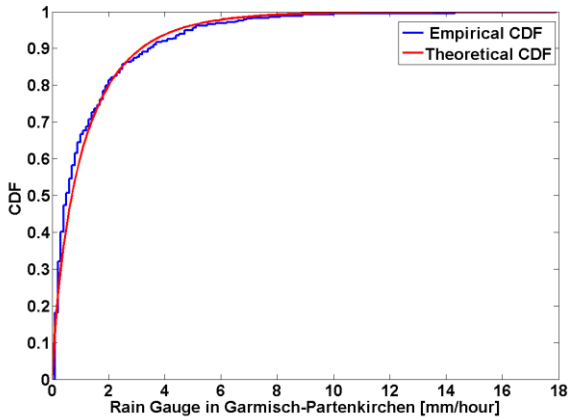
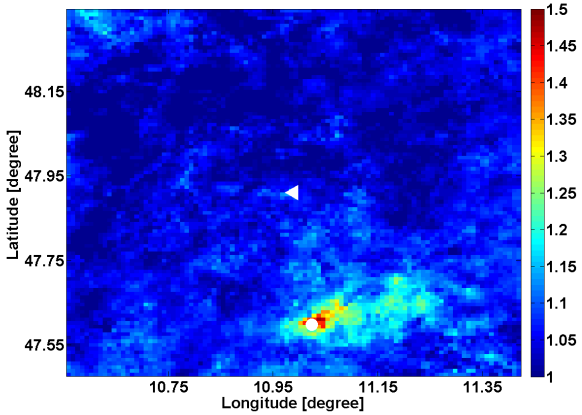
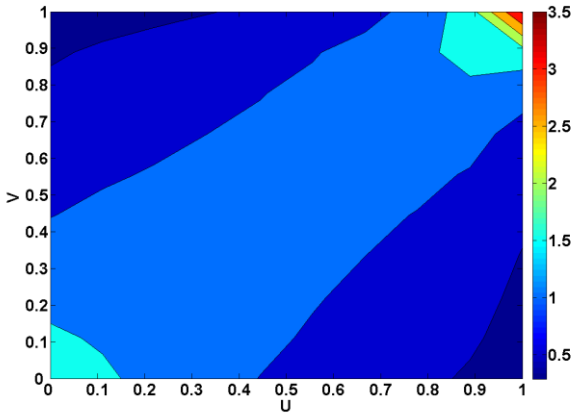
Copula-based assimilation of radar and gauge information to derive bias corrected precipitation fields

Wei Qiu, S. Vogl, P. Laux, G. Mao & H. Kunstmann
IMK-IFU, Karlsruhe Institute of Technology
27.04.2012 Vienna

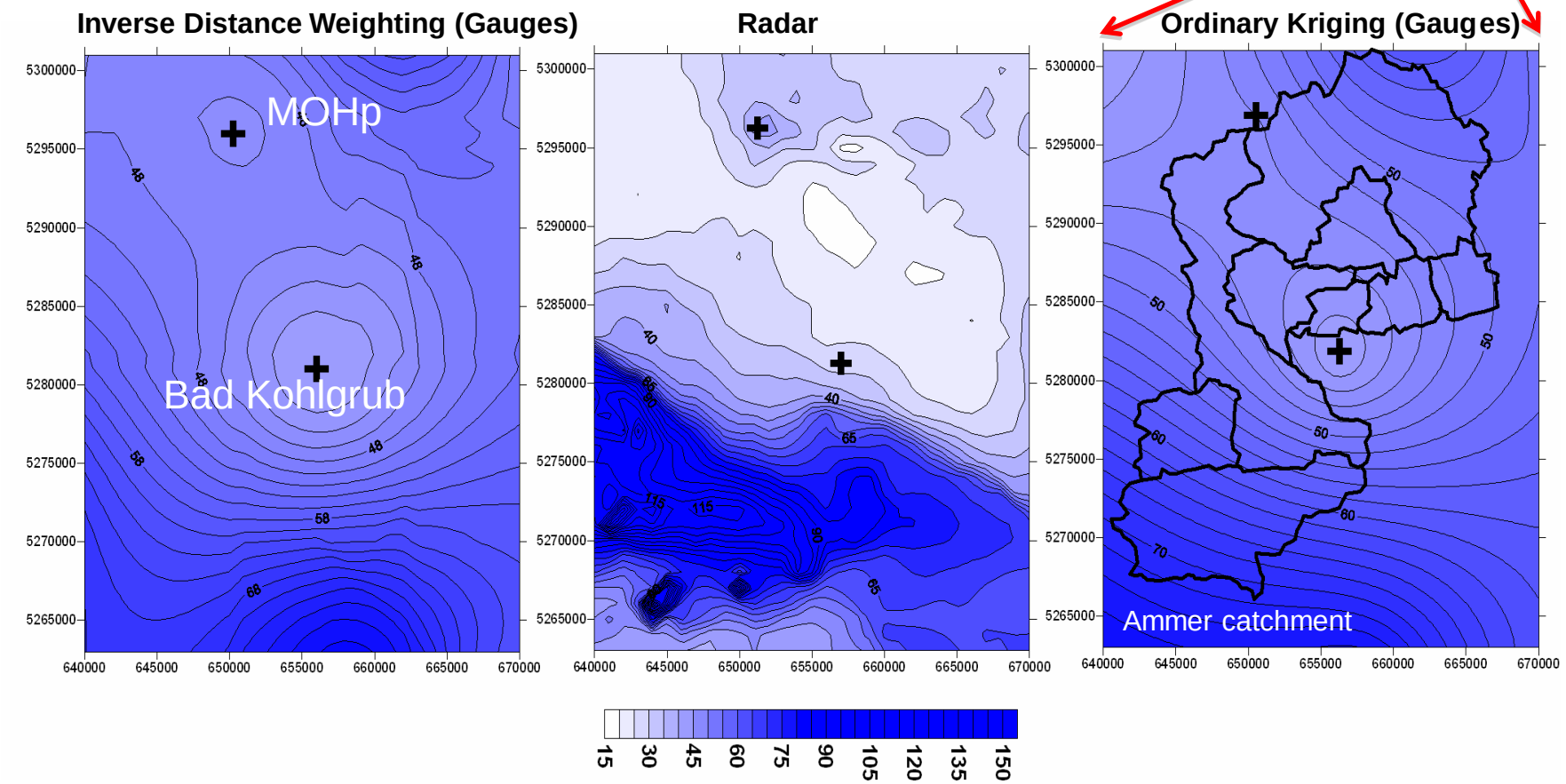






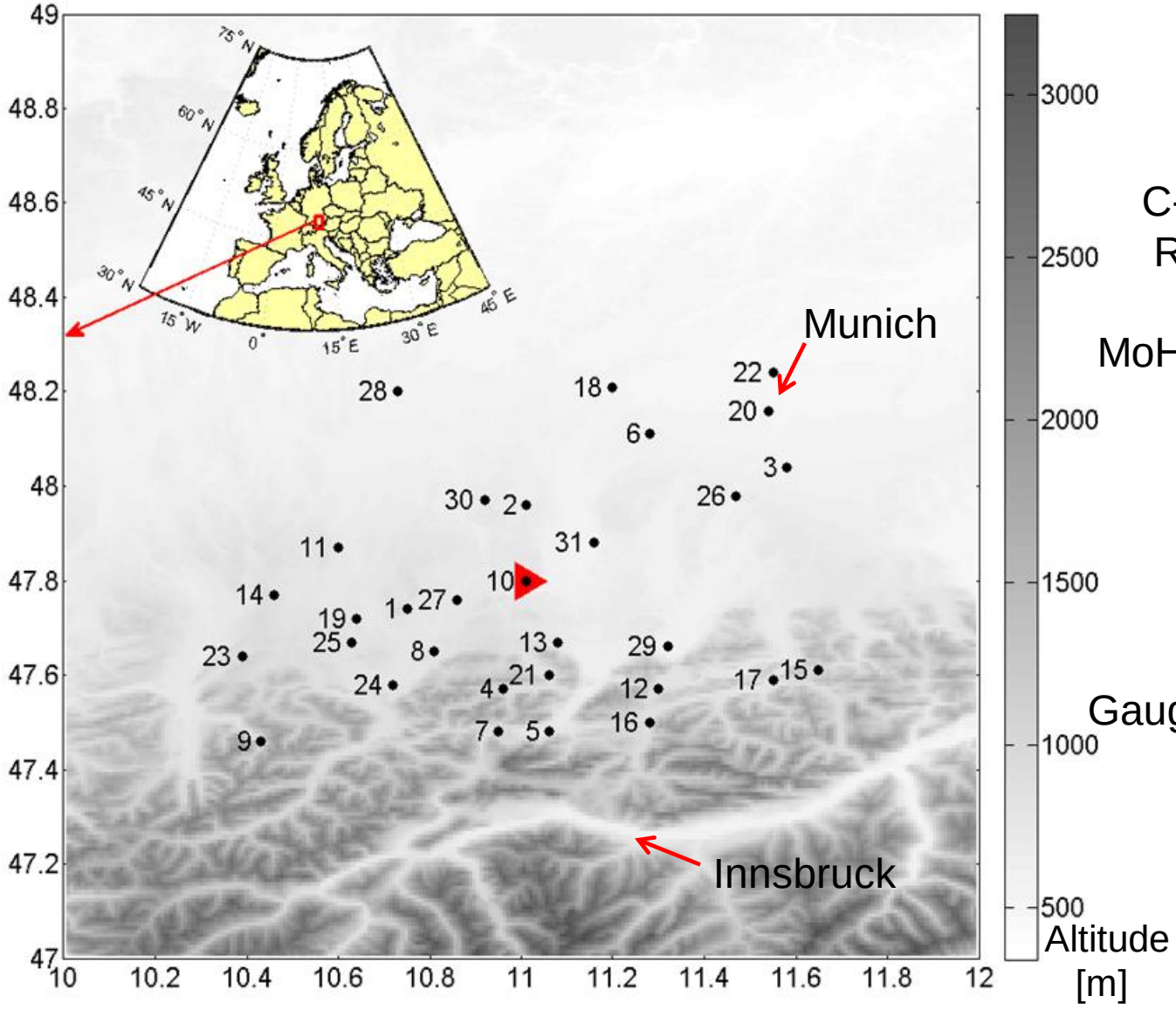
Interpolated Gauge – Radar fields

Large discrepancies between interpolation methods and spatial rainfall field!



