tau \left(\frac{\sigma_{eq}(t')}{\sigma_0} \right)^n dt'$$

Calculation of $P_f(t)$ by FE-Postprocessing (self-developed STAU postprocessor)

- non-linear problem!

The local risk of rupture – a design tool

Consider partial volume V_t

$$\pi(\vec{x}) = \lim_{V_t \to 0} \frac{P(\text{critical flaw in } V_t)}{P(\text{critical flaw in } V)} = \lim_{V_t \to 0} \frac{P(FK)}{P(K)}$$

using the following events:

F: given flaw is located in V_t $P(F) = V_t / V$

and K: given flaw is critical $P(K) = Q_1$

We need the probability P(F|K):

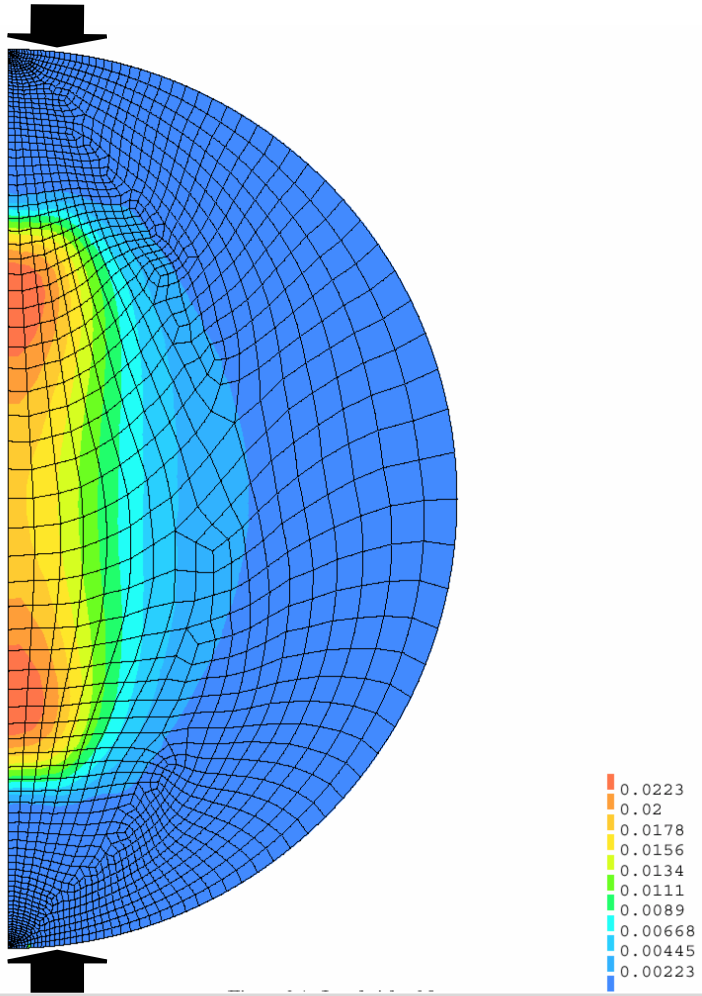
P(F|K) = P(FK) / P(K) (Bayes' theorem)

We obtain the probability P(FK) from:

$$P(FK) = \frac{1}{V} \int_{V_t} \frac{1}{4\pi} \int_{\Omega} (1 - F_a(a_c))\, d\Omega\, dV.$$

and thus:

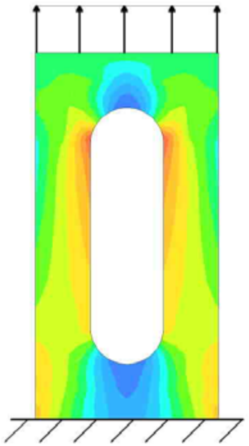
$$\pi(\vec{x}) = \frac{\frac{1}{4\pi} \int_{\Omega} \left(\frac{\sigma_{eq}}{\sigma_0}\right)^m d\Omega}{\frac{1}{V} \int_V \frac{1}{4\pi} \int_{\Omega} \left(\frac{\sigma_{eq}}{\sigma_0}\right)^m d\Omega\, dV}$$



The basic idea of STAU (FE interface)

stress analysis

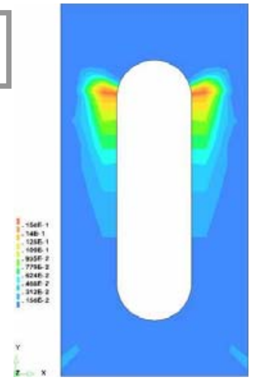
Finite Elements



Weibull-Theory

failure description

fracture mechanics

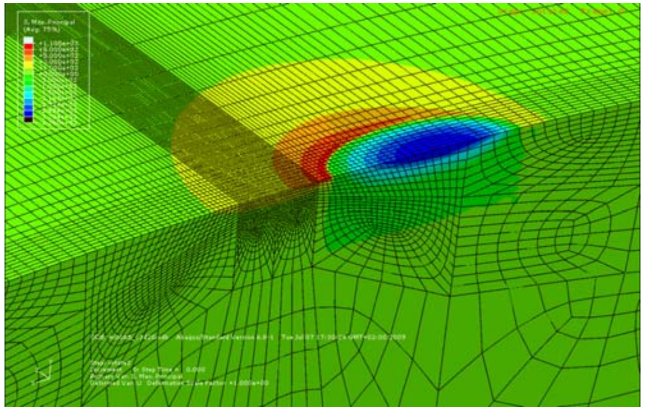


reliability / failure probability / local risk of fracture

The interface to Finite Elements

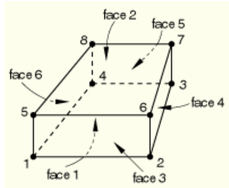
what we have to calculate (simple case):

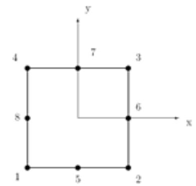
$$P_f = 1 - \exp\left[-\frac{1}{V_0} \int_V \frac{1}{4\pi} \int_\Omega \left(\frac{\sigma_{eq}(x,y,z,\alpha,\beta)}{\sigma_0} \right)^m d\Omega \, dV \right]$$

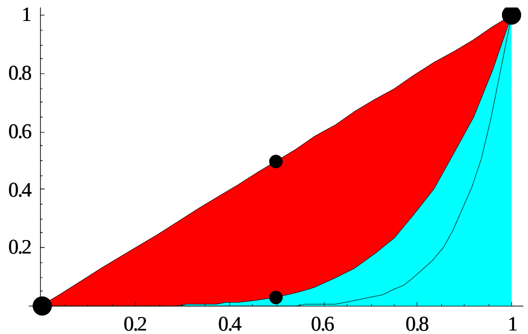


from Finite Element stress analysis we obtain stress tensor at node *n*:

$$\sigma_{ij}(x_n, y_n, z_n)$$







large difference (red) between linear interpolation and m=5...10 curves!

