

## Theoretical considerations on the energy balance closure

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# Overview

## The energy balance closure problem

- Heterogeneity at the landscape scale
- Possible causes for the imbalance
- Secondary circulations

Heat fluxes and averages

Near-surface energy budget



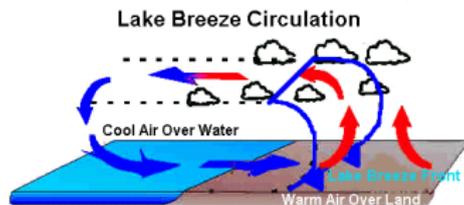
# Heterogeneity affects boundary layer processes

Mesoscale circulations. . . what about smaller scales?

Lake breeze, valley wind

Oasis effect, urban heat island

Leading edge effect



Why has a zebra stripes? (Ruxton, *Mammal Rev* 2002)

Predator/parasite avoidance, social benefits, thermoregulation

“rotary breezes could be created by differential heating”

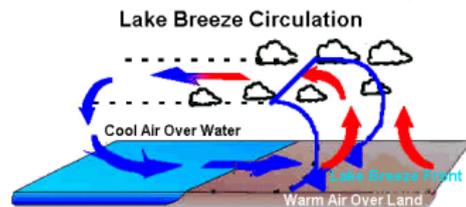
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Of course, we aim for realistic simulations:



How do these patched heterogeneous landscapes influence the surface energy budget?

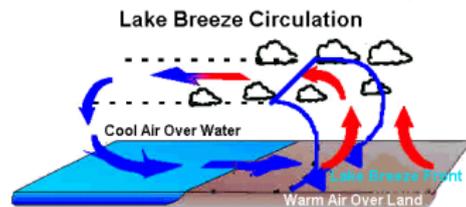
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Turbulent fluxes of latent and sensible heat often add up to only 70–90 % of the available energy

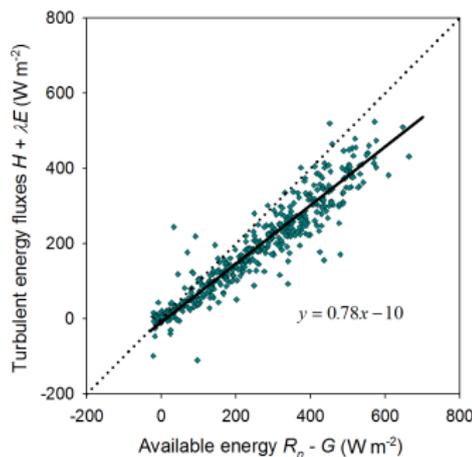
Surface energy budget:  $R_n - G = LE + H$

Turbulent fluxes measured by eddy covariance towers, scintillometry, aircraft data

**Closure problem:** eddy covariance underestimates turbulent fluxes

LSM etc. use surface fluxes as boundary conditions  
Partitioning of missing flux?

(Stoy and Mauder 2011)

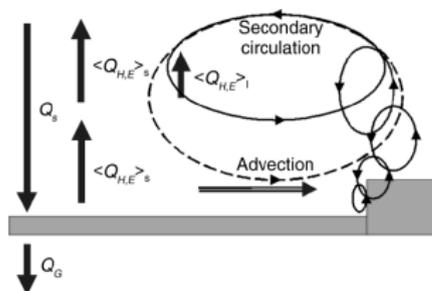


# Possible causes for the underestimation of turbulent fluxes

**Storage terms** give phase lag (Leuning et al '12)

Small remainder of nonclosure possibly from flux-divergence

**Instrumental errors** (Frank et al '12; Kochendorfer et al '13)  
for non-orthogonal sonic anemometers



**Advection effects** (Foken 2008)

quasi-stationary secondary circulations  
in heterogeneous terrain

EC towers cannot capture mean flow

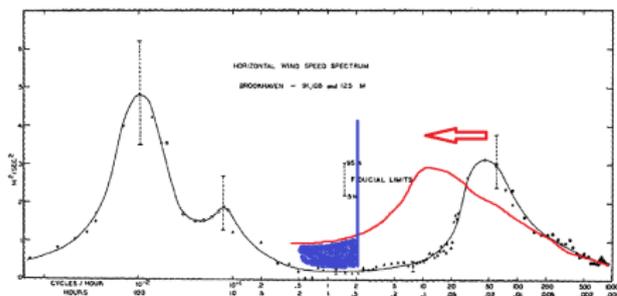
Correlation between terrain characteristics and air circulation  
creates systematic underestimation of the energy budget

