Joint International Conference on Mathematical Methods and Supercomputing in Nuclear Applications
(M & C + SNA 1993), Karlsruhe, Germany, April 19-23, 1993

Contributions to Turbulence Modelling of Natural Convection in Liquid Metals by Direct Numerical Simulation

M. Wörner, G. Grötzbach

Kernforschungszentrum Karlsruhe
Institut für Reaktorsicherheit
7500 Karlsruhe, F.R.G.

Introduction

- **Problem**

  fast breeder reactor

  - passive decay heat removal

- **Procedure**

experiment	reactor
- model (1:20, 1:5)	1:1
- water	liquid sodium
- mainly laminar flow	turbulent convection

  computer codes

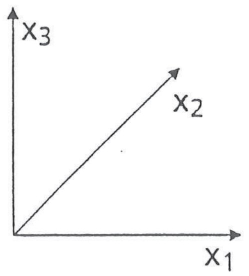
  - statistical turbulence models
  - calibration for new application

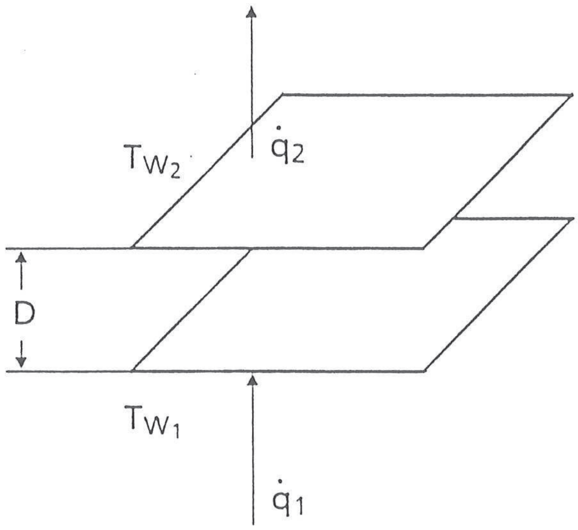
- **Objectives**

  - preparation of statistical turbulence data from direct numerical simulation
  - calibration of turbulence models
  - suggestions for model improvements

Rayleigh-Bénard convection

- geometry





- dimensionless numbers

  - Rayleigh-number:

$$Ra = \frac{g\beta\left(T_{W1} - T_{W2}\right) D^3}{\nu\,\kappa}$$

  - Prandtl-number: $Pr = \nu/\kappa$

    air: Pr = 0.71, sodium: Pr = 0.006

  - Grashof number: $Gr = Ra/Pr$

- convection: $Ra \geq 1708$

Direct simulation method

- full 3d, time-dependent conservation equations
  - mass

$$\frac{\partial u_i}{\partial x_i} = 0$$

  - momentum

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i \; u_j)}{\partial x_j} = - \frac{\partial p}{\partial x_i} - \left(T_{ref} - T \right) \delta_{i3} + \frac{1}{\sqrt{Gr}} \frac{\partial^2 u_i}{\partial x_j^2}$$

  - thermal energy

$$\frac{\partial T}{\partial t} + \frac{\partial (T \; u_j)}{\partial x_j} = \frac{1}{Pr\sqrt{Gr}} \frac{\partial^2 T}{\partial x_j^2} .$$

- resolve all scales

→ no model assumptions
no parameters

only low turbulence levels

Computer code TURBIT

- spatial discretization
  - second order central finite differences
  - staggered grid

- time integration

  momentum equation

  - explicit Euler-Leapfrog scheme

$$\Delta t \leq \left(\frac{|u_i|_{max}}{\Delta x_i} + \frac{4}{\sqrt{Gr}\,\Delta x_i^2} \right)^{-1}$$

  thermal energy equation

  - semi-implicit Leapfrog-Crank-Nicholson scheme
  - factor of time step increase: 20-40
  - additional computational expense: 10-20%