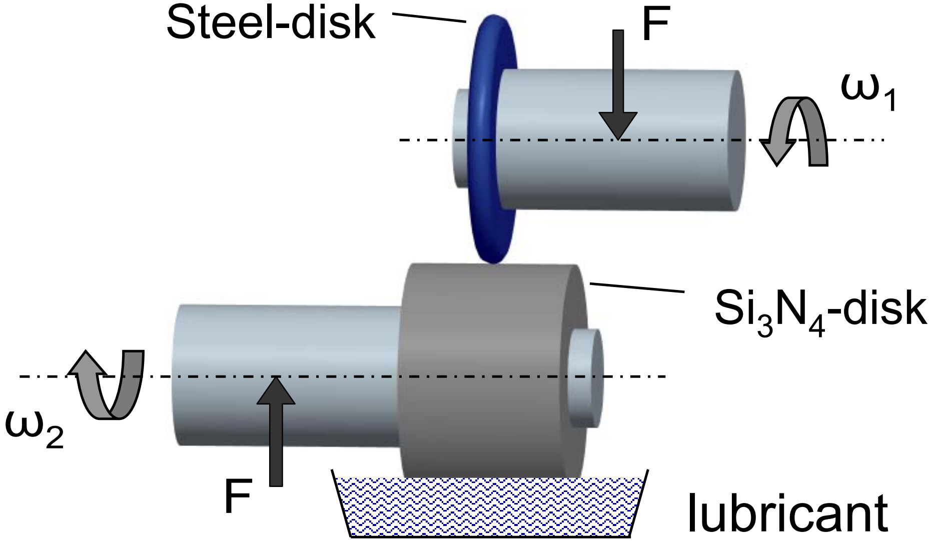
➔ integration of σ^m would be very inaccurate!

we use the interpolation functions to generate additional integration points

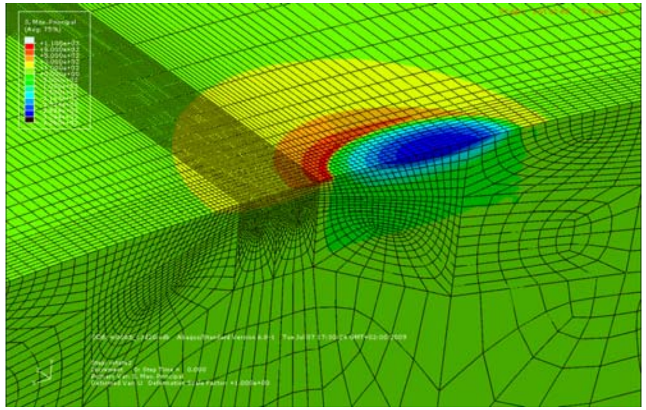
The options for Finite Element models

geometry:	2D model (plane stress, plane strain) 3D model axisymmetric model
analysis:	surface flaws volume flaws interfaces
options:	spontaneous fracture sub-critical crack propagation cyclic crack propagation stress gradients R-curve behaviour *(not all to combine!)*

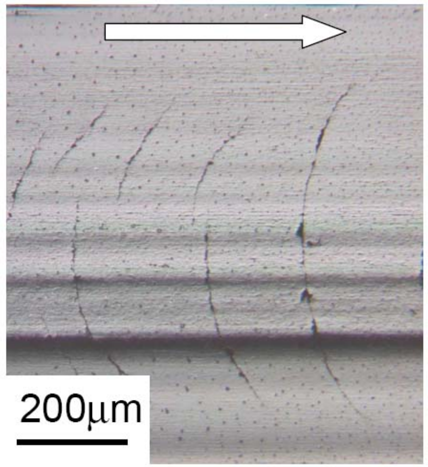
Results: Rolling contact fatigue (RCF) test



Stress distribution



Damage after 10^5 rotations

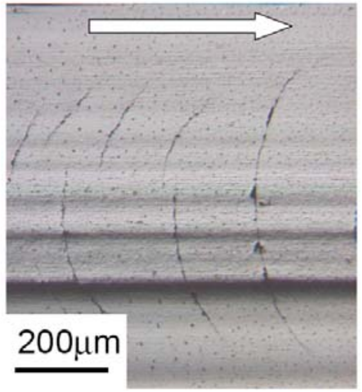


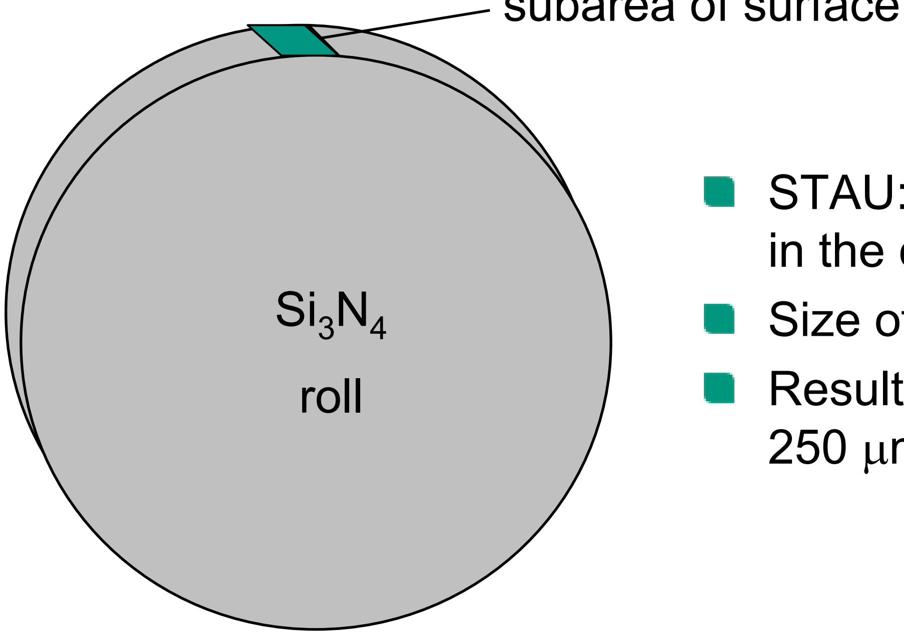
- lubricant: friction coefficient μ=0.085
- F=1700N
- relative slip: ~22%
- max. principal stress: ~1100 MPa