# Secondary circulations in heterogeneous terrain

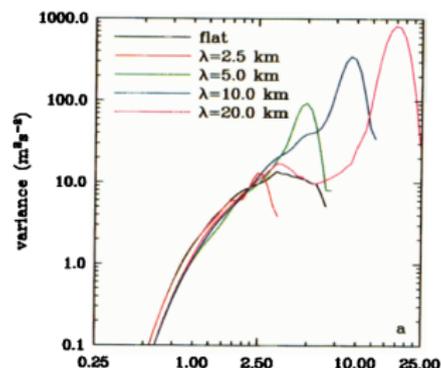
Inagaki et al (2006)  
turbulent mesoscale circulations  
which carry part of the imbalance

Stoy et al (2013)  
correlation between non-closure  
and terrain heterogeneity

Shift from turbulent to mean transport:



Baidya Roy et al (2002):



Nonclosure issues:

- At measurement heights?
- Quantification

# Overview

The energy balance closure problem

Heat fluxes and averages

- Webb, Pearman and Leuning (1980)

Near-surface energy budget

## Heat fluxes as defined by Webb, Pearman & Leuning (1980)

No net vertical transport of dry air:  $\overline{\rho_d w} = 0$

Latent heat flux from mixing ratio:  $\lambda E = \lambda \overline{\rho w' r'}$

$$H = c_{pd} \overline{\rho_d w (T - T_b)} + c_{pv} \overline{\rho_v w (T - T_b)} \approx c_p \bar{\rho} \overline{w' T'}$$

*“Here  $T_b$ , taken as constant at any given height, represents roughly an assumed initial “base” temperature from which each element of air is warmed (or cooled) during the vertical transfer of heat supplied (or removed) at the underlying surface. Even though  $T_b$  is not amenable to precise specification, it is included because the heat imparted to and carried by each parcel of air is represented by the temperature change,  $T - T_b$ , not the temperature  $T$  itself.”*

$T_b$  drops out in homogeneous terrain, what when heterogeneous?

# Overview

The energy balance closure problem

Heat fluxes and averages

Near-surface energy budget

- Expression from first principle
- Base temperature and base humidity

# Energy balance closure in a formula

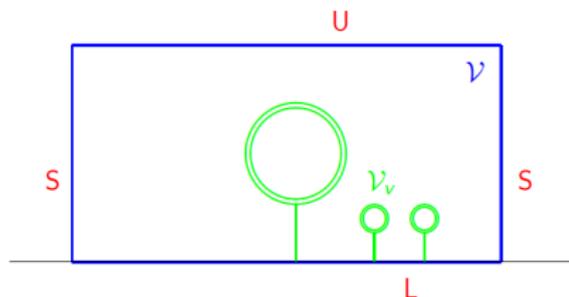
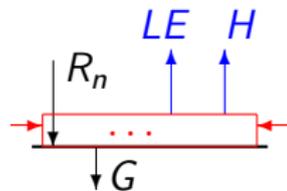
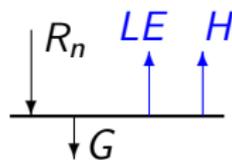
Commonly:  $R_n - G = LE + H$

Yet experiments find  $R_n - G \geq LE + H$

True expression **above the surface**

$$R_n - G = LE_{(t)} + H_{(t)} + \Delta$$

Additional advection and accumulation terms



Control volume (no air parcels!)

