

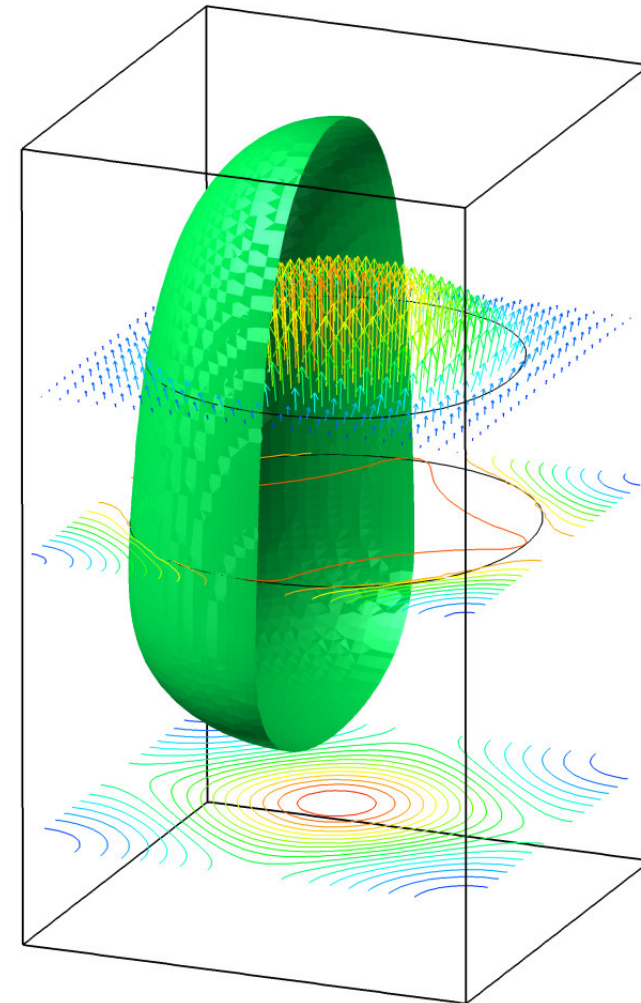
# Mehrphasenströmungen und Transportvorgänge in der Mikroverfahrenstechnik

- *simulieren*
- *verstehen*
- *modellieren*

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Seminarvortrag am IMM, 9. Juli 2013

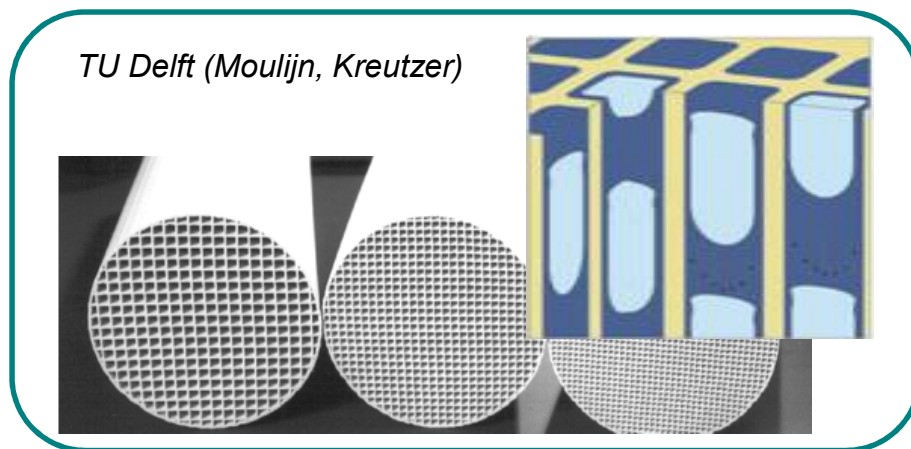
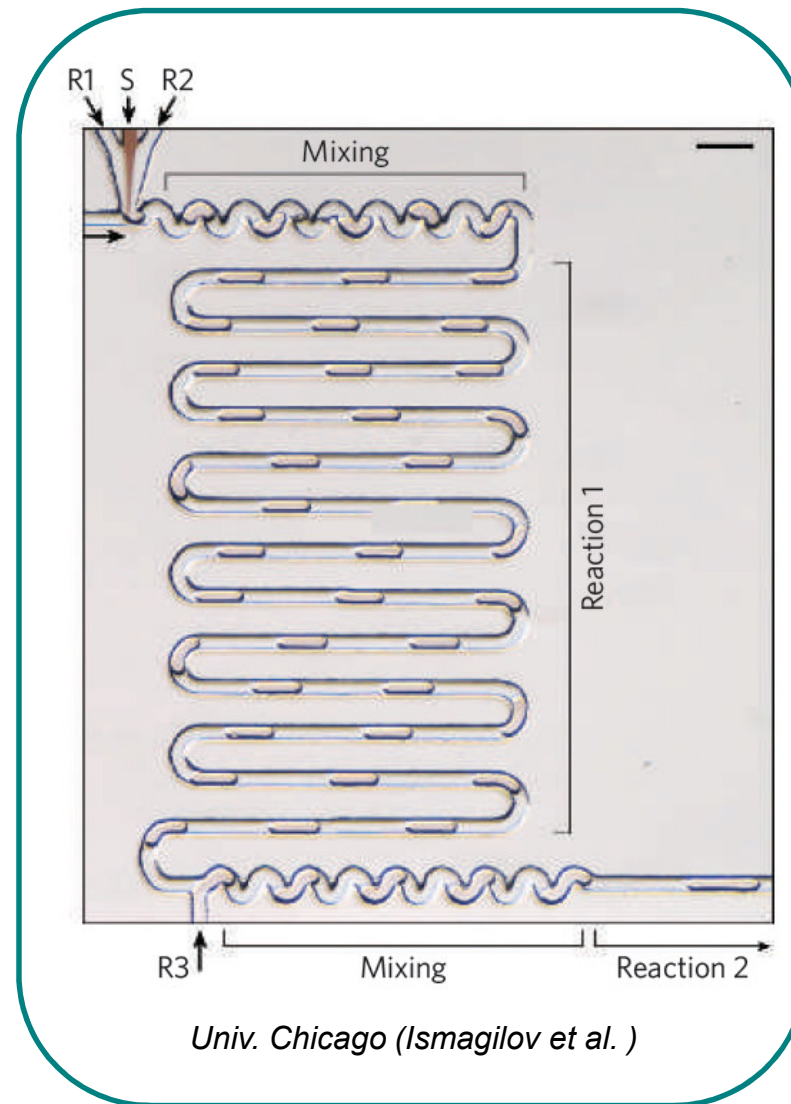
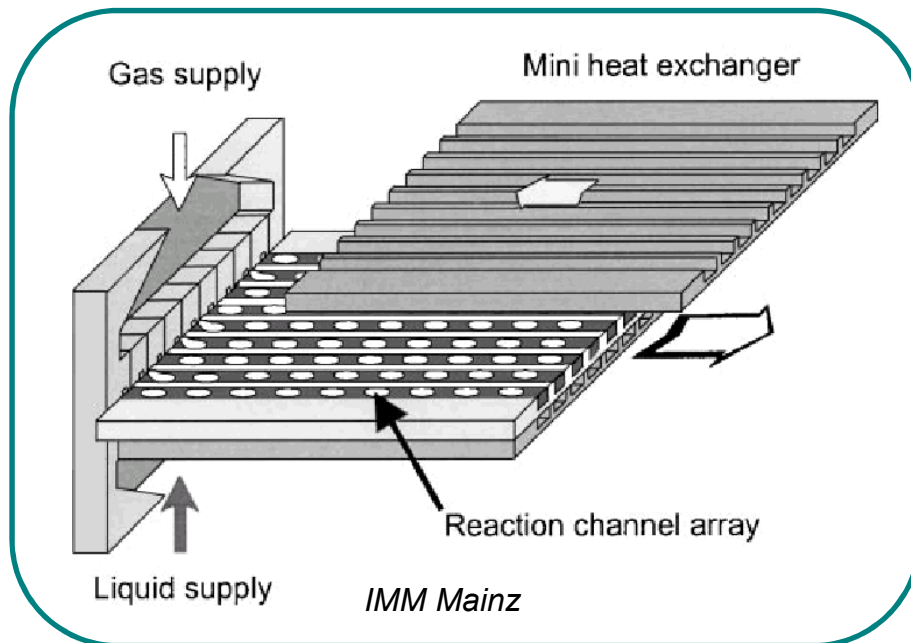


Taylor-Strömung mit  
Stofftransport in einem  
quadratischen Mini-Kanal

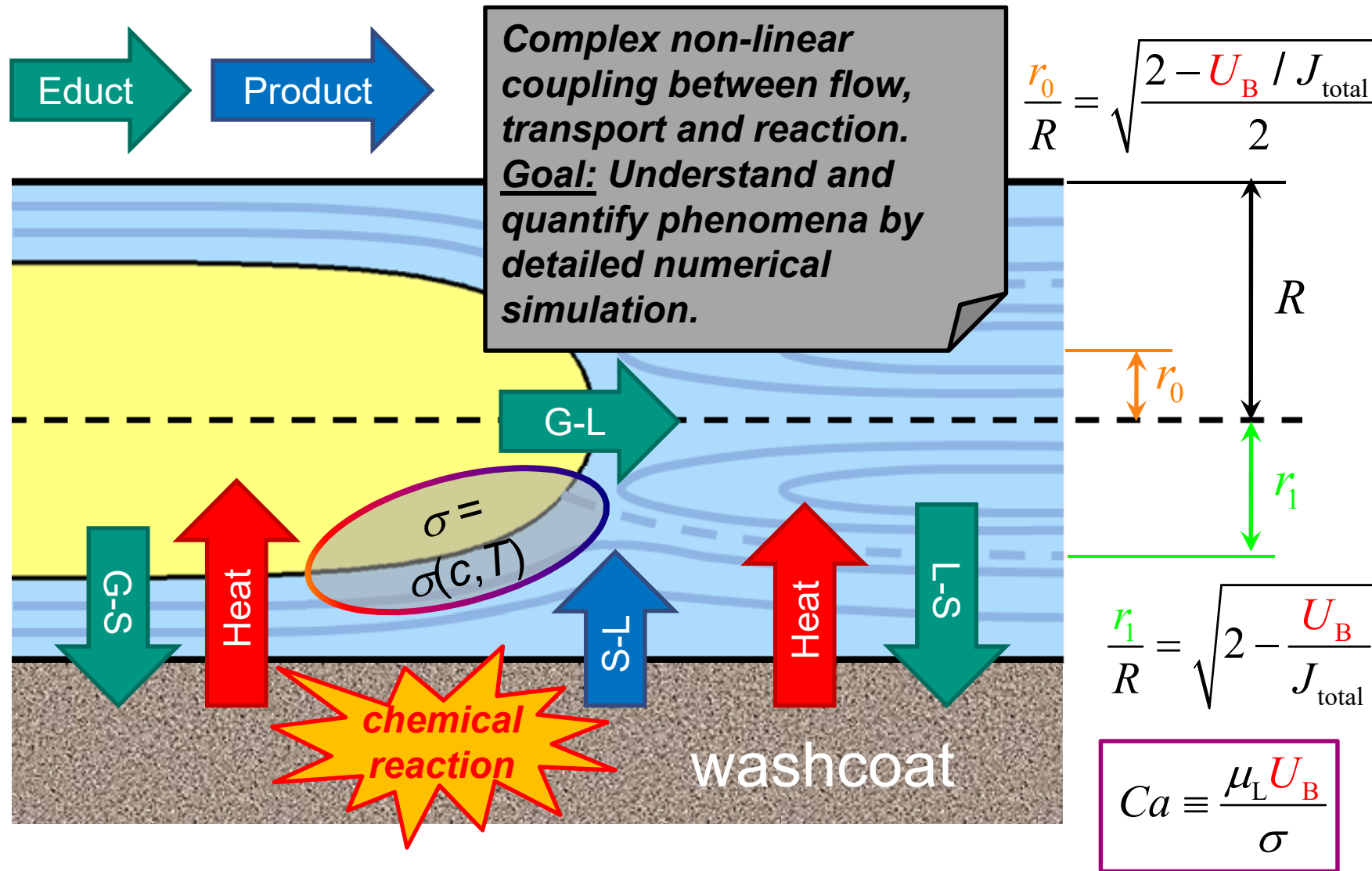
# Outline

- Introduction
- Computational fluid dynamics in micro process engineering
  - Status and developments
- Taylor flow in square capillaries
  - Numerical method and computational setup
  - Insight by scale-resolving simulations
  - Development of engineering models
    - Pressure drop
    - Residence time distribution
- Short survey on further ongoing projects in the group
- Summary

# Multiphase micro reactors



# Flow, mass transfer, and reaction



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# Single-field Navier-Stokes eqs.