RCF tests: Iyas Khader, Fraunhofer Institute IWM, Freiburg

Results: STAU analysis

- Contact damage: initiation of macroscopic flaws
- Predicting of a certain flaw density on the surface

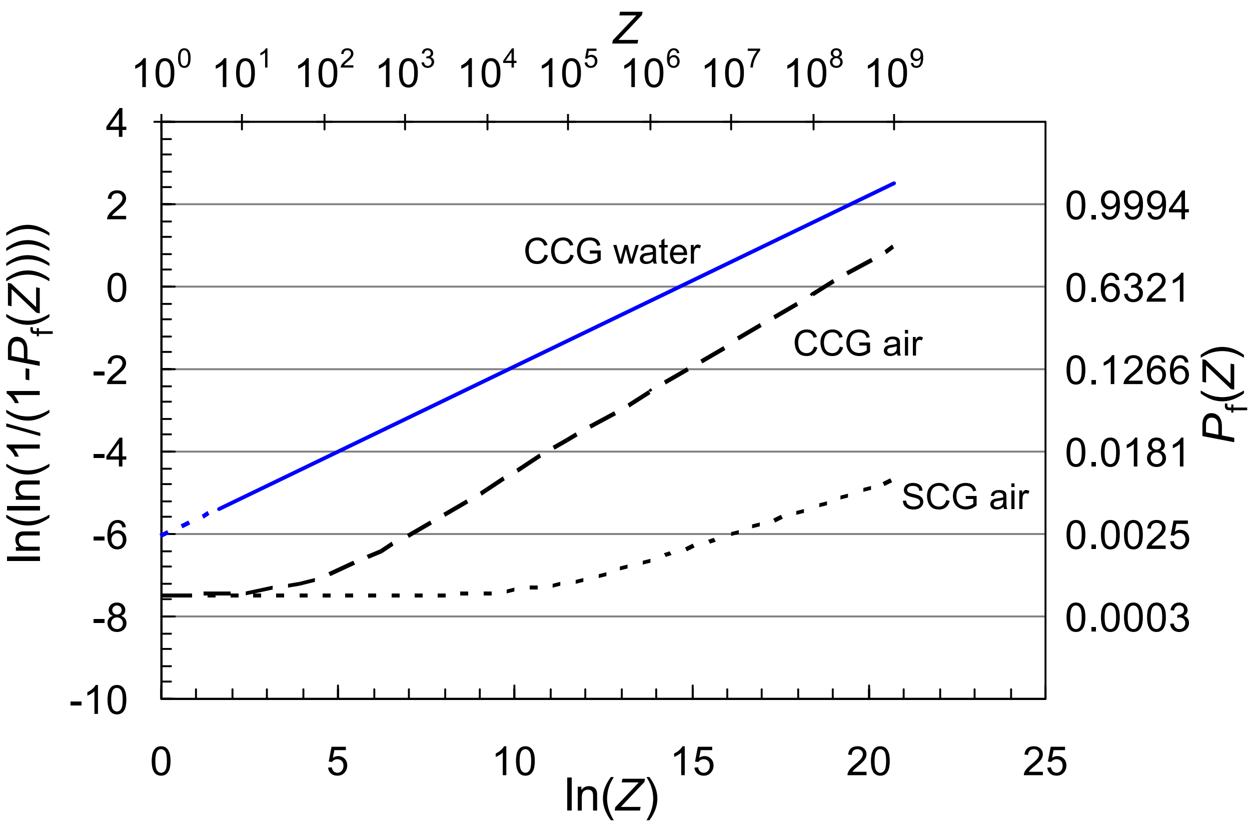




- STAU: probability of the initiation of <u>one</u> flaw in the considered subarea
- Size of subarea ↔ crack density
- Results refer to crack density of 1 crack per 250 µm along the circumference

Results: Failure probability

- Probability to initiate one macroscopic crack every 250 µm



- Weibull CDF

$$P_{\mathrm{f}} = 1 - \exp\left[-\left(\frac{Z}{N_0}\right)^m\right]$$

m – slope of the curve

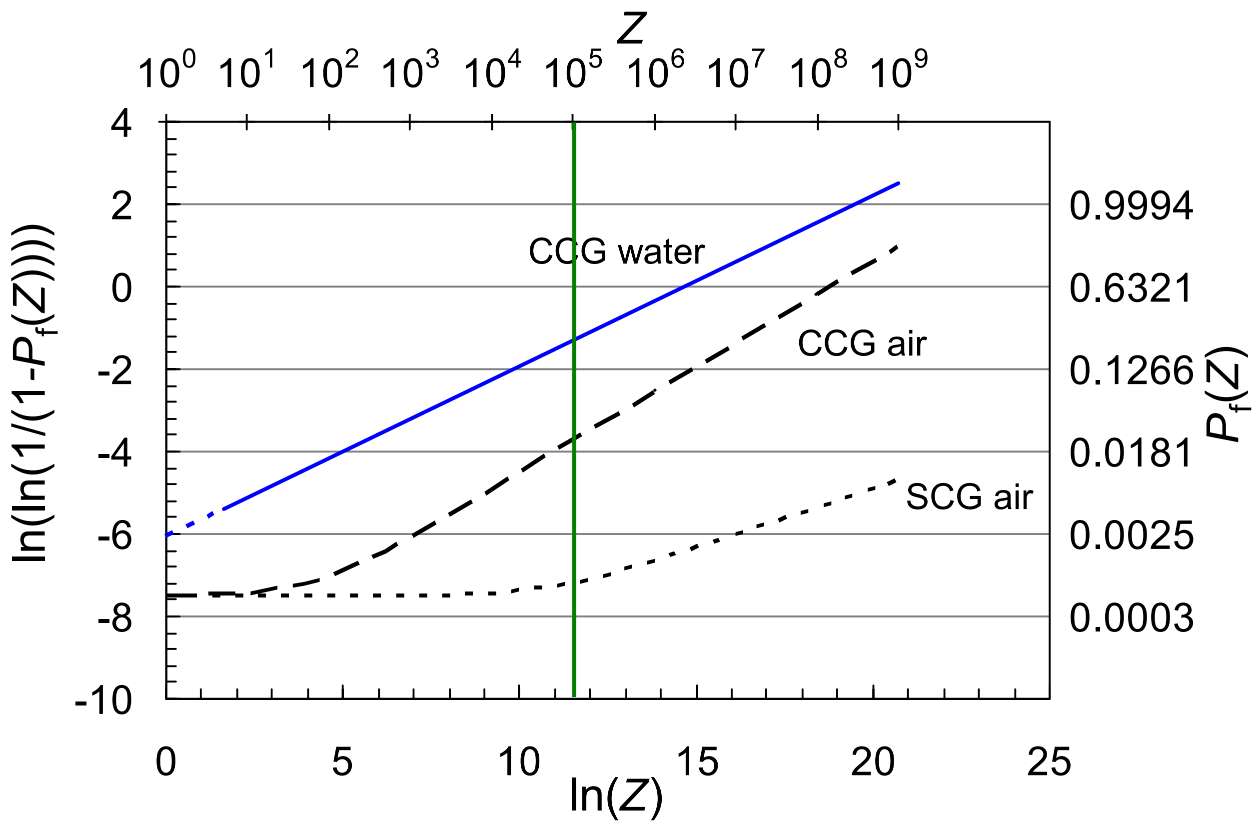
N_0 - characteristic lifetime (63%-quantile)