Air + canopy

Energy conservation

Boussinesq approximation

Surface layer (EC towers)

## An expression for the (time-averaged) imbalance $\Delta$

$$H_m + \lambda E_m$$

mean upward fluxes

$$H_{\parallel} + \lambda E_{\parallel}$$

laterally advected fluxes

$$\frac{1}{\delta t} \left[ \int \rho c_v T dz \right]_t^{t+\delta t}$$

thermal energy accumulation

$$gz_v E_v$$

potential energy accumulation

$$\frac{1}{\delta t} [K_v]_t^{t+\delta t}$$

kinetic energy accumulation

$$-\int \mathbf{v} \cdot \nabla p dz$$

minor rest term

$$L_p (F_{pg} - F_p - F_{p\parallel})$$

CO<sub>2</sub> flux due to photosynthesis

$$\frac{1}{\delta t} \left[ \int \rho (\lambda q - L_p q_p) dz \right]_t^{t+\delta t}$$

water and CO<sub>2</sub> accumulation

Necessary to estimate magnitude of these terms, with:

$$\bar{w} \sim 1 \text{ cm/s} ; u \sim 4 \text{ m/s} ; \nabla p \sim 0.1 \text{ Pa/m}$$

$$z_m \sim 2 \text{ m} ; \delta T / \delta t \sim 2 \text{ K/hr} ; \bar{q} \sim 3 \text{ g/m}^3$$

Advection terms largely cancel each other out  
but leave a remainder of the order of the imbalance

$\frac{1}{\delta t} \left[ \int \rho c_v T dz \right]_t^{t+\delta t}$	1 W/m <sup>2</sup> in air ~ 50 W/m <sup>2</sup> (Leuning et al 2012)
$gz_v E_v$	10 <sup>-5</sup> λE (cf Oncley et al 2007)
$\frac{1}{\delta t} [K_v]_t^{t+\delta t}$	same order as next term
$-\overline{\int \mathbf{v} \cdot \nabla p dz}$	1 W/m <sup>2</sup>
$L_p \left( F_{pg} - F_p - F_{p\parallel} \right.$	10% (R <sub>n</sub> - G) under very productive
$\left. - \frac{1}{\delta t} \left[ \int \rho q_p dz \right]_t^{t+\delta t} \right)$	circumstances (Meyers & Hollinger 2004)
$\frac{1}{\delta t} \left[ \int \rho \lambda q dz \right]_t^{t+\delta t}$	0.6 W/m <sup>2</sup>

$H_m \sim 4000 \text{ W/m}^2$  ;  $\lambda E_m \sim 80 \text{ W/m}^2$  ;  $H_{\parallel} + \lambda E_{\parallel}$   
 Simulations:  $H. \sim 10^4 \text{ W/m}^2$  ;  $H_m + H_{\parallel} \sim 10 - 10^2 \text{ W/m}^2$

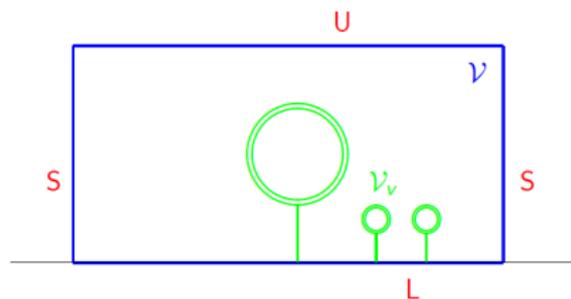
## The base temperature incorporates advection effects

Sum of sensible heatflux through upper boundary and advected flux

$$\mathcal{H} = \int_U \rho c_p T (\mathbf{v} \cdot d\mathbf{A}) + \int_S \rho c_p T (\mathbf{v} \cdot d\mathbf{A})$$

Rewrite as

$$\mathcal{H} = \int_U \rho c_p (T - T_b) (\mathbf{v} \cdot d\mathbf{A})$$



with base temperature:

$$T_b = - \frac{\int_S \rho c_p T (\mathbf{v} \cdot d\mathbf{A})}{\int_U \rho c_p (\mathbf{v} \cdot d\mathbf{A})}$$

