Analysis of the transport equation of temperature variance dissipation rate by direct numerical simulation data of natural convection

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Introduction

Statistical modelling of turbulent buoyant flows

Closure assumptions are based on

- mechanical turbulence time scale $\tau = \dfrac{k}{\varepsilon}$
- thermal turbulence time scale $\tau_\theta = \dfrac{\overline{T'^2}/2}{\varepsilon_T}$

$k - \varepsilon - \overline{T'^2}$ - Model

- τ calculated from transport equations for k and ε
- τ_θ calculated from
  - transport equation for $\overline{T'^2}$
  - prescribed constant time scale ratio $R = \tau_\theta / \tau$
- shortcomings: $R \neq$ const., but depends on
  - Prandtl number of fluid
  - importance of buoyancy
  - turbulence level
  - type of thermal boundary conditions

$\Rightarrow$ **need to solve transport equation for** ε_T

Objective

Analytical transport equation for $\varepsilon_T = \kappa \overline{\frac{\partial T'}{\partial x_j}\frac{\partial T'}{\partial x_j}}$

$$\frac{D\varepsilon_T}{Dt} = -2\kappa\overline{\frac{\partial T'}{\partial x_j}\frac{\partial u_l'}{\partial x_j}}\frac{\partial \overline{T}}{\partial x_l} - 2\kappa\overline{u_l'\frac{\partial T'}{\partial x_j}}\frac{\partial^2 \overline{T}}{\partial x_l \partial x_j}$$
$$-2\kappa\overline{\frac{\partial T'}{\partial x_j}\frac{\partial T'}{\partial x_l}}\frac{\partial \overline{u}_l}{\partial x_j} - 2\kappa\overline{\frac{\partial T'}{\partial x_j}\frac{\partial u_l'}{\partial x_j}\frac{\partial T'}{\partial x_l}}$$
$$-2\kappa^2\overline{\left(\frac{\partial^2 T'}{\partial x_j \partial x_l}\right)^2} - \frac{\partial}{\partial x_l}\left(\overline{\varepsilon_T' u_l'} - \kappa\frac{\partial \varepsilon_T}{\partial x_l}\right)$$

- unknown correlations need to be modelled
- no experimental information about ε_T-equation

Present contribution:

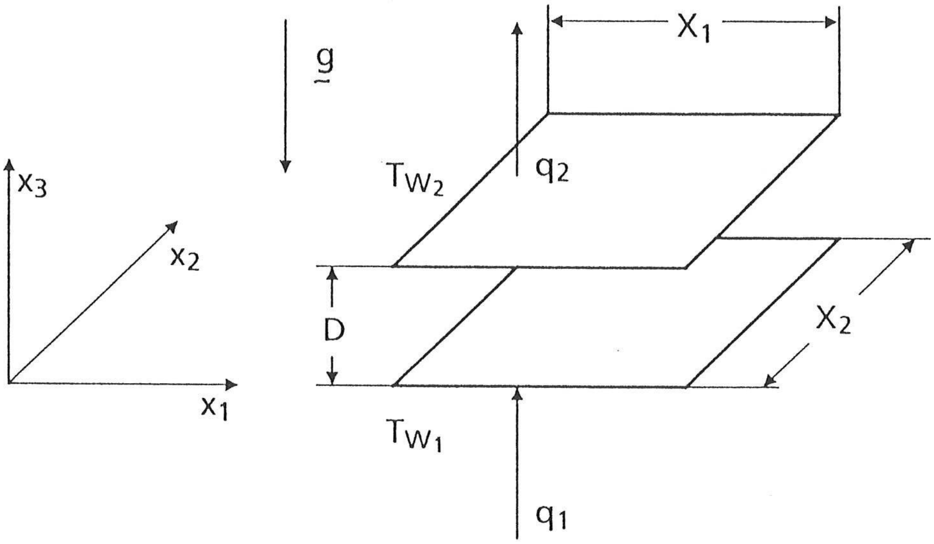
Use Direct Numerical Simulation data of turbulent natural convection to

- investigate relevance of different terms
- investigate performance of model assumptions

Physical Model

Rayleigh-Bénard convection:

Natural convection in fluid layer heated from below



Dimensionless numbers:

- Rayleigh number

$$Ra = \frac{g\beta\Delta T_{Wall} D^3}{\nu\kappa}$$

- Prandtl number $Pr = \nu/\kappa$
  - here: air $Pr = 0.71$, sodium $Pr = 0.006$
- Grashof number $Gr = Ra/Pr$
- Boussinesq number $Bo = Ra \cdot Pr$