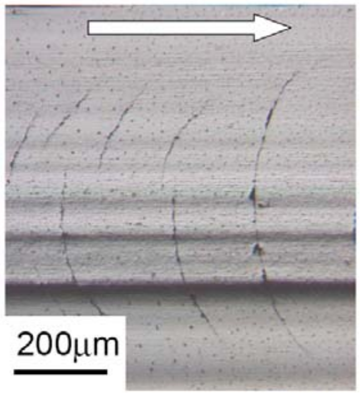
Z – no. of rotations

- Highest failure probability obtained for fatigue parameters in water.

Results: Failure probability

- Relation with experimental crack density after 10 h

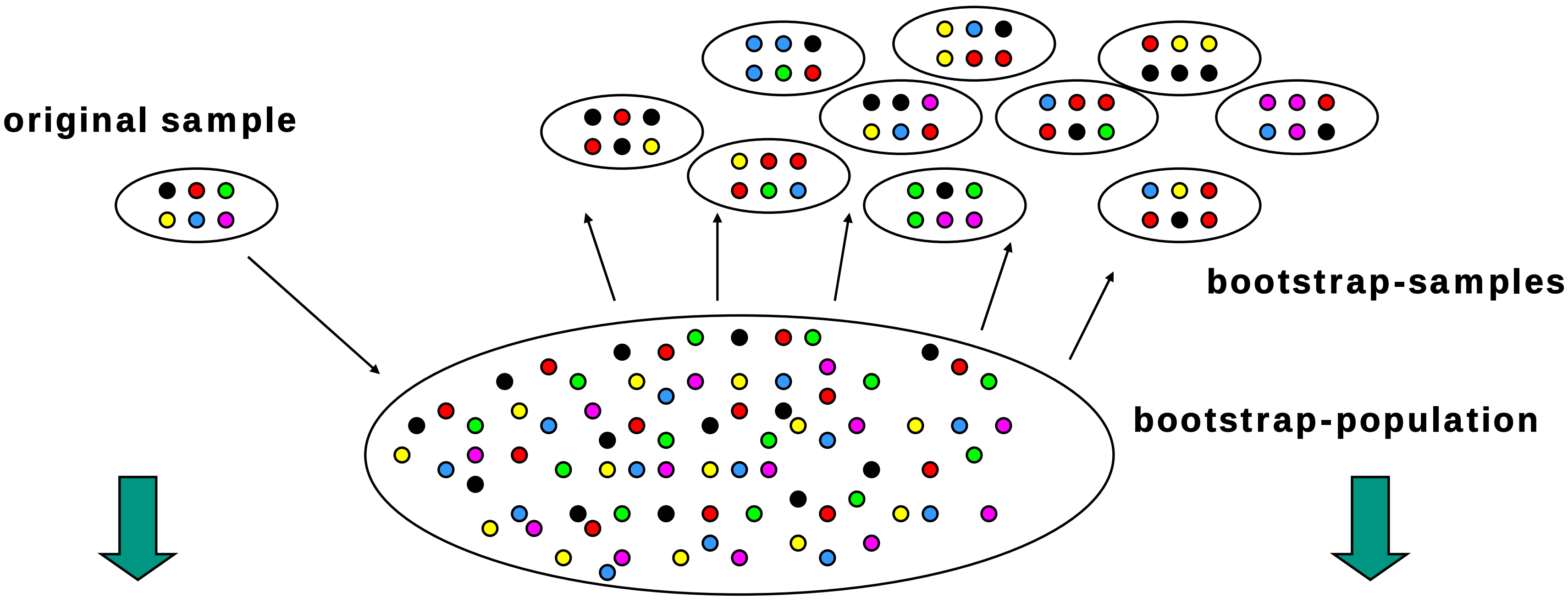




- Initiation probability low for parameters in (air)

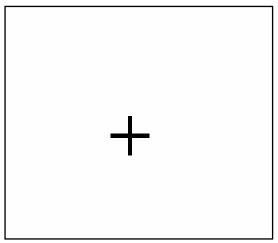
Bootstrapping: assessing the uncertainty by resampling