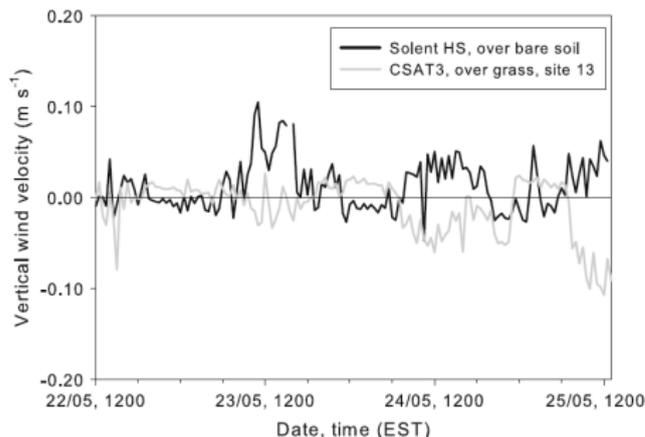
From conservation of air and incompressibility:  $T_b \approx \langle T \rangle_S$

Similar procedure for base humidity  $q_b$

# Advection effects important when secondary air circulation driven by local temperature differences

$\mathcal{H} = \overline{\rho c_p w (T - T_b)}$  means correlation of

- vertical wind
- temperature difference between upward and advected air



Difficult in practice

cf. Mauder et al ('08, '10)

- spatiotemporal average over nearby stations
- better results for sensible than for latent heat

From simulation:  $\text{Var}[q_b(t)]$  stronger than  $\text{Var}[T_b(t)]$

$T_b(t)$  different from constant  $T_b$  in WPL  $\Rightarrow w' T_b'$

# Theoretical considerations on the energy balance closure

Additional terms appearing in **near-surface energy budget**  
Importance of advection and storage

Interpretation to the base temperature of WPL (1980)  
Average temperature over lateral sides of control-volume  
Allows – in principle – to account for advection effects

**Secondary circulations in heterogeneous terrain**  
as a cause for the **non-closure** of the energy balance  
especially when storage/NPP is low

# Overview

The energy balance closure problem

Heat fluxes and averages

- Webb, Pearman and Leuning (1980)
- Kowalski (2012)

Near-surface energy budget

## Averaging procedures: a matter of taste?

Kowalski (2012) seeks alternative “correct” averages that satisfy physical laws without corrections

*“Boundary layer meteorology [...] clearly suffers from a grave and persistent fault. The inability to close the surface energy budget [...] suggests possible errors in basic methodology, within which accurate averaging procedures are critical.”*

*“For studies of eddy transport, and micrometeorology in particular, [...] imprecisely determined averages of state and flow variables bias the perturbation variables over the entire averaging domain and thereby skew estimates of mass, heat, and momentum exchange.”*

For example, ideal gas law (e.g. Stull '88):  $\bar{p} = \mathcal{R}\bar{\rho}\bar{T} + \mathcal{R}\overline{\rho'T'}$

Define  $\tilde{T} = \bar{T} + \overline{\rho'T'}/\bar{\rho}$  such that  $\bar{p} = \mathcal{R}\bar{\rho}\tilde{T}$

## Averaging procedures: a matter of taste?

Kowalski raises valid remarks about sample & ensemble means  
BUT

(1) Impossible to satisfy multiple laws/definitions at once

$$\tilde{T} = \bar{T} + \frac{\overline{\rho' T'}}{\bar{\rho}} \neq \bar{T} + \frac{\overline{(\rho c_p)' T'}}{(\rho c_p)}$$

=> corrections always needed

When corrections are taken into account  
traditional averages remain equally valid

(2) Necessary to go beyond Boussinesq approximation?

Otherwise only triple correlation  $H = c_p \overline{\rho' T' w'}$  +  $q$ -terms