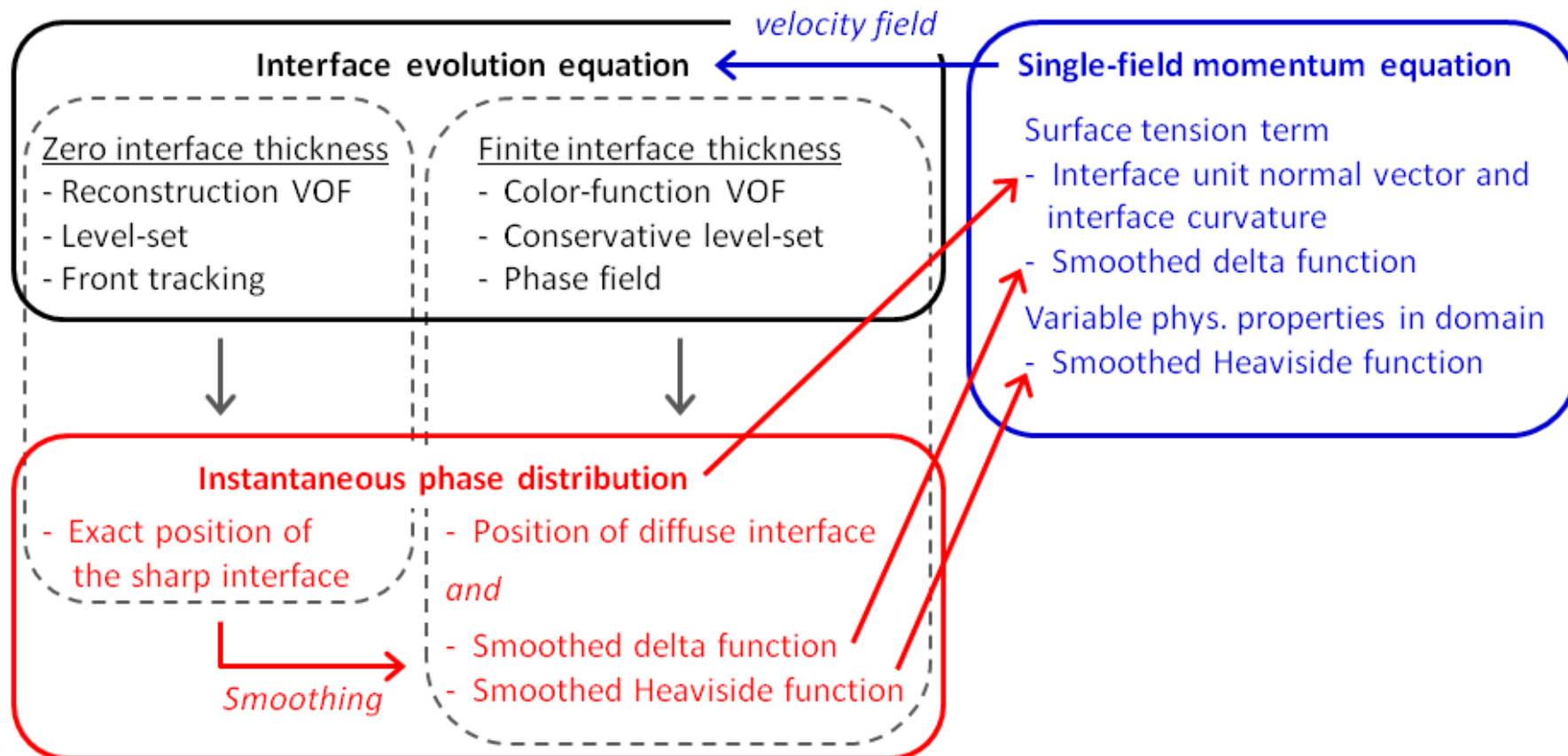
- Two incompressible Newtonian fluids with constant viscosities

$$\left. \begin{aligned} \nabla \cdot \mathbf{v} &= 0 \\ \frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) &= -\nabla p + \nabla \cdot \boldsymbol{\mu} (\nabla \mathbf{v} + (\nabla \mathbf{v})^T) \\ &+ \rho \mathbf{g} + (\sigma \kappa \hat{\mathbf{n}}_\Gamma + \nabla_\Gamma \sigma) \delta_\Gamma \end{aligned} \right\} \mathbf{x} \in \Omega \neq \Omega(t)$$

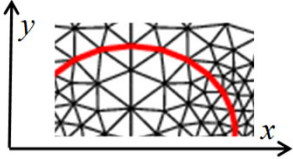
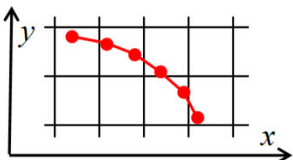
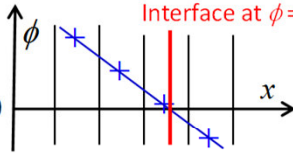
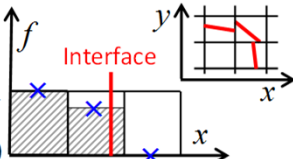
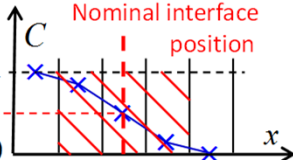
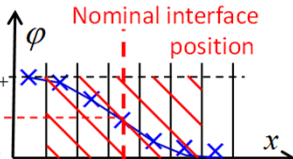
*Eqs. are valid in entire domain*

- Density  $\rho(\mathbf{x}, t)$  and viscosity  $\boldsymbol{\mu}(\mathbf{x}, t)$  are piecewise constant in each phase but are discontinuous at the interface
- **Surface tension term**
  - $\kappa$  = interface curvature
  - $\delta_\Gamma$  = Dirac delta function with support on the interface
- *A method for describing the phase distribution is required!*

# Coupling between interface evolution and momentum eq.



W., Microfluid Nanofluid **12** (2012) 841-886

		Interface representation	Interface evolution
Lagrangian type	Moving mesh		Lagrangian movement of interface (unstructured grid)
	Front-tracking		Lagrangian movement of interface marker points (•) within structured grid
Zero interface thickness			Advection equation for signed distance function $\phi$
Eulerian type	IR-VOF		Geometric evaluation of phase fluxes across mesh cell faces
	CF-VOF and C-LS		Advection Eq. (11) for color function C (in C-LS followed by a compression step)
	Phase field		Cahn-Hilliard Eq. (14) for order parameter $\phi$

### ■ Moving mesh method

- + Interface does not cut cells but separates phases
- Adaptive grid required; difficulty in handling break-up and coalescence

### ■ Front-tracking method

- + Does not impose artificial coalescence as VOF and LS method often do
- Complexity in handling break-up/coalescence

### ■ Level-set method

- + Avoids interface reconstruction; flexibility in handling topological changes
- Special measures necessary to reduce mass conservation errors

### ■ Volume-of-fluid method with interface reconstruction

- + Very good mass conservation
- Complex interface reconstruction/advection algorithms in 3D

### ■ Color-function VOF and conservative level-set meth.

- + Methods are rather easy to implement
- Special measures necessary to keep the interface thickness constant and uniform

### ■ Phase-field method

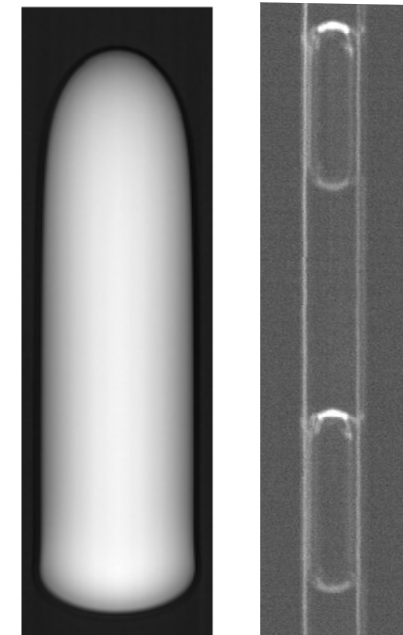
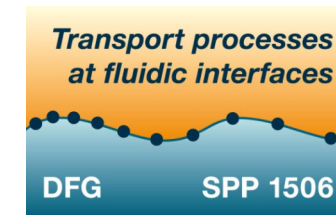
- + Allows for effective control of interface thickness; can handle moving contact lines
- For accurate results the diffuse interface region must be adequately resolved

W., Microfluid Nanofluid **12** (2012) 841-886



# SPP 1506 Benchmark Taylor Flow

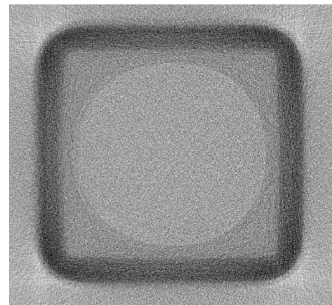
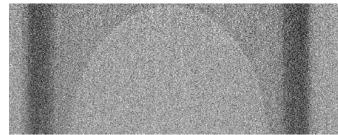
- SPP 1506 “Transport Processes at fluidic interfaces”
  - Guiding measure Taylor flow
  - Goal: establish benchmarks for detailed quantitative validation of CFD method and codes
- Measurements by experimental groups
  - $d_h = 2$  mm capillary with circular / square cross section
  - water-glycerine solution / air; co-current upward flow
  - Single Taylor bubble: Boden/Hampel HZDR
  - Taylor flow: Meyer/Schlüter TUHH
- **TBSC = Taylor Bubble Square Channel (3D)**
  - contour of bubble shape (lateral and diagonal plane)
- **TFSC = Taylor Flow Square Channel (3D)**
  - velocity profiles from  $\mu$ PIV in liquid slug and liquid film
- **TBCC = Taylor Bubble Circular Channel (2½D)**
  - contour of bubble shape



HZDR

TUHH

# Case Taylor Bubble Square Channel

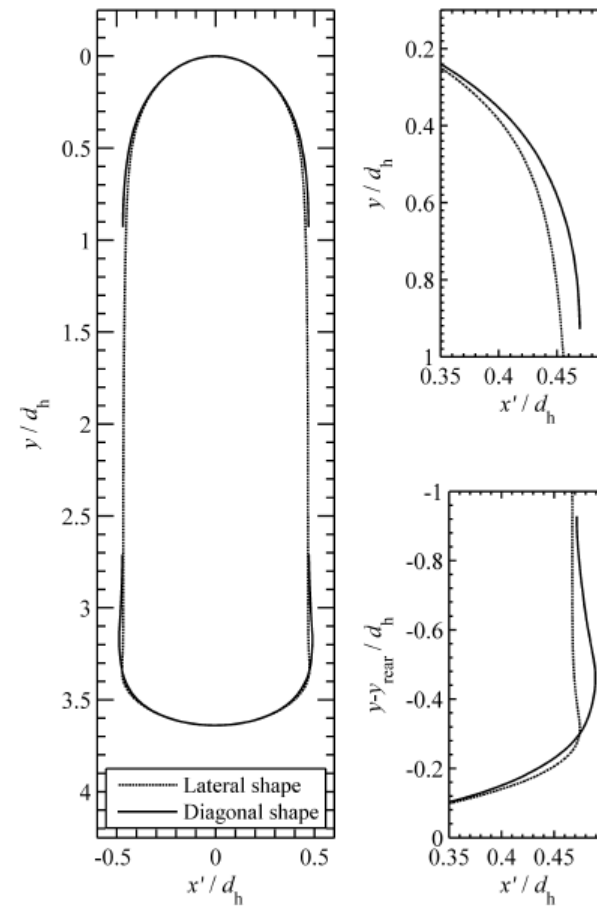


**HZDR**

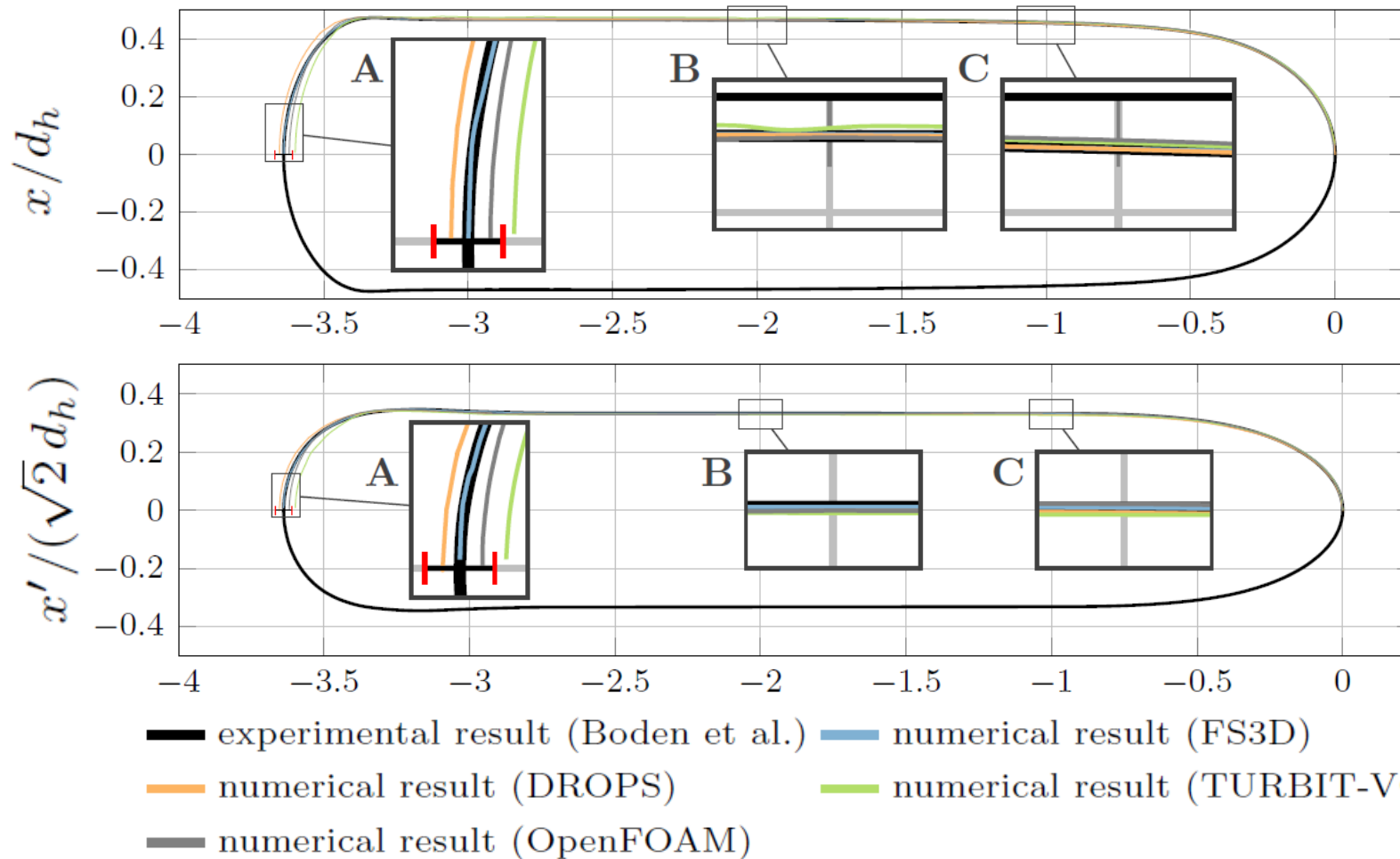
**HELMHOLTZ  
ZENTRUM DRESDEN  
ROSSENDORF**

S. Boden,  
U. Hampel

*Bubble is not axi-symmetric!*



# Comparison of numerical methods<sup>\*,#</sup>



\*Marschall, Boden, Lehrenfeld, Falconi, Hampel, Reusken, W., Bothe; Computers & Fluids (accepted)

#For 2D test case see Aland et al., Int. J. Num. Meth. Fluids (accepted, available online)

# Interface resolving simulations: status and ongoing developments

- Hydrodynamics of two-phase flow including bubble and drop formation ✓
- *Coalescence phenomena* ✂
- *Mass transfer (realistic Schmidt numbers)* ✂
- *Interfacial transport of soluble and insoluble surfactants* ✂
- *Coupling of two-phase flow and transport with chemical kinetics* ✂

