Theoretical considerations on the energy balance closure

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Overview

The energy balance closure problem

- Heterogeneity at the landscape scale
- Possible causes for the imbalance
- Secondary circulations

Heat fluxes and averages

Near-surface energy budget
Heterogeneity affects boundary layer processes

Mesoscale circulations...what about smaller scales?

Lake breeze, valley wind
Oasis effect, urban heat island
Leading edge effect

In simulations, often idealized heterogeneities:

Chessboards and zebra patterns
Heterogeneity affects boundary layer processes

Mesoscale circulations...what about smaller scales?

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Why has a zebra stripes? (Ruxton, *Mammal Rev 2002*)
Predator/parasite avoidance, social benefits, thermoregulation
“rotary breezes could be created by differential heating”
Heterogeneity affects boundary layer processes

Mesoscale circulations…what about smaller scales?

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Of course, we aim for realistic simulations:

How do these patched heterogeneous landscapes influence the surface energy budget?
Heterogeneity affects boundary layer processes

Mesoscale circulations... what about smaller scales?

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Of course, we aim for realistic simulations:

How do these patched heterogeneous landscapes influence the surface energy budget?
Turbulent fluxes of latent and sensible heat often add up to only 70–90 % of the available energy.

Surface energy budget: \[ R_n - G = LE + H \]

Turbulent fluxes measured by eddy covariance towers, scintillometry, aircraft data

Closure problem: eddy covariance underestimates turbulent fluxes

LSM etc. use surface fluxes as boundary conditions

Partitioning of missing flux?

(Stoy and Mauder 2011)
Possible causes for the underestimation of turbulent fluxes

Storage terms give phase lag (Leuning et al '12)  
Small remainder of nonclosure possibly from flux-divergence

Instrumental errors (Frank et al '12; Kochendorfer et al '13)  
for non-orthogonal sonic anemometers

Advection effects (Foken 2008)  
quasi-stationary secondary circulations  
in heterogeneous terrain  
EC towers cannot capture mean flow

Correlation between terrain characteristics and air circulation  
creates systematic underestimation of the energy budget
Secondary circulations in heterogeneous terrain

turbulent mesoscale circulations which carry part of the imbalance

Stoy et al (2013)
correlation between non-closure and terrain heterogeneity

Shift from turbulent to mean transport:


Nonclosure issues:
• At measurement heights?
• Quantification
Overview

The energy balance closure problem

Heat fluxes and averages


Near-surface energy budget
Heat fluxes as defined by Webb, Pearman & Leuning (1980)

No net vertical transport of dry air: \( \overline{\rho_d w} = 0 \)

Latent heat flux from mixing ratio: \( \lambda E = \lambda \overline{\rho w' r'} \)

\[
H = c_{pd} \overline{\rho_d w (T - T_b)} + c_{pv} \overline{\rho_v w (T - T_b)} \approx c_p \overline{\rho w' T'}
\]

“Here \( T_b \), taken as constant at any given height, represents roughly an assumed initial “base” temperature from which each element of air is warmed (or cooled) during the vertical transfer of heat supplied (or removed) at the underlying surface. Even though \( T_b \) is not amenable to precise specification, it is included because the heat imparted to and carried by each parcel of air is represented by the temperature change, \( T - T_b \), not the temperature \( T \) itself.”

\( T_b \) drops out in homogeneous terrain, what when heterogeneous?
Overview

The energy balance closure problem

Heat fluxes and averages

Near-surface energy budget

- Expression from first principle
- Base temperature and base humidity
Energy balance closure in a formula

Commonly: \( R_n - G = LE + H \)

Yet experiments find \( R_n - G \geq LE + H \)

True expression above the surface

\[
R_n - G = LE(t) + H(t) + \Delta
\]

Additional advection and accumulation terms

Control volume (no air parcels!)
Air + canopy
Energy conservation
Boussinesq approximation
Surface layer (EC towers)
An expression for the (time-averaged) imbalance $\Delta$

\[
H_m + \lambda E_m \\
H_{\parallel} + \lambda E_{\parallel} \\
\frac{1}{\delta t} \left[ \int \rho c_v T \, dz \right]^{t+\delta t}_t \\
g z_v E_v \\
\frac{1}{\delta t} \left[ K_v \right]^{t+\delta t}_t \\
- \int \mathbf{v} \cdot \nabla p \, dz \\
L_p \left( F_{pg} - F_p - F_{p\parallel} \right) \\
\frac{1}{\delta t} \left[ \int \rho \left( \lambda q - L_p q_p \right) \, dz \right]^{t+\delta t}_t \\
\text{mean upward fluxes} \\
\text{laterally advected fluxes} \\
\text{thermal energy accumulation} \\
\text{potential energy accumulation} \\
\text{kinetic energy accumulation} \\
\text{minor rest term} \\
\text{CO}_2 \text{ flux due to photosynthesis} \\
\text{water and CO}_2 \text{ accumulation}
\]

