Analyse und Verständnis lokaler drei-dimensionaler Effekte bei der Taylor-Strömung in einem quadratischen Mini-Kanal

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Outline
- Introduction
- Experiment
- Numerical simulations
- Results
- Discussion
- Conclusions
Guiding measure Taylor flow

- Taylor flow in millimeter size channels is of practical technical relevance and of fundamental physical interest
- **Goal:** Provide detailed experimental data under well controlled conditions which allow for a quantitative validation of numerical methods and computer codes
- Three cases from two experimental groups
  - **TBCC = Taylor Bubble Circular Channel**
    - Boden et al. Exp Fluids 55 (2014) 1
  - **TBSC = Taylor Bubble Square Channel**
    - Boden et al. Exp Fluids 55 (2014) 1
    - Marschall et al. Comp Fluids 102 (2014) 336
  - **TFSC = Taylor Flow Square Channel**
    - Falconi et al. Phys Fluids submitted

Taylor flow experiment

**Geometry** Square vertical mini-channel (2.1 mm side length)

**Flow direction** Co-current upward

**Gas phase** Air

**Liquid phase** Water/glycerol mixture

**Gas volume fraction** 0.37±0.02

**Bubble length** 3.26±0.18 mm

**Liquid slug length** 1.33±0.09 mm

**Unit cell length** 4.59±0.27 mm

**Bubble velocity** 135.9±1.9 mm/s

**Capillary number** 0.1 \( Ca = U_B \mu / \sigma \)

**Reynolds number** 7.0 \( Re = \rho D h U_B / \mu \)

**Measurements** Velocity field in liquid slug and liquid film (µPIV)

Estimation of bubble shape from µPIV observations

This very regular Taylor flow is realized by a special injection valve combined with a compensation pipe for eliminating pressure fluctuations

Experiment

Refractive index matching

Equations and computer codes

- Single-field formulation for two immiscible Newtonian fluids with constant density, viscosity and surface tension (sharp interface limit)

\[ \nabla \cdot \mathbf{u} = 0, \quad \partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \nabla \cdot \eta \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) + \rho \mathbf{g} + \sigma \kappa \mathbf{n} \nabla \chi \]

- Three academic in-house codes for 3D interface capturing
  - FS3D (ITLR Stuttgart, TU Darmstadt)
    - Volume-of-Fluid method (PLIC geometrical reconstruction, split advection)
    - Finite volume discretization on fixed staggered Cartesian grid
    - Advanced surface tension force model (balanced CSF)
  - TURBIT-VOF (KIT)
    - Similar to FS3D but un-split advection and less advanced surface tension model
  - DROPS (RWTH Aachen)
    - Level-set method with reinitialization
    - Finite element method with adaptive multilevel mesh hierarchy
    - Laplace Beltrami technique for surface tension, discontinuous pressure field

Marschall et al., Computers & Fluids 102 (2014) 336–352
Computational set-up

From experiment
- Side length $D_h = 2.076$ mm
- Unit cell length $L_{UC} = 4.59$ mm
- Gas volume fraction $\varepsilon = 0.375$

Target value for simulations
- Experimental bubble velocity
- Simulations are transient but only quasi-steady results are analyzed here

Physical properties

<table>
<thead>
<tr>
<th></th>
<th>Liquid</th>
<th>Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>1197.2</td>
<td>1.3</td>
</tr>
<tr>
<td>Viscosity</td>
<td>0.0481</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>Surface tension</td>
<td>0.0624</td>
<td></td>
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</table>

