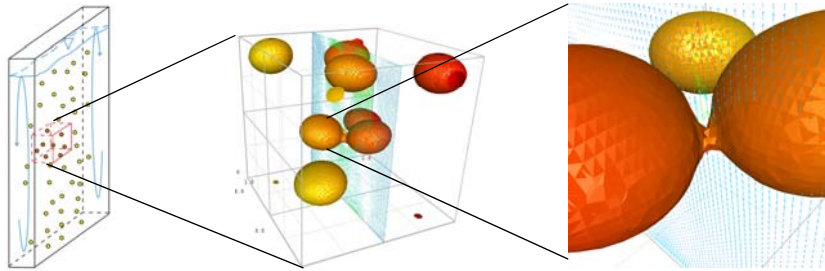


## Critical assessment of statistical turbulence models for bubble-driven flows

Martin Wörner, Milica Ilić, Sercan Erdogan  
84<sup>th</sup> Annual Meeting of the GAMM  
Novi Sad, Serbia, March 18-22, 2013

INSTITUTE OF CATALYSIS RESEARCH AND TECHNOLOGY



KIT – University of the State of Baden-Wuerttemberg and  
National Research Center of the Helmholtz Association

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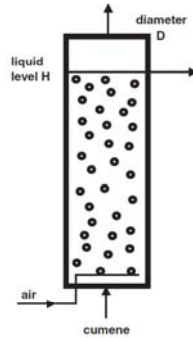
## Outline

- Introduction
- CFD methods for bubble columns
- Model limitations
  - Turbulence in bubbly flows
  - Interaction between closure relations
- Development of improved models by DNS
  - Previous results for  $k_L$  equation
  - Ongoing work within BMBF project Multiphase
- Outlook

# Introduction



- Applications/devices with bubble-driven gas-liquid flows
  - Waste water treatment
  - Nuclear power plants
  - Chemical reactors
    - Air lift reactors
    - Bubble column reactors
- Bubble column reactors
  - Volume 0.01 – 3000 m<sup>3</sup>
  - Diameter 0.2 – 20 m
  - Height to diameter ratio 3 – 10
  - High pressure, high temperature
  - Reactions of organic liquids
    - Oxidations, hydrogenations, ...
    - Example: oxidation of cumene for production of phenol (a precursor to plastics)



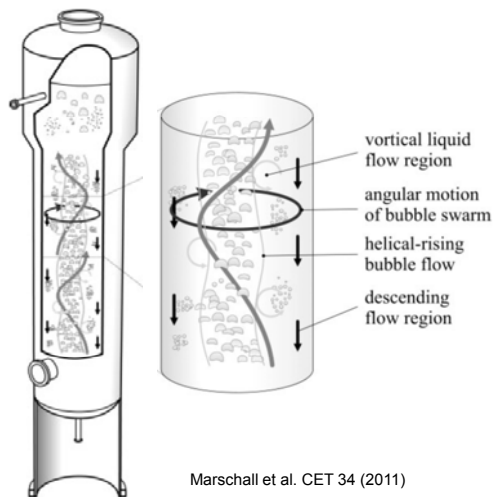
M. Weber, CET 25 (2002)



# Flow features in bubble columns



- Rising bubbles drive recirculating liquid flow (upward in center, downward near wall) and generate bubble-induced turbulence (also called pseudo-turbulence)
- Recirculating liquid flow generates shear-induced turbulence which interacts with the pseudo-turbulence
- Turbulence influences collision and breakup of bubbles which results in a spectrum of bubble sizes (heterogeneous regime)



Marschall et al. CET 34 (2011)

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## CFD methods for bubble columns