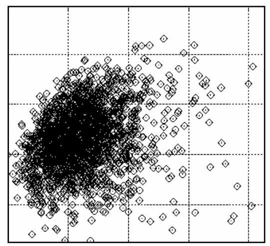
Use original sample to get confidence intervals for failure probability



estimator (m, σ_u) or P_f

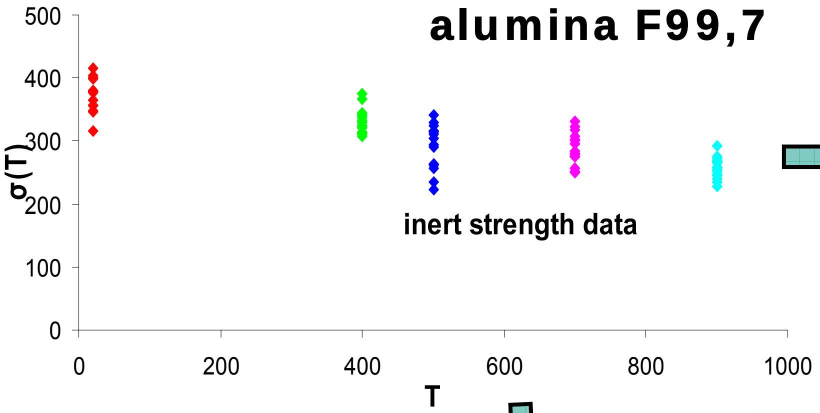
bootstrap-replications for estimator



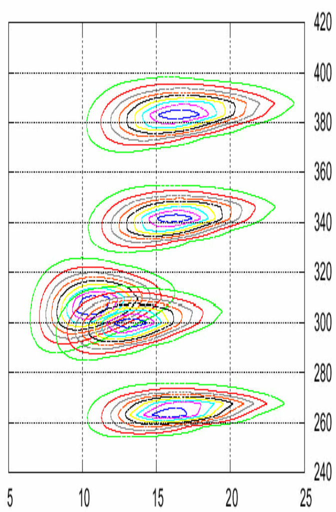


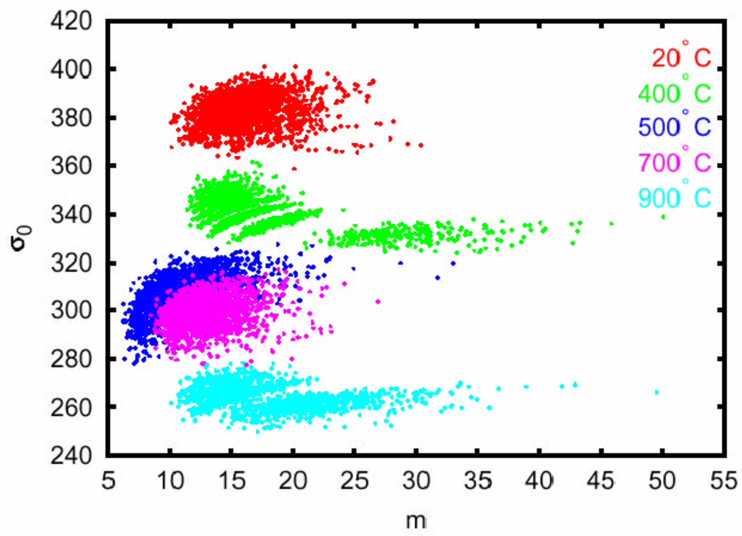
H. Riesch-Oppermann et al., Engng. Fract. Mech.

Assessment of material uncertainties – statistical evaluation



bootstrap resampling



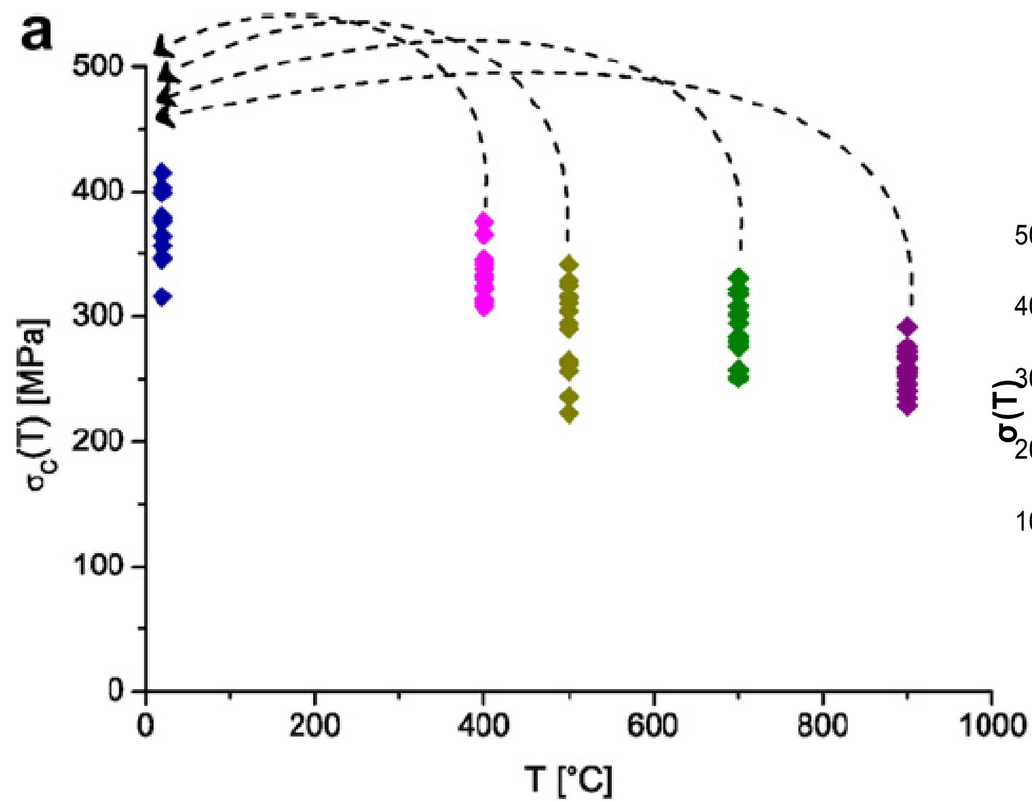


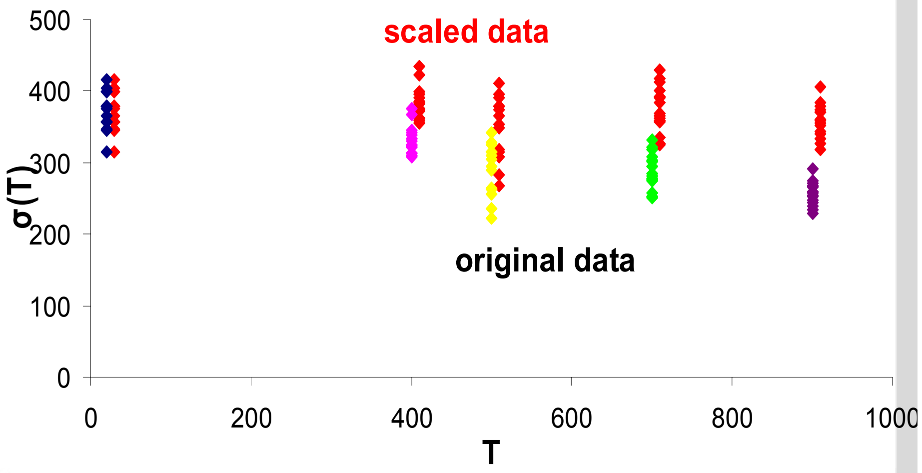
Bayesian posteriori analysis

(generalized Maximum-Likelihood method)