Grid and integral simulation results

<table>
<thead>
<tr>
<th>Domain</th>
<th>FS3D</th>
<th>TUV $V_{UC}/4$</th>
<th>DROPS $V_{UC}/8$</th>
<th>Dev$_{v} [%]$</th>
<th>Exp.</th>
<th>Dev$_{i} [%]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grid* (for $V_{UC}$)</td>
<td>64x64x128</td>
<td>100x100x220</td>
<td>64x64x128</td>
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<tr>
<td>$U_B$ [mm/s]</td>
<td>135.9</td>
<td>135.9</td>
<td>135.0</td>
<td>0.4</td>
<td>135.9±1.9</td>
<td>0.2</td>
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<tr>
<td>$L_B$ [mm]</td>
<td>3.355</td>
<td>3.328</td>
<td>3.329</td>
<td>0.5</td>
<td>3.26±0.18</td>
<td>2.4</td>
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<tr>
<td>$L_S$ [mm]</td>
<td>1.235</td>
<td>1.262</td>
<td>1.261</td>
<td>1.4</td>
<td>1.33±0.09</td>
<td>5.8</td>
</tr>
<tr>
<td>$\Delta p/L_{UC}$ [Pa]</td>
<td>117.8</td>
<td>119.2</td>
<td>121.2</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J$ [mm$^3$/s]</td>
<td>377.2</td>
<td>391.6</td>
<td>382.6</td>
<td>2.0</td>
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<td></td>
</tr>
</tbody>
</table>

$J = $ total superficial velocity (volumetric flow rate of the two-phase flow)

*Grid resolution corresponds to about 3 mesh cells per lateral film width (smallest length scale of the flow)

$$\text{Dev}_{v} = \frac{\max_{i,j,k} |C_{v_{CFD}} - C_{v_{Exp}}|}{C_{v_{max}}}$$

$$\text{Dev}_{i} = \frac{|C_{i_{Exp}} - C_{i_{CFD}}|}{C_{i_{Exp}}}$$
### Bubble shape

![Diagram of bubble shape](image)

\[
\frac{Z_B}{L_B} \equiv \frac{Z_B}{L_B} \\
\frac{Z_S}{L_S} \equiv \frac{Z_S}{L_S}
\]

---

### Quantitative comparison of bubble shape

<table>
<thead>
<tr>
<th>Cut</th>
<th>( z_B/L_B )</th>
<th>FS3D</th>
<th>TUV</th>
<th>DROPS</th>
<th>Dev. [%]</th>
<th>Exp.</th>
<th>Dev. [%]</th>
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<tbody>
<tr>
<td>L</td>
<td>0.25</td>
<td>952</td>
<td>961</td>
<td>960</td>
<td>0.6</td>
<td>946±8</td>
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<td></td>
<td>0.5</td>
<td>918</td>
<td>925</td>
<td>922</td>
<td>0.4</td>
<td>940±8</td>
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<td></td>
<td>0.75</td>
<td>824</td>
<td>825</td>
<td>824</td>
<td>0.1</td>
<td>847±8</td>
<td>2.7</td>
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<tr>
<td>D</td>
<td>0.25</td>
<td>983</td>
<td>991</td>
<td>993</td>
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<td>949</td>
<td>953</td>
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<td>845</td>
<td>839</td>
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<tr>
<td>L</td>
<td>0.25</td>
<td>85</td>
<td>77</td>
<td>78</td>
<td>0.1</td>
<td>92±8</td>
<td>12.7</td>
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<td></td>
<td>0.5</td>
<td>120</td>
<td>113</td>
<td>116</td>
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<td>98±8</td>
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<td></td>
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<td>214</td>
<td>213</td>
<td>214</td>
<td>0.3</td>
<td>18±1</td>
<td>11.9</td>
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<tr>
<td>D</td>
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<td>485</td>
<td>477</td>
<td>475</td>
<td>1.3</td>
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<td></td>
<td>0.5</td>
<td>519</td>
<td>515</td>
<td>514</td>
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<td>623</td>
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<tr>
<td>L</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>Extreme value</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td>( R_{BL,\max} )</td>
<td>959</td>
<td>973</td>
<td>967</td>
<td>0.8</td>
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<tr>
<td></td>
<td>( \delta_{BL,\max} )</td>
<td>79</td>
<td>65</td>
<td>71</td>
<td>10.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( z_B/L_B )</td>
<td>0.273</td>
<td>0.270</td>
<td>0.275</td>
<td>0.3</td>
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<td></td>
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<tr>
<td></td>
<td>( R_{BL,\max-D} )</td>
<td>985</td>
<td>991</td>
<td>994</td>
<td>0.5</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>( \delta_{BL,\max-D} )</td>
<td>483</td>
<td>477</td>
<td>474</td>
<td>1.1</td>
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</table>
Velocity profiles in liquid slug