- Length scales
  - Column diameter  $d_C = 0.2 - 20$  m
  - Bubble diameter  $d_B = 1 - 10$  mm  
→  $d_C / d_B = 200 - 2000$→ *Direct numerical simulation is impossible (and also not meaningful)*
- Time scales
  - Liquid recirculation time and bubble residence time are much larger than the time scale of bubble motion  $d_b/U_B$→ *Large-eddy-simulations is impossible (a huge problem time is required to achieve reliable statistics)*
- Gas holdup up to 40% → *Euler-Lagrange approach is not meaningful*
- Typical simulation turn around time requested by industry 1 - 3 days
- Euler-Euler approach based on Reynolds-averaged Navier-Stokes equations is the only approach that can meet industrial demands
  - Reynolds stress models
  - Two-equation eddy viscosity models ( $k-\varepsilon$ ,  $k-\omega$ , SST, ...)
- *CFD is not yet used as tool for design of industrial scale bubble columns*

## Common model approaches



- Modeled  $k_L$  equation is solved (with or without two-phase specific interfacial term) while turbulence in gas phase is neglected
- Both, the modeled  $k_L$  and  $k_G$  equations are solved (with or without two-phase specific interfacial term)
- The modeled  $k_L$  equation is solved without interfacial term and the bubble-induced turbulence is taken into account by an extra contribution to the eddy viscosity (e.g. by the model of Sato)
- It is common practice to adopt single phase closure laws with single phase coefficient sets (e.g. standard, RNG, realizable  $k$ - $\epsilon$ , low  $Re$ , ...)
  - Motivation: in limit  $\alpha_G \rightarrow 0$  the model must reduce to single phase version
  - However, model coefficients are not universal even for single phase flow
- Status of model development
  - Computations with model variants are compared with experimental data (mean flow is often well described, but not turbulence quantities)
  - Useful to identify which models performs best for a certain experiment but hardly useful for development of physically sound improved closure relations

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# Turbulence in bubbly flows



## Pool of stagnant liquid

- Rising bubbled induce pseudo-turbulence (displacement of liquid, bubble wakes)
- Non-Gaussian probability density function
- Spectrum follows a power law with slope -3 (this is attributed to bubble wakes)

## Turbulent pipe flow

- Wall or core peaking  $\Rightarrow$  modulation of mean liquid velocity by bubbles
- Bubbles can enhance or attenuate liquid turbulence as compared to single phase flow with same liquid flow rate
- Spectrum exhibits different ranges (-5/3, -3)

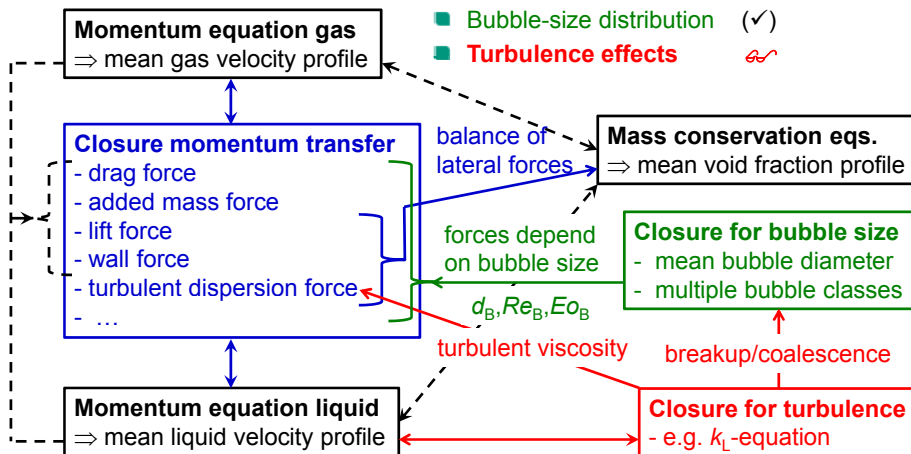
- Neither pure pseudo-turbulence (bubble-induced turbulence) is fully understood nor how it modifies shear turbulence
- In bubble columns, there is an inherent non-linear interaction between bubble-induced and shear-induced turbulence

# Interaction of closure relations



## Closure relations in two-fluid model

- Interfacial transfer terms ✓
- Bubble-size distribution (✓)
- Turbulence effects  $\omega$