Pooling: reduction in uncertainty by combining samples

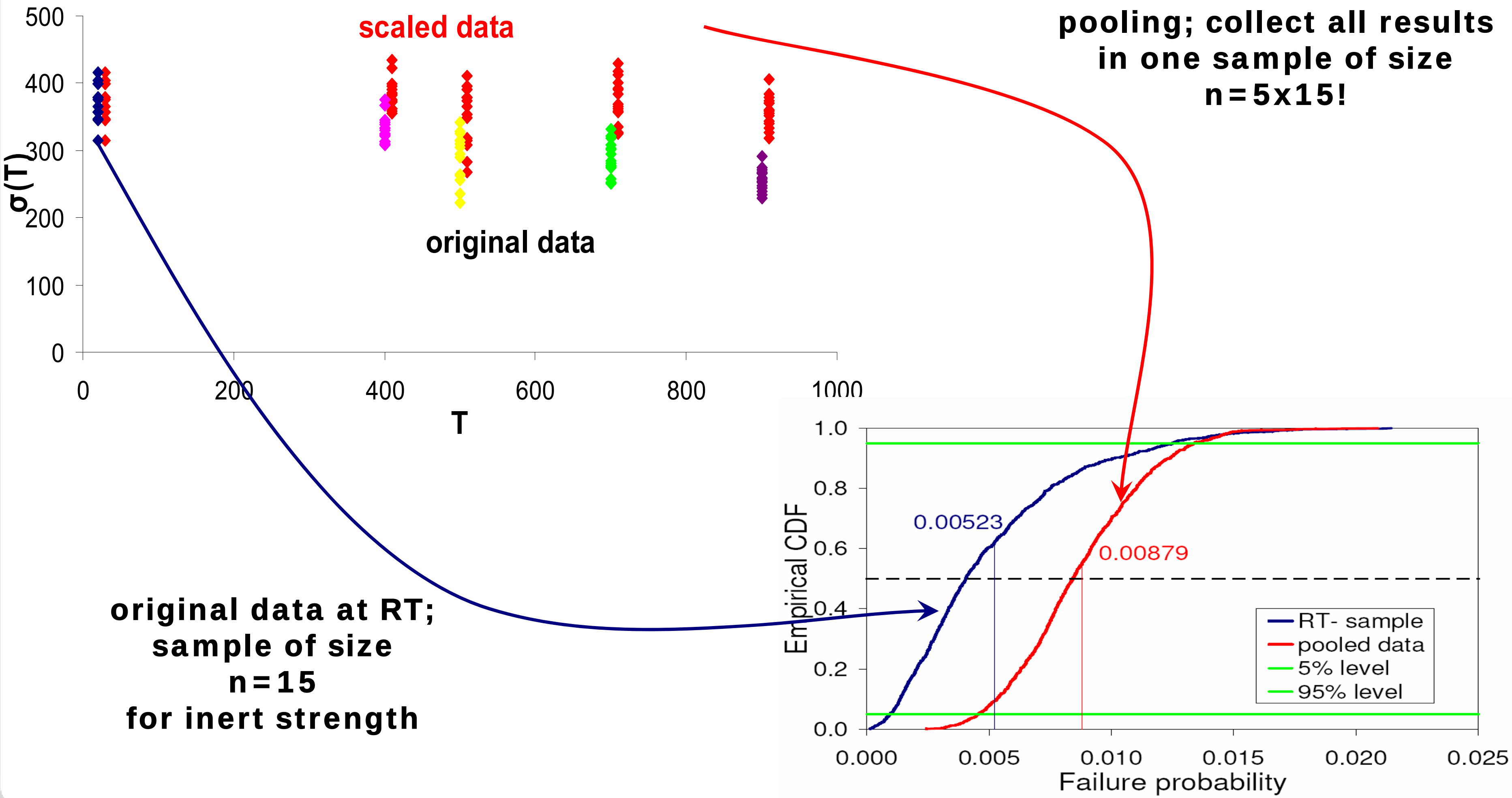
Scaling of strength data to ambient temperature