Velocity profiles in bubble and film
Quantitative comparison of velocity profiles

Centerline axial velocity

<table>
<thead>
<tr>
<th>$z/S$</th>
<th>FS3D</th>
<th>TUV</th>
<th>DROPS</th>
<th>Dev. [%]</th>
<th>Exp.</th>
<th>Dev. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid slug [mm/s]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td>146.4</td>
<td>152.7</td>
<td>146.1</td>
<td>2.9</td>
<td>141.3</td>
<td>5.0</td>
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<tr>
<td>0.5</td>
<td>169.0</td>
<td>174.8</td>
<td>171.2</td>
<td>1.8</td>
<td>178.3</td>
<td>3.7</td>
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<tr>
<td>0.1</td>
<td>148.9</td>
<td>144.9</td>
<td>149.0</td>
<td>1.8</td>
<td>151.2</td>
<td>2.4</td>
</tr>
<tr>
<td>$z/B$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bubble [mm/s]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>199.0</td>
<td>192.8</td>
<td>211.4</td>
<td>5.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>270.9</td>
<td>264.0</td>
<td>284.7</td>
<td>4.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>250.6</td>
<td>245.0</td>
<td>250.6</td>
<td>1.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Flow field (Results of FS3D)

- Streamlines (moving ref. frame)
  - Gas phase
    - One main vortex
    - Two counter-rotating vortices at bubble nose and rear
    - Positions of center of main vortex is different in lateral and diagonal cut → flow in bubble is three-dimensional
  - Liquid phase
    - Region with circulation flow (channel center) and bypass flow (near wall) are separated by the dividing streamline
  - Vertical velocity $u_z$ (fixed frame)
    - Regions with negative velocity in rear part of the liquid film → local backflow

Flow field diagram showing streamlines and velocity profiles.
Local backflow in liquid film

Isosurface $u_z = -0.016 \text{ m/s}$

Local backflow in co-current flow

- Local backflow results in a temporal inversion of the wall shear stress at a fixed position during passage of a Taylor bubble
- Important for various applications with Taylor flow
  - Heat and mass transfer
  - Cleaning of membranes for ultrafiltration
  - Synthesis of nanoparticles
  - Biological and medical applications with living cells

Literature status
- The phenomenon of possible shear stress reversal is known for some time
- Local backflow in co-current Taylor flow has not been measured before
- It has been found in computations for circular tubes
  - S.P. Quan, Co-current flow effects on a rising Taylor bubble, IJMF 37 (2011) 888

Questions related to present results for square channel
- Why differ axial locations with backflow in lateral film and corner region?
- Can we give criteria when this newly discovered phenomenon occurs?
Sufficient condition for backflow

Liquid mass balance in moving frame of reference yields

\[ \dot{V}_{l,infr} = (J - U_B)A = \left[U_T(z_B) - U_B\right]A_f(z_B) \]

From liquid mass balance it follows

\[ \frac{U_f(z_B)}{J} = \frac{1 - \frac{U_B A_f(z_B)}{J A}}{1 - \frac{A_f(z_B)}{A}} \Rightarrow \text{backflow occurs where} \ \frac{U_B A_f(z_B)}{J A} > 1 \]

For an axisymmetric bubble it is

\[ U_f(z_B) = \frac{J - U_B}{4} \frac{\pi D^2_B(z_B)}{D_h} \]

This relation can be used to compute the mean velocity in the liquid film, \( U_f \), from the axial profile of the bubble diameter.
Evaluation from L/D bubble profiles

- Predicted regions with backflow differ as observed
- L/D profiles differ → bubble is not axisymmetric. Why?

