Analysis of second order transport equations by numerical simulations of turbulent convection in liquid metals

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Introduction

Problems

Natural convection in fast breeder reactors

model experiments in water
numerical analysis for sodium

Requirements for statistical turbulence models

purely buoyant convection
stable and unstable stratification
liquid metals

→ need for turbulence data

Objectives

Provide information by direct num. simulations

predict turbulence in liquid metals, $Pr << 1$

analyse turbulence models, $f(Pr)$

terms of k-ε-g model

Numerical Simulation of Convection

Conservation equations for

- mass
- momentum
- thermal energy

Difficulties due to features of turbulence

- time-dependent
- three-dimensional
- spectrum of length scales

Direct numerical simulations

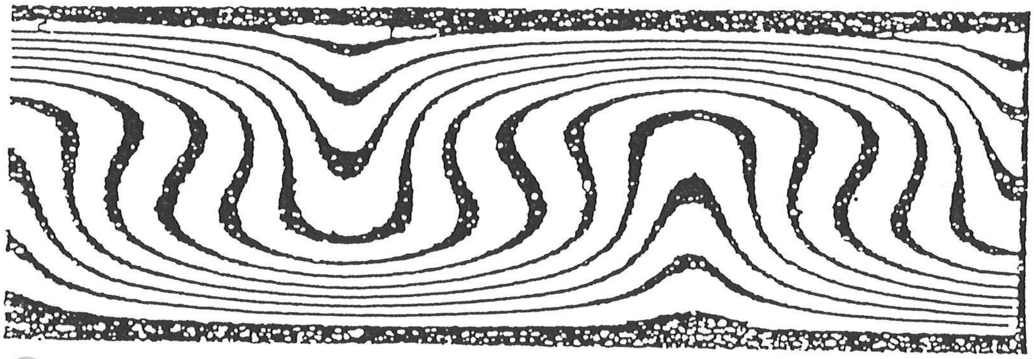
- solve complete conservation equations 3d, f(t)
- resolve all spatial scales
  - large - small structures
  - boundary layers

→ no models → no parameters

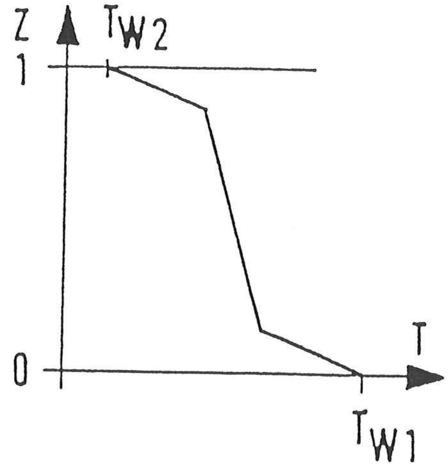
TURBIT - 4 code

- 2nd order finite difference scheme in space
- semi-spectral direct Poisson solver
- semi-implicit time integration scheme ($Pr << 1$)