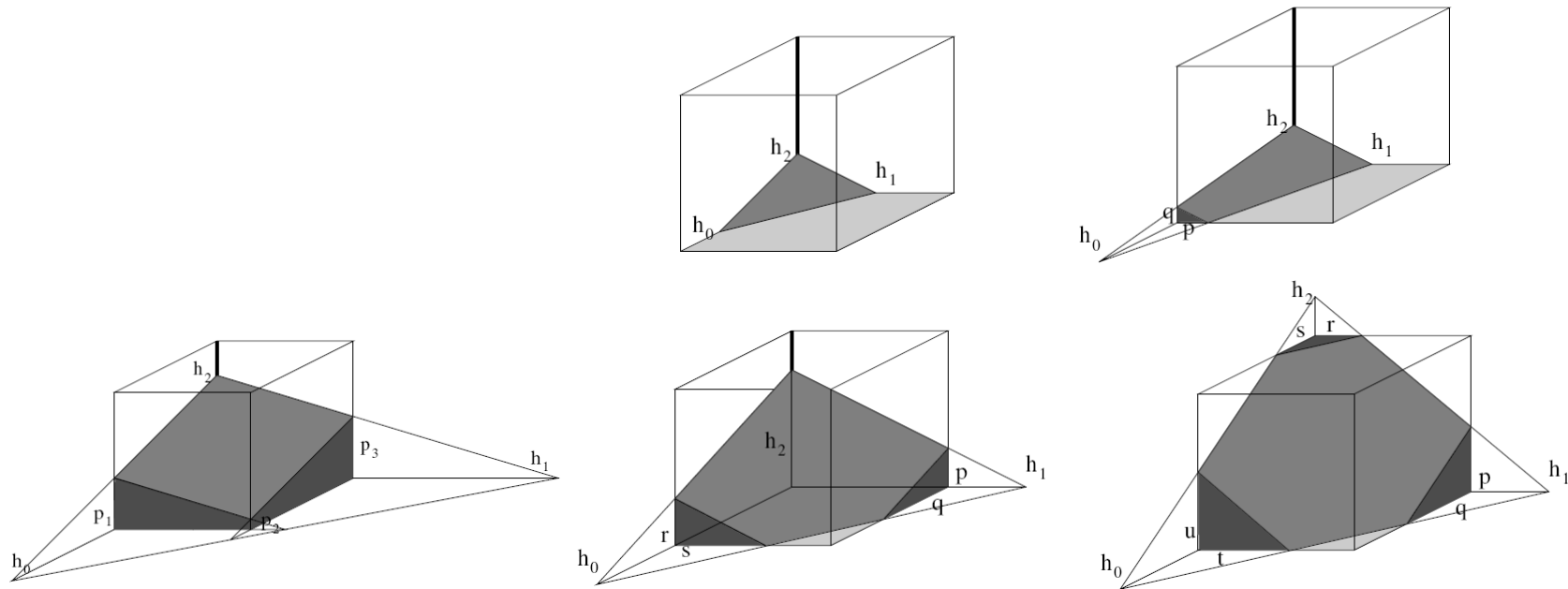
*New SPP 1740 Reactive Bubbly Flows (Prof. M. Schlüter, TUHH)*

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# Simulation method\*

- In-house computer code TURBIT-VOF  
for details see Öztaskin et al. Phys. Fluids **21** (2009) 042108
- Volume-of-fluid method with piecewise linear interface reconstruction



- Finite volume discretization on a staggered 3D Cartesian grid
- Projection method for pressure-velocity coupling
- Explicit 3<sup>rd</sup> order Runge-Kutta time integration scheme

# Computational set-up for Taylor flow

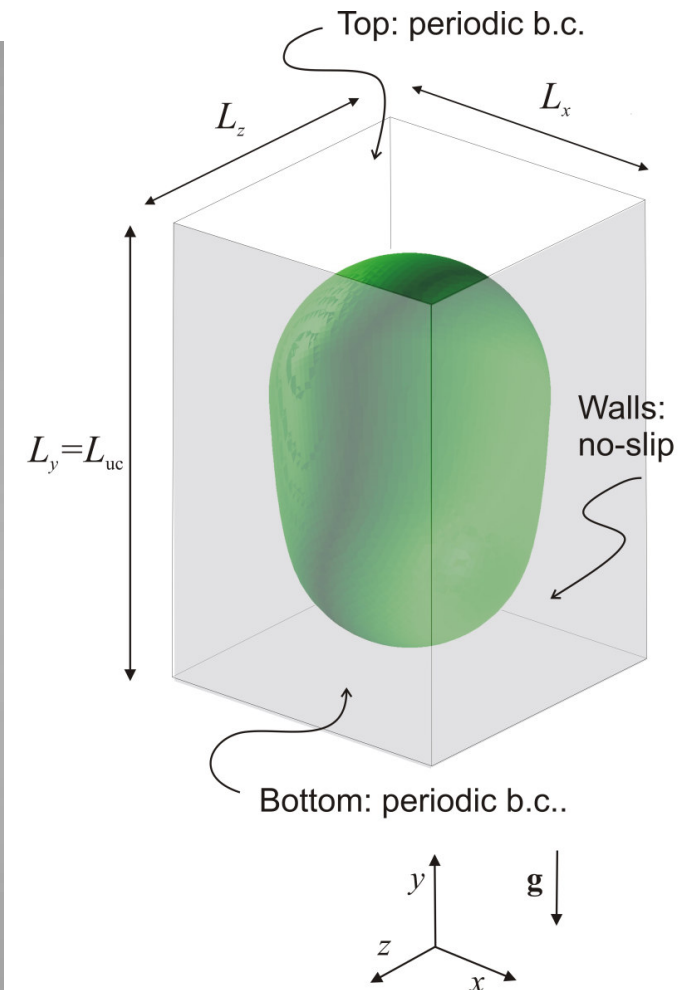
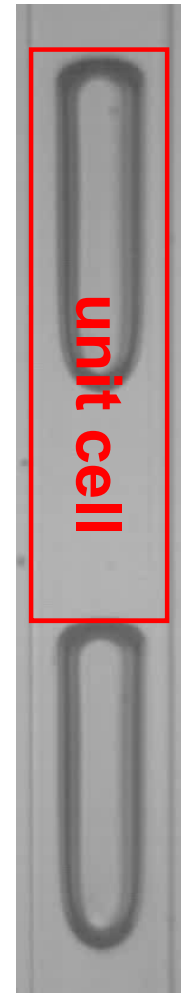
## ■ Set-up

- Co-current downward flow of squalane ( $C_{30}H_{62}$ ) and nitrogen
- Square channel (hydraulic diameter  $D_h = 0.5, 1, 2, 4$  mm)
- Consideration of **one unit cell**
- Periodic boundary conditions in vertical (axial) direction

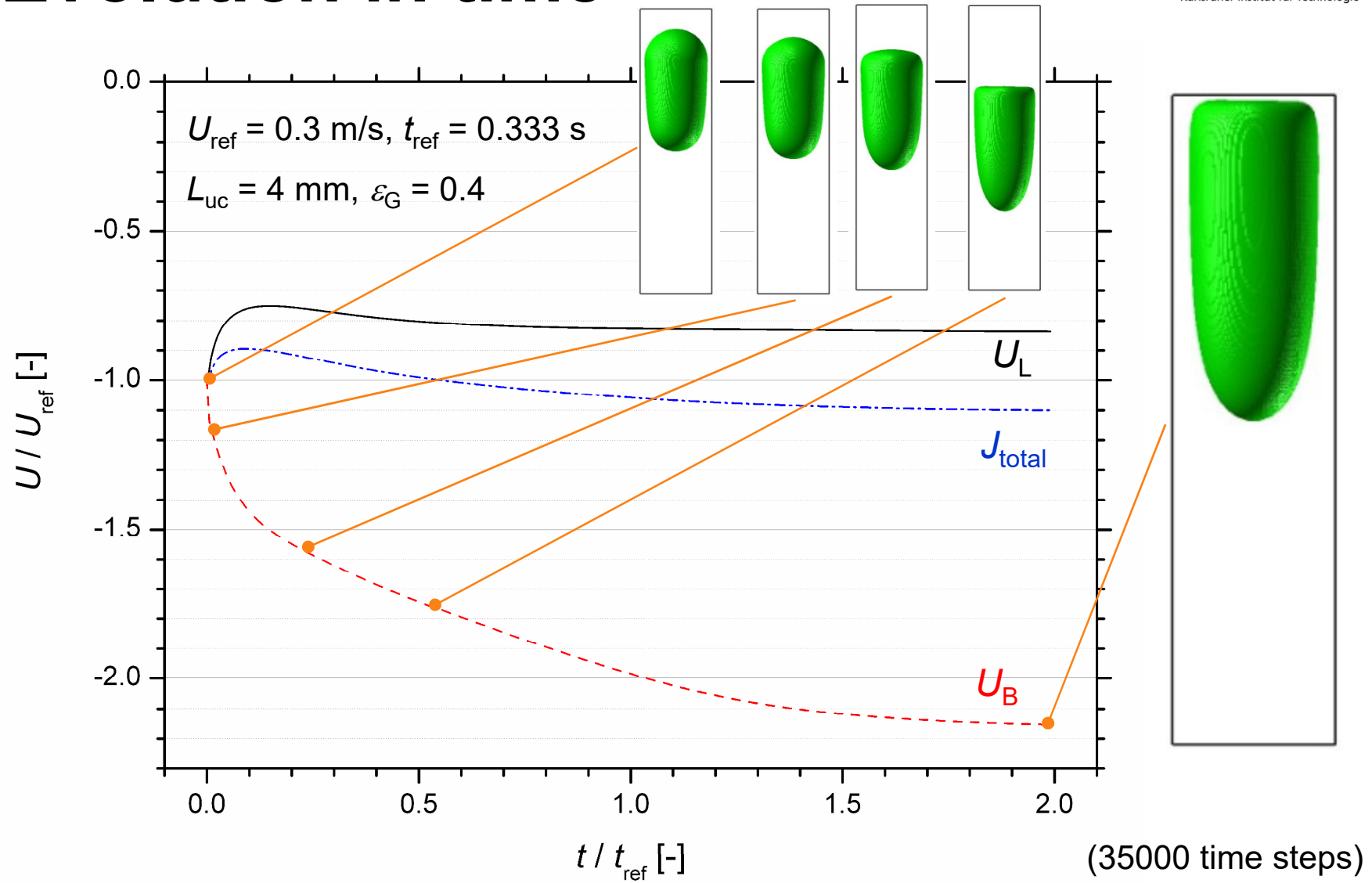


## ■ Prescribed parameters

- Gas content:  $\varepsilon_G = 0.2$  or  $0.4$
- Unit cell length:  $L_y / D_h = 4$  or  $6$  (grid up to  $80 \times 480 \times 80$  cells)
- Pressure drop across the unit cell  $Eu_{ref} = \Delta p_{uc} / (\rho_L U_{ref}^2)$



# Evolution in time





# Steady bubble shape for $D_h = 1 \text{ mm}$

$1 \text{ mm} \times 1 \text{ mm}$

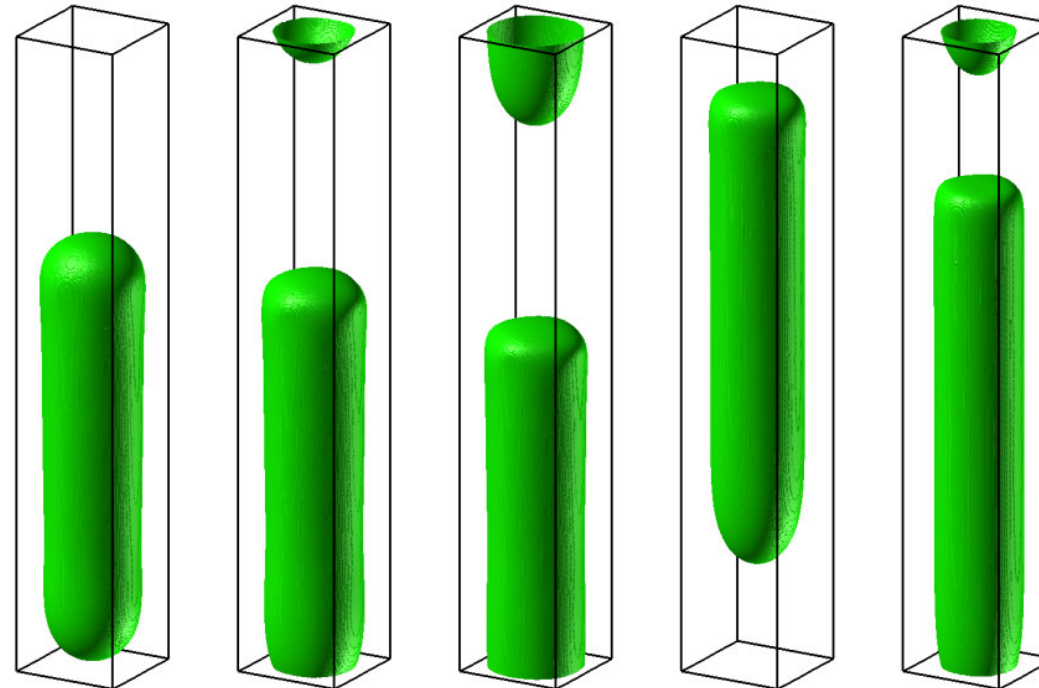
$$L_{uc} = 6 \text{ mm}$$

$$\varepsilon_G = 0.4$$

$$Ca_B = \frac{\mu_L U_B}{\sigma}$$

$$Re_B = \frac{\rho_L D_h U_B}{\mu_L}$$

$$La = \frac{\sigma \rho_L D_h}{\mu_L^2} = 27.27$$



$$Ca_B = 0.045$$

$$0.12$$

$$0.17$$

$$0.26$$

$$0.49$$

$$Re_B = 1.22$$

$$3.19$$

$$4.64$$

$$7.16$$

$$13.4$$

Increase of  $Ca_B$  and  $Re_B = La Ca_B$

Increase of  $\delta_F$ ,  $L_B$ ,  $\kappa_{\text{front}}$  (curvature of front meniscus)  
 Decrease of  $D_B$ ,  $L_{\text{slug}}$ ,  $\kappa_{\text{rear}}$  (curvature of rear meniscus)

