Analysis of Diffusion of Turbulent Kinetic Energy by Numerical Simulations of Natural Convection in Liquid Metals

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Introduction

- **Motivation**

  advanced liquid metal cooled reactors

  - **passive decay heat removal by natural convection**

  theoretical investigation of flow phenomena

  - **statistical turbulence models**

    e.g.: k-ε model $\nu_t \sim k^2/\varepsilon$

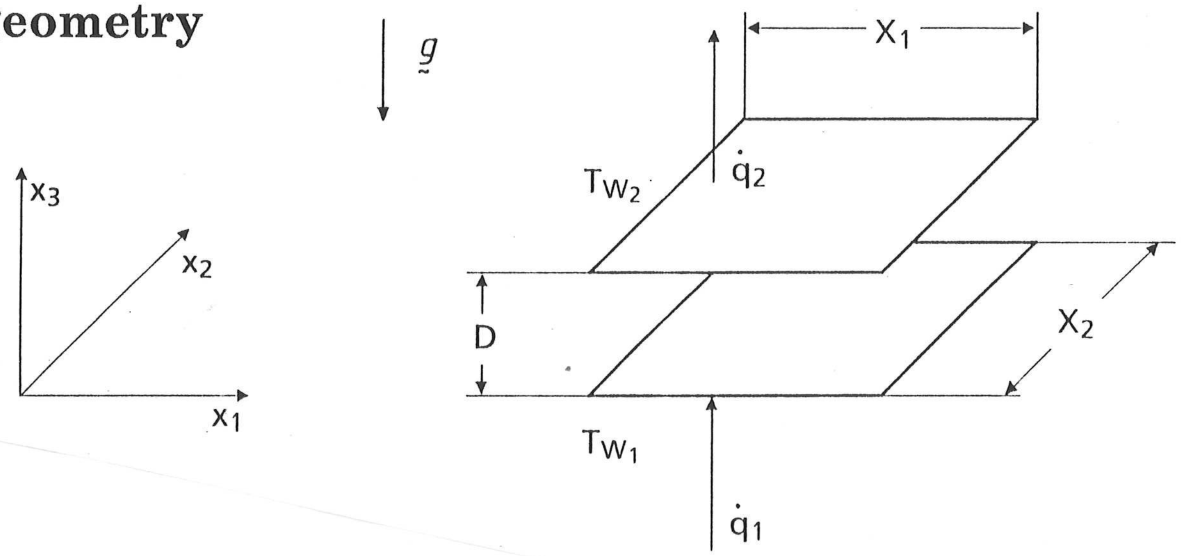
  - **transport equations for k, ε**
  - **closure of unknown correlations**

- **Objective**

  - **detailed analysis of all terms in k-equation**
  - **turbulent natural convection in liquid sodium**
  - **evaluation of direct simulation results**

Rayleigh-Bénard convection

- **geometry**



- **dimensionless numbers**
  - **Rayleigh-number:**

$$Ra = \frac{g\beta \left(T_{W1} - T_{W2} \right) D^3}{\nu \, \kappa}$$

  - **Prandtl-number:** $Pr = \nu / \kappa$

    liquid sodium: $Pr = 0.006$

  - **Grashof number:** $Gr = Ra/Pr$

Direct simulation method

- full conservation equations for
  - mass
  - momentum
  - energy

- three-dimensional, time-dependent

- resolve all scales

→ no model assumptions
  no parameters

Case specifications

Case	Pr	Ra	Gr	grid
A	0.006	3,000	$5 \cdot 10^5$	$128 \cdot 128 \cdot 31$
B	0.006	6,000	10^6	$200 \cdot 200 \cdot 31$
C	0.006	12,000	$2 \cdot 10^6$	$250 \cdot 250 \cdot 39$
D	0.006	24,000	$4 \cdot 10^6$	$250 \cdot 250 \cdot 39$

- **boundary conditions:**
  - periodic in horizontal direction ($X_{1,2} = 8$)
  - walls: no slip condition
    constant wall temperatures