Case specifications

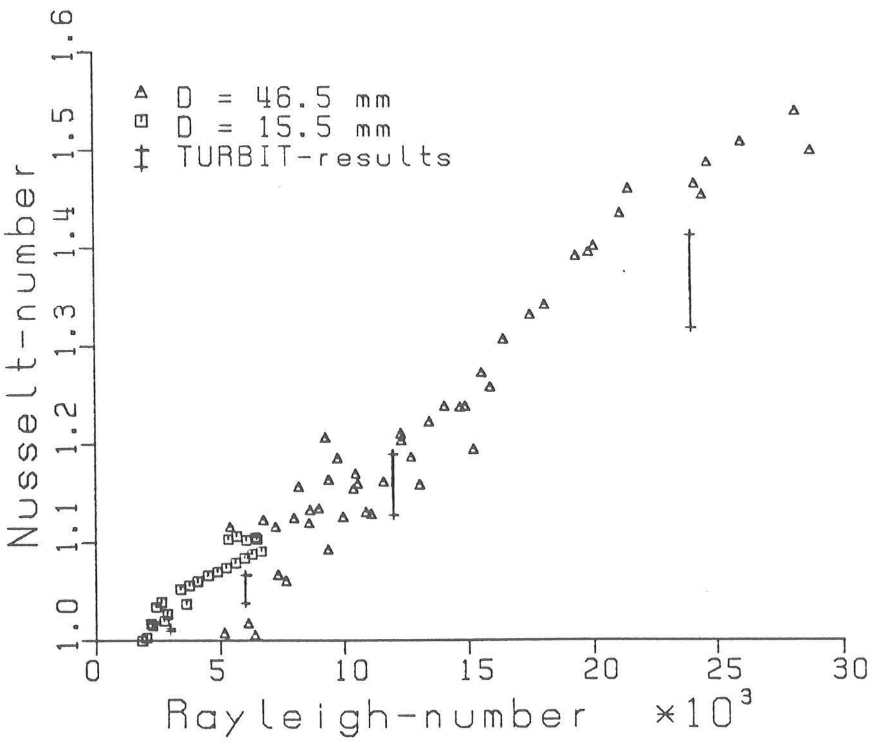
Pr	Ra	Gr	$N_{1,2}$	N_3
0.006	3,000	$0.5 \cdot 10^6$	128	31
0.006	6,000	$1.0 \cdot 10^6$	200	31
0.006	12,000	$2.0 \cdot 10^6$	250	39
0.006	24,000	$4.0 \cdot 10^6$	250	39
0.71	630,000	$0.9 \cdot 10^6$	200	39

- boundary conditions:
  - periodic in horizontal directions ($X_{1,2} = 8$)
  - walls: no slip condition
    constant wall temperature
- initial conditions
  - fluid at rest
  - final data of simulations with lower Ra