Precipitation Sum17.-21.06.2001 [mm]

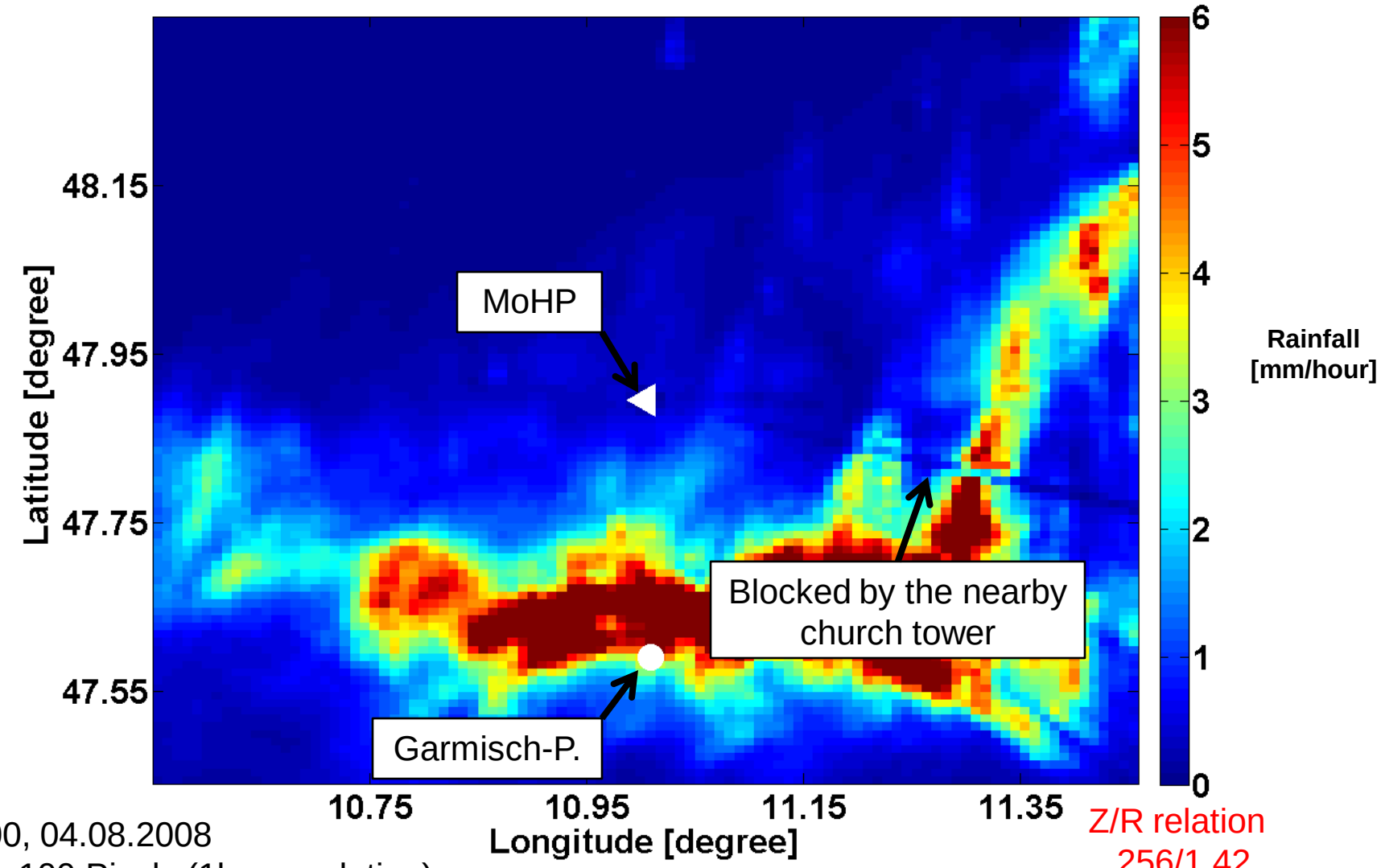
Study Area and Data Sources



C-band Radar at MoHP/DWD

31 Gauges/DWD

Example C-Band Radar Product at MoHP



13:00, 04.08.2008
100 x100 Pixels (1km resolution)

Z/R relation
256/1.42

Introduction to Copulas

Sklar (1959): For any bivariate distribution function $F_{XY}(x,y)$ with univariate marginal distribution functions $F_X(x)$ and $F_Y(y)$ there exists a copula C such that

$$F_{XY}(x,y) = C(F_X(x), F_Y(y)) \qquad x, y \in \mathbb{R} \quad u, v \in [0,1]$$
$$= C(u,v) \qquad C : [0,1]^2 \to [0,1]$$

When pdf c is known, pdf of bi/multi-variate F(x,y) is given by

$$f(x,y) = c(F_X(x), F_Y(y)) \cdot f_X(x) \cdot f_Y(y)$$

: Canonical representation of multivariate pdf

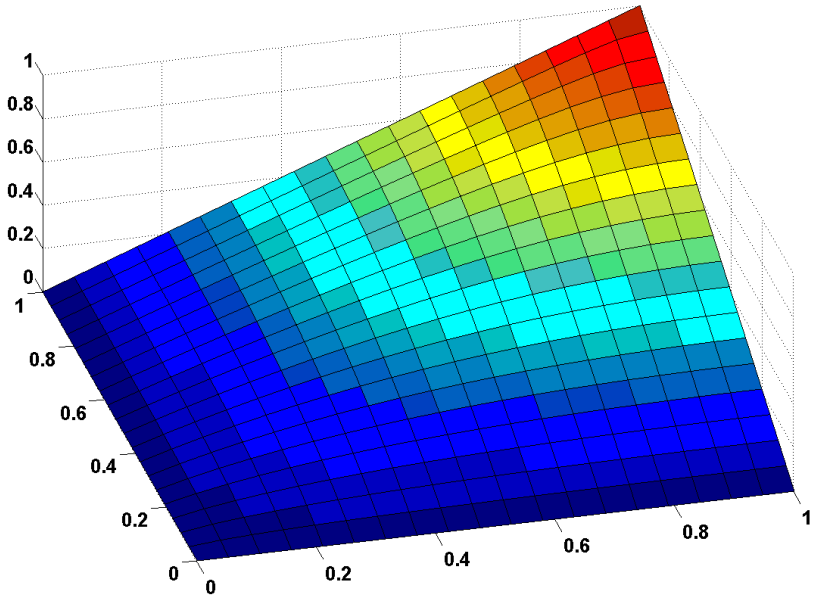
pdf of Copula (copula density)

pdfs of marginal distributions

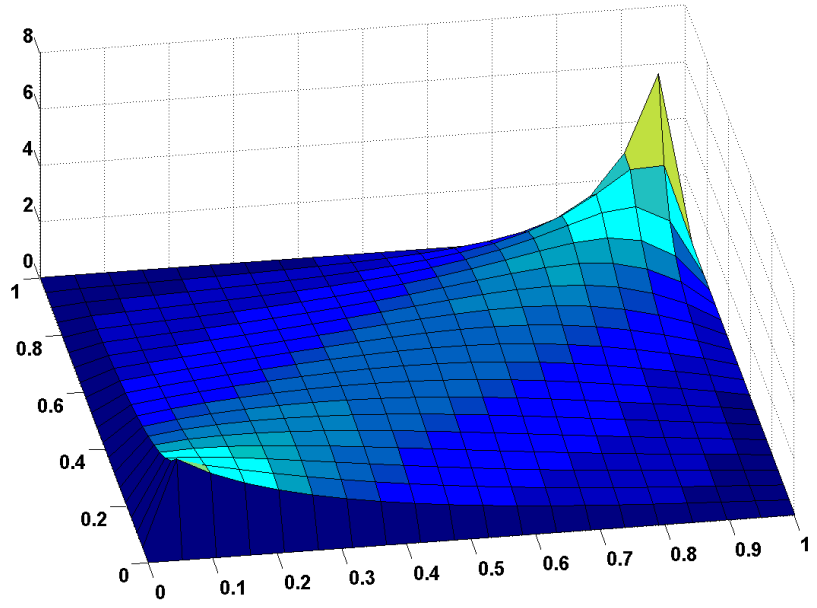
Joint density can be written as product of marginal densities and copula density ($\Rightarrow$ c is called“dependence function”)