## Exact analytical $k_L$ equation



$$k_L = \frac{1}{2} \overline{\mathbf{u}_L \cdot \mathbf{u}_L}$$

$$\frac{\partial}{\partial t} (\alpha_L k_L) + \nabla \cdot (\alpha_L k_L \overline{\mathbf{u}_L}) = \underbrace{\frac{1}{Re_{ref}} \nabla \cdot (\alpha_L \overline{\boldsymbol{\tau}_L \cdot \mathbf{u}_L}) - \nabla \cdot \left[ \alpha_L \left( \overline{p_L \mathbf{u}_L} + \frac{1}{2} \overline{\mathbf{u}_L^2 \mathbf{u}_L} \right) \right]}_{\text{DIFFUSION}} + \underbrace{-\alpha_L \overline{\mathbf{u}_L \mathbf{u}_L} : \nabla \overline{\mathbf{u}_L}}_{\text{PRODUCTION BY SHEAR}} - \underbrace{\frac{1}{Re_{ref}} \alpha_L \overline{\boldsymbol{\tau}_L} : \nabla \overline{\mathbf{u}_L}}_{\text{DISSIPATION}} + \underbrace{\left[ \frac{1}{Re_{ref}} \overline{\boldsymbol{\tau}_L} - p_L \mathbb{I} \right] \cdot \overline{\mathbf{u}_L} \cdot \hat{\mathbf{n}}_{L,i} a_i}_{\text{PRODUCTION BY INTERFACIAL TERM}}$$

Kataoka & Serizawa (1989)

- All terms on the right hand side must be modeled
- All terms involve correlations between various fluctuating quantities or their gradients which can hardly be measured in non-dilute bubbly flow
- Here: use DNS to obtain insight in budget of  $k_L$  and perform a-priori tests of performance of models for individual closure terms

## Outline

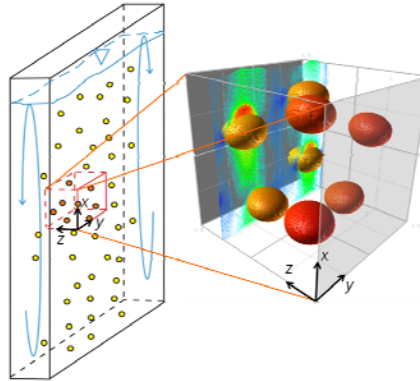


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## Set-up in our DNS studies



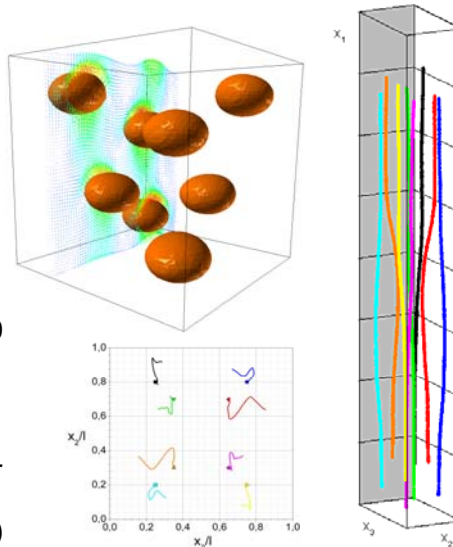
- Considering walls is essential
  - In triple-periodic domains the liquid recirculation typical for bubble columns is absent and production of  $k_L$  and dissipation are in local equilibrium
  - In wall-bounded flows there is no local equilibrium but a redistribution of  $k_L$  by diffusion
- Computational domain
  - Part of a flat bubble column
  - Two lateral side walls and periodic boundary conditions in vertical and transverse direction
- Computer code (in-house)
  - Incompressible Navier-Stokes eqs. in single-field formulation
  - Volume-of-fluid method with piecewise linear interface reconstruction