Specifications for Rayleigh-Bénard convection



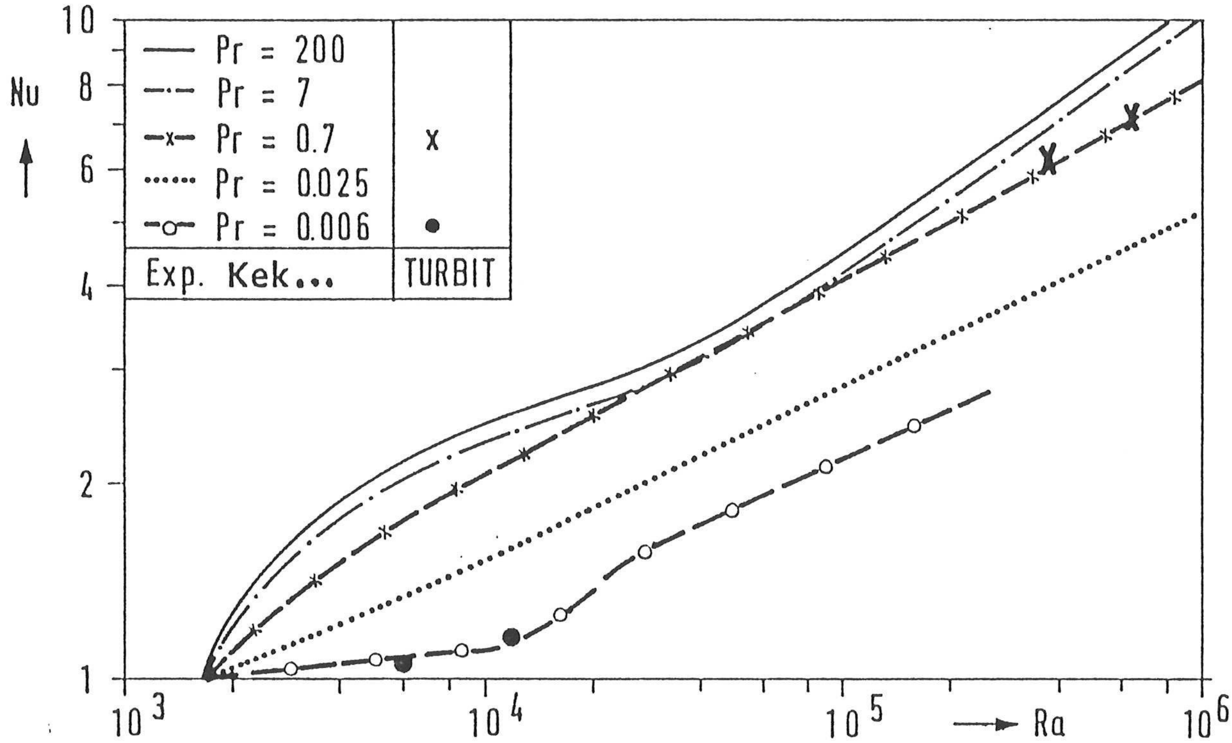
Pr = 0.71, Jahn 1975



0.71	Pr	0.006
$220 \cdot Ra_{cr}$	Ra	$3.5 \cdot Ra_{cr}$
180·180·32	mesh cells	200·200·31
49.120	time steps	116.000

initial data: $\underline{u} = 0$, $\langle T \rangle$ = linear, T' = random