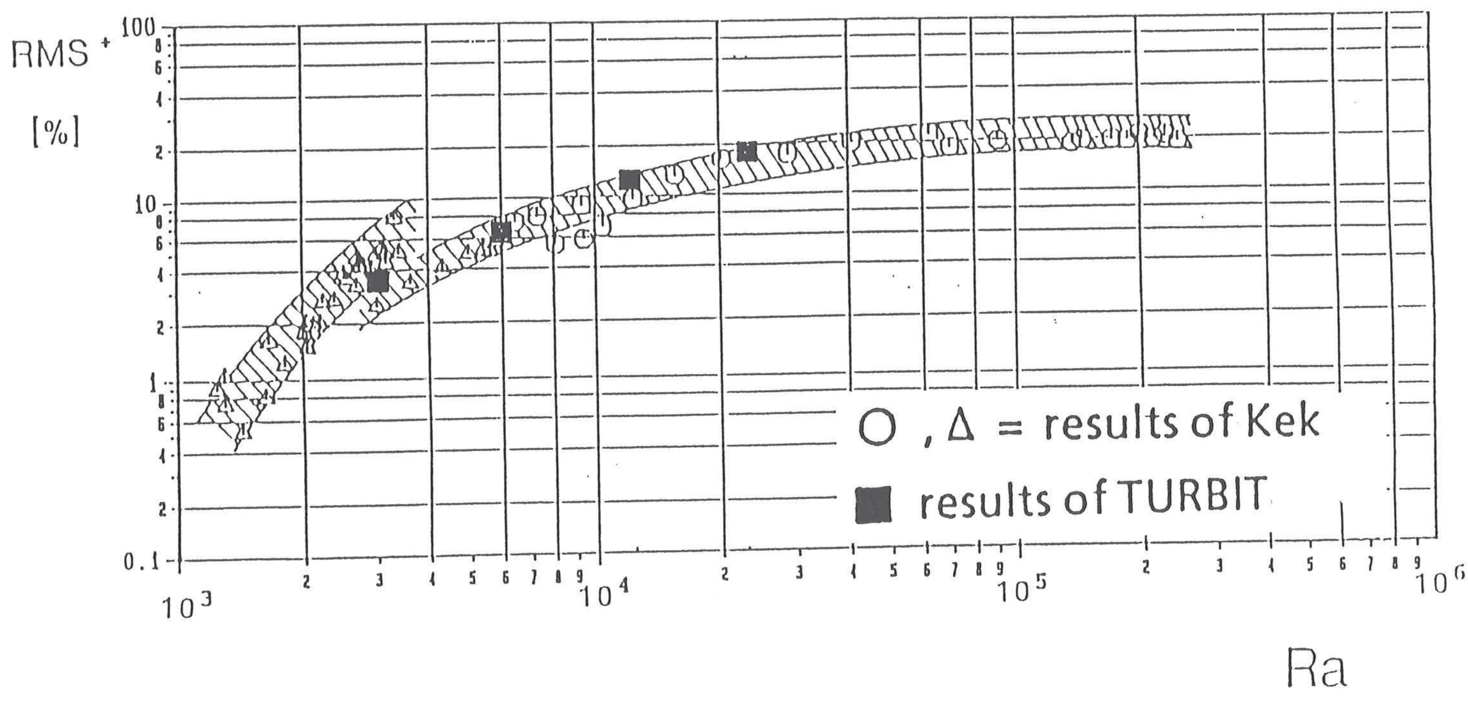
Verification (sodium)