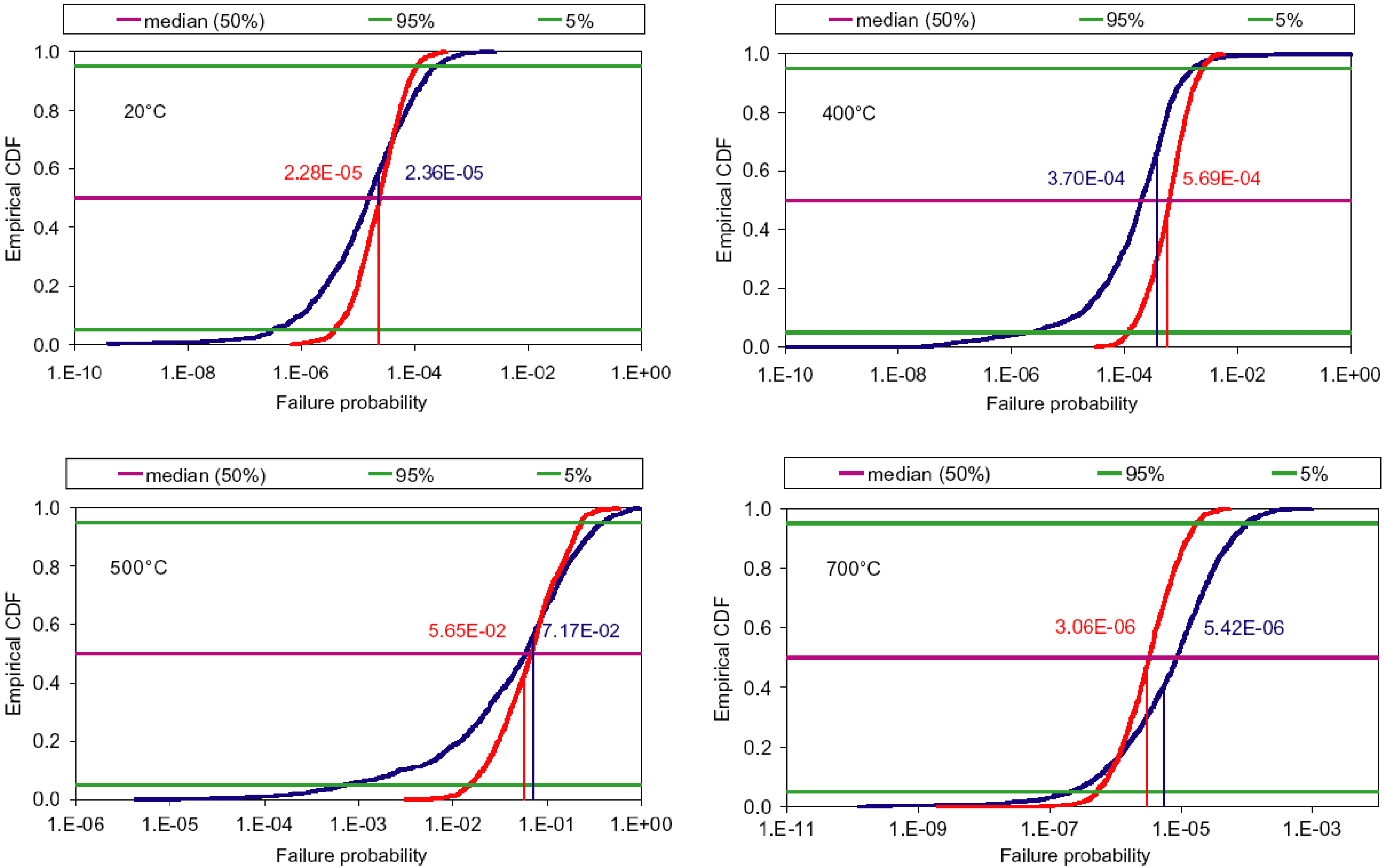


Sample size enlarged by pooling

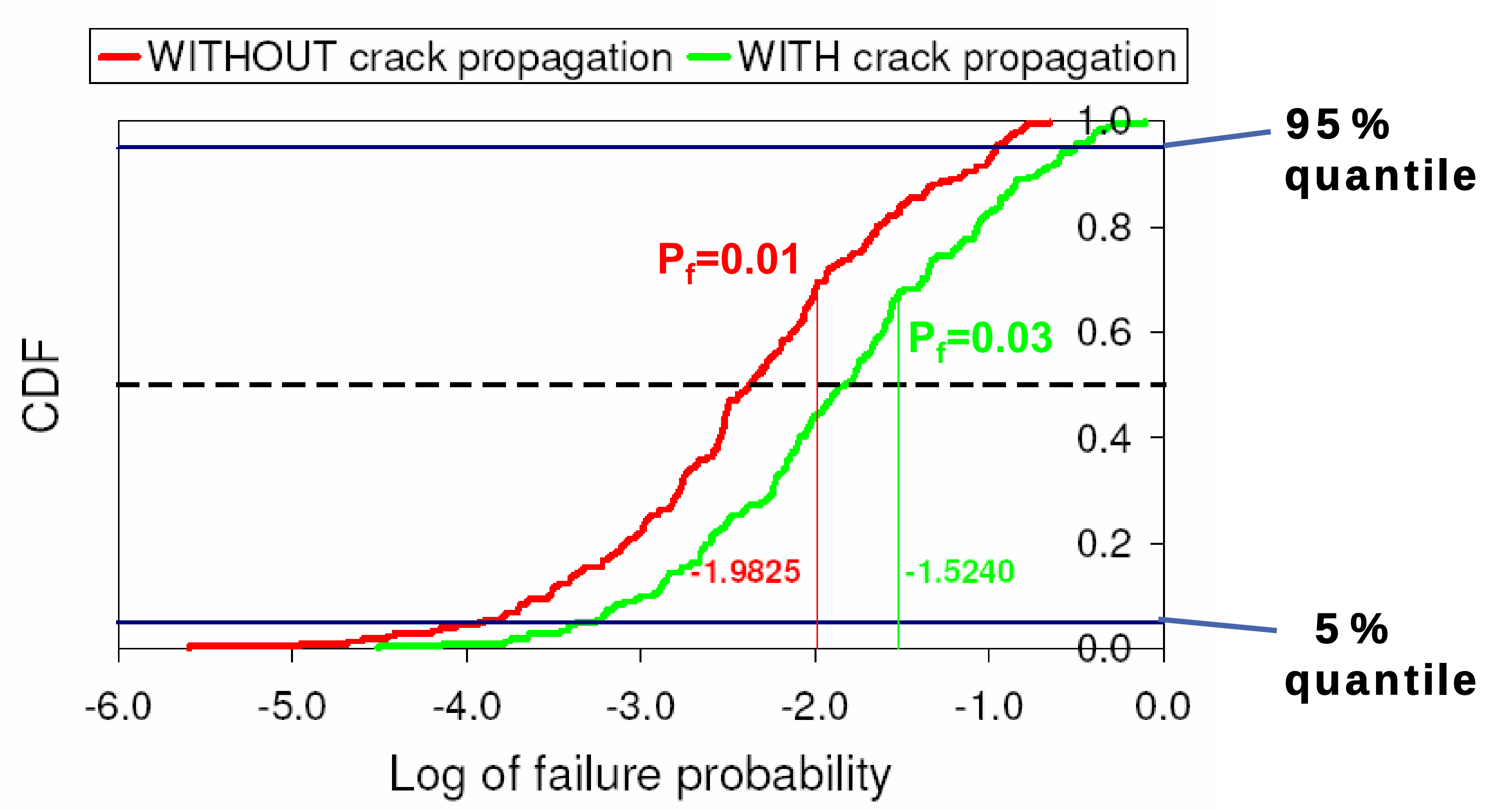
Reduction of scatter by intelligent pooling – ceramic component