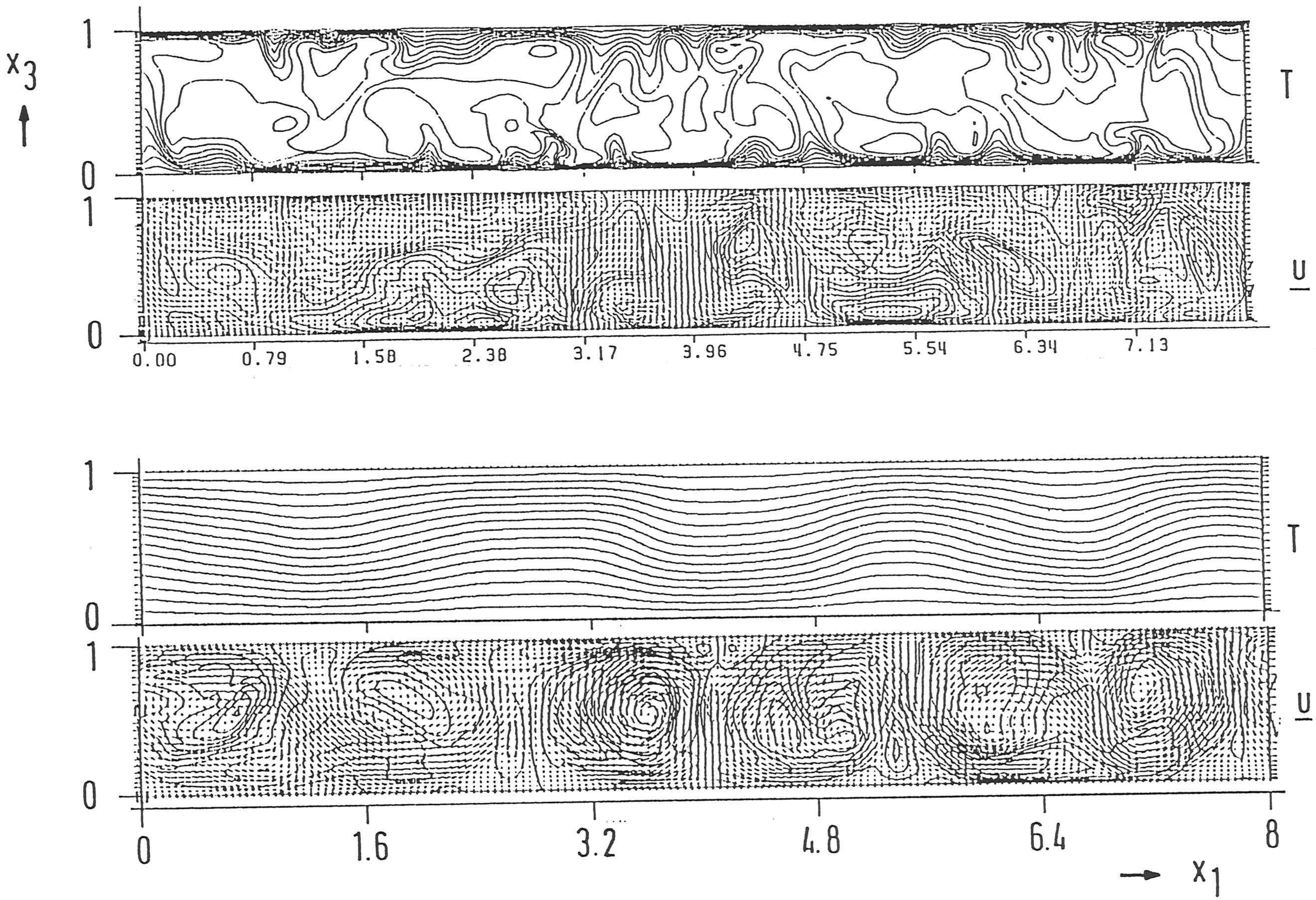
Verification



rms (T') see paper Fig. 1

no other data in open literature for Pr ≈ 0.006

Instantaneous temperature and velocity fields



Pr = 0.7

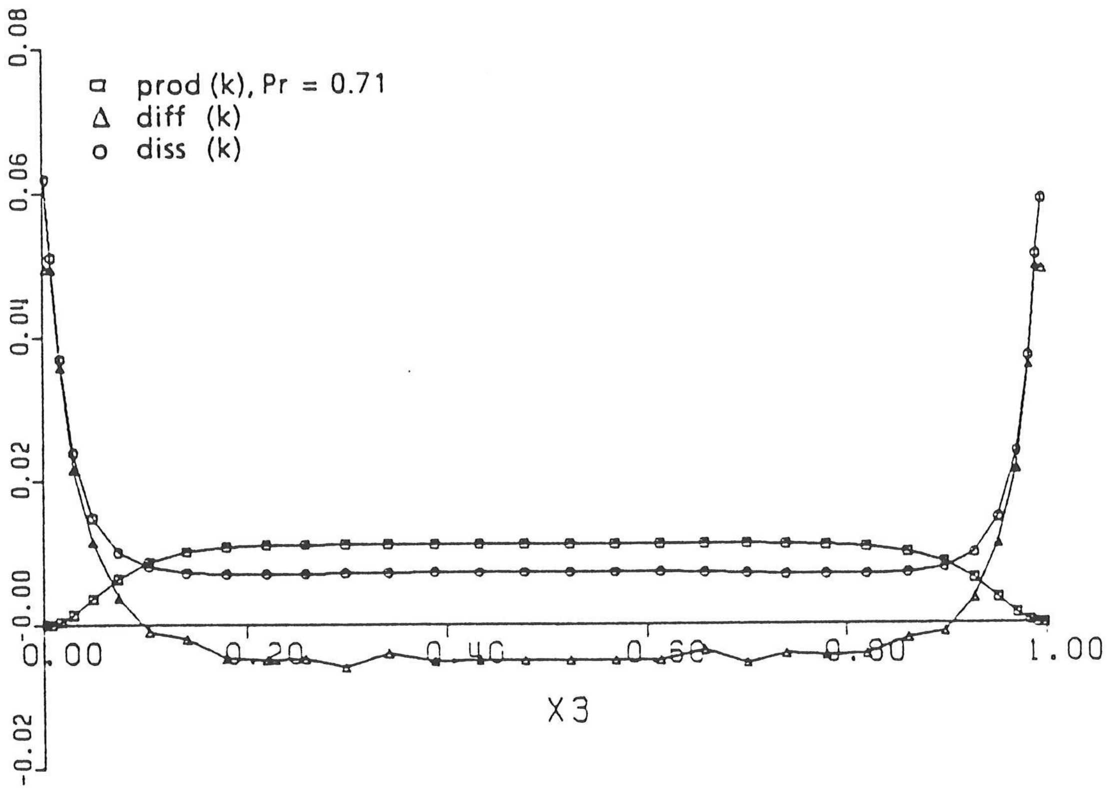
Pr = 0.006

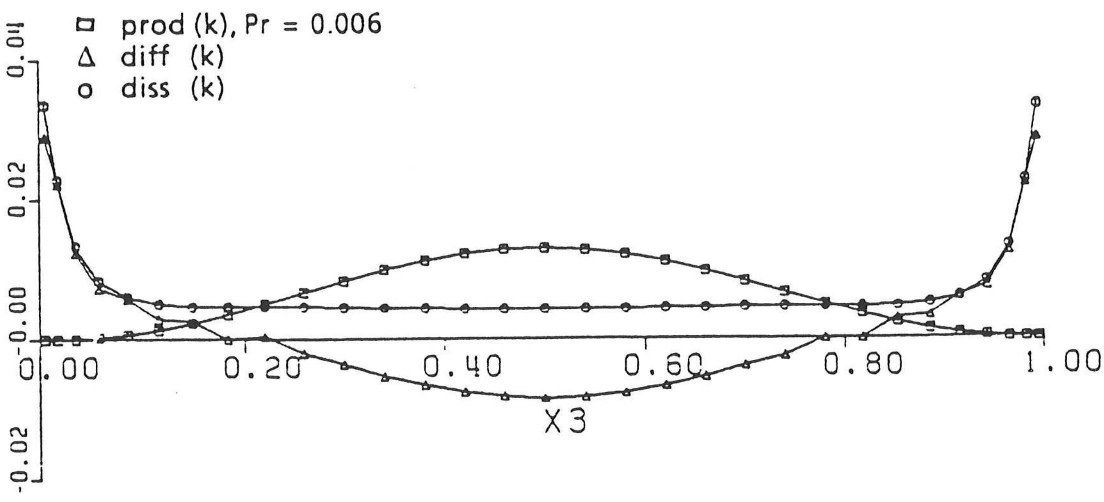
Turbulent momentum transfer

$$\partial k/\partial t = - Gr/Re_o^2 \, \overline{\underline{u}'T'} - \mathrm{div}\, [\overline{\underline{u}'k} + \overline{\underline{u}'p'}] + \nu \, \mathrm{div\, grad}\ k - \varepsilon$$

prod. turb.diff. visc. diff.

$$\varepsilon = \nu \, (\overline{(\mathrm{rot}\, u')^2} + 2\, \overline{\mathrm{div}\, (u'\, \mathrm{grad}\, u')}).$$



