Analysis of thermal variance equation for natural convection of air and sodium

M. Wörner, G. Grötzbach
Kernforschungszentrum Karlsruhe
Institut für Reaktorsicherheit
D-76021 Karlsruhe
Germany

International Symposium on
Turbulence, Heat and Mass Transfer
Lisbon, Portugal, August 9-12, 1994

Introduction

- **Motivation**

  turbulent heat transfer in buoyant flows

  k-ε turbulence model

  $$\overline{u_i'T'} = -\kappa_t \frac{\partial \overline{T}}{\partial x_i} \qquad \kappa_t = \frac{\nu_t}{Pr_t}$$

  - inadequate for buoyant flows

  full differential models for $\overline{u_i'T'}$ and $\overline{T'^2}$

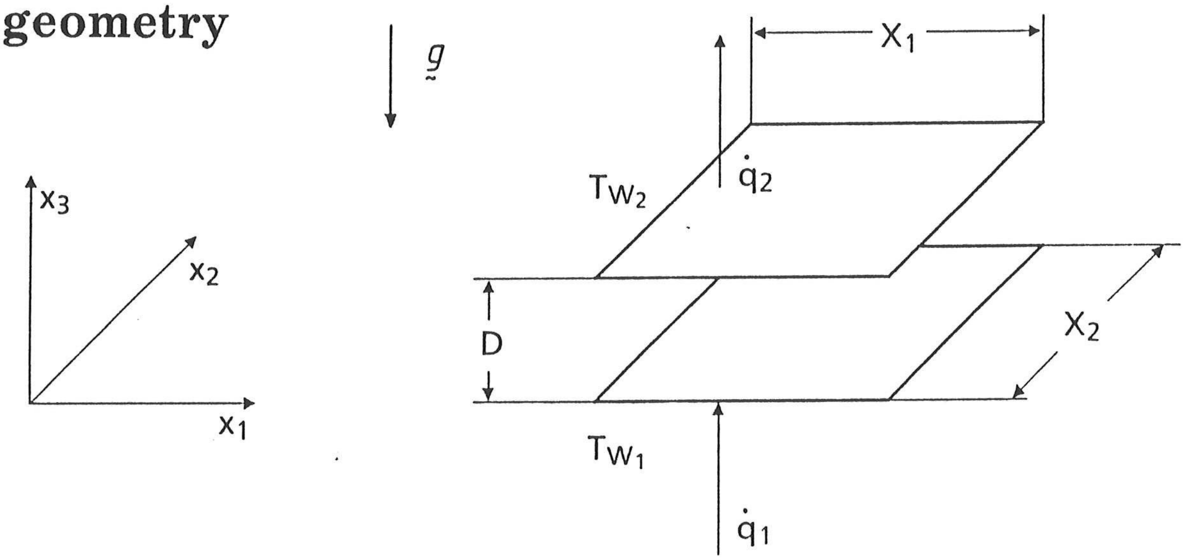
  - closure of unknown correlations

- **Objective**

  - detailed analysis of all terms in $\overline{T'^2}$-equation
  - turbulent natural convection in air and sodium
  - evaluation of direct simulation results

Rayleigh-Bénard convection

- geometry



- dimensionless numbers
  - Rayleigh-number:

$$Ra = \frac{g\beta\left(T_{W1} - T_{W2}\right) D^3}{\nu\,\kappa}$$

  - Prandtl-number: $Pr = \nu/\kappa$

    air: $Pr = 0.71$, sodium: $Pr = 0.006$

  - Grashof number: $Gr = Ra/Pr$

Direct simulation method

- full conservation equations for
  - mass
  - momentum
  - energy

- three-dimensional, time-dependent

- resolve all scales

→ no model assumptions
no parameters

Computer code TURBIT

- finite volume method

- spatial discretization
  - finite differences
  - staggered grid

- time integration
  - momentum equation

    explicit Euler Leapfrog scheme
    projection method of Chorin

  - thermal energy equation

    semi-implicit Leapfrog-Crank-Nicholson scheme

- verified for natural and forced convection in various fluids

Case specifications

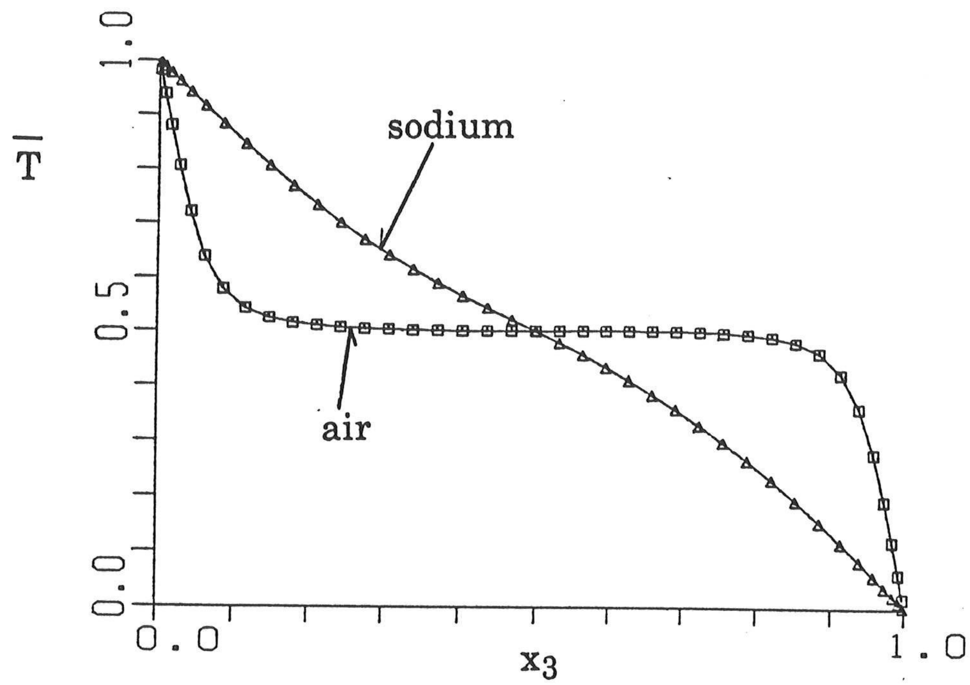
Fluid	Ra	Gr	grid
air	630,000	$0.9 \cdot 10^6$	$200 \cdot 200 \cdot 39$
sodium	24,000	$4 \cdot 10^6$	$250 \cdot 250 \cdot 39$

- boundary conditions:
  - periodic in horizontal direction ($X_{1,2} = 8$)
  - walls: no slip condition
    constant wall temperatures

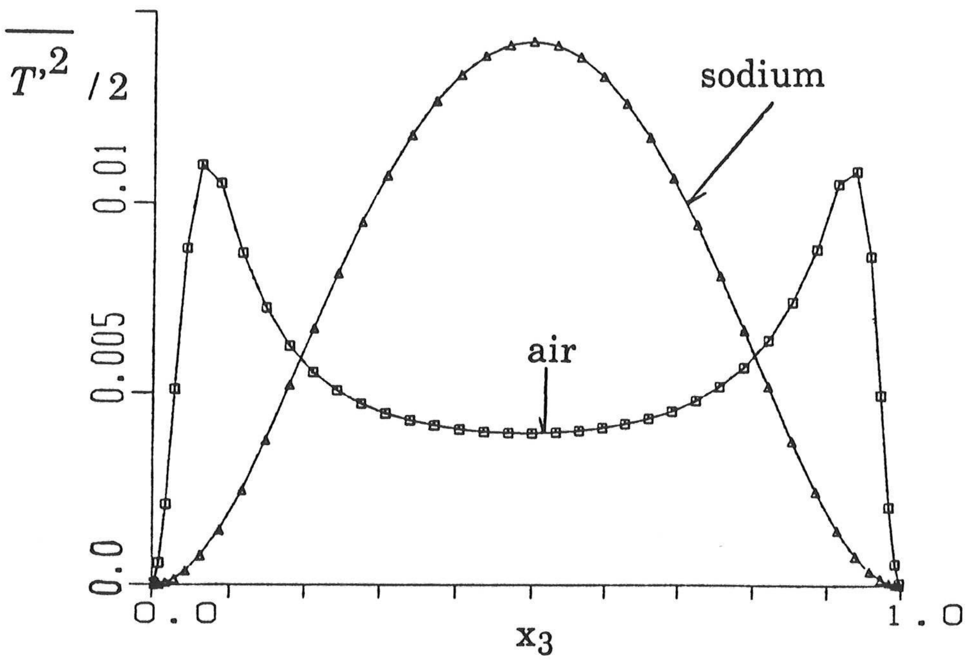
- initial conditions
  - final data of air / sodium simulations
    with lower Ra