## Simulation results PhD thesis M. Ilić (2006)



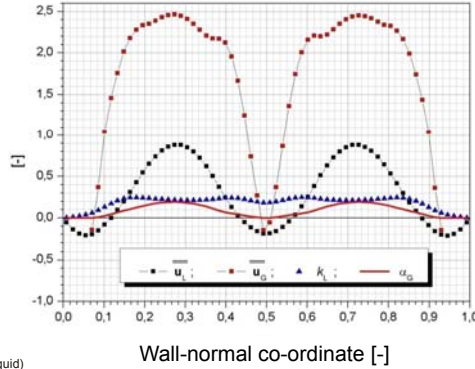
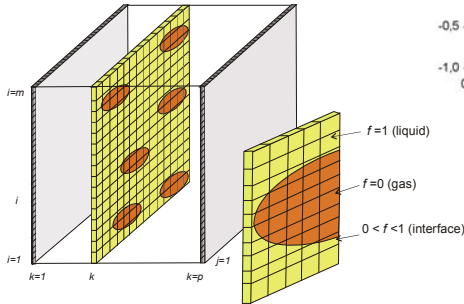
- Cubic domain,  $d_B / d_{wall} = 1/4$
- $\rho_G / \rho_L = 1/2, \mu_G / \mu_L = 1$
- 1 - 8 bubbles
- Gas content 0.8 - 6.5 %
- Eötvös number = 3.065
- Three different values of the Morton number
  - $M = 3 \times 10^{-2}, 3 \times 10^{-4}, 3 \times 10^{-6}$
- Bubble Reynolds no.  $Re_B < 100$
- Here: results for  $M = 3 \times 10^{-6}$  with 8 bubbles ( $\alpha_G = 6.5\%$ )
- Study is now extended to smaller values of  $M$  and larger number of bubbles (BMBF project *Multiphase*, see below)



# Averaging of simulation results



- Averaging over planes parallel to the side walls (statistically homogeneous)
- Additional averaging over different instants in time
- ⇒ wall normal profiles



Phase-average      Phase-fluctuation

$$\bar{A}_{L:k} = \frac{\sum_{i,j} f_{i,j,k} A_{L:i,j,k}}{\sum_{i,j} f_{i,j,k}} \quad A'_{L:i,j,k} = A_{L:i,j,k} - \bar{A}_{L:k}$$

# Budget of exact $k_L$ -equation



$$\frac{\partial}{\partial t} (\alpha_L k_L) + \nabla \cdot (\alpha_L k_L \bar{\mathbf{u}}_L) =$$

$$\frac{1}{Re_{ref}} \nabla \cdot (\alpha_L \overline{\mathbb{T}}_L \cdot \bar{\mathbf{u}}_L) - \nabla \cdot \left[ \alpha_L \left( \overline{p_L \mathbf{u}_L} + \frac{1}{2} \overline{(\mathbf{u}_L \cdot \mathbf{u}_L) \mathbf{u}_L} \right) \right]$$

DIFFUSION

$$-\alpha_L \overline{\mathbf{u}_L \mathbf{u}_L} : \nabla \bar{\mathbf{u}}_L$$

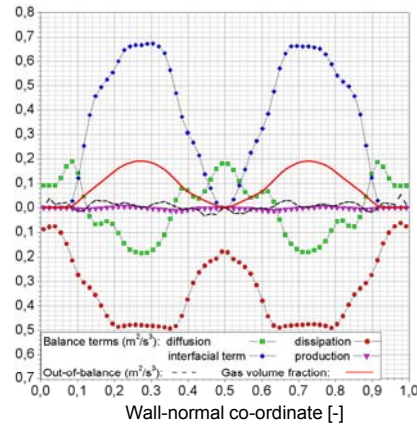
PRODUCTION

$$-\frac{1}{Re_{ref}} \alpha_L \overline{\mathbb{T}}_L : \nabla \bar{\mathbf{u}}_L$$

DISSIPATION

$$+ \left[ \frac{1}{Re_{ref}} \overline{\mathbb{T}}_{L:in} - \overline{p_{L:in} \mathbb{I}} \right] \cdot \bar{\mathbf{u}}_{L:in} \cdot \bar{\mathbf{n}}_{L:in} a_{in}$$

INTERFACIAL TERM



- Interfacial term is main source (production by shear is negligible here)
- No local equilibrium between production and dissipation
- Redistribution from core to wall by diffusion (pressure term > triple correl.)