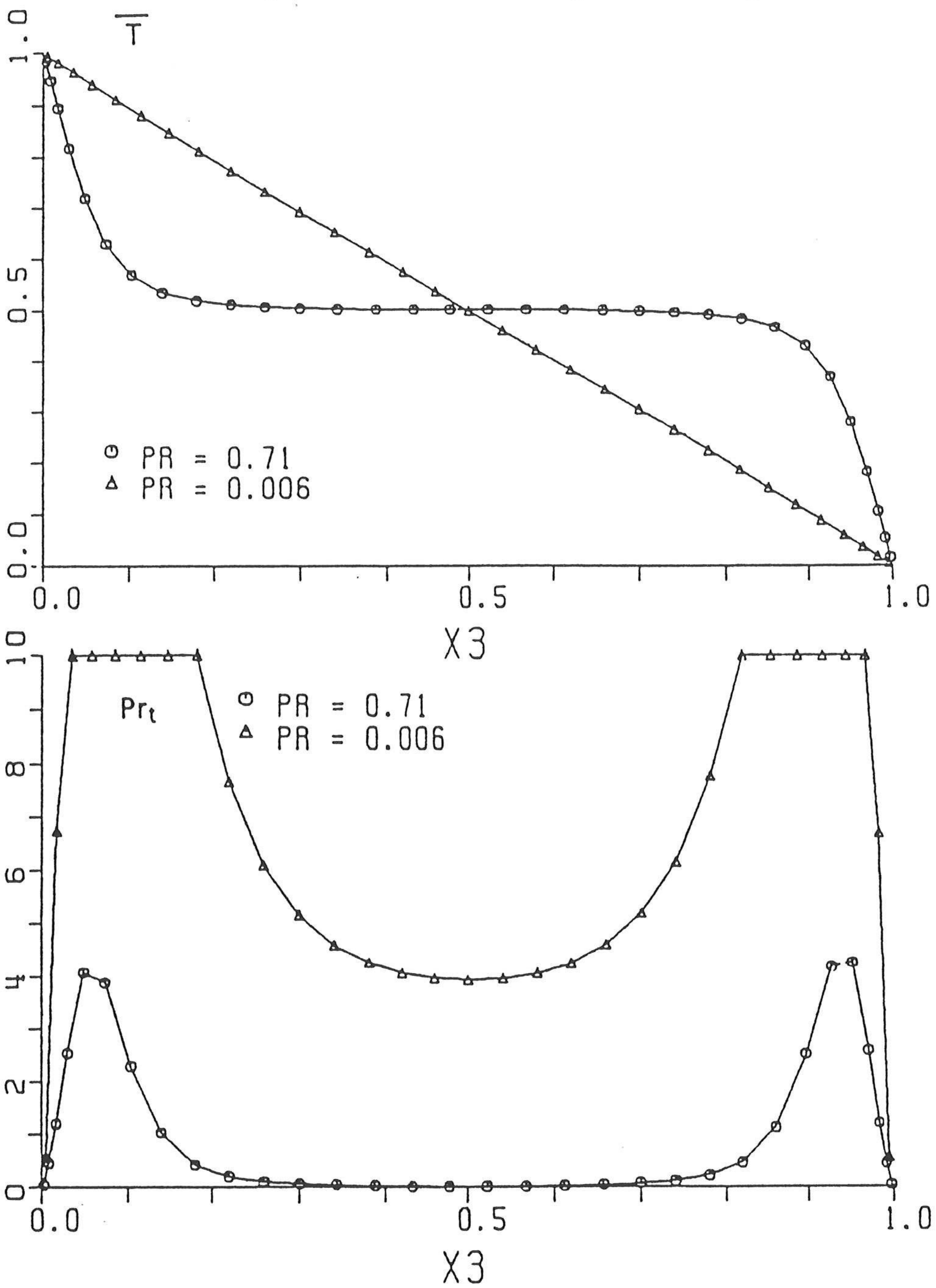


similar u-field → similar ε-distribution

different T-field → different prod. + diff.