Example Gumbel Copula

$$C_\theta(u,v) = \exp(-((-\ln u)^\theta + (-\ln v)^\theta)^{1/\theta}), \theta \geq 1$$



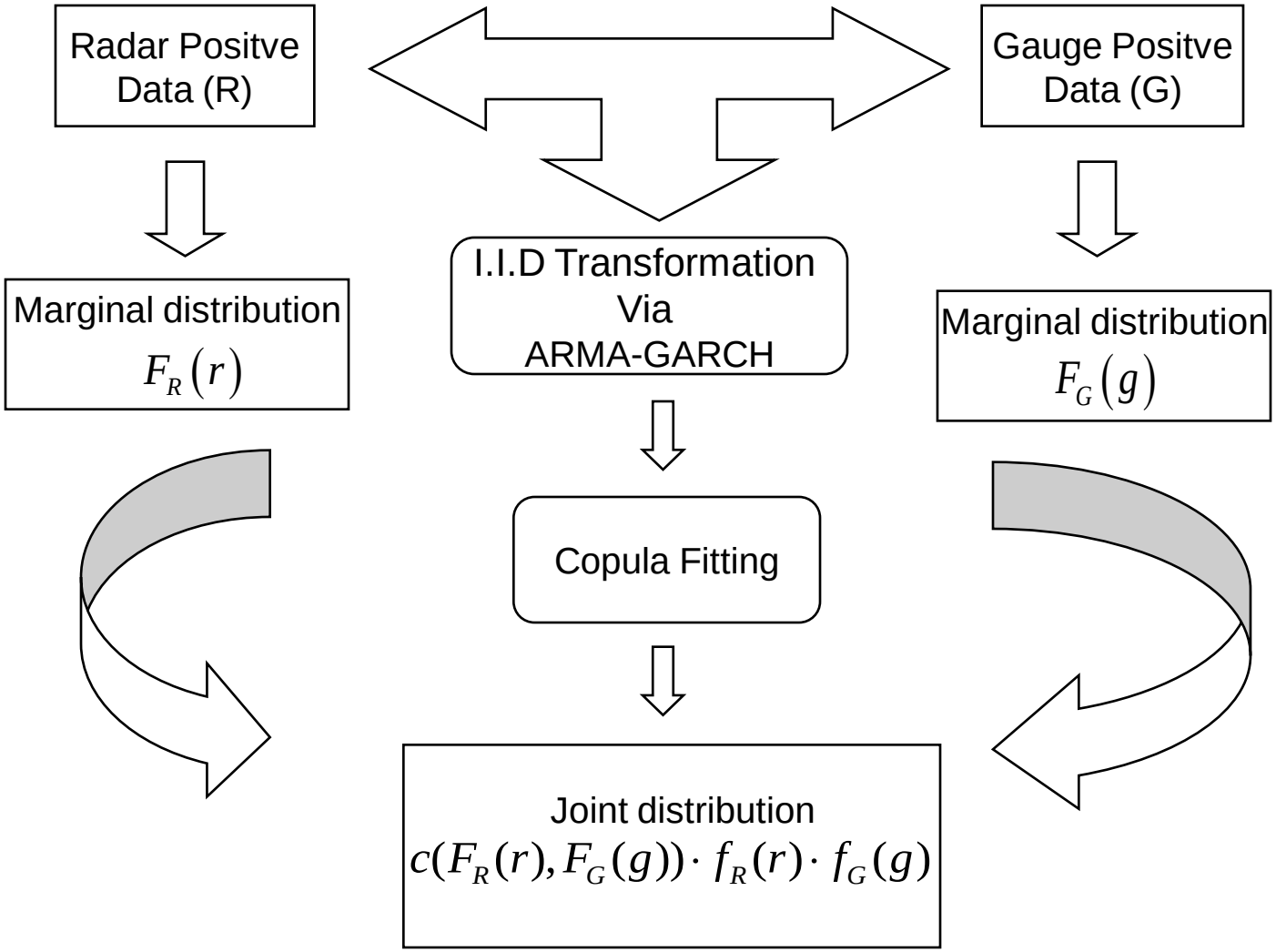
Gumbel Copula CDF C



Gumbel Copula PDF c

$$\theta = 1.9$$

Procedure of Copula-Based Analysis

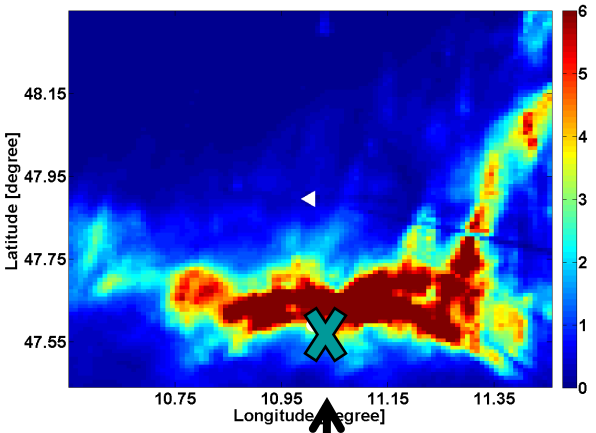


How to assimilate different data sources?

$$\theta = f(\text{distance}, \text{direction})$$

Produce Copula Parameter Map

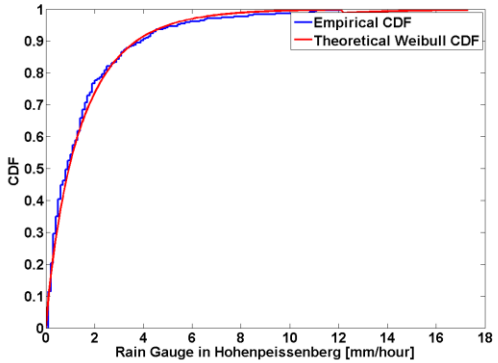
Step 1: For one specific rain gauge and the corresponding radar grid cell (positive pairs), the Gumbel Copula parameter theta can be calculated.



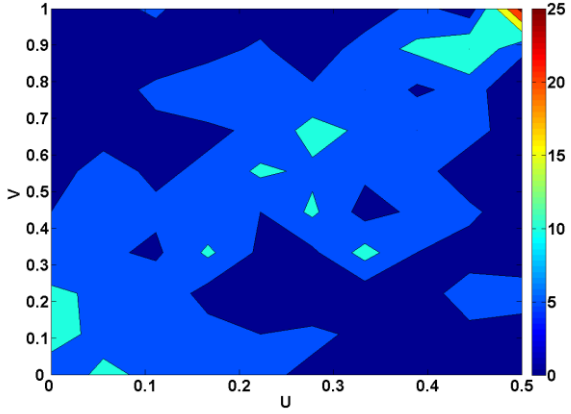
Rainfall [mm/hour]