- experiment of Kek (1989)
  Pr = 0.006 $\quad$ 1,500 ≤ Ra ≤ 250,000

  - Nusselt number



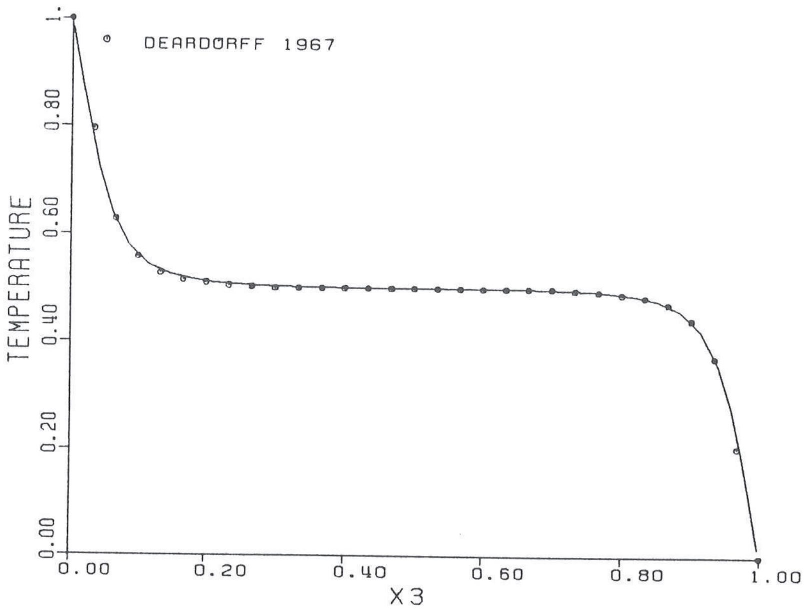
  - T_{rms} in channel midwidth



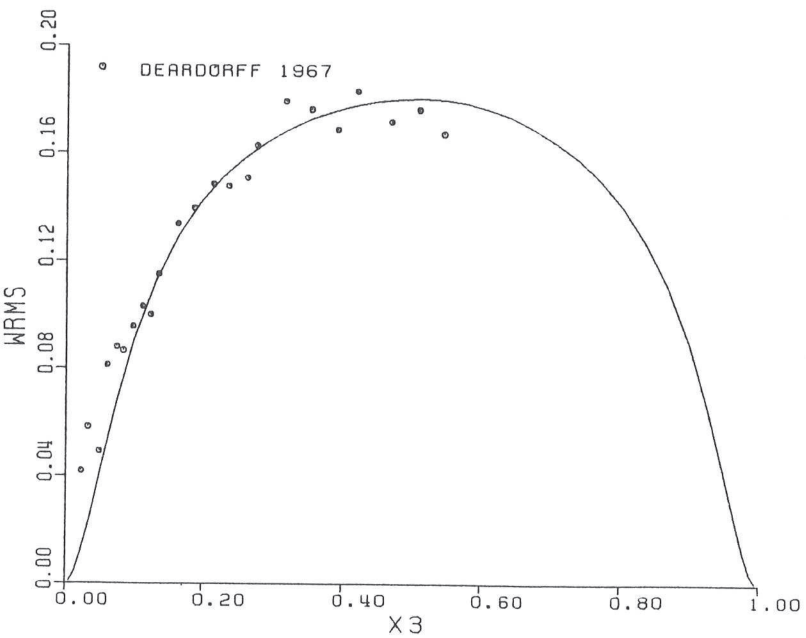
Verification (air)