Turbulent heat transfer

- eddy conductivity concept -



$$\mathbf{Pr_t = v_t/a_t}$$

$$v_{tj} = -\overline{u'_3\,u'_j}\,/(\partial\overline{u_j}/\partial x_3),$$

$$a_t = -\overline{u'_3T'}/(\partial\overline{T}/\partial x_3)$$

$$\rightarrow \mathbf{Pr_t = f\,(x_3, Pr, \ldots)}$$

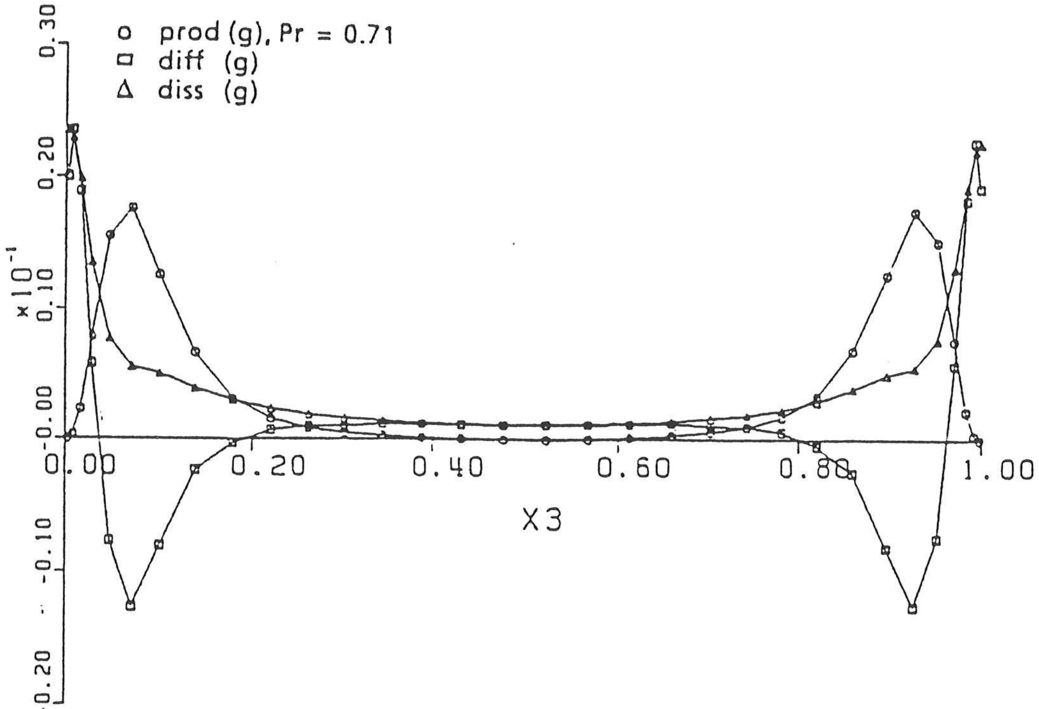
Turbulent heat transfer

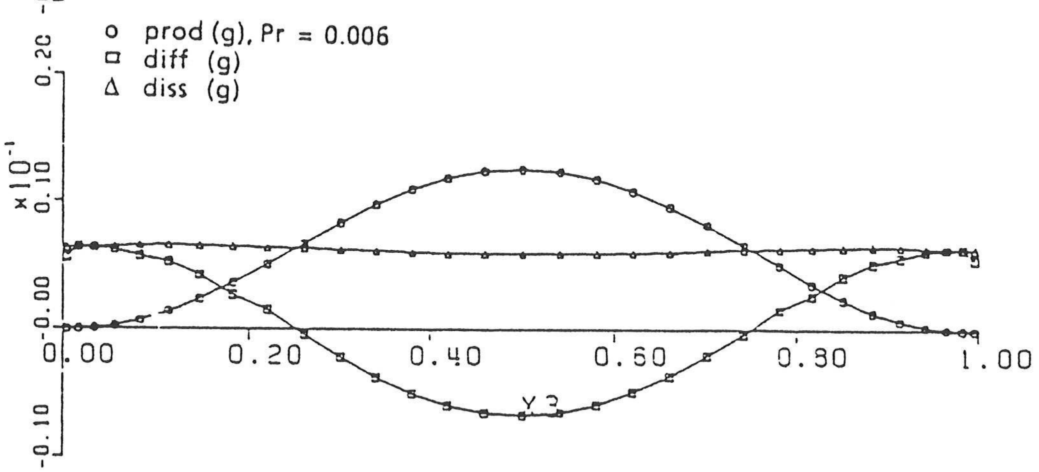
- three-equation concept -

$$a_t = C_H \, k \, g / \varepsilon_g \qquad (g = \tfrac{1}{2} \overline{T'^2})$$

$$\partial g/\partial t = - \overline{\underline{u}'T'} \, \text{grad} \, \overline{T} - \text{div} \, \overline{\underline{u}'g} + a \, \text{div grad} \, g - a \, \overline{(\text{rot} \, T')^2}$$