Location:Gauge Garmisch

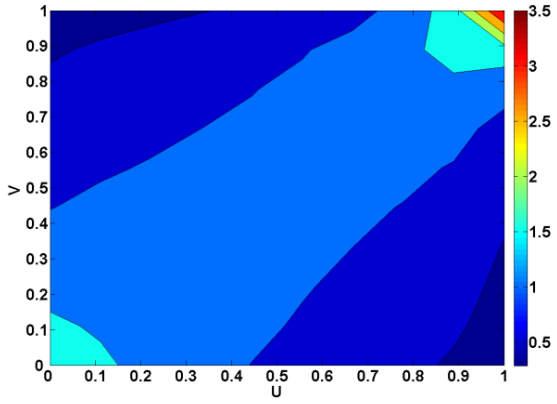
Gauge: Weibull



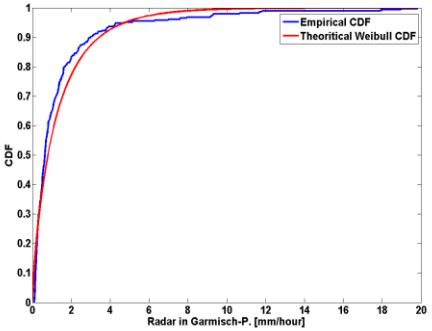
Empirical Copula



Fitted theoretical Gumbel Copula



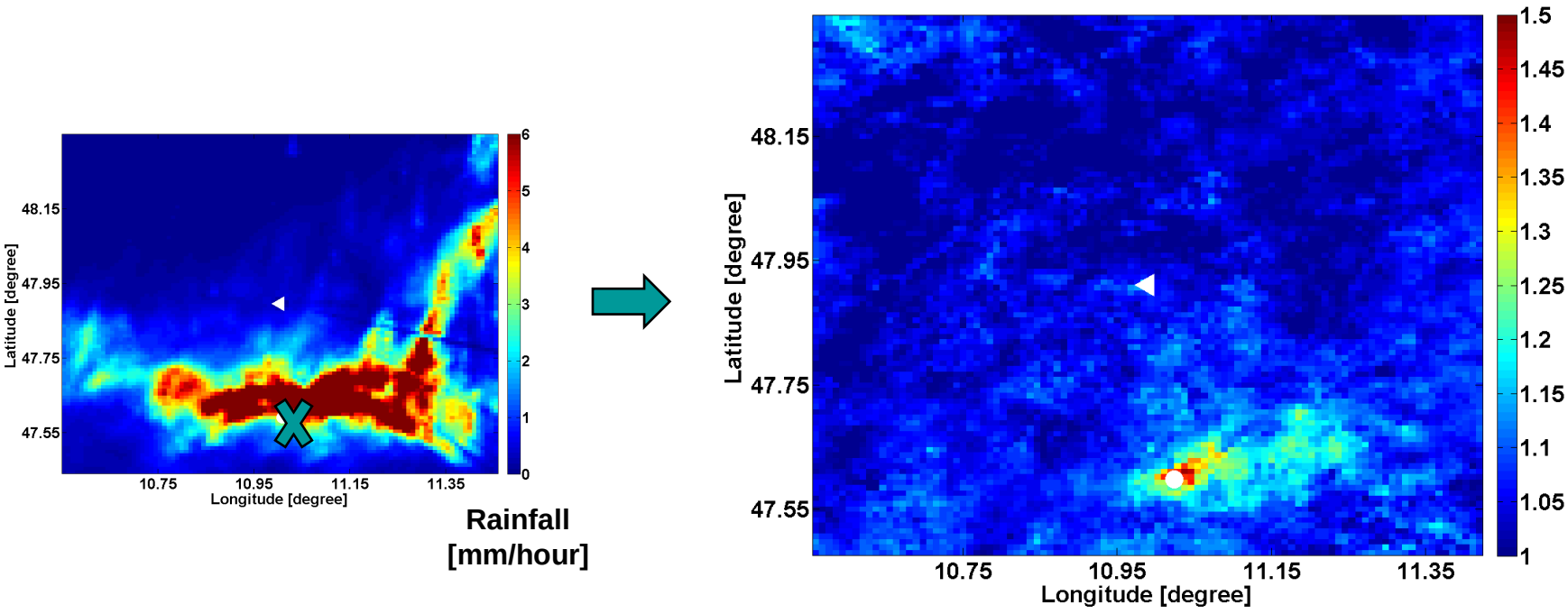
Radar: Weibull



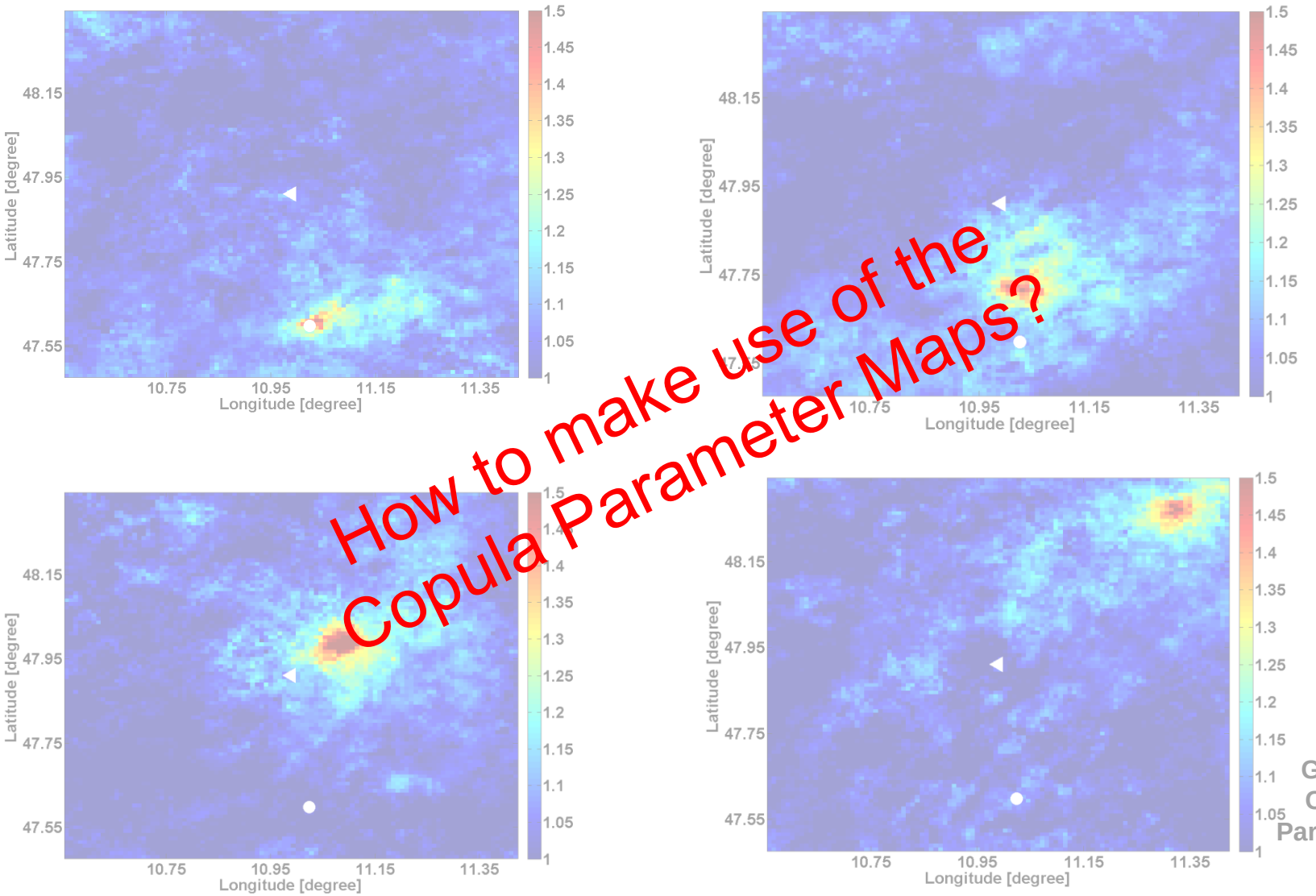
u: CDF value of radar
v: CDF value of gauge

Produce Copula Parameter Map

Step 2: For one specific rain gauge and ***all the other*** radar grid cells (positive pairs), the Gumbel Copula parameter can also be calculated to produce the Copula map from this station.

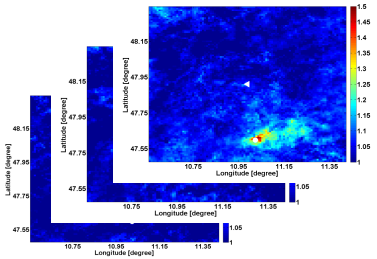


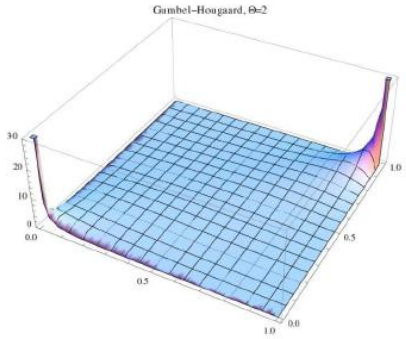
Garmisch-Partenkirchen

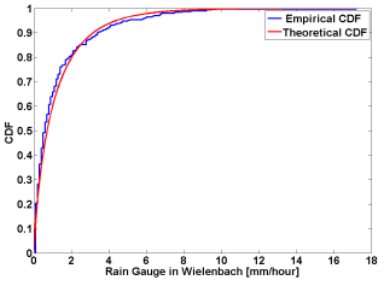

How to make use of the Copula Parameter Maps?
Latitude [degree]
48.15
47.95
47.75
47.55
Longitude [degree]
10.75
10.95
11.15
11.35
1.5
1.45
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
Gumbel Copula Parameter θ

Recipe for Maximum Theta

Copula maps







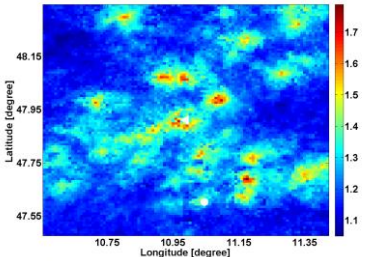
31 Theta maps from 31 gauges

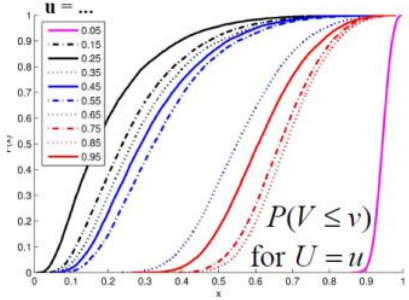
Sample of mean pseudo-observations (rank space)

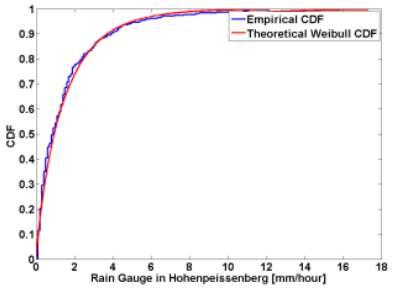
Back Transformation (data space)