- experiment of Deardorff (1967)
  Pr = 0.71   Ra = 630,000

  - mean temperature



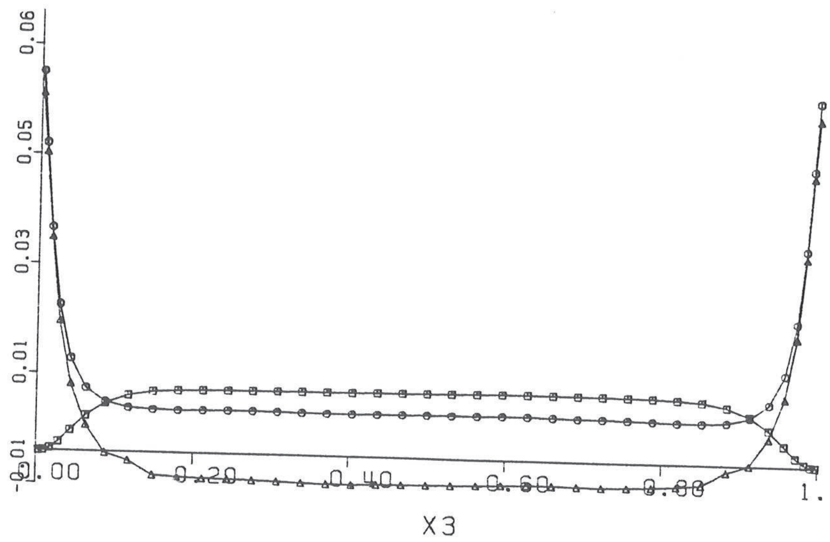
  - rms-value of vertical velocity



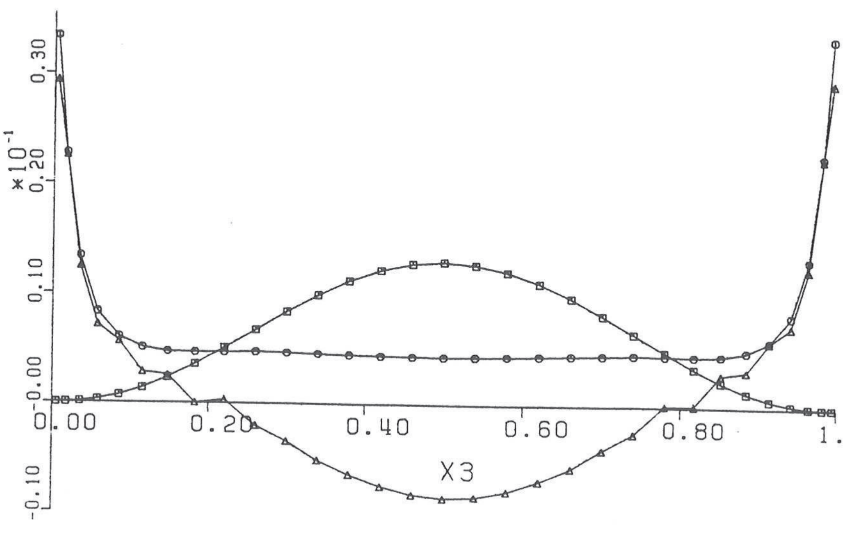
Balance of turbulent kinetic energy

$$0 = \underbrace{\frac{\partial}{\partial x_3}\left[- \overline{u_3' \frac{u_i' u_i'}{2}} - \overline{u_3' p'} + \frac{1}{\sqrt{Gr}} \frac{\partial k}{\partial x_3} \right]}_{\triangle\ D} \underbrace{+ \overline{u_3' T'}}_{\square\ G} \underbrace{- \frac{1}{\sqrt{Gr}} \overline{\frac{\partial u_i'}{\partial x_l} \frac{\partial u_i'}{\partial x_l}}}_{\bigcirc\ \varepsilon}$$

air: Ra = 630,000



sodium: Ra = 6,000

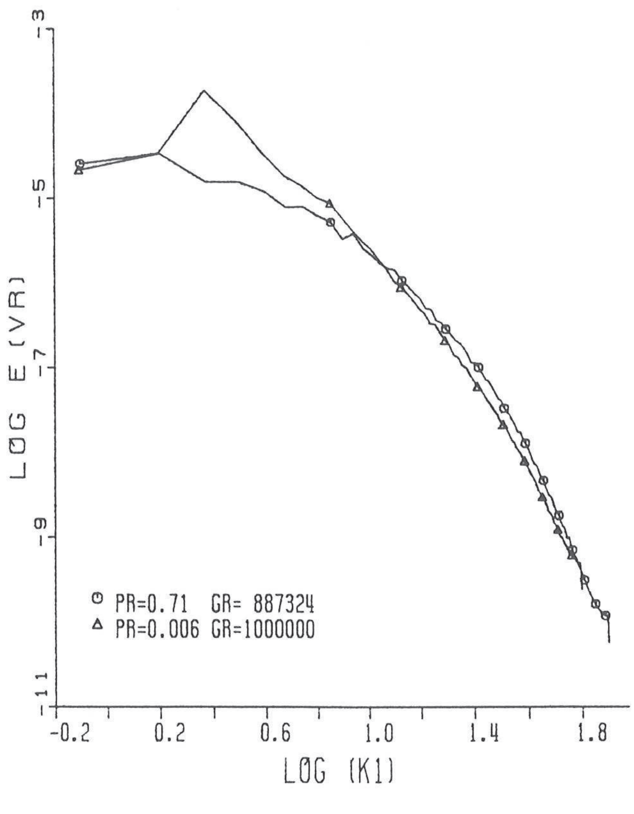


Grashof analogy

- momentum equation

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i\ u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} - \left(T_{ref} - T \right) \delta_{i3} + \frac{1}{\sqrt{Gr}} \frac{\partial^2 u_i}{\partial x_j^2}$$