# Channel size effects in Taylor flow

- Size  $0.5 \times 0.5$  mm

$$D_h = 0.5 \text{ mm}$$

$$Ca_B = 0.202$$

$$Re_B = 2.75$$

$$Eö = 0.067$$

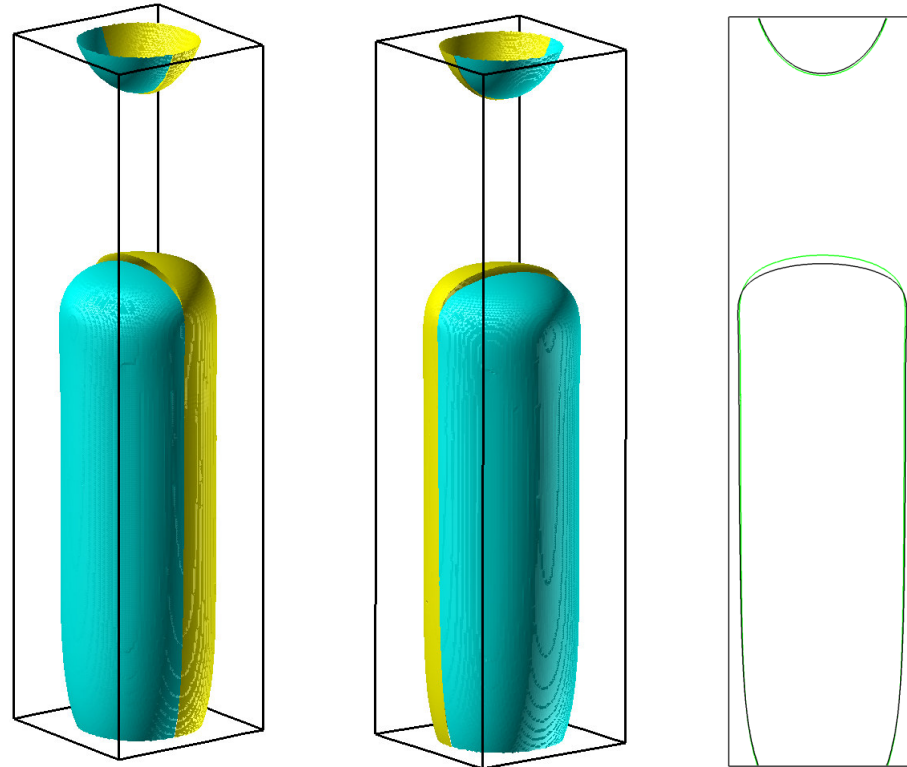
- Size  $2 \times 2$  mm

$$D_h = 2.0 \text{ mm}$$

$$Ca_B = 0.213$$

$$Re_B = 11.6 \text{ (inertia)}$$

$$Eö = 1.068 \text{ (buoyancy)}$$



- Combined influence of  $Re_B$  and  $Eö$  for fixed value of  $Ca_B$

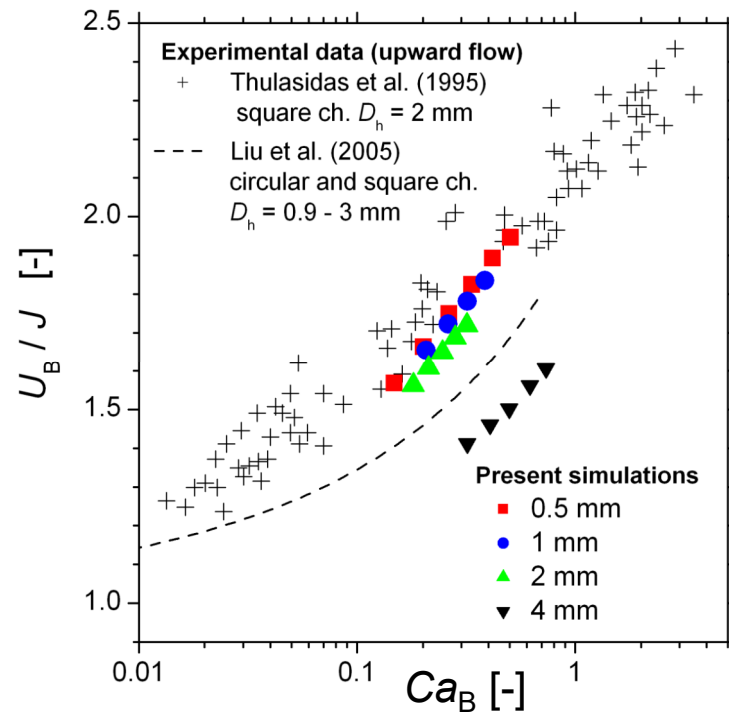
- Very small influence on  $\delta_F/D_h$  (film thickness) and  $\kappa_{\text{front}} D_h$

- Notable influence on  $\kappa_{\text{rear}} D_h$  (inertial effect, known in literature)

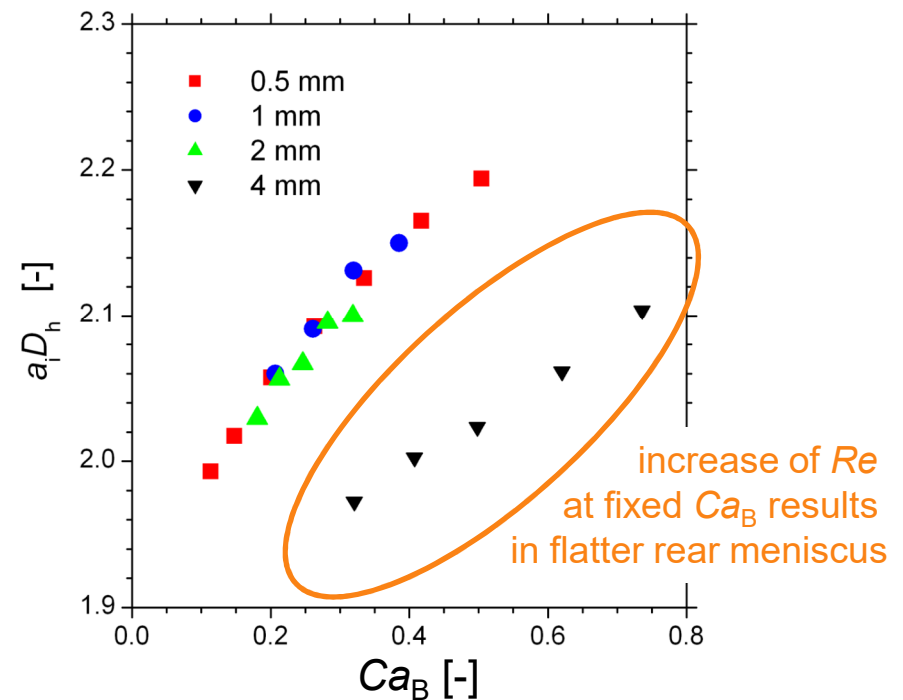
W., J. Chem. Eng. Japan **46** (2013) 335

# Influence of channel size

## Non-dimensional bubble velocity



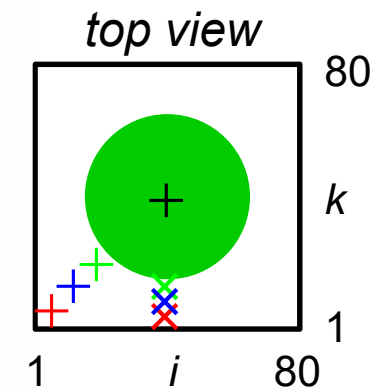
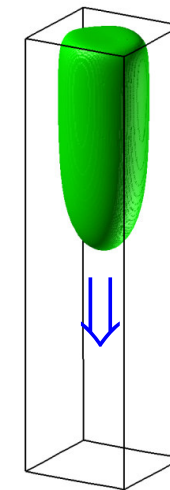
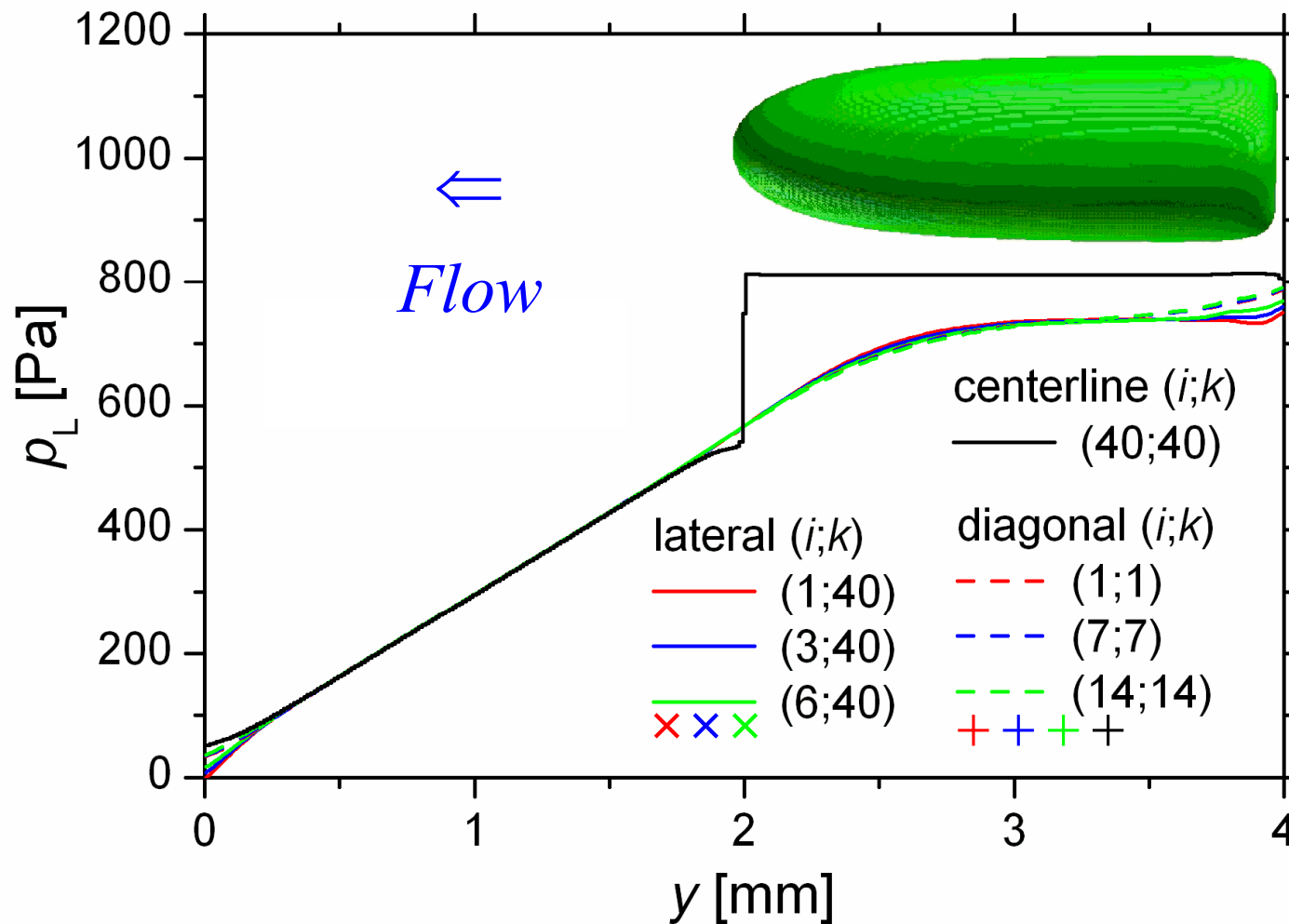
## Non-dim. specific interfacial area



- For  $D_h \leq 2$  mm the influence of inertia ( $Re_B$ ) and buoyancy ( $E\ddot{o}$ ) is very small  $\rightarrow$  **scaling with  $Ca_B$**
- For  $D_h > 2$  mm the influence of inertia and buoyancy become important

# Axial profiles of liquid pressure

$$D_h = 1 \text{ mm}, L_{uc} / D_h = 4, \varepsilon_G = 0.4, Ca_B = 0.66, Re_B = 18.0$$



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# Pressure drop models in literature

*Kreutzer et al.\**

*Warnier et al.#*

- Hydrodynamic pressure drop in a unit cell

$$-\left(\frac{dP}{dz}\right)_{uc} = -\left(\frac{dP}{dz}\right)_{slug} \left(\frac{L_{slug}}{L_{bubble} + L_{slug}}\right)$$

$$-\left(\frac{dP}{dz}\right)_{uc} = -\left(\frac{dP}{dz}\right)_{slug} \left(\frac{L_{slug} + D_B / 3}{L_{bubble} + L_{slug}}\right)$$

- Frictional pressure drop in the liquid slug

$$-\left(\frac{dP}{dz}\right)_{slug} = 32 \frac{\mu_L J_{total}}{D_h^2} \left(1 + a \frac{D_h}{L_{slug}} La^{0.33}\right)$$

$$-\left(\frac{dP}{dz}\right)_{slug} = 32 \frac{\mu_L J_{total}}{D_h^2} \left(1 + a \frac{D_h}{L_{slug} + D_B / 3} La^{0.33}\right)$$

$$J_{total} = J_G + J_L, \quad La = \sigma \rho_L D_h / \mu_L^2$$

*Models do not involve  $Ca_B$  or  $Re_B$  (only the ratio of both,  $La$ )*

- Fitting coefficient

$a = 0.07$  numerical

$a = 0.17$  experimental

$a = 0.1$  experimental

\*Kreutzer et al. AIChE J. **51** (2005) 2428

#Warnier et al. Microfluid Nanofluid **8** (2010) 33

# Pressure drop model Boran & W.

- Goal: development of a “mechanistic” model for the hydrodynamic pressure drop along one unit cell
- The model shall involve only “a priori” known quantities
  - Physical properties  $\rho_L, \mu_L, \sigma, g$
  - Channel hydraulic diameter  $D_h$
  - Superficial velocities  $J_L, J_G \rightarrow J_{\text{total}} = J_G + J_L$
  - The following non-dimensional numbers can be defined

$$\beta = \frac{J_G}{J_{\text{total}}}, \quad Ca_J = \frac{\mu_L J_{\text{total}}}{\sigma}, \quad La = \frac{\sigma \rho D_h}{\mu_L^2}$$

- The pressure drop along a unit cell consists of two parts
  - Pressure drop in the liquid slug
  - Pressure drop in the liquid film along the bubble

$$\frac{\Delta P_{\text{uc}}}{L_{\text{uc}}} = \frac{\Delta P_{\text{slug}}}{L_{\text{uc}}} + \frac{\Delta P_{\text{film}}}{L_{\text{uc}}}$$

# Pressure drop in the liquid slug

■ Analogous to Kreutzer et al.:

- Round channel:  $C_f = 64$
- Square channel:  $C_f = 56.9$

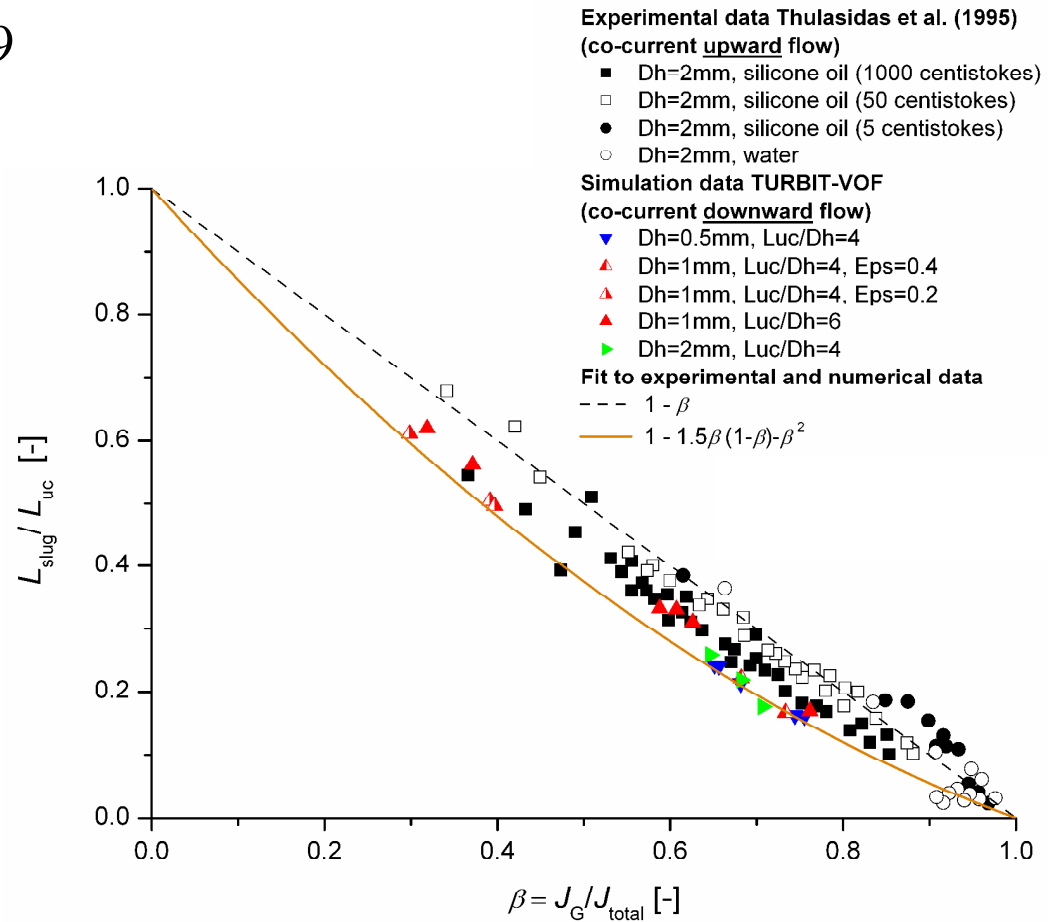
■ The ratio  $L_{slug}/L_{uc}$  is modeled by fitting of experimental and numerical data

$$\frac{L_{slug}}{L_{uc}} = 1 - C_{slug} \beta (1 - \beta) - \beta^2$$

$$C_{slug} = 1 \rightarrow \frac{L_{slug}}{L_{uc}} = 1 - \beta$$

Here:  $C_{slug} = 1.5$

$$\frac{\Delta P_{slug}}{L_{uc}} = \left( \frac{dP}{dy} \right)_{slug} \frac{L_{slug}}{L_{uc}} = \frac{C_f}{2} \frac{\mu_L J_{total}}{D_h^2} \frac{L_{slug}}{L_{uc}}$$





# Pressure drop along liquid film

- Young-Laplace equation at the front and rear meniscus

$$p_{B,\text{front}} \approx p_{L,\text{front}} + \sigma \kappa_{\text{front}}, \quad p_{B,\text{rear}} \approx p_{L,\text{rear}} + \sigma \kappa_{\text{rear}}$$

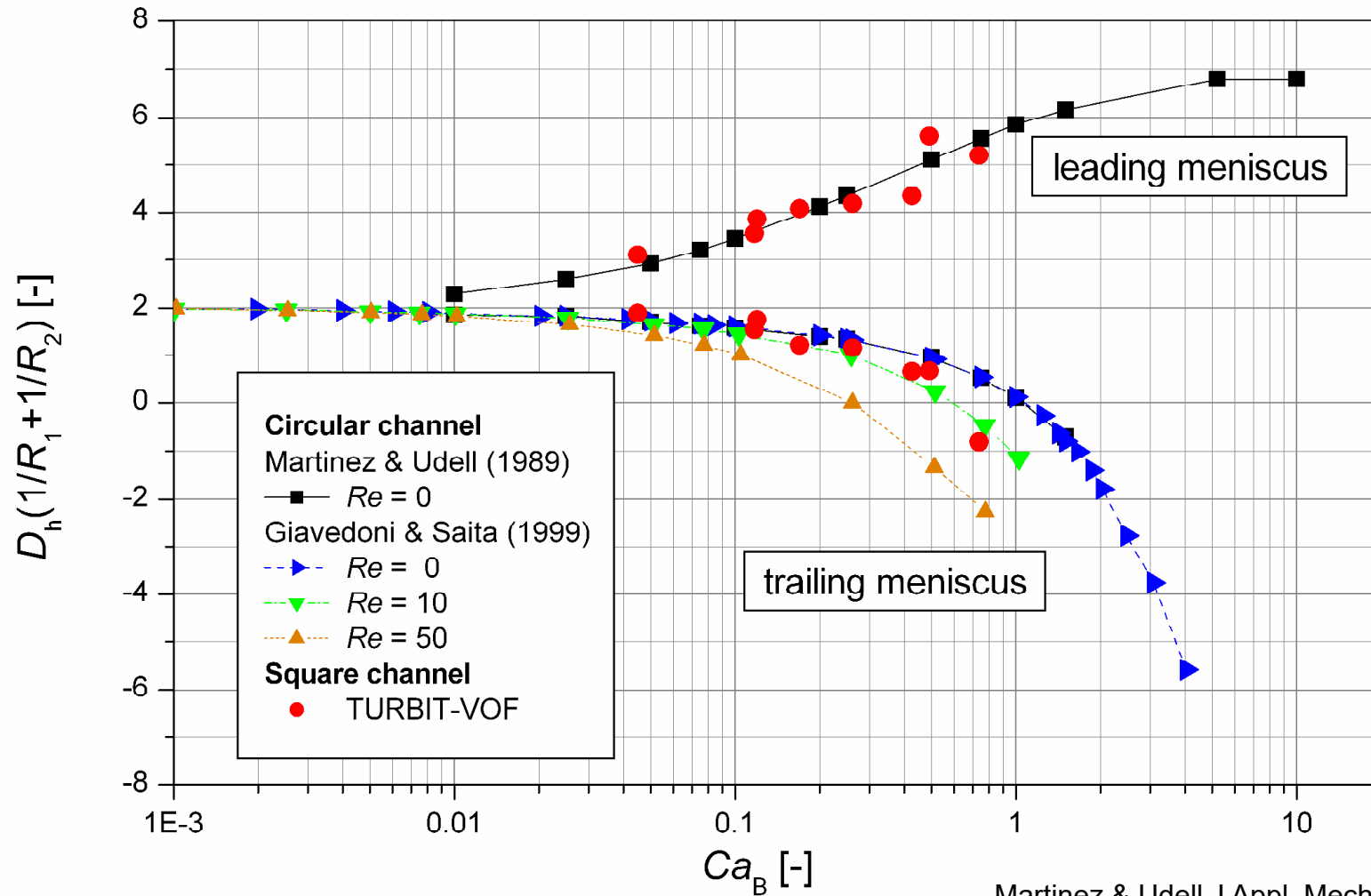
- The pressure in the bubble is about constant so that

$$\Delta P_{\text{film}} = p_{L,\text{rear}} - p_{L,\text{front}} = \sigma (\kappa_{\text{rear}} - \kappa_{\text{front}})$$

$$\frac{\Delta P_{\text{film}}}{L_{\text{uc}}} = (\kappa_{\text{rear}} - \kappa_{\text{front}}) \frac{\sigma}{L_{\text{uc}}} = (D_h \kappa_{\text{rear}} - D_h \kappa_{\text{front}}) \frac{\sigma}{D_h L_{\text{uc}}}$$

→ we need models for the non-dimensional bubble front and rear curvatures  $D_h \kappa_{\text{rear}}$  and  $D_h \kappa_{\text{front}}$

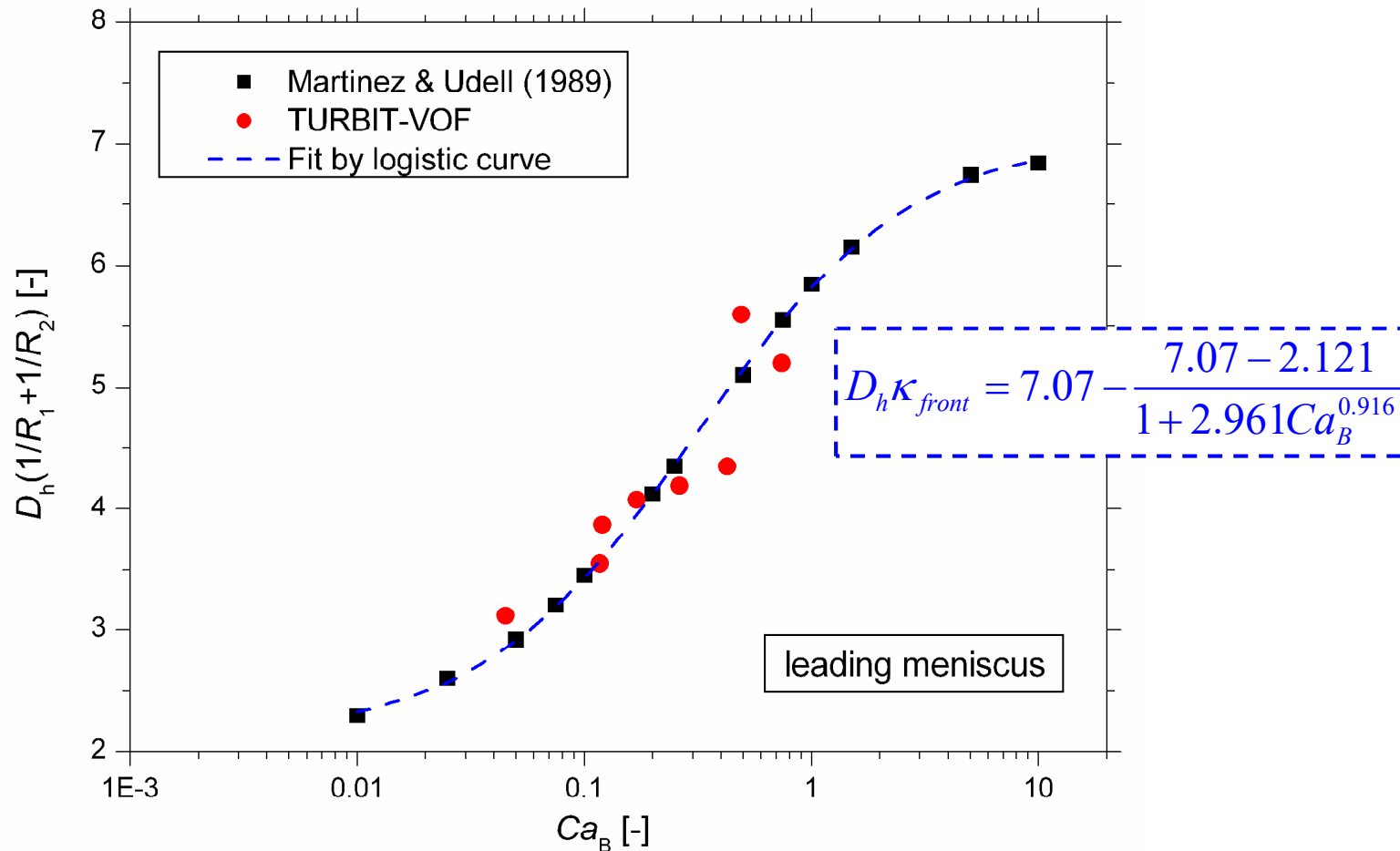
# Data for front and rear curvature



Martinez & Udell J Appl. Mech. **56** (1989) 211  
 Giavedoni & Saita Phys. Fluid **11** (1999) 786