Simulated pseudo-observation field of precipitation (data space)

Maximum Theta

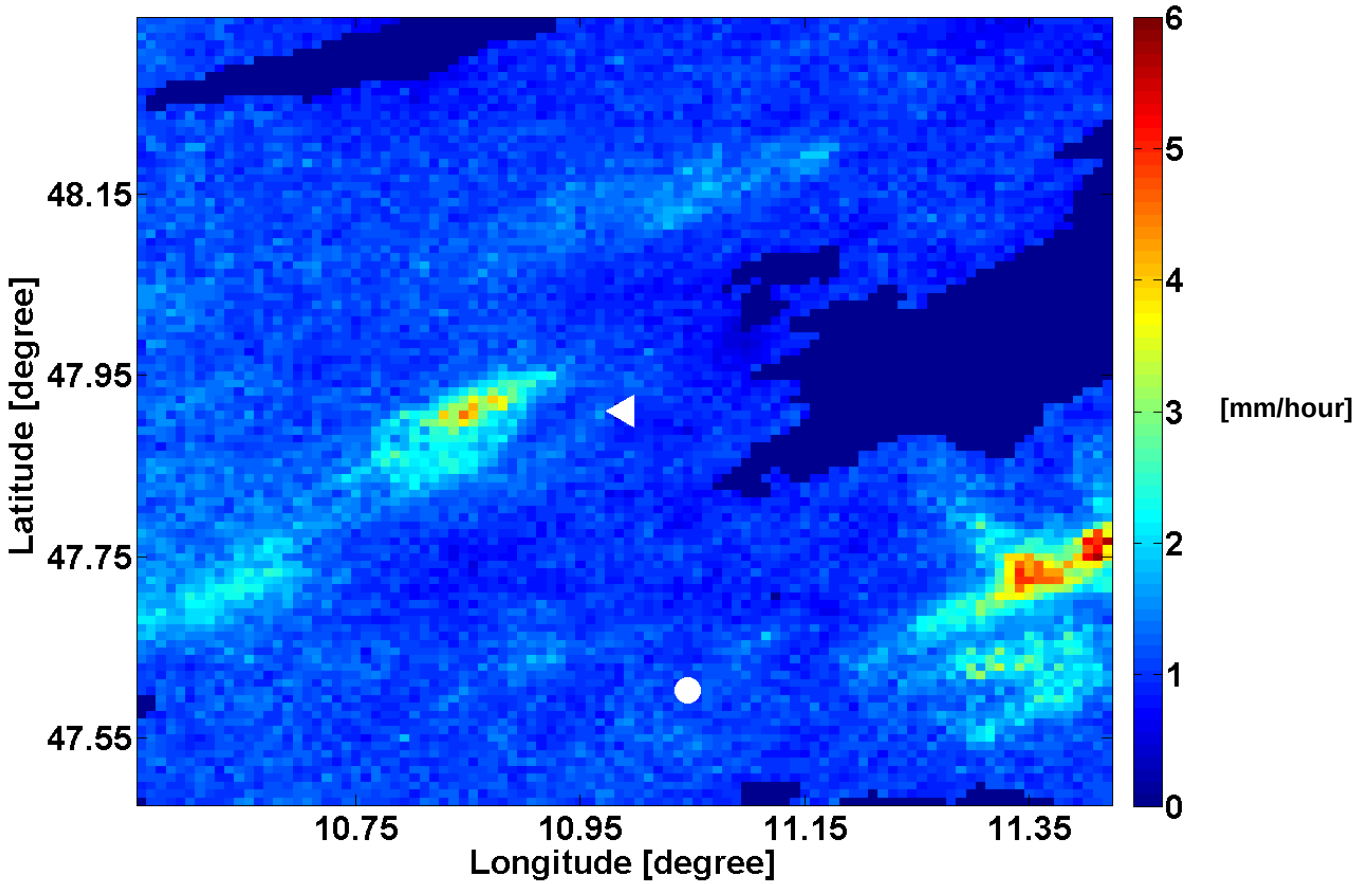






Maximum Theta➔Back Transf.

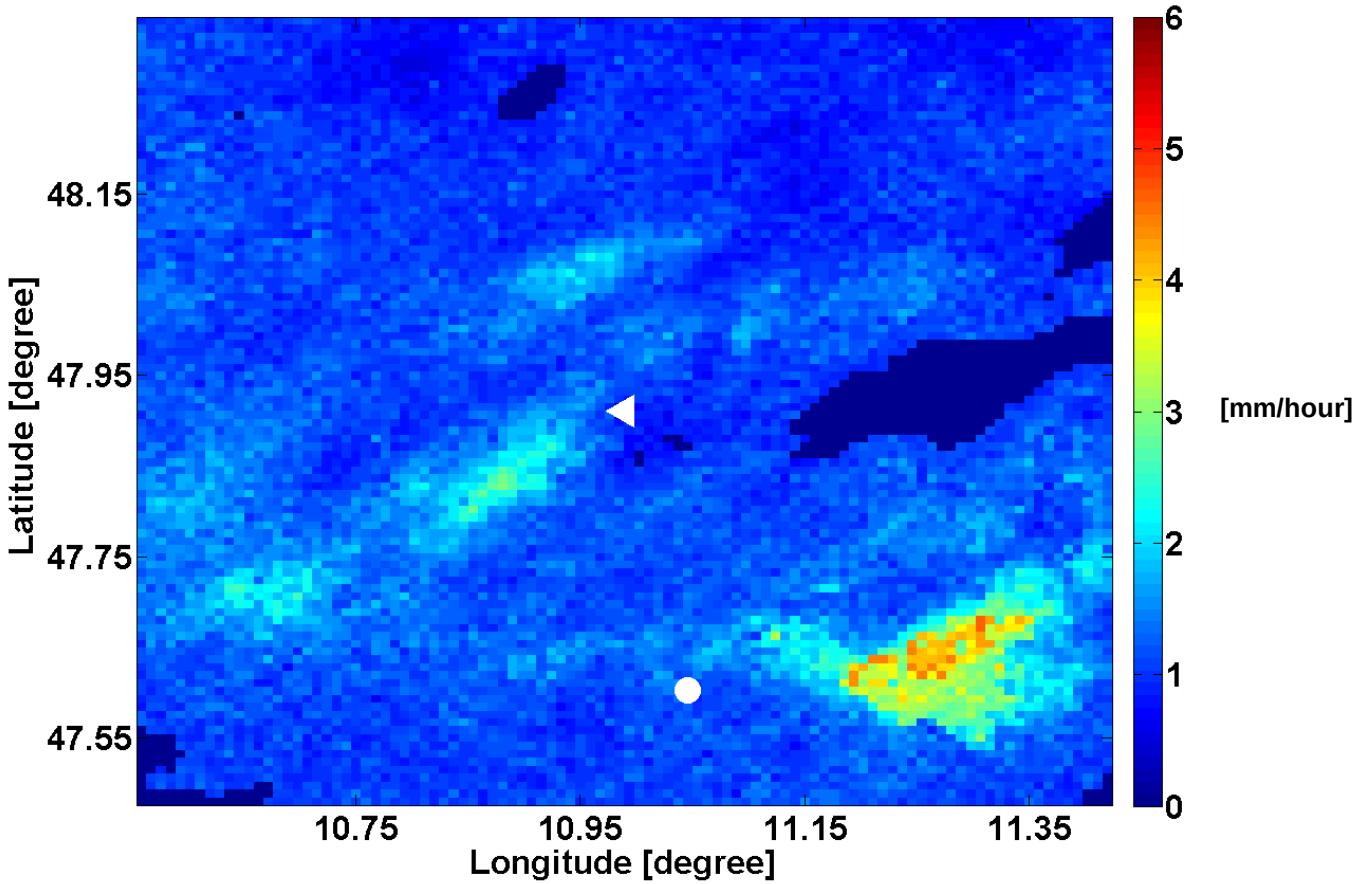
Simulated pseudo-observations



08:00, 04.08.2008

Maximum Theta➔Back Transf.