- powerspectra of vertical velocity
  - air: Ra = 630,000 Gr = $0.9 \cdot 10^6$
  - sodium: Ra = 6,000 Gr = $1.0 \cdot 10^6$



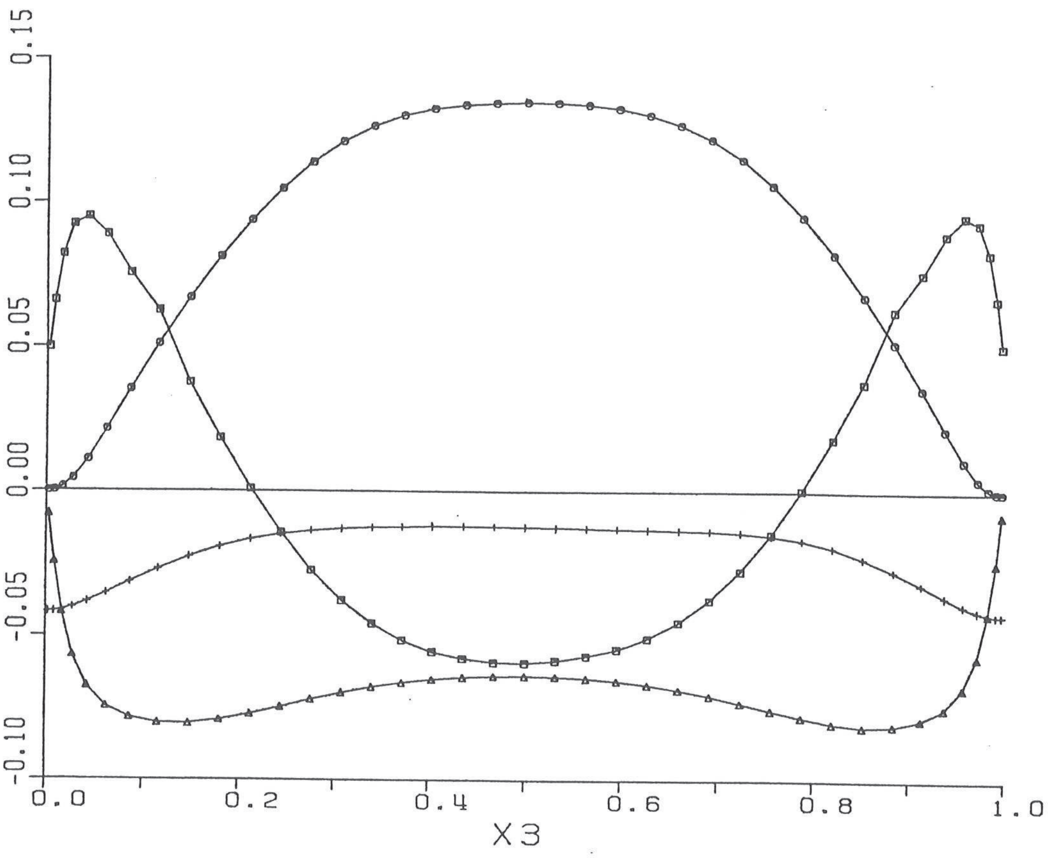
- ε-equation
  - no need for model modifications expected for liquid sodium

Balance of turbulent heat flux

$$0 = -\underbrace{\frac{\partial}{\partial x_3}\left(\overline{u_3'^2 T'} + \overline{p'T'} - \frac{1}{Pr\sqrt{Gr}}\,\overline{u_3' \frac{\partial T'}{\partial x_3}} - \frac{1}{\sqrt{Gr}}\,\overline{T' \frac{\partial u_3'}{\partial x_3}} \right)}_{\square\ D}$$