# A priori test of closure assumptions



- For each closure term, the profile predicted by different models is compared with the exact profile of the closure term as evaluated from the DNS data
- Main findings of the evaluation of model assumptions by Ilić
  - Interfacial term:** modeling as work of drag force together with Tomiyama correlation for  $C_D$  shows good performance for all Morton numbers

# Models for interfacial term

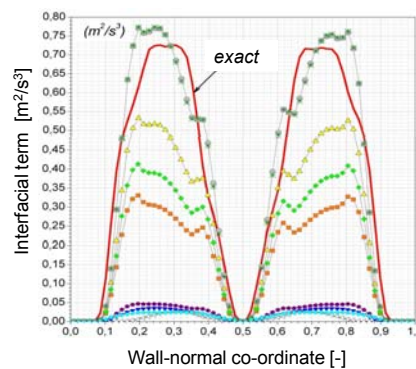


**exact:**  $-\rho_{L,ij} u_{L,ij} n_{L,i} \bar{\theta}_{in} + \tau_{L,ij} u_{L,ij} n_{L,i} \bar{\theta}_{in}$

CLOSURE ASSUMPTIONS		
DRAG CONTRIBUTION Defined in the form of:	Other contributions	Model of:
<b>Mean quantities:</b>		
<b>As power of drag force</b>	$M_{vm} \bar{u}_t$	Morel ●
$W_D = 0.75 C_D \alpha_G \rho_L \bar{u}_t^3 / d_B$	none	Troshko&Hassan ✱
<b>As part of power of drag force</b>		
$0.05 \alpha_G W_D$	none	Boisson et al. ●
$0.75 W_D$	none	Olmos et al. ▲
$1.44 W_D$	none	Pfleger et al. ■
$0.075 W_D$	$\alpha_G \rho_L k_L^{2/3} / d_B$	Kataoka et al. ◁
<b>Drag force not explicitly included:</b>		
$0.25 \alpha_G \rho_L [1 + C_D^{4/3}] \bar{u}_t^3 / d_B$	none	Lahey et al. ◆
<b>Mean and turbulent quantities:</b>		
<b>Only liquid turbulence properties</b>	$2.53 \alpha_G \alpha_L \Pi$	Sheng et al. ▼
$0.45 C_D \alpha_G \rho_L k_L / d_B$		
<b>Turbulence properties of both phases</b>		
$\frac{3}{4} C_D \frac{\bar{u}_t^3}{d_B} \left[ 2 \alpha_G \rho_L (C_i - 1) k_L - \frac{v^k \bar{u}_t \alpha_G}{\alpha_L \alpha_G} \right]$	none	Hill et al. ●

$C_D$  = drag coefficient (different formulations used)

$$C_i = u'_G u'_G / u'_L u'_L \propto f(l_e, Re_i, \alpha_G, d_B, \bar{u}_t, C_D)$$



Modeling of interfacial term as power of drag force gives good results in combination with  $C_D$  model of Tomiyama

$$C_D = \max \left[ \min \left[ \frac{16}{Re_B} (1 + 0.15 Re_B^{0.687}), \frac{48}{Re_B} \right], \frac{8}{3} \frac{E \bar{\theta}_B}{E \bar{\theta}_B + 4} \right]$$

## A priori test of closure assumptions



- For each closure term, the profile predicted by different models is compared with the exact profile of the closure term as evaluated from the DNS data
- Main findings of the evaluation of model assumptions by Ilić
  - Interfacial term: modeling as work of drag force together with Tomiyama correlation for  $C_D$  shows good performance for all Morton numbers
  - Production term and diffusion term: poor performance of standard single-phase type models (shear production term is strongly overestimated, diffusion term is strongly underestimated)
- The impact of any potential model improvement derived from DNS data is hard to assess in engineering CFD computations where the results are influenced by the non-linear interaction between models for bubble forces, bubble size distribution and turbulence. Development of improved models for BIT is an iterative process and requires detailed experimental data for various scale bubble columns for validation.  
⇒ **BMBF project Multi-Phase**

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## BMBF project Multiphase



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- **Multiscale modeling of multiphase reactors**  
(coordinated by Dr. M. Becker, Evonik industries)