- **initial conditions**
  - A, B, C: fluid at rest
  - D: final data of C

Computer code TURBIT

- finite volume method

- spatial discretization
  - finite differences
  - staggered grid

- time integration
  - momentum equation

    explicit Euler Leapfrog scheme
    projection method of Chorin

  - thermal energy equation

    semi-implicit Leapfrog-Crank-Nicholson scheme

- verified for natural and forced convection in various fluids

Evaluation of numerical results

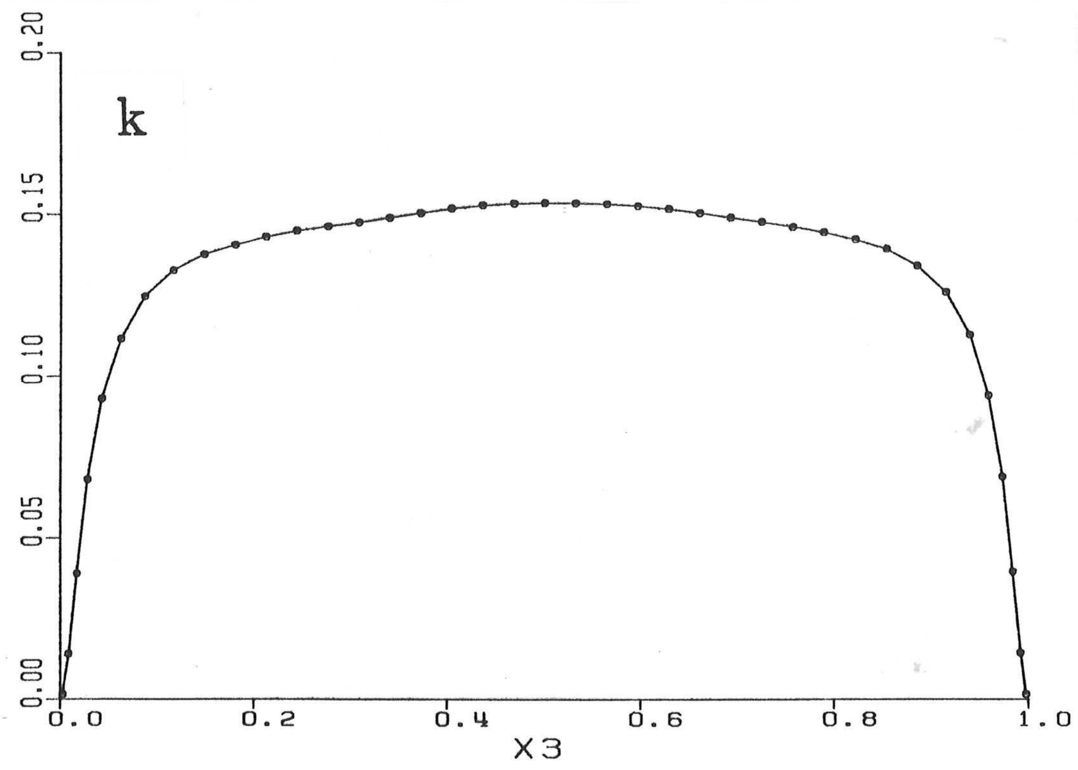
- **3d, time-dependent data for u_1, u_2, u_3, p, T**

- **kinetic turbulence energy**

$$k = \frac{1}{2}\,\overline{u_i' u_i'} = \frac{1}{2}\left(\overline{u_1'^2} + \overline{u_2'^2} + \overline{u_3'^2}\right)$$

where $u_i' = u_i - \overline{u_i}$

- **vertical profile of k**



Transport equation for k

- turbulent Rayleigh-Bénard convection
  - no mean velocity $\overline{u_i} = 0$
  - no gradients in horizontal directions
  - fully developed flow

$$0 = \underbrace{\frac{\partial}{\partial x_3}\left[- \overline{u_3' \frac{u_i' u_i'}{2}} - \overline{u_3' p'} + \frac{1}{\sqrt{Gr}} \frac{\partial k}{\partial x_3} \right]}_{D}$$