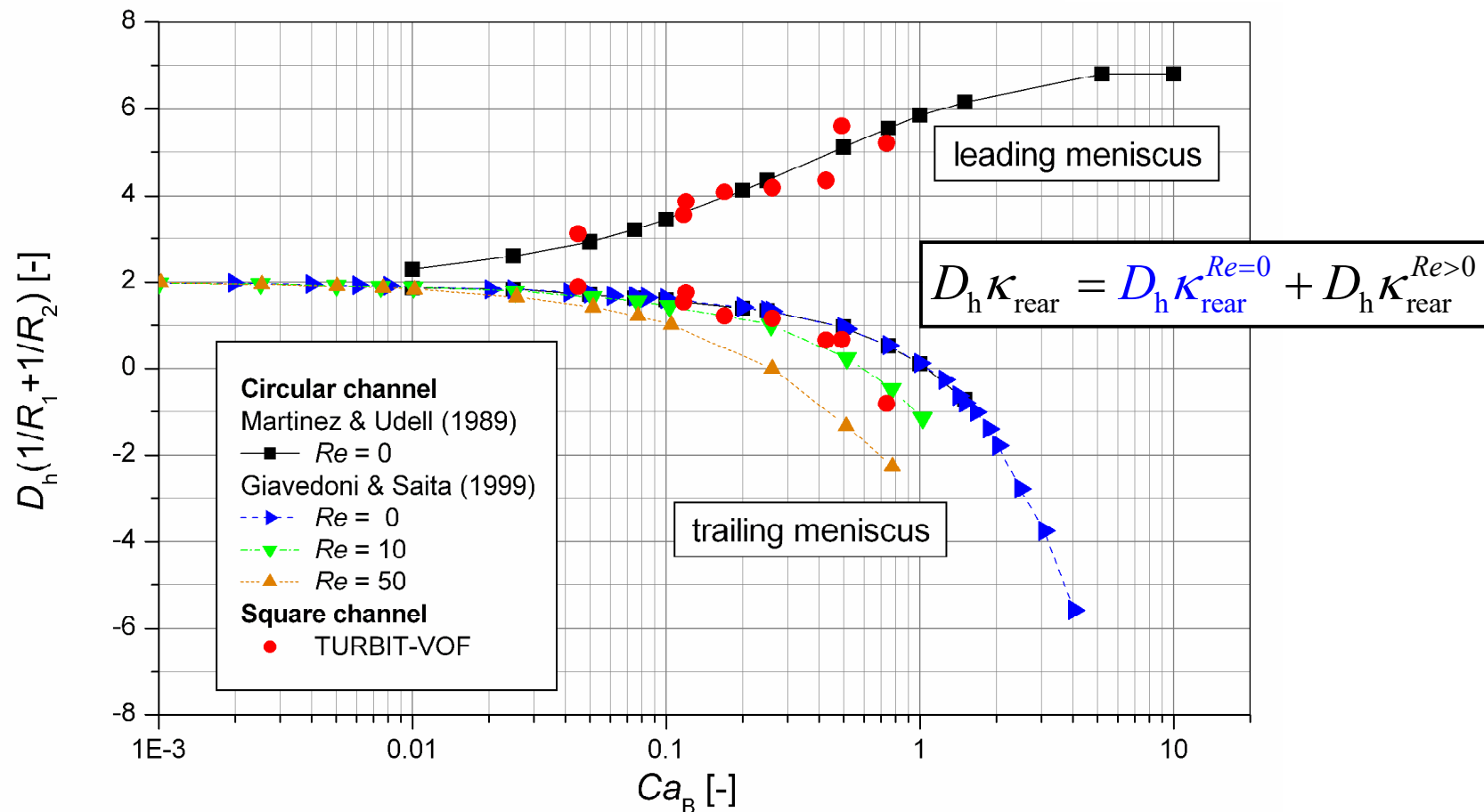
# Modeling of front curvature

- The non-dimensional front curvature depends on  $Ca_B$  only

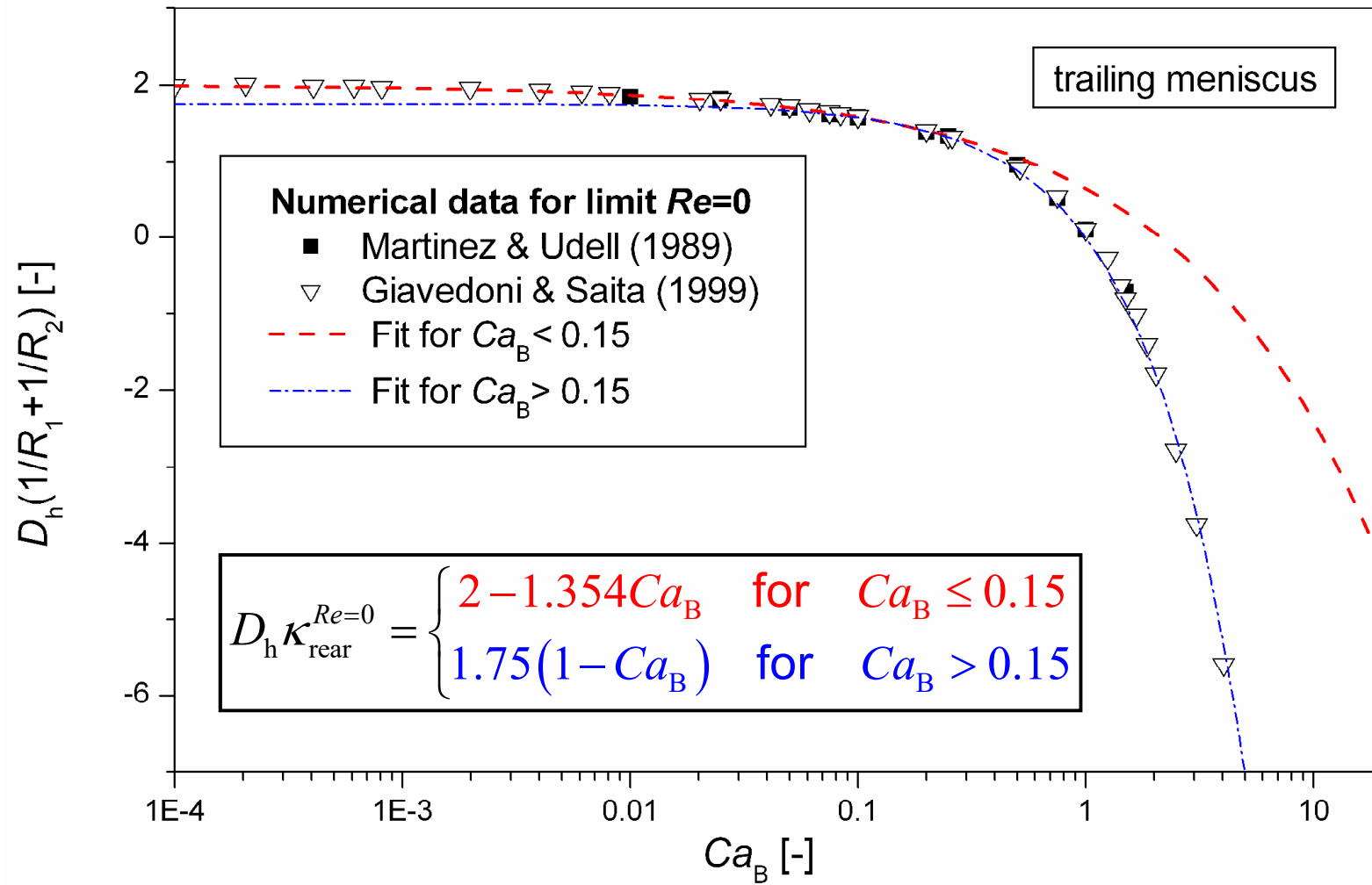


# Modeling of rear curvature

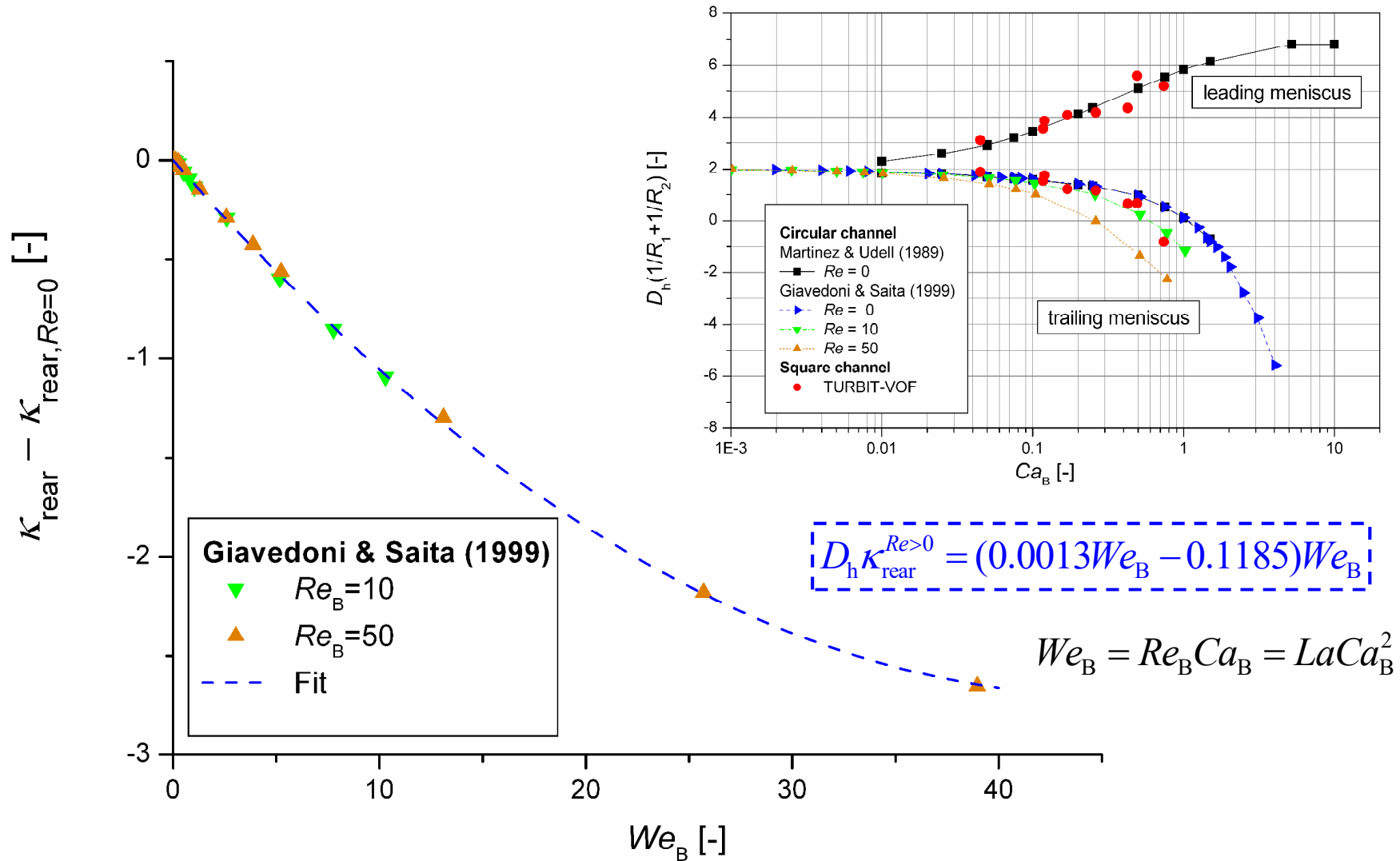
- The non-dimensional rear curvature depends on  $Ca_B$  and  $Re_B$
- It is modeled as the sum of two contributions:  $\kappa_{\text{rear}} = \kappa_{\text{rear}}^{Re=0} + \kappa_{\text{rear}}^{Re>0}$



# Rear curvature in limit $Re_B = 0$



# Rear curvature for $Re_B > 0$



# Closure of the model

- At this stage, the model is not closed since the front and rear curvatures depend on  $Ca_B$  which involves the unknown bubble velocity

$$Ca_B = \frac{\mu_L U_B}{\sigma}$$

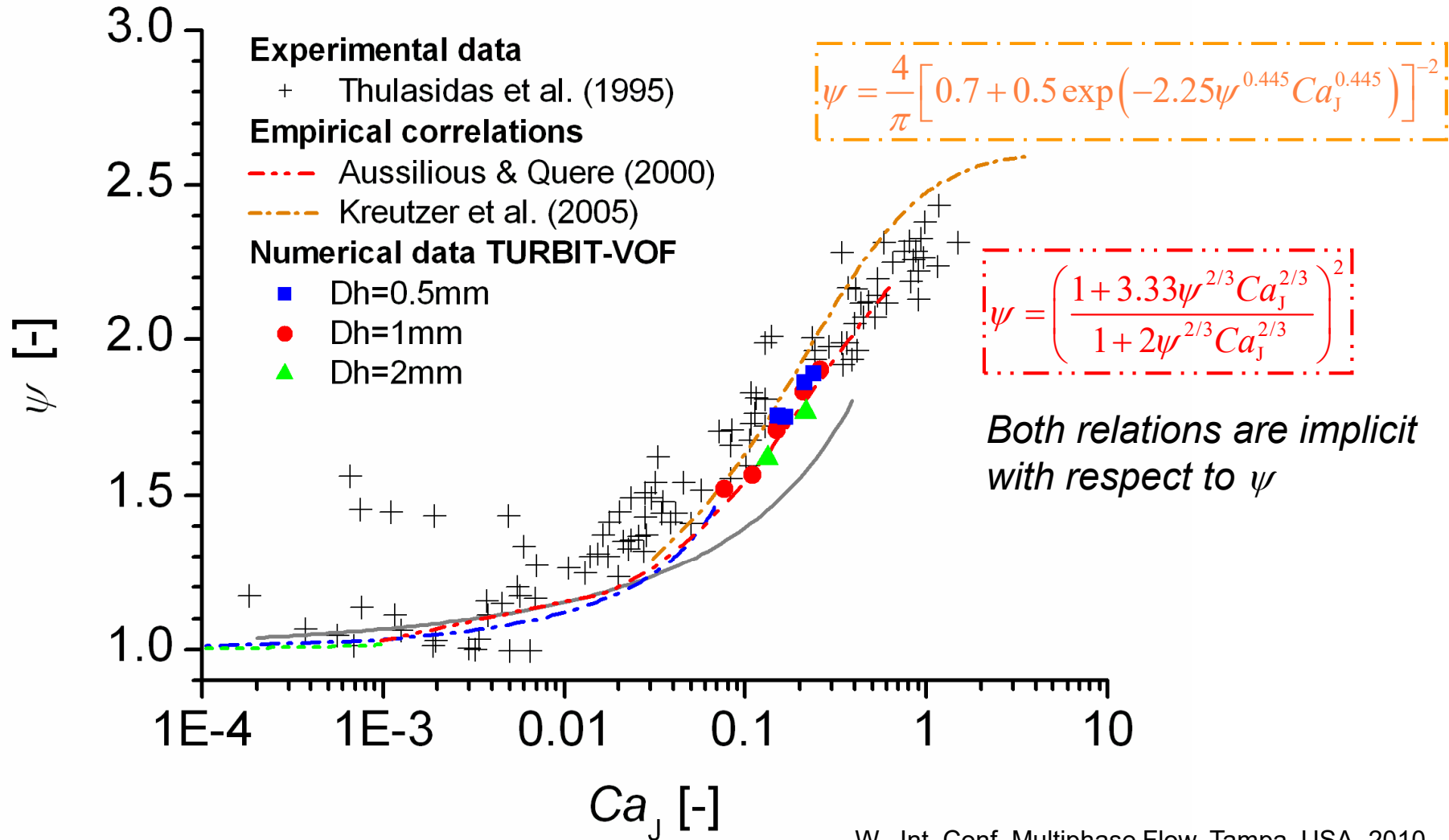
- As a remedy we relate  $U_B$  to the total superficial velocity  $J_{\text{total}}$

$$\psi = \frac{U_B}{J_{\text{total}}} \quad \rightarrow \quad Ca_B = \frac{U_B}{J_{\text{total}}} \frac{\mu_L J_{\text{total}}}{\sigma} = \psi Ca_J$$

- The ratio  $\psi$  is obtained by fitting experimental and numerical data

$$\psi = \psi(Ca_J)$$

# Relation between $\psi$ and $Ca_J$



W., Int. Conf. Multiphase Flow, Tampa, USA, 2010



# Summary of pressure drop model

Input:  $\rho_L, \mu_L, \sigma, g, D_h, L_{uc}, J_L, J_G \rightarrow \beta, La, Ca_J$

$$\rightarrow \psi = \left( \frac{1 + 3.33\psi^{2/3} Ca_J^{2/3}}{1 + 2\psi^{2/3} Ca_J^{2/3}} \right)^2$$

Model:

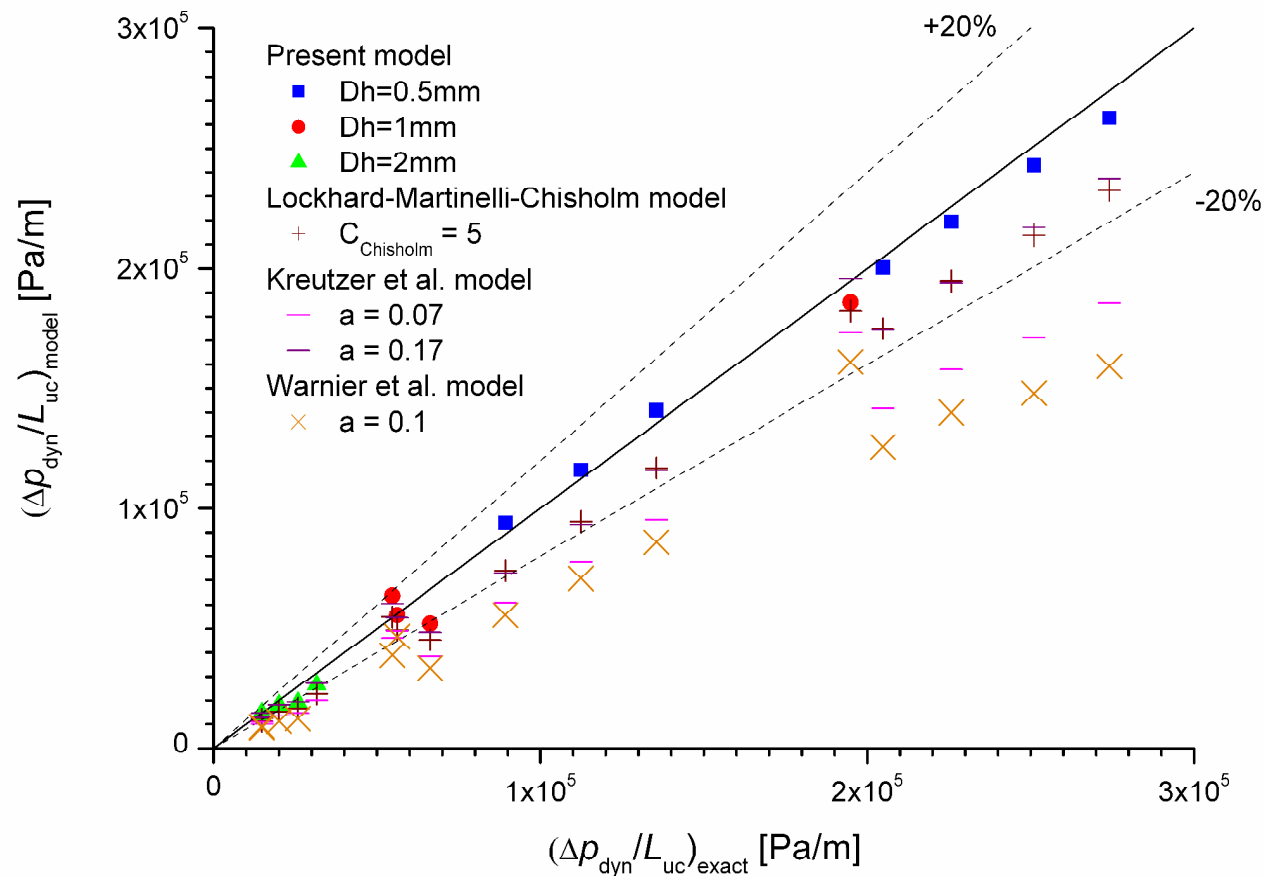
$$\frac{\Delta P_{uc}}{L_{uc}} = \frac{C_f}{2} \frac{\mu_L J_{total}}{D_h^2} \left[ 1 - 1.5\beta(1-\beta) - \beta^2 \right] + \left( D_h \kappa_{rear}^{Re=0} + D_h \kappa_{rear}^{Re>0} - D_h \kappa_{front} \right) \frac{D_h}{L_{uc}} \frac{\sigma}{D_h^2}$$

$$D_h \kappa_{rear}^{Re=0} = \begin{cases} 2 - 1.354\psi Ca_J & \text{for } \psi Ca_J \leq 0.15 \\ 1.75(1 - \psi Ca_J) & \text{for } \psi Ca_J > 0.15 \end{cases}$$

$$D_h \kappa_{rear}^{Re>0} = (0.0013\psi^2 La Ca_J^2 - 0.1185)\psi^2 La Ca_J^2 \quad \text{for } \psi^2 La Ca_J^2 \leq 40$$

$$D_h \kappa_{front} = 7.07 - \frac{7.07 - 2.121}{1 + 2.961\psi^{0.916} Ca_J^{0.916}}$$

# Comparison of pressure drop models



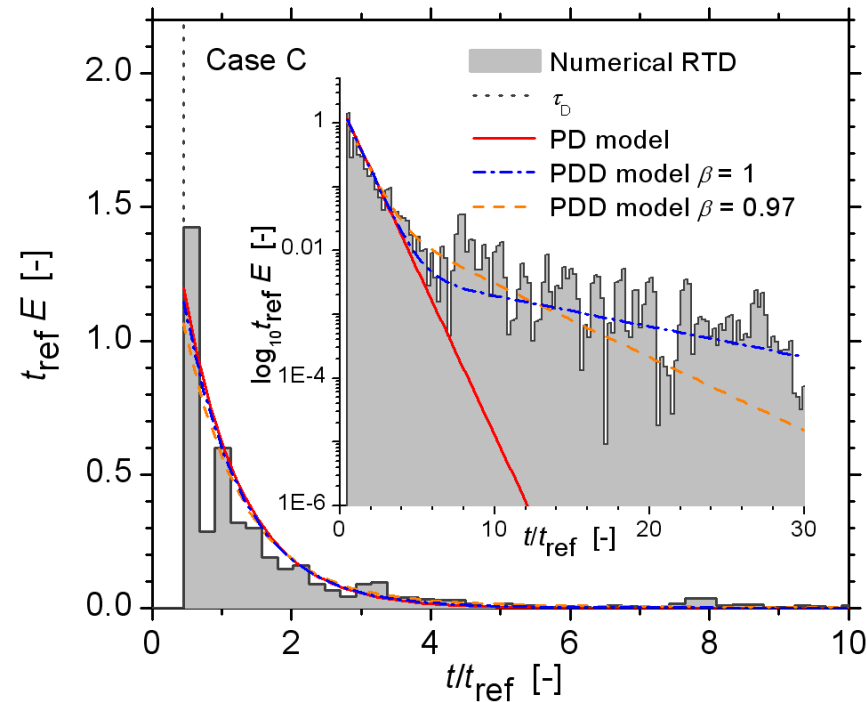
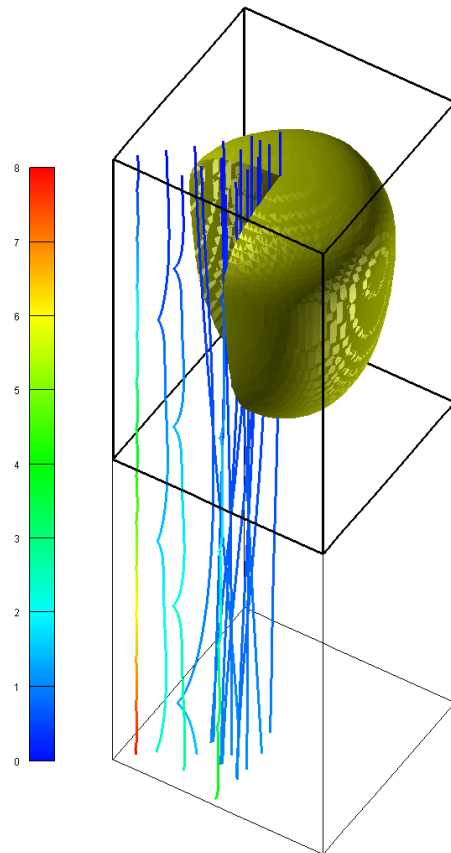
- The new model is in good agreement with DNS data (not surprising)
  - compare model with experimental pressure drop data

# Residence time distribution

- Liquid flows in micro-channels are usually laminar and species diffusivities are usually low (high Schmidt number)
- The RTD is therefore significantly affected by the non-uniform velocity distribution → the RTD is rather broad
  - Conventional RTD models (PFR, CSTR, axial-dispersed plug flow model) are often not appropriate
  - Influence of channel shape on diffusion-free RTD (analytical)  
W., CES **65** (2010) 3499, Erdogan & W., CEJ **227** (2013) 158
- Segmented flow (Taylor flow) narrows the RTD (A. Günther)

# RTD in Taylor flow

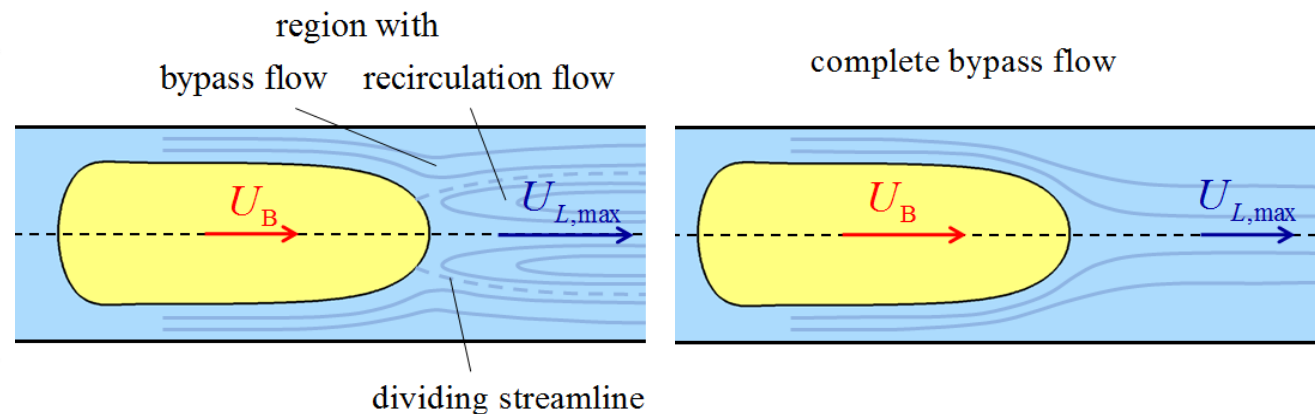
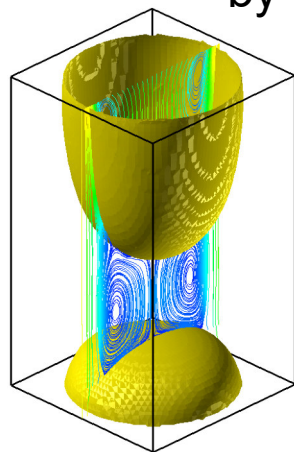
- Evaluation of the diffusion-free RTD from simulation results by tracking a set of virtual particles and recording the time the particle needs to travel a certain axial distance




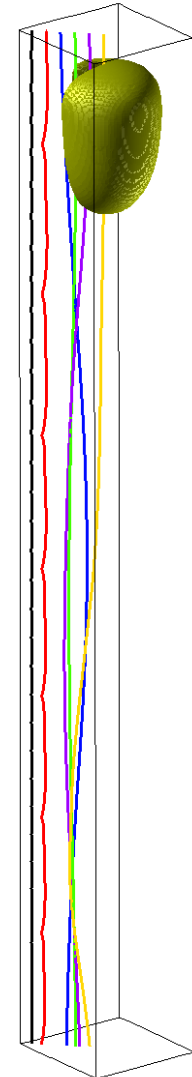
Erdogan, W., Soyhan, **200-202** CEJ (2012) 380

# RTD in Taylor flow

- RTD of one unit cell be modeled as PFR-CSTR in series
- Determining the RTD of multiple unit cells by convoluting the single unit-cell RTD fails
  - Reason: the liquid in a unit cell consists of two regions: by-pass flow and recirculation flow (Thulasidas et al. 1995)



- In absence of diffusion there is no exchange across the dividing streamline → neighboring units cells are not independent
- The size of both regions depends on  $\psi = U_B / J_{\text{total}}$   
for rectangular channels see Kececi, W., Onea, Soyhan; Catalysis Today **147S** (2009) S125
- A reliable RTD model should account for this 



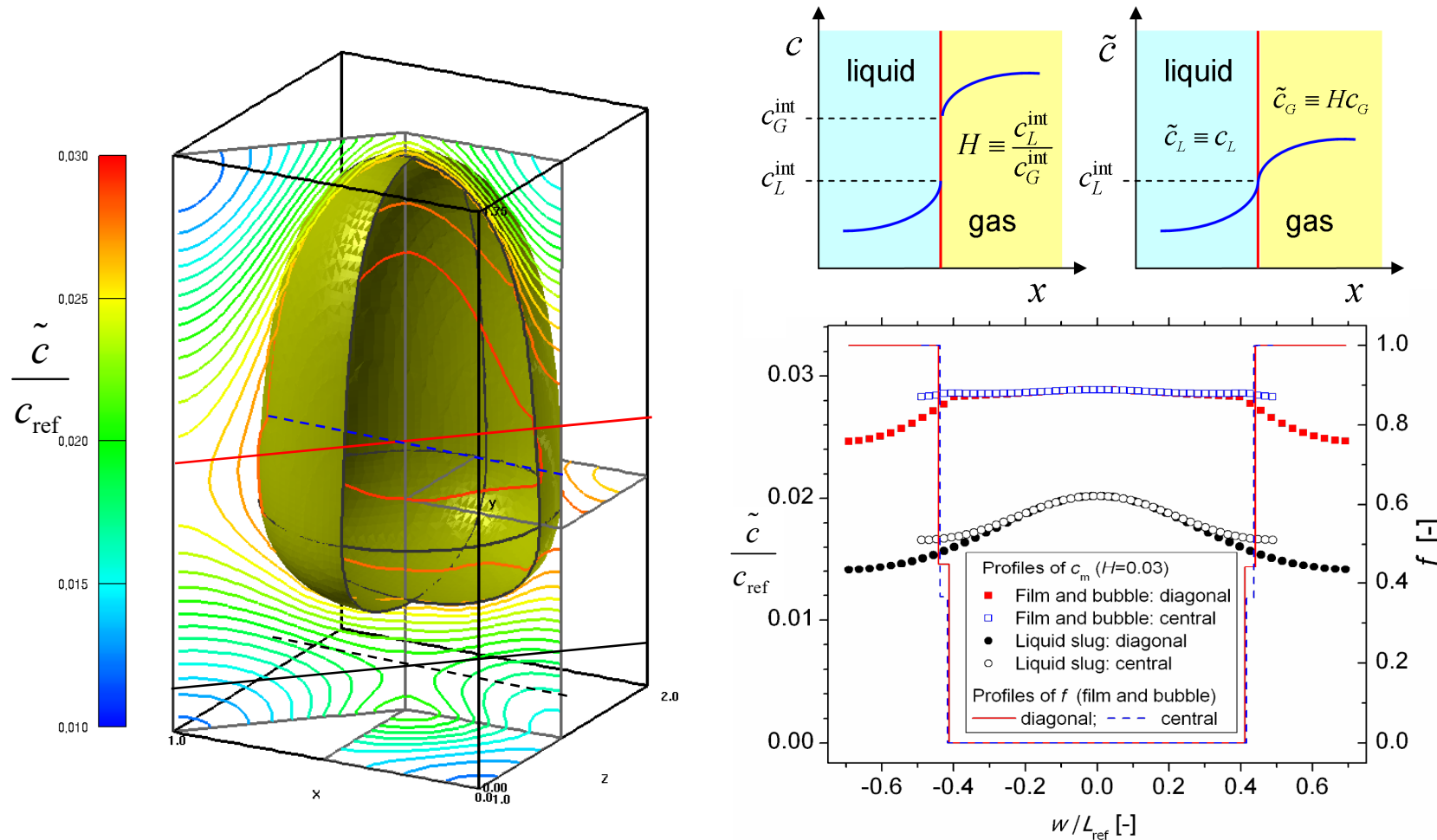
# Outline

- Introduction
- Computational fluid dynamics in micro process engineering
  - Status and developments
- Taylor flow in square capillaries
  - Numerical method and computational setup
  - Insight by scale-resolving simulations
  - Development of engineering models
    - Pressure drop
    - Residence time distribution
- Short survey on further ongoing projects in the group
- Summary