Evaluated results

- **mean temperature $\overline{T}$**



- **temperature variance $\overline{T'^2}/2$**



Transport equation for $\overline{T'^2}/2$

- turbulent Rayleigh-Bénard convection
  - no mean velocity $\overline{u_i} = 0$
  - no gradients in horizontal directions

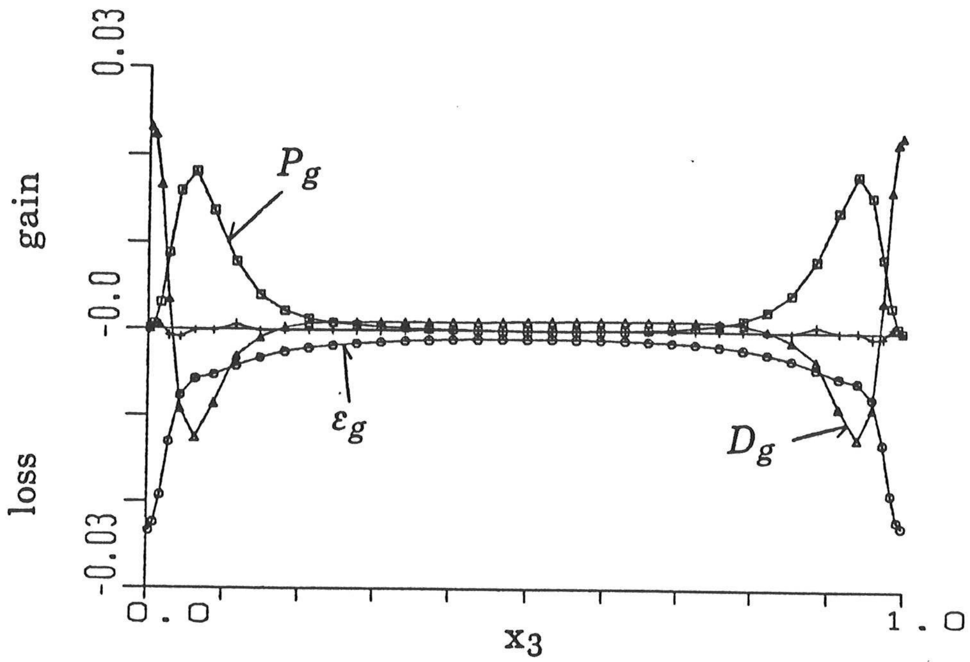
$$\frac{\partial\left(\overline{T'^2}/2\right)}{\partial t} = \underbrace{-\,\overline{u_3'T'}\,\frac{\partial \overline{T}}{\partial x_3}}_{P_g}$$

$$\underbrace{-\,\frac{\partial}{\partial x_3}\left(\overline{u_3'T'^2/2} - \frac{1}{Pr\sqrt{Gr}}\,\frac{\partial\left(\overline{T'^2}/2\right)}{\partial x_3}\right)}_{D_g}$$

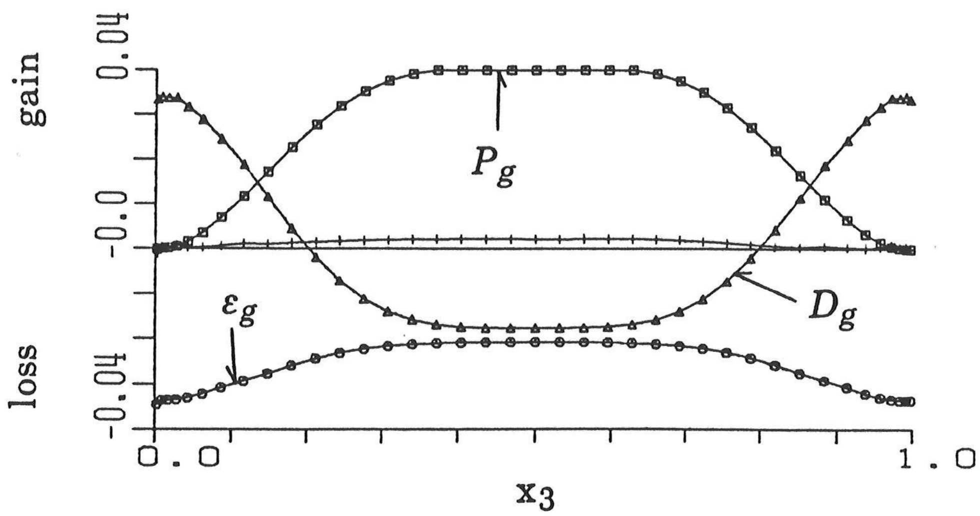
$$\underbrace{-\,\frac{1}{Pr\sqrt{Gr}}\,\overline{\frac{\partial T'}{\partial x_i}\cdot\frac{\partial T'}{\partial x_i}}}_{\varepsilon_g}$$