- One of the main goals of the project: Development of reliable multi-scale models which allow the numerical investigation and optimization of industrial scale multiphase reactors

## BMBF project Multiphase

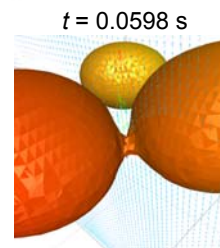
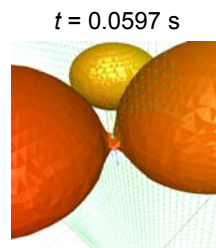
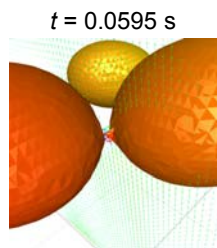
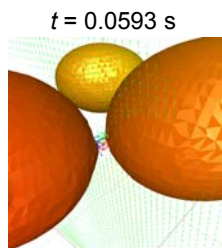
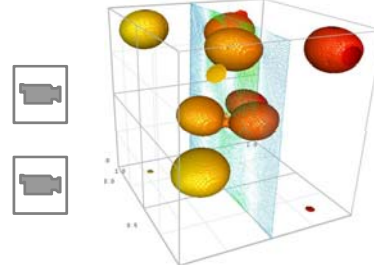
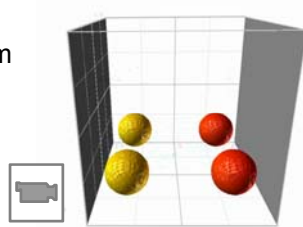


- Contribution of KIT (subcontractor of TU Hamburg Harburg):  
Development of improved turbulence models for bubbly flows using direct numerical simulations
  - A-priori testing of existing models and proposal of model improvements
  - A-posteriori testing by Euler-Euler simulations with OpenFOAM
  - Validation by experimental data for lab scale, pilot scale and industrial scale bubble columns
- Focus is on mono-disperse bubbly flows in organic liquids, Morton number  $M$  in range  $10^{-7} - 10^{-10}$
- Status of PhD project of S. Erdogan (started in 2/2012)
  - $d_B / d_{wall} = 1/4 - 1/6$ , Eötvös number in range 0.25 - 2.5
  - Grid independent results when bubble diameter is resolved by 20 cells
  - $\rho_G / \rho_L = 1/25$  gives results independent on  $\rho_G$
  - Comparison with single bubble experiments of TUHH is underway
  - Problem: undesired coalescence in bubble swarm simulations

## Coalescence: physical or not?



- $M = 10^{-8}$
- $d_B = 1\text{mm}$



## DNS methods and coalescence



- Front tracking methods (where the bubble is represented by a set of Lagrangian marker particles) suppress any coalescence unless a special merge condition is implemented
- Level-set methods and volume-of-fluid methods tend to merge bubbles automatically (and possibly unphysical) until a special prevention algorithm is implemented (e.g. representing each bubble by a own volume-fraction field)
- $\Rightarrow$  all current methods are not predictive regarding coalescence
- To judge whether coalescence in numerical simulations are physical or not, well designed experiments for bubbles in pure systems would be useful which indicate the maximum bubble diameter where no coalescence occurs

## Conclusions



- Flows in bubble columns are characterized by an inherent non-linear interaction between bubble-induced and shear-induced turbulence which is physically not fully understood
- One of the weakest points in Euler-Euler CFD computations of bubbly flows concerns adequate closures for turbulence
- Adapted single-phase two-equations models can provide reasonable results for the mean flow but not for turbulence quantities
  - Turbulence quantities are essential for predicting bubble size distribution
- DNS for insight (e.g. in the budget of  $k_L$ ) and a-priori testing of closures
  - Modeling of interfacial term as work of drag force ✓
  - Closures for shear production and diffusive transport fail
  - Problem of reliably handling coalescence phenomena
- Combined theoretical, experimental and numerical efforts by the community are required to develop physically sound and general turbulence models for bubble-driven flows

## Acknowledgement



- Dr. M. Ilić (from Serbia)
- M.Sc. S. Erdogan

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*Thank you four your attention*