prod. **turb.diff.** **therm.diff.** **dissip.**



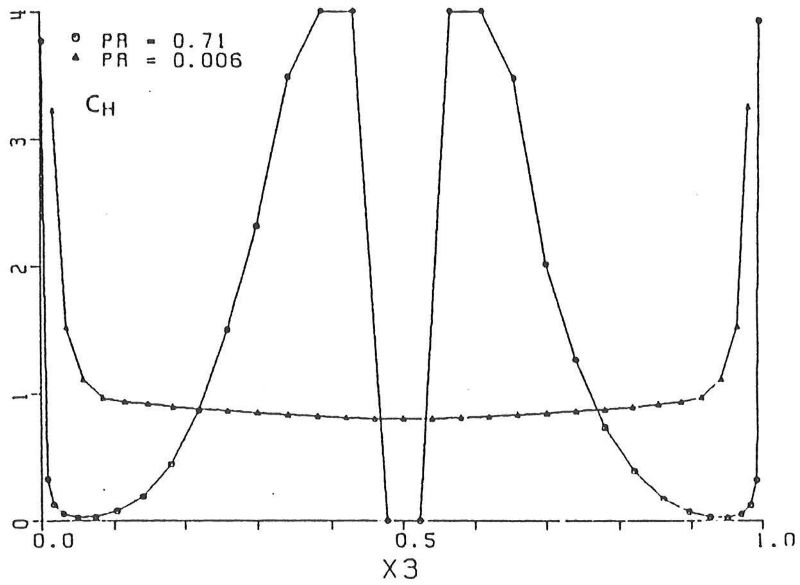


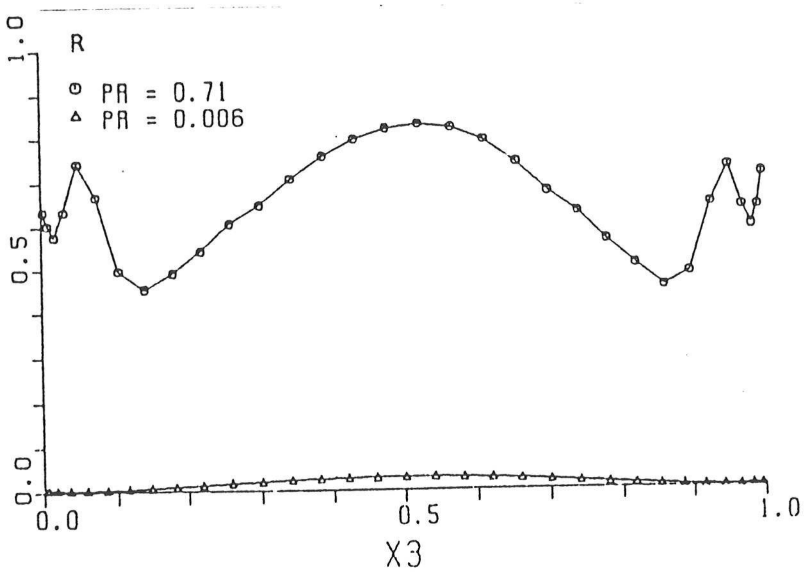
→ **g, f (g) = f (Pr ...)**

Coefficients for the three-equation model

$$a_t = C_H\, k\, g / \varepsilon_g = -\overline{u'_3 T'} / (\partial \overline{T} / \partial x_3)$$

$$\varepsilon_g = \varepsilon\, g / (k \cdot R)$$





→ no gradient diffusion at Pr ≈ 1

$C_H = f(Pr, x_3, \ldots)$

$R \;= f(Pr, x_3, \ldots) \sim Pr$

→ extend current models

Conclusions

Direct simulation method

no model parameters
resolve all scales

Rayleigh-Bénard convection of air and sodium

Verification for sodium: Nu, rms (T')

Flow phenomena f(Pr)

similarity of velocity = f (Gr = Ra/Pr)
very different temperature fields

Turbulence data

similar dissipation profiles
strong variations in Pr_t
coefficients in g-equation = f (Pr, x_3 ...)

Outlook

Simulations for larger Ra

Use of data for model improvements

Extend analysis to low-Re-models