Budget of $\overline{T'^2}/2$

- air ($Pr = 0.71$)



- sodium ($Pr = 0.006$)



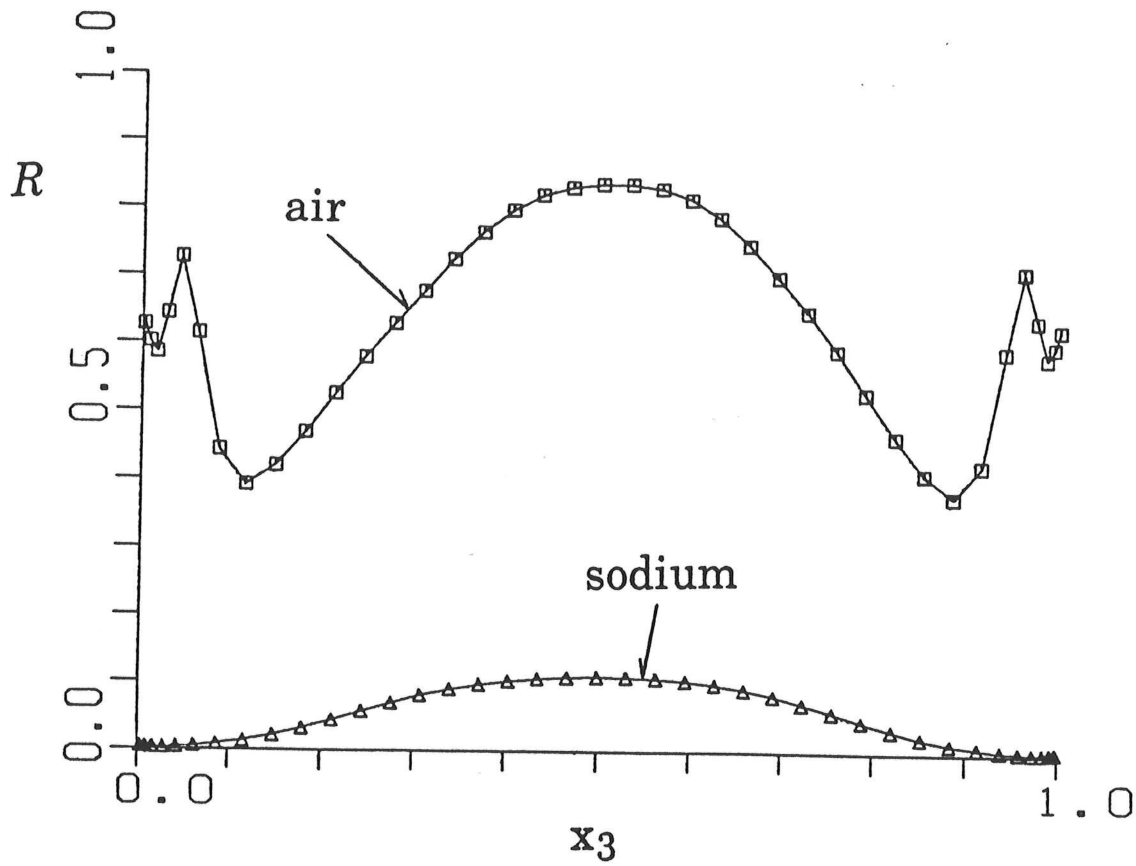
Closure of sink term ε_g

- Launder (1975)