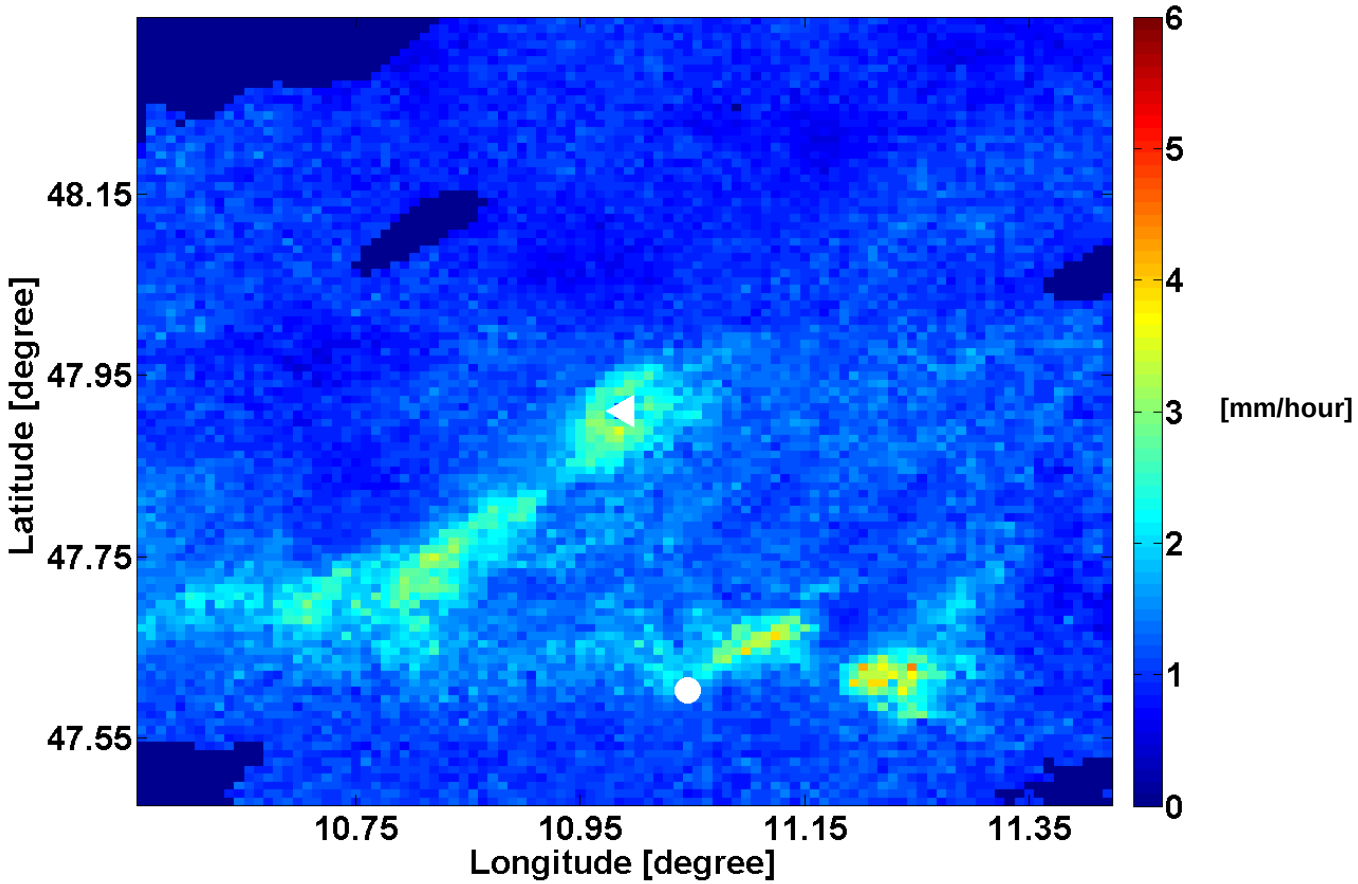
Simulated pseudo-observations



09:00, 04.08.2008

Maximum Theta➔Back Transf.

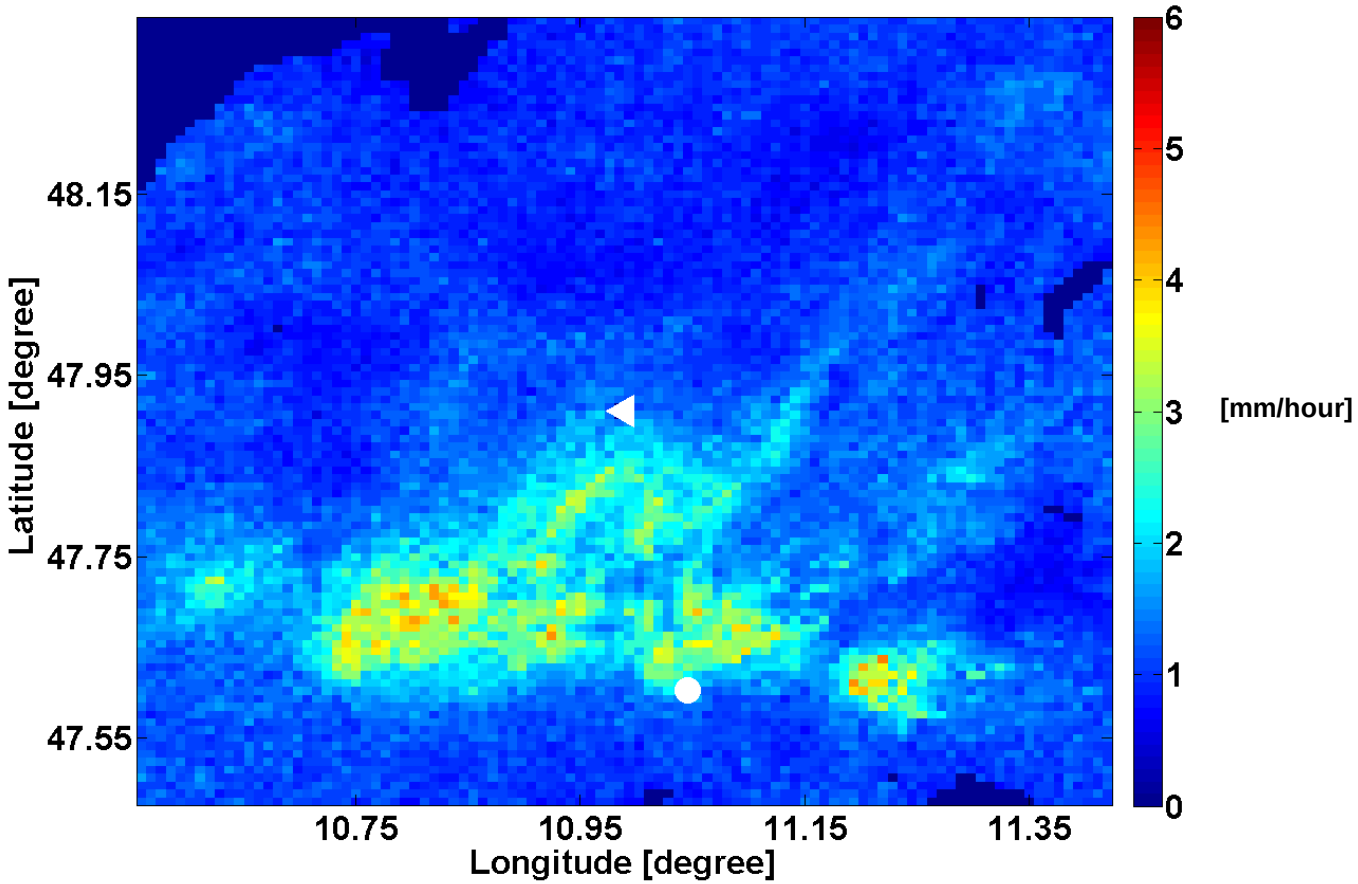
Simulated pseudo-observations



10:00, 04.08.2008

Maximum Theta➔Back Transf.