Uncertainty assessment and reduction by pooling & resampling



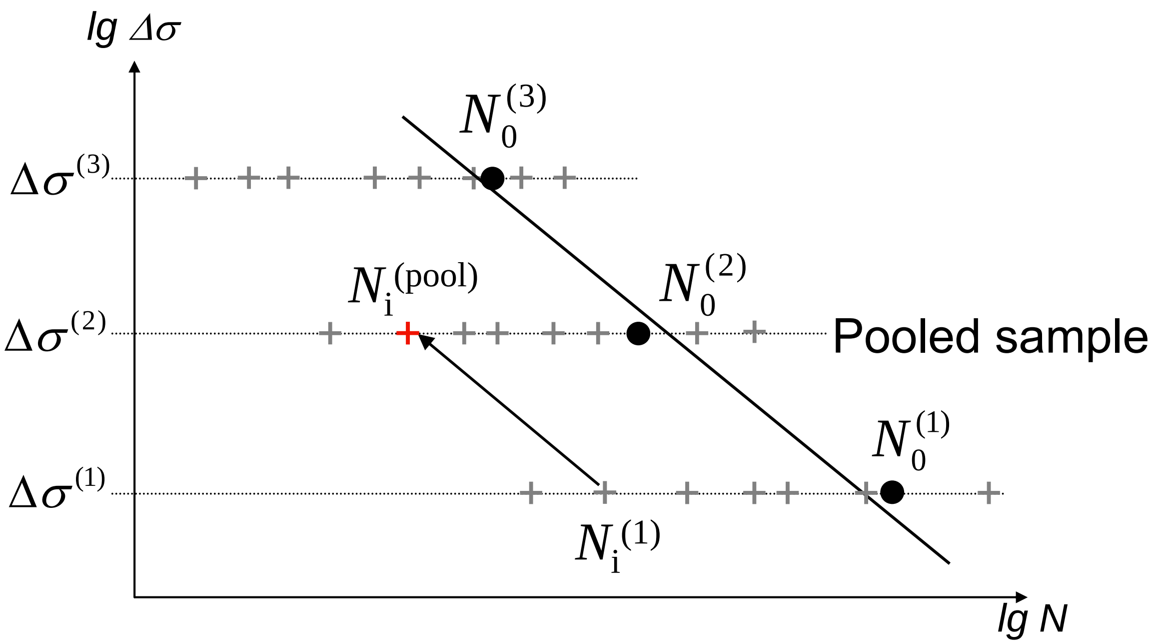
Prediction of scatter in Pf – bootstrap results



note: large impact of crack propagation and 2-3 mag's uncertainty in results!

Pooling: reduction in uncertainty by combining samples

- Increase number of data available for one load level



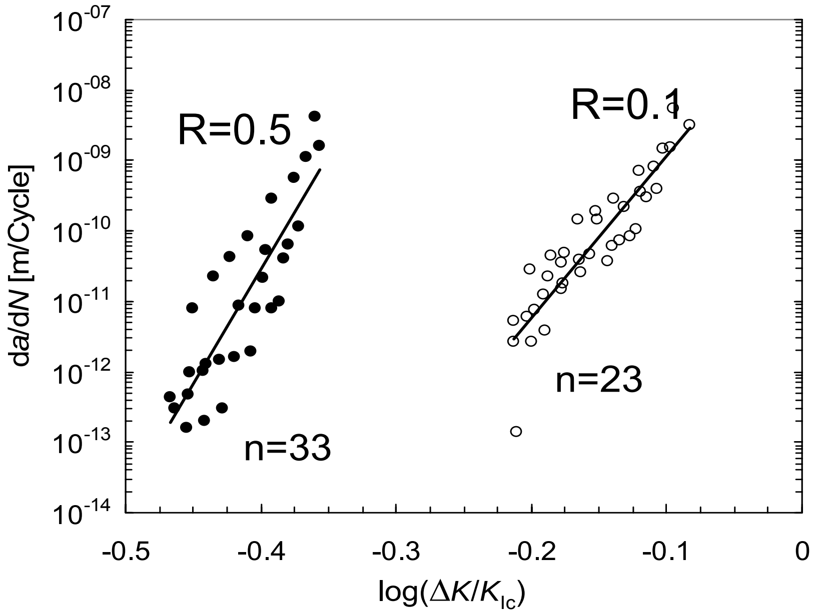
- Conversion using Weibull-distribution:

$$N_i^{(pool)} = N_i^{(1)} \left(\frac{N_0^{(pool)}}{N_0^{(1)}} \right)$$

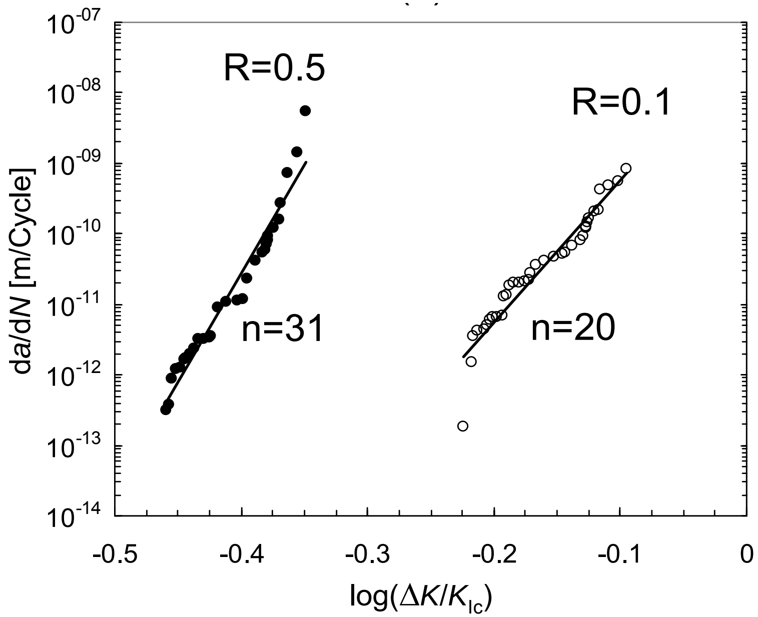
- Relation between $\Delta\sigma$ and N_0 follows from S-N-curve fit

Pooled crack growth curves

unpooled



pooled



- Scatter (uncertainty in *n*) is decreased by pooling
- variation of *n* with *R* remains

M. Härtelt et al., J.Am.Ceram.Soc., (2011), in press

Summary

- Ceramics
- WLT statistics
- fracture mechanics
- the role of microstructure

- pooling: using data efficiently
  getting an idea about possible inhomogeneities

- uncertainties: (bootstrap) and Bayes

- (modelling uncertainties)

Acknowledgements

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High performance
sliding and friction systems
based on advanced ceramics