# Gas-liquid mass transfer

■ Transient simulations (“qualitative”)  $Sc_L = v_L/D_L = 0.8$ ,  $H = 0.03$

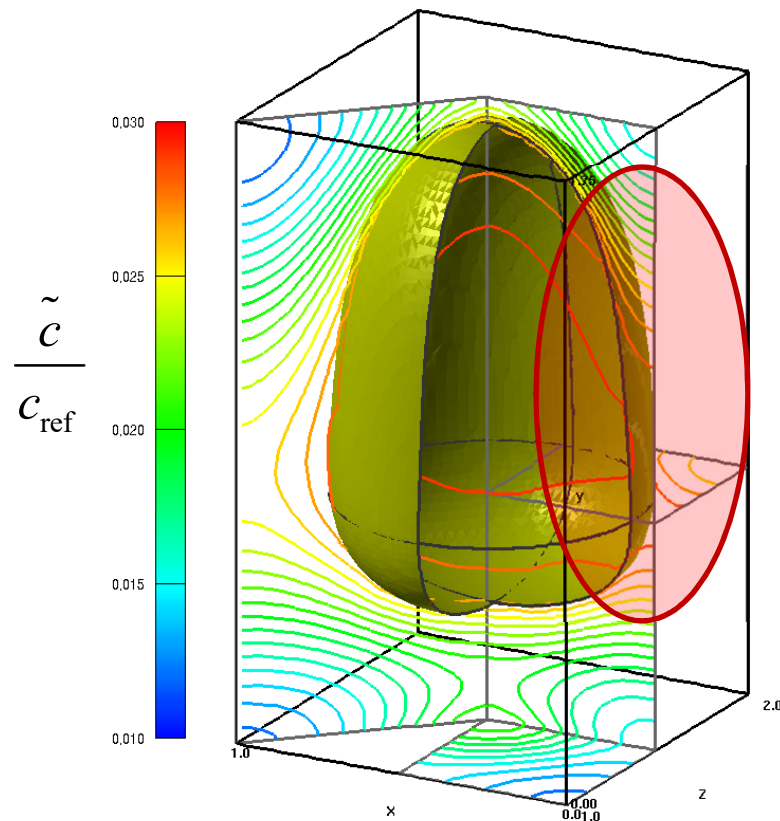
Onea et al. Chem. Eng. Sci. 64 (2009) 1416 – 1435



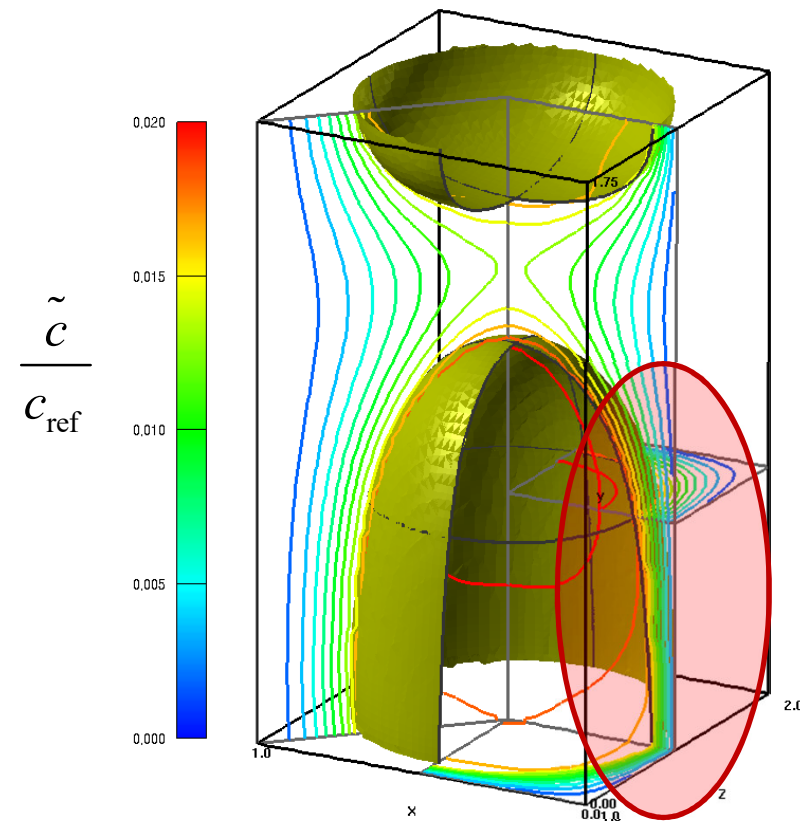
# Mass transfer and chemical reaction

■ Without chemical reaction

■ Fast heterog. reaction (1<sup>st</sup> o.)



Short bubbles are more efficient



Long bubbles are more efficient



# Scale separation in mass transfer

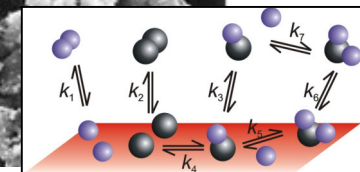
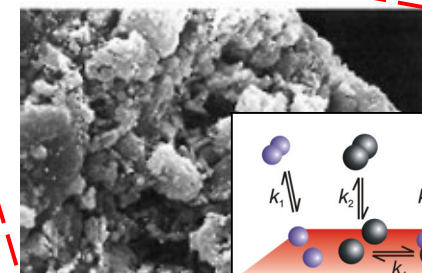
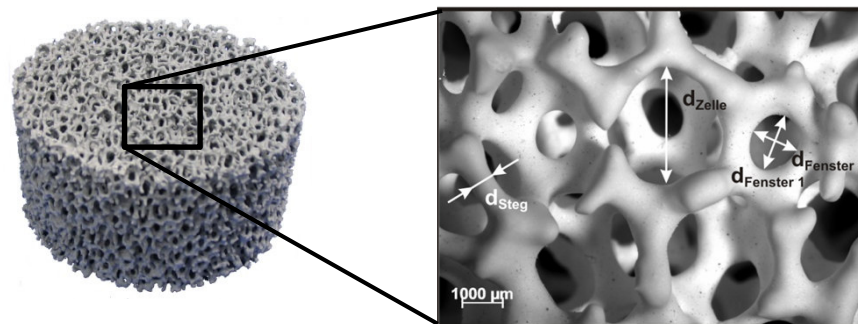
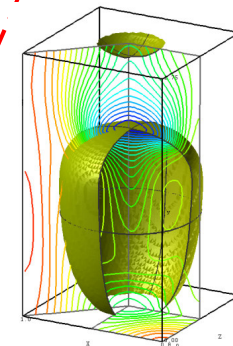
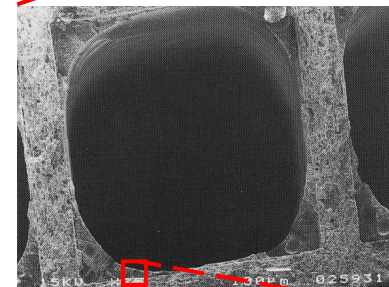
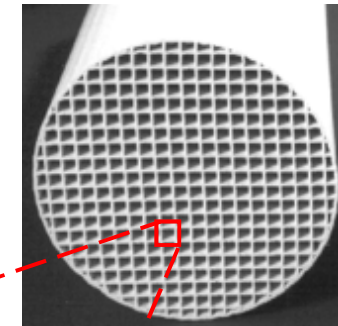
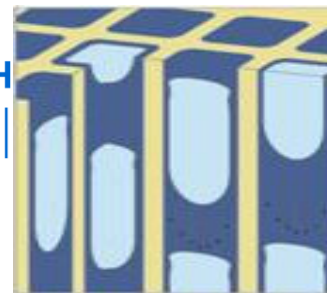
- Ratio of the smallest scales in the concentration and velocity field and the thickness of the boundary layers

$$\frac{\lambda_c}{\lambda_u} \propto \frac{(\nu D^2 / \varepsilon)^{1/4}}{(\nu^3 / \varepsilon)^{1/4}} \propto \sqrt{\frac{D}{\nu}} = \frac{1}{\sqrt{Sc}} \quad \frac{\delta_c}{\delta_u} \propto \frac{L / \sqrt{Pe}}{L / \sqrt{Re}} = \frac{1}{\sqrt{Sc}}$$

- Liquids:  $Sc = O(1000)$  → the grid required to resolve all scales in the concentration field is 30 times finer than for the velocity field (in 3D this corresponds to  $30^3 = 27\,000$ )
- This makes numerical methods that use the same grid for the velocity and concentration field very inefficient  
⇒ use of a hierarchical grid, i.e. a finer grid for the concentration field than for the velocity field (PhD C. Falconi)

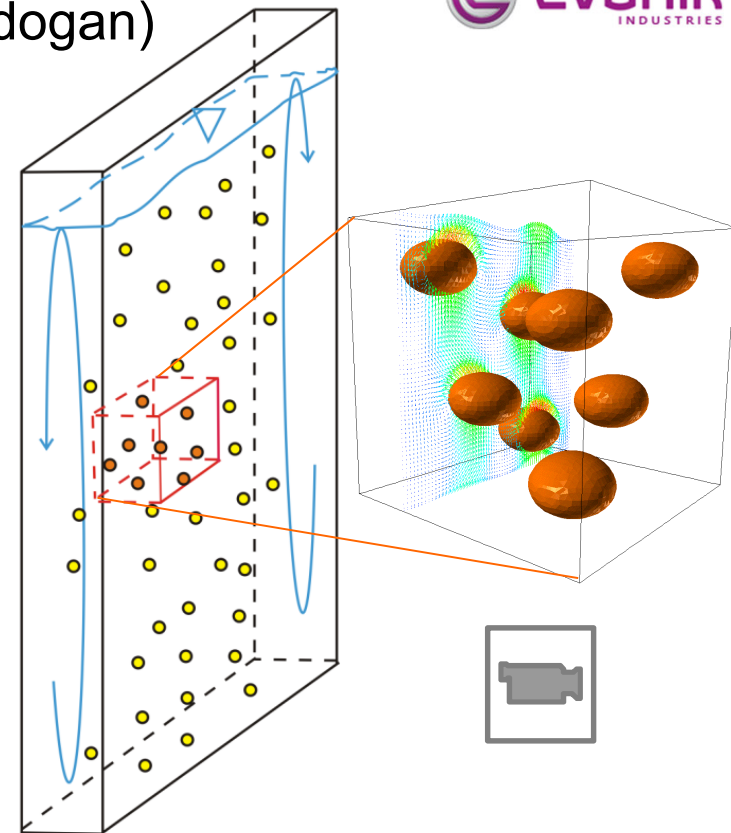
# Helmholtz Alliance „Energy efficient chemical multiphase processes”

- Scale resolving simulations of catalytic multi-phase flows in structured reactors
  - Oxidation of isobutane
  - Hydrogenation of nitrobenzene
- Single channel of a monolith reactor
  - Coupling TURBIT-VOF / DETCHEM
- Portion of a solid sponge
  - Phase field simulations (OpenFOAM)



# DNS of bubble swarm flows

- BMBF Project “Multiscale Modelling of Multiphase Reactors”
  - Development of scale-up strategies that will allow a model based design of industrial scale multiphase reactors
- Contributions of KIT (PhD project of S. Erdogan)
  - Investigate pseudo-turbulence in bubble swarm flows by DNS
  - Evaluation of all terms in the transport equation for liquid phase turbulent kinetic energy from DNS data
  - Analyze, assess and improve engineering models for bubble-induced turbulence in two-fluid model
  - Implementation of improved models in OpenFOAM, validation by experiments of project partners



# Conclusions

- High potential of Taylor flow for intensification of heterogeneously catalyzed gas-liquid processes in microreactors
- Complicate interaction between two-phase flow hydrodynamics, transport of mass and heat, and chemical reaction
- Understanding and quantification of this non-linear interaction (which involves various length and time scales) is a key issue toward the design and optimization of multi-phase micro reactors
- Detailed numerical investigations provide a promising tool for understanding the phenomena and for developing engineering models
- Current status of interface resolving numerical simulation methods
  - Hydrodynamics of two-phase flow including bubble and drop formation ✓
  - *Coalescence phenomena* ✘
  - *Mass transfer (realistic Schmidt numbers  $Sc = 1000$ )* ✘
  - *Interfacial transport of soluble and insoluble surfactants* ✘
  - *Coupling of two-phase flow and transport with detailed chem. kinetics* ✘

# Acknowledgements

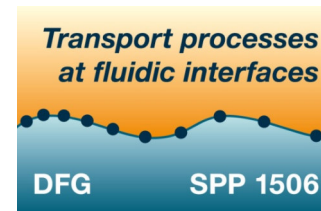
## ■ PhD students

- W. Sabisch (2000)
- B. Ghidersa (2004)
- M. Ilić (2005)
- A. Onea (2007)
- X. Cai, C. Falconi,  
S. Erdogan, M. Woo

## ■ Erasmus students from Sakarya University (Turkey)

- F. Özkan (2006)
- Ö. Keskin (2007)
- M. Öztaskin (2007)
- S. Kececi (2008)
- A. Boran (2010)

## ■ Funding



GEFÖRDERT VOM



Bundesministerium  
für Bildung  
und Forschung

## ■ Cooperation