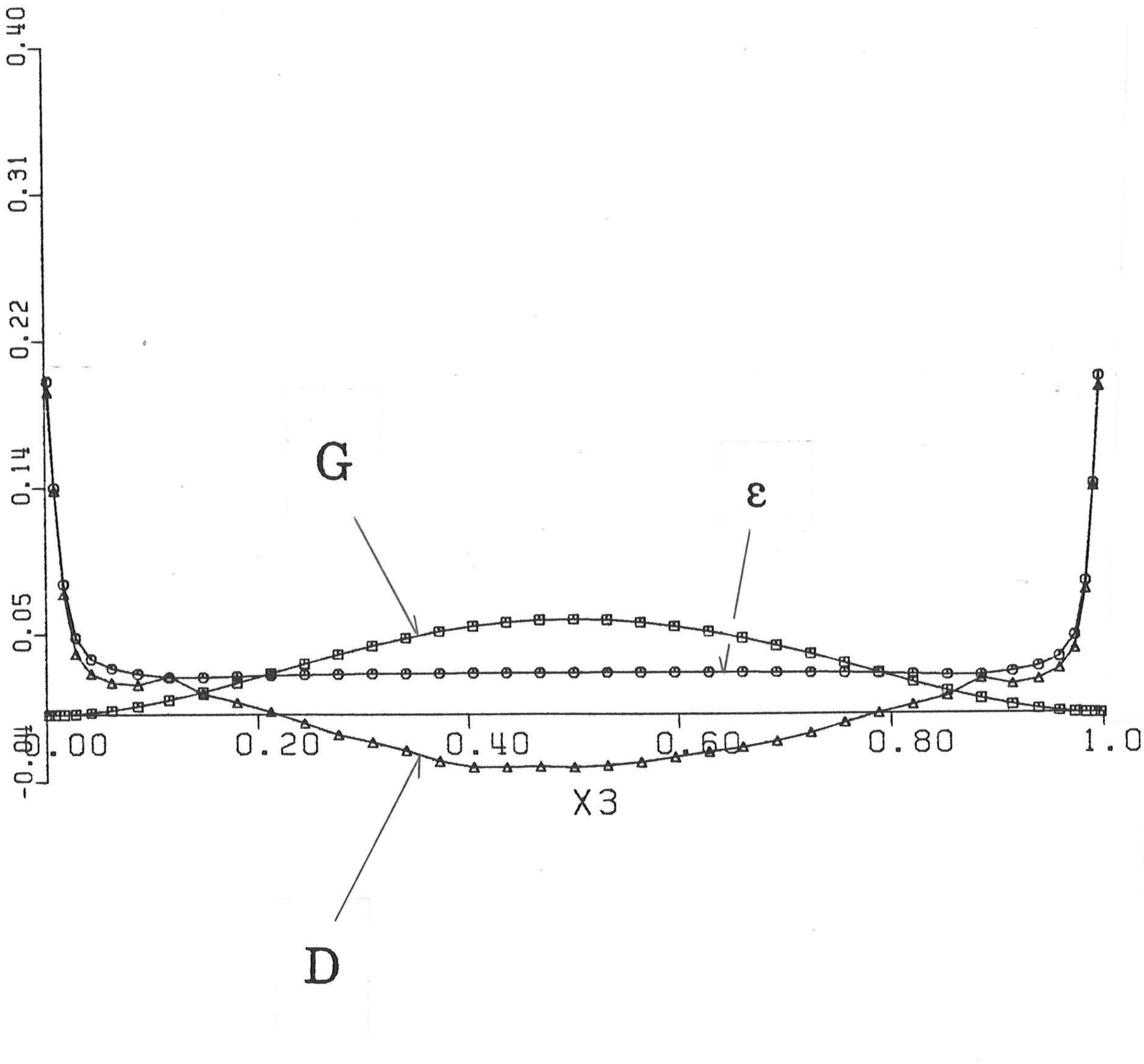
$$\underbrace{+ \overline{u_3' T'}}_{G}$$

$$\underbrace{- \frac{1}{\sqrt{Gr}} \overline{\frac{\partial u_i'}{\partial x_l} \frac{\partial u_i'}{\partial x_l}}}_{\varepsilon}$$