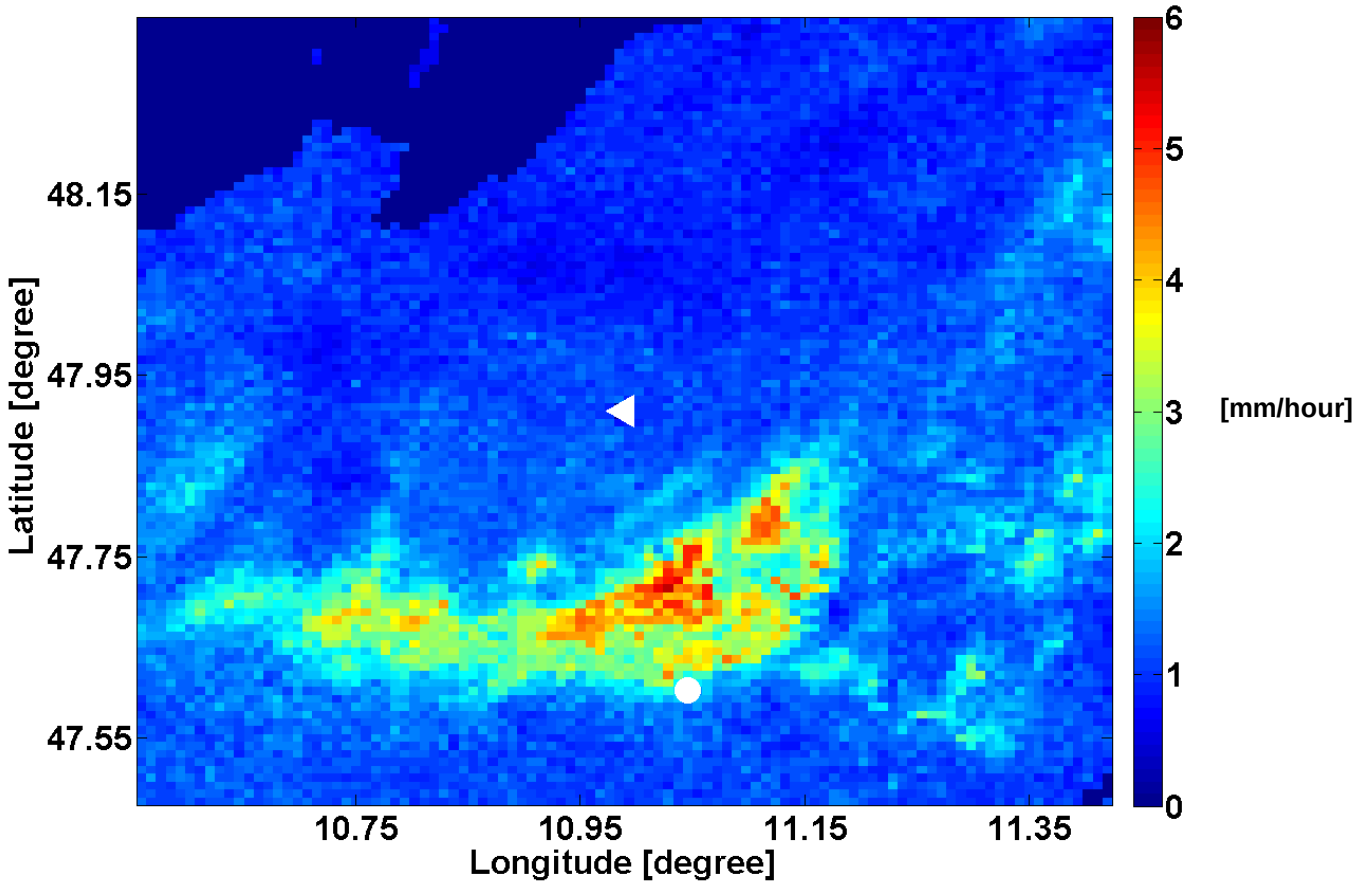
Simulated pseudo-observations



11:00, 04.08.2008

Maximum Theta➔Back Transf.

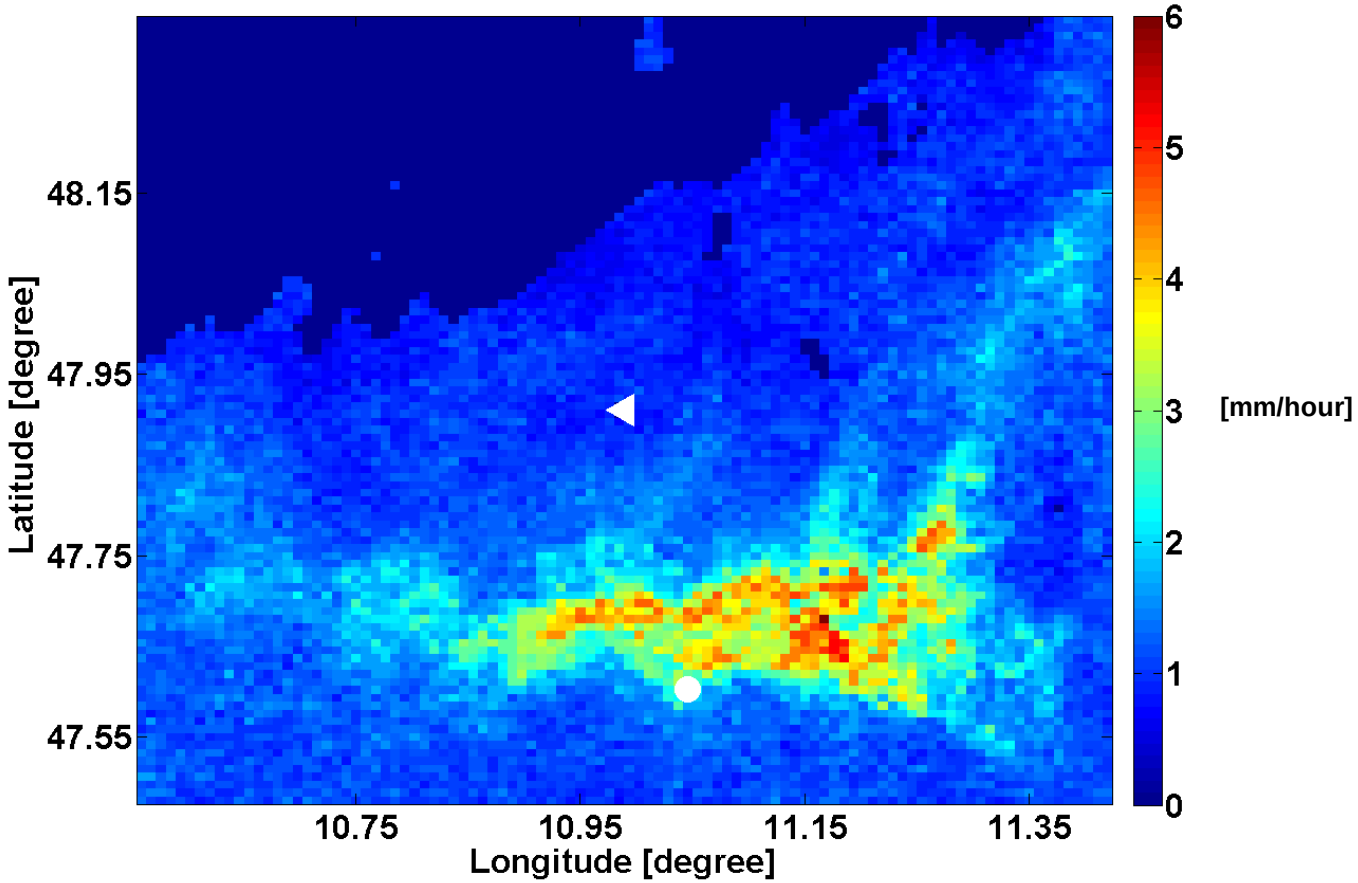
Simulated pseudo-observations



12:00, 04.08.2008

Maximum Theta➔Back Transf.

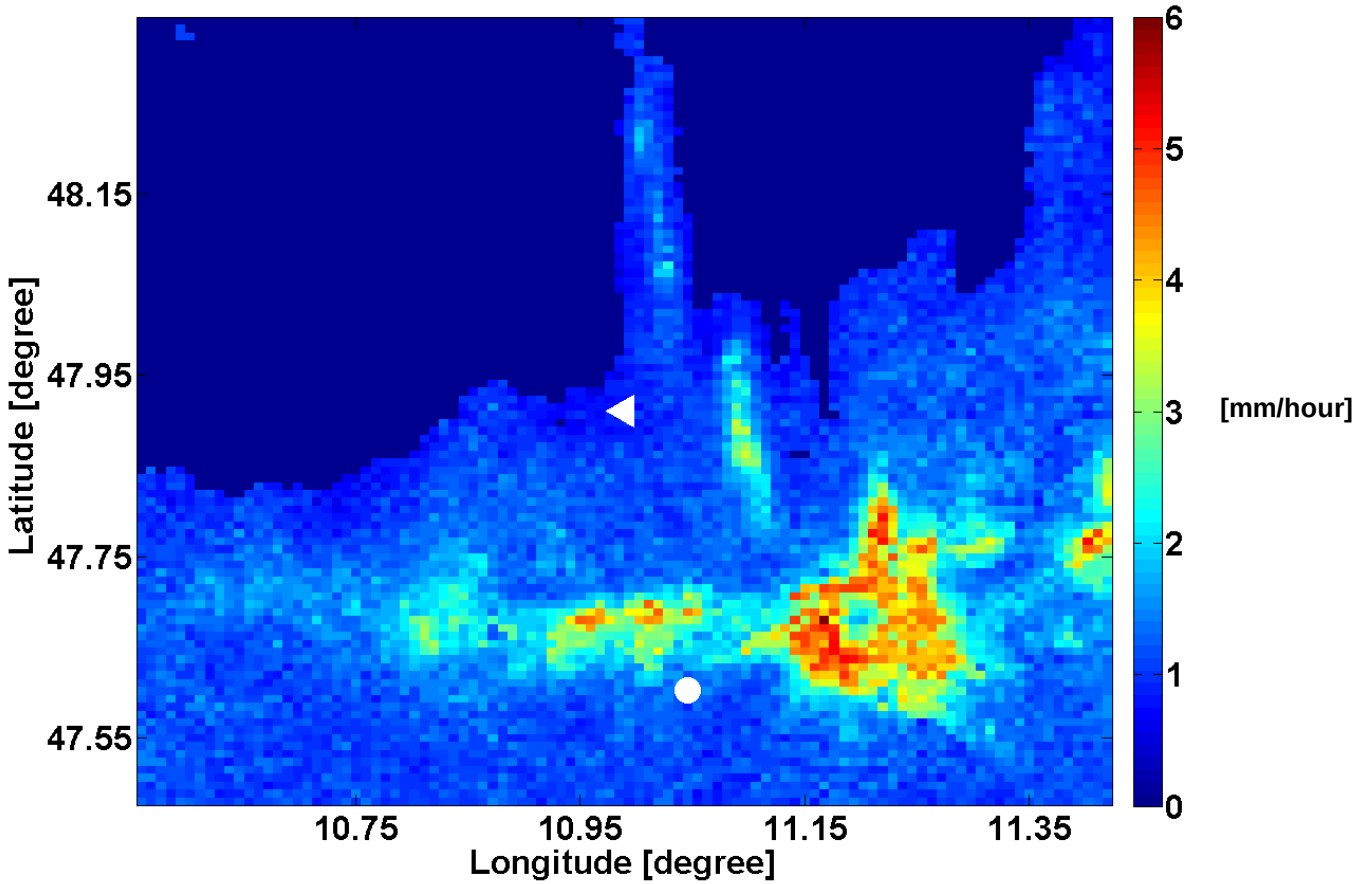
Simulated pseudo-observations



13:00, 04.08.2008

Maximum Theta➔Back Transf.