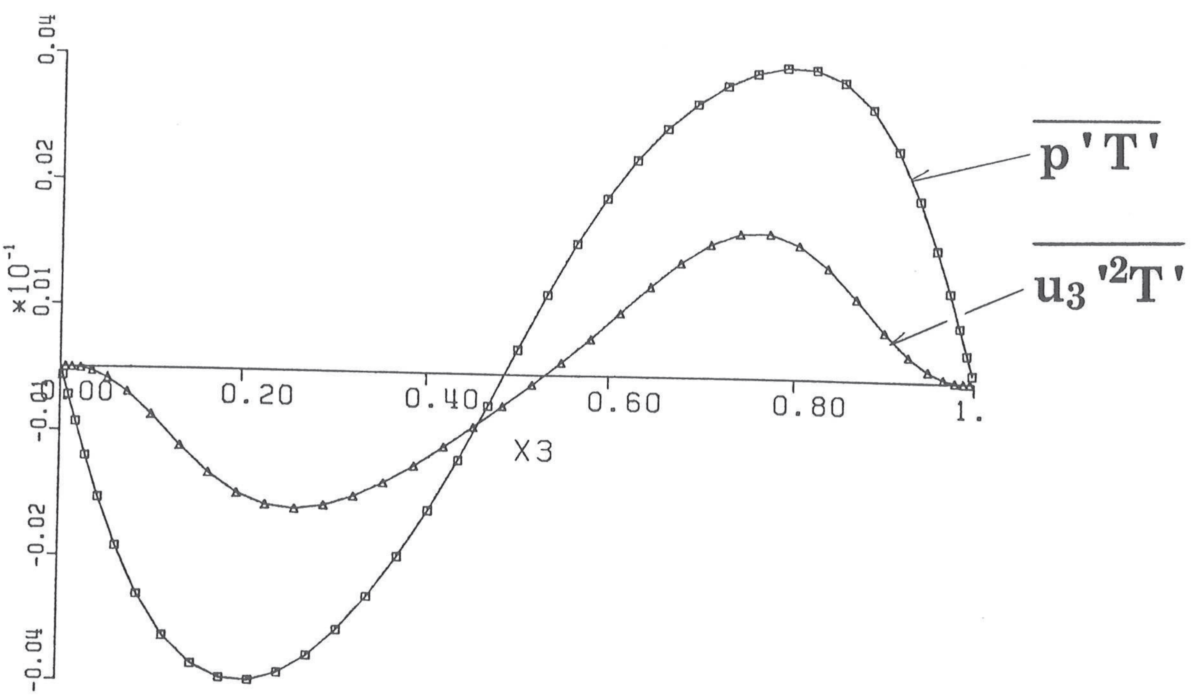
$$\underbrace{-\overline{u_3'^2}\,\frac{\partial \overline{T}}{\partial x_3} + \overline{T'^2}}_{\bigcirc\ P} + \underbrace{\overline{p'\frac{\partial T'}{\partial x_3}}}_{+\ PS} \underbrace{- \frac{1}{\sqrt{Gr}}\left(1+\frac{1}{Pr}\right)\overline{\frac{\partial u_3'}{\partial x_i}\cdot\frac{\partial T'}{\partial x_i}}}_{\triangle\ MD}$$

sodium: Ra = 24,000



Turbulent diffusion

- sodium: Ra = 24,000
  - □ $\overline{p'T'}$
  - Δ $\overline{u_3'^2T'}$



Conclusions

- Direct numerical simulation
  - turbulent Rayleigh-Bénard convection
  - sodium and air
  - verification by experimental data

- Results

  k-equation (sodium and air)
  - redistribution of k by diffusion
  - no local equilibrium

  ε-equation
  - Grashof analogy
  - no need for model modifications expected for liquid sodium

  $\overline{u_3'T'}$-equation (sodium)
  - molecular destruction is important sink
  - redistribution by diffusion
  - turbulent diffusion mainly due to pressure fluctuations

- standard models neglet:
  - molecular diffusion
  - pressure diffusion
  - molecular destruction

- natural convection of liquid metals:
  - modelling of these terms is essential