$$\varepsilon_g = \frac{1}{R} \frac{\overline{T'^2}/2}{k} \varepsilon$$

$$R \approx 0.8$$

- evaluation of R



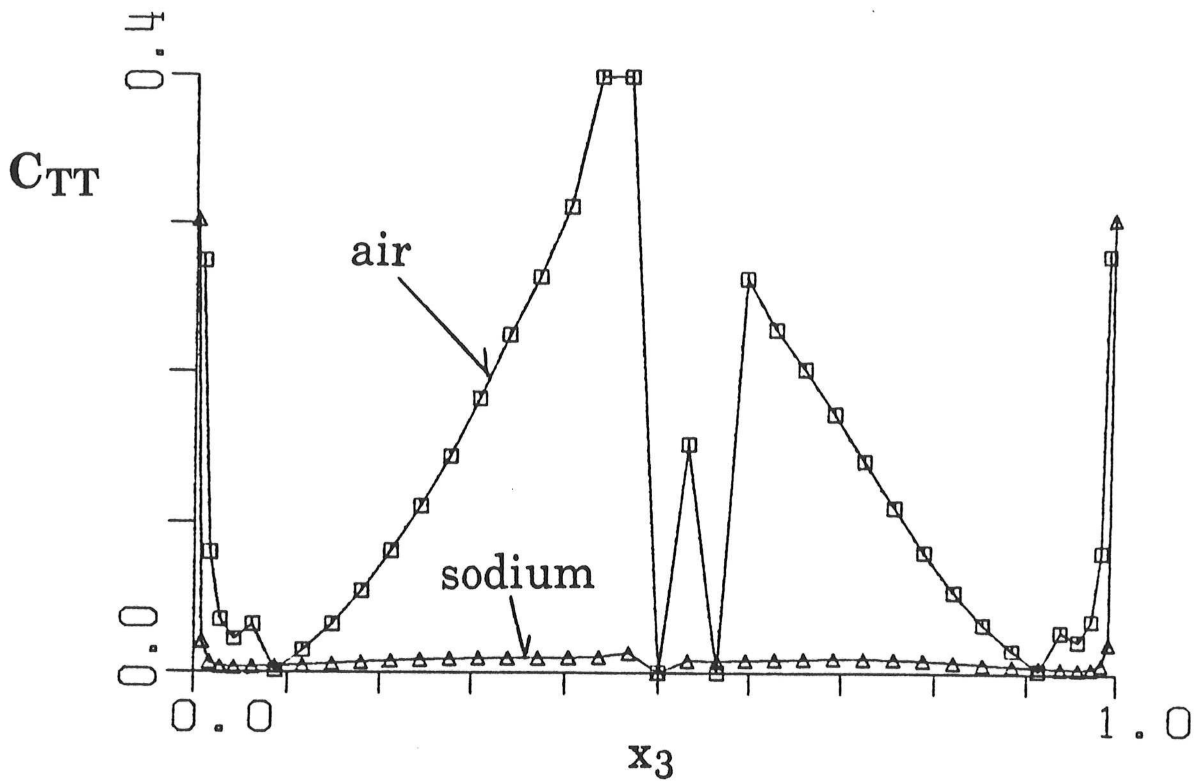
Closure of diffusion term

- Spalding (1971)

$$-\overline{u_3 ' T'^2 /2} = C_{TT} \frac{k^2}{\varepsilon} \frac{\partial \left(\overline{T'^2} /2 \right)}{\partial x_3}$$

$$C_{TT} \approx 0.3$$

- evaluation of C_{TT}



Conclusions

- **Direct numerical simulation**
  - turbulent Rayleigh-Bénard convection
  - air and sodium

- **Balance of thermal variance**
  - no local equilibrium $P_g \neq \varepsilon_g$
  - redistribution of $\overline{T'^2}$ by diffusion

- **Modelling of sink term ε_g**
  - time scale ratio $R = \mathrm{f}\,(Pr, Gr)$
  - → transport equation for ε_g

- **Modelling of diffusion D_g**
  - core region: $\partial\,(\overline{T'^2}/2)/\partial x_3 = 0$, but $D_g \neq 0$
  - → D_g model not based on gradient assumption