Necessary to estimate magnitude of these terms, with:

$\bar{w} \sim 1 \text{ cm/s } ; u \sim 4 \text{ m/s } ; \nabla p \sim 0.1 \text{ Pa/m}$
$z_m \sim 2 \text{ m } ; \delta T/\delta t \sim 2 \text{ K/hr } ; \bar{q} \sim 3 \text{ g/m}^3$
Advection terms largely cancel each other out but leave a remainder of the order of the imbalance

\[
\frac{1}{\delta t} \left[ \int \rho c_v \, T \, dz \right]_{t+\delta t} - \frac{1}{\delta t} \left[ K_v \right]_t - \int v \cdot \nabla p \, dz + \frac{1}{\delta t} \left[ \int \rho q_p \, dz \right]_{t+\delta t} + \frac{1}{\delta t} \left[ \int \rho \lambda q \, dz \right]_{t+\delta t}
\]

1 W/m² in air

\sim 50 W/m² (Leuning et al 2012)

10⁻⁵ λE (cf Oncley et al 2007)

same order as next term

1 W/m²

gz_v E_v

L_p \left( F_{pg} - F_p - F_{p\parallel} \right)

10% (R_n - G) under very productive circumstances (Meyers & Hollinger 2004)

0.6 W/m²

\[ H_m \sim 4000 \text{ W/m}^2 ; \lambda E_m \sim 80 \text{ W/m}^2 ; H_{\parallel} + \lambda E_{\parallel} \]

Simulations: \[ H_{\parallel} \sim 10^4 \text{ W/m}^2 ; H_m + H_{\parallel} \sim 10 - 10^2 \text{ W/m}^2 \]
The base temperature incorporates advection effects

Sum of sensible heatflux through upper boundary and advected flux

\[ \mathcal{H} = \int_U \rho c_p T (v \cdot dA) + \int_S \rho c_p T (v \cdot dA) \]

Rewrite as

\[ \mathcal{H} = \int_U \rho c_p (T - T_b) (v \cdot dA) \]

with base temperature:

\[ T_b = -\frac{\int_S \rho c_p T (v \cdot dA)}{\int_U \rho c_p (v \cdot dA)} \]

From conservation of air and incompressibility: \( T_b \approx \langle T \rangle_S \)

Similar procedure for base humidity \( q_b \)
Advection effects important when secondary air circulation driven by local temperature differences

\[ \mathcal{H} = \rho c_p w (T - T_b) \]

means correlation of

- vertical wind
- temperature difference between upward and advected air

**Difficult in practice**

- spatiotemporal average over nearby stations
- better results for sensible than for latent heat

From simulation: \( \text{Var}[q_b(t)] \) stronger than \( \text{Var}[T_b(t)] \)

\( T_b(t) \) different from constant \( T_b \) in WPL \( \Rightarrow w' T'_b \)
Theoretical considerations on the energy balance closure

Additional terms appearing in near-surface energy budget
Importance of advection and storage

Interpretation to the base temperature of WPL (1980)
Average temperature over lateral sides of control-volume
Allows – in principle – to account for advection effects

Secondary circulations in heterogeneous terrain
as a cause for the non-closure of the energy balance
especially when storage/NPP is low
Overview

The energy balance closure problem

Heat fluxes and averages

- Kowalski (2012)

Near-surface energy budget
Averaging procedures: a matter of taste?

Kowalski (2012) seeks alternative “correct” averages that satisfy physical laws without corrections

“Boundary layer meteorology [...] clearly suffers from a grave and persistent fault. The inability to close the surface energy budget [...] suggests possible errors in basic methodology, within which accurate averaging procedures are critical.”

“For studies of eddy transport, and micrometeorology in particular, [...] imprecisely determined averages of state and flow variables bias the perturbation variables over the entire averaging domain and thereby skew estimates of mass, heat, and momentum exchange.”

For example, ideal gas law (e.g. Stull ’88): \( \bar{p} = R \bar{\rho} \bar{T} + R \bar{\rho'} \bar{T}' \)

Define \( \tilde{T} = \bar{T} + \bar{\rho'} \bar{T}' / \bar{\rho} \) such that \( \bar{p} = R \bar{\rho} \tilde{T} \)
Averaging procedures: a matter of taste?

Kowalski raises valid remarks about sample & ensemble means

BUT

(1) Impossible to satisfy multiple laws/definitions at once

$$\tilde{T} = \bar{T} + \frac{\rho' \bar{T'}}{\bar{\rho}} \neq \bar{T} + \frac{(\rho c_p)' \bar{T}'}{(\rho c_p)}$$

=> corrections always needed

When corrections are taken into account
traditional averages remain equally valid

(2) Necessary to go beyond Boussinesq approximation?
Otherwise only triple correlation $H = c_p \rho' \bar{T}' w' + q$-terms