Budget of kinetic turbulence energy

- **sodium, Ra = 24,000**

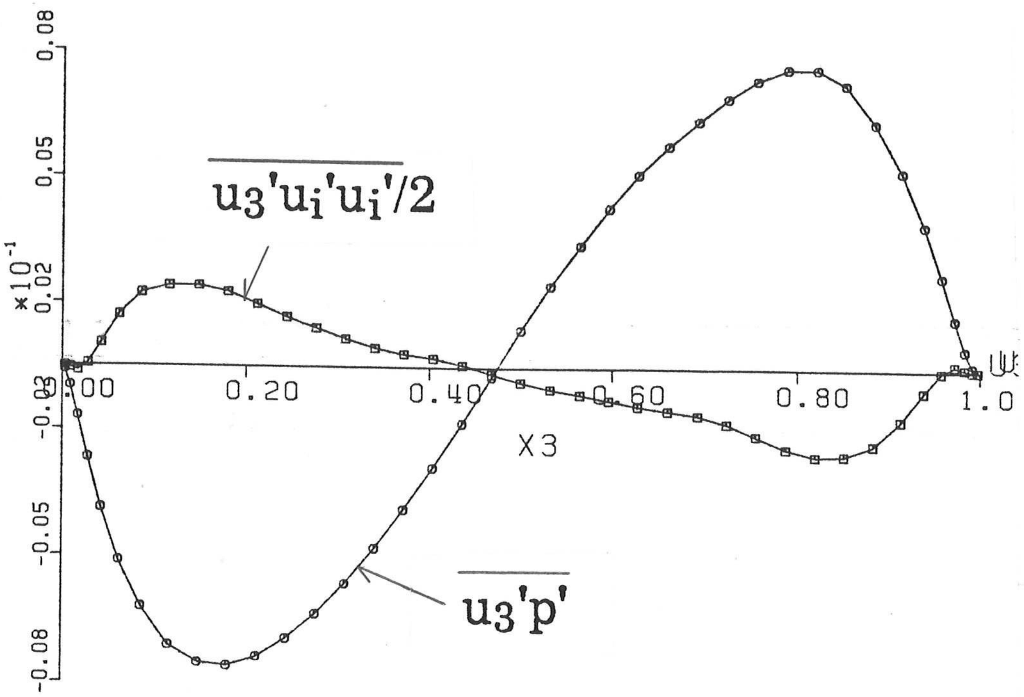
$$0 = G + D - \varepsilon$$



Analysis of diffusion of k

$$D = \frac{\partial}{\partial x_3}\left[\overline{u_3' \frac{u_i' u_i'}{2}} + \overline{u_3' p'} - \frac{1}{\sqrt{Gr}} \frac{\partial k}{\partial x_3}\right]$$

- **sodium, Ra = 24,000**



- **standard turbulence models**
  - **neglect molecular diffusion**
  - **neglect pressure diffusion**
  - **model triple correlation**

- **proposal of Lumley:** $\overline{u_3'p'} = -1/5\ \overline{u_3' u_i' u_i'}$

Conclusions

- **Direct numerical simulation**
  - **turbulent Rayleigh-Bénard convection**
  - **liquid sodium**

- **Balance of kinetic turbulence energy**
  - **no local equilibrium $G \neq \varepsilon$**
  - **redistribution of k by diffusion**
  - **turbulent diffusion mainly due to $\overline{u_i'p'}$**

- **Standard turbulence models**
  - **neglect pressure diffusion**