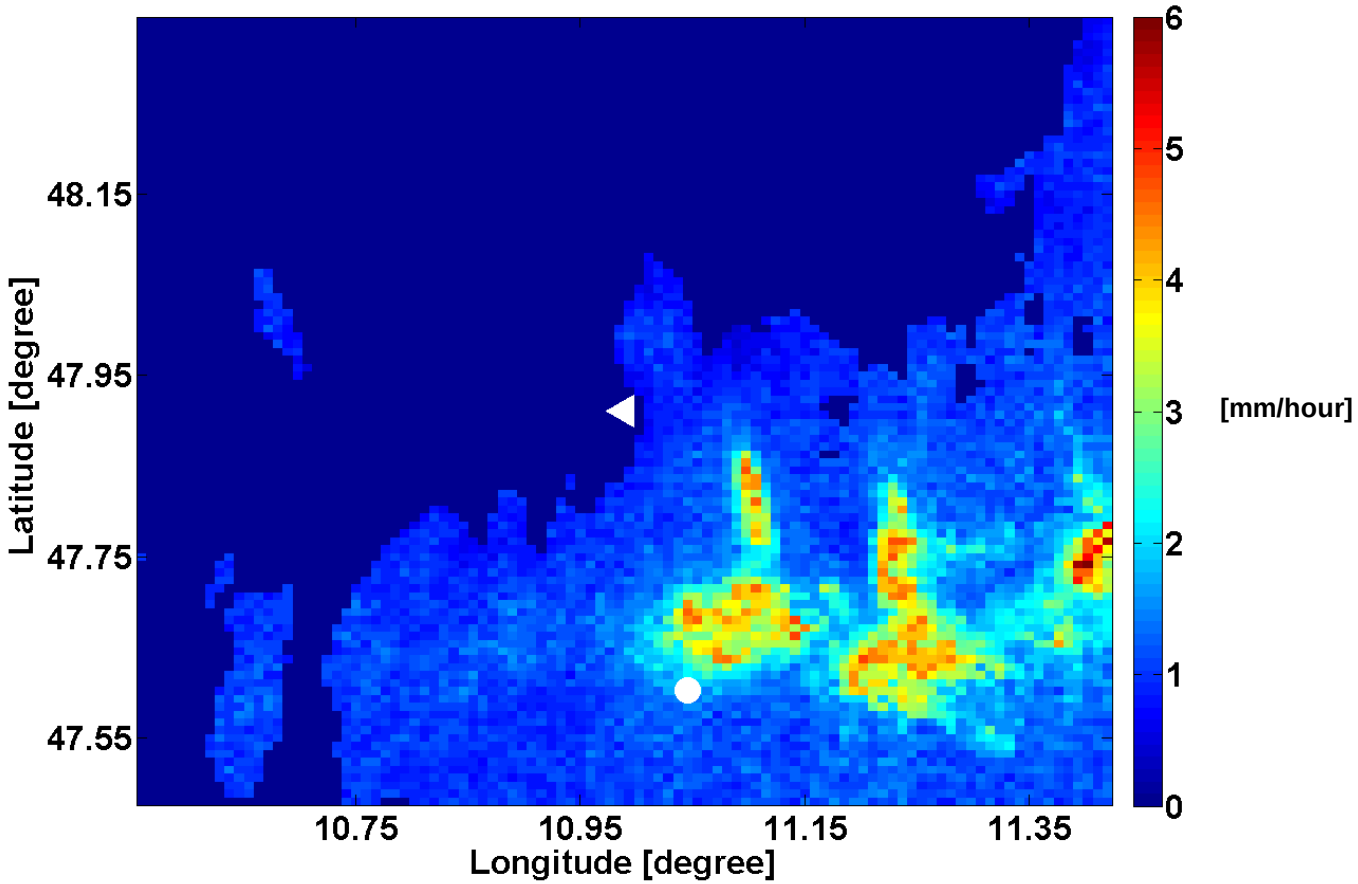
Simulated pseudo-observations



14:00, 04.08.2008

Maximum Theta➔Back Transf.

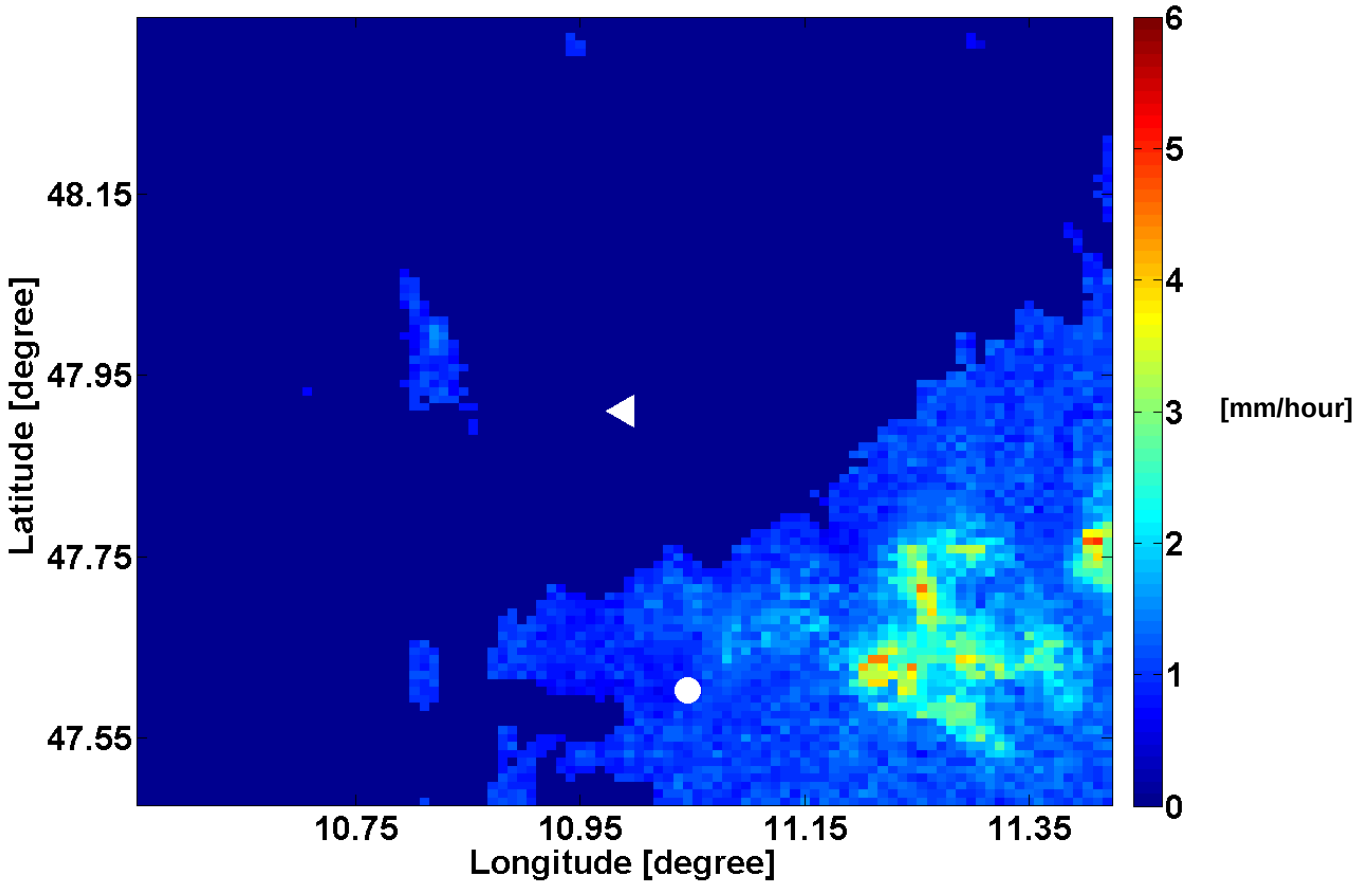
Simulated pseudo-observations



15:00, 04.08.2008

Maximum Theta➔Back Transf.

Simulated pseudo-observations



16:00, 04.08.2008

Point Wise Validation-Maximum Theta

Station Name	Pearson‘s Correlation (for positive pairs)	RMSE [mm/hour] (for positive pairs)
Garmisch-P.	0.59	1.84
Oberammergau	0.65	1.81
Wielenbach	0.53	1.69
Munich City	0.56	1.87

Analysis done for altogether 31 stations ... summer, 05 to 08, wet days only

Results and Conclusions

Copula-based approach for simulation of **precipitation pseudo-observation fields** assimilating data from radar and gauge.

Reasonable representation of precipitation: Simulated fields combine advantages of radar spatial patterns and qualified point information from gauges.

Purely statistical approach: can be used for **data sparse regions** and **complex terrain**

Results strongly affected by estimation of **marginal distributions**, here: fixed distribution used

Thank you!