Bubble aspect ratio

- Minimum diagonal film thickness
- Minimum lateral film thickness
Pressure field (Results of TURBIT-VOF)

- Non-dimensional (periodic) pressure field in two horizontal cross-sections
- In front part of the bubble the pressure is higher in the lateral film than in the corner film
- In the rear part of the film it is opposite

Force balance normal to the interface

- Local force balance normal to the interface in a horizontal cross-section

\[ -(p_L - p_G) + \left[ \mu_L (\nabla v_L + \nabla v_L^T) - \mu_G (\nabla v_G + \nabla v_G^T) \right] \hat{n}_L \cdot \hat{n}_L = 2H\sigma \]

- Neglecting normal viscous stresses \( \Rightarrow \) the pressure jump is locally balanced by capillary forces

\[ p_G - p_L \approx 2H\sigma \quad \sigma = \text{const.} \quad p_G = p_B \approx \text{const.} \]

- For the given azimuthal variation of \( p_L \) this can only be achieved by a change of the local interface curvature

\[ H = \frac{1}{2} \left( \frac{1}{R_{\min}} + \frac{1}{R_{\max}} \right) \approx \frac{p_B - p_L}{2\sigma} \]

- This causes the deviation of the bubble shape from rotational symmetry
Refined liquid mass balance analysis

Splitting the liquid flux into two parts through lateral and diagonal film

\[(J - U_B)\, A = [U_{F,L}(z_B) - U_B] \, A_{F,L}(z_B) + [U_{F,D}(z_B) - U_B] \, A_{F,D}(z_B)\]

We are interested in the position where the axial velocity in L/D film is minimal \(\rightarrow\) derivative with respect to \(z_B\) gives

\[A_{F,L} \frac{\partial U_{F,L}}{\partial z_B} + A_{F,D} \frac{\partial U_{F,D}}{\partial z_B} = \left[U_B - U_{F,D}(z_B)\right] \frac{\partial A_{F,D}}{\partial z_B} + \left[U_B - U_{F,L}(z_B)\right] \frac{\partial A_{F,L}}{\partial z_B}\]

\[A_{F,D} \frac{\partial U_{F,D}}{\partial z_B} = U_B \frac{\partial A_L}{\partial z_B} - A_{F,L} \frac{\partial U_{F,L}}{\partial z_B}\]

\[A_f = A_{F,L} + A_{F,D}\]

Region I \(>0\) \(>0\) \(>0\)

Region II \(>0\) \(=0\) \(>0\)

Region III \(<0\) \(<0\) \(<0\)

In the lower part of region II the derivatives have opposite signs so that the position of velocity minimum in the L and D film differ.

Thinnest lateral film

Thinnest diagonal film

\(\text{cf. local profiles of } u_z\)
Approximate criteria for backflow

- Backflow occurs for
  \[ \frac{U_B}{J} \frac{A_B}{A} > 1 \]
  Both ratios depend on the capillary number
  \[ Ca \uparrow \frac{U_B}{J} \uparrow \frac{A_B}{A} \downarrow \]

- Circular channels
  - Correlation of Liu et al. (2005) or Abiev (2013) for \( \frac{U_B}{J} \)
  - Correlation of Aussilious & Quere (2000) for \( \frac{\delta_b}{A} \)

- Square channels
  - Correlation of Liu et al. (2005) for \( \frac{U_B}{J} \)
  - Correlation of Kreutzer et al. (2005) for \( \frac{D_B, D_L}{A} \)
  - If the Taylor bubble is not axisymmetric then the local backflows in the lateral and diagonal film occur in different axial regions
  - Transition occurs at \( Ca \approx 0.04 - 0.1 \) (for larger values bubble is axisymmetric)

Conclusions and outlook

- Quantification of relative deviations between codes
  - Bubble diameter 0.6% ✓
  - Minimum liquid film thickness 10% (model for surface tension force ✗)
  - Centerline velocity liquid slug 3% ✓
  - Centerline velocity within bubble 5% ✓

- Analysis of local backflow in rear part of the liquid film
  - Temporal reversal of wall shear stress during passage of a Taylor bubble
  - Location of backflow region in lateral and diagonal film (square channel)
    - For non-axisymmetric bubbles, the local backflow occurs at different axial positions in the lateral and corner film
    - Reason: a cross-over of the diagonal/lateral bubble aspect ratio
    - Variation of bubble aspect ratio has strong implications on local flow in rear film
  - Approximate criteria when phenomena may occur in practice

- Outlook
  - Influence of \( Ca \) and \( Re \) on size of backflow region
  - Trapezoidal and triangular channels
We gratefully acknowledge the funding by the DFG

Thank you for your attention!