Numerical Simulations

- Governing equations (dimensionless)

$$\frac{\partial u_j}{\partial x_j} = 0$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{\sqrt{Gr}} \frac{\partial^2 u_i}{\partial x_j \partial x_j} - (T_{ref} - T)\delta_{i3}$$

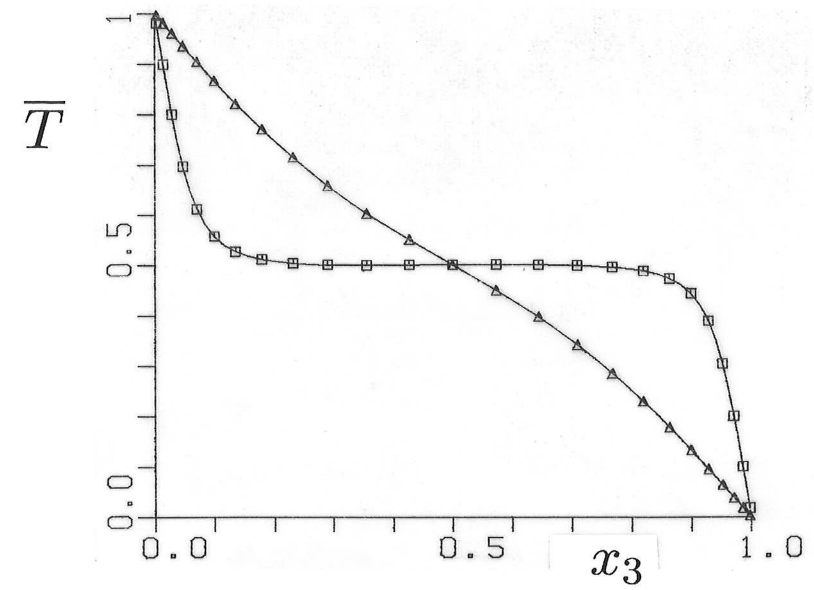
$$\frac{\partial T}{\partial t} + \frac{\partial (T u_j)}{\partial x_j} = \frac{1}{\sqrt{Bo}} \frac{\partial^2 T}{\partial x_j \partial x_j}$$

- Direct Numerical Simulation (DNS) $\Rightarrow$ use spatial discretization which resolves all scales of turbulence
- Computer code TURBIT (Finite Volume Method)
- Parameter and grid data of simulations

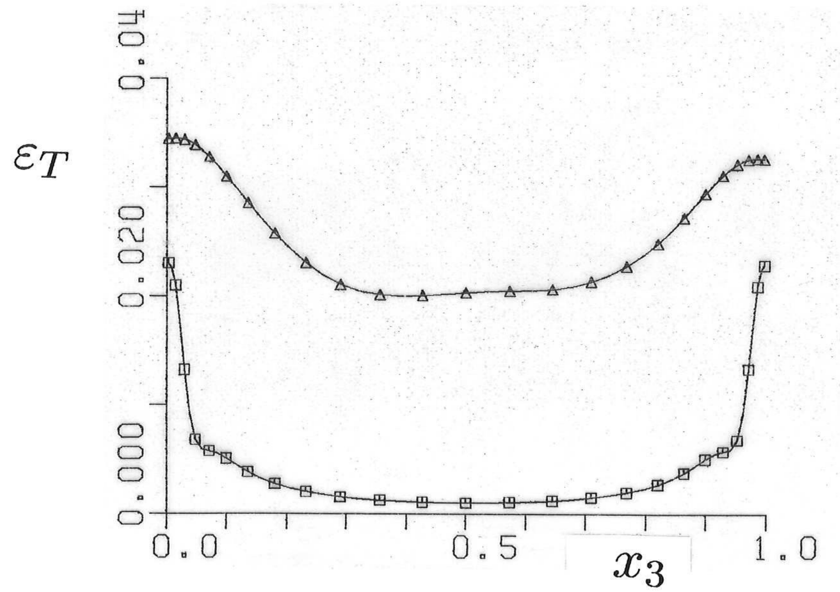
	air	sodium
Pr	0.71	0.006
Ra	630,000	24,000
Bo	447,300	144
Gr	887,324	4,000,000
$X_{1,2}$	7.92	8.0
$N_{1,2}$	200	250
N_3	49	49

Results (□ =air, △ =sodium)[1]

- Vertical profile of mean temperature $\overline{T}$



- Vertical profile of $\varepsilon_T = \frac{1}{\sqrt{Bo}} \overline{\frac{\partial T'}{\partial x_j} \frac{\partial T'}{\partial x_j}}$



[1] overbar denotes ensemble and time averaging of quantity

Transport equation for ε_T

- here: $\overline{u_3} = 0, \quad \frac{\partial \overline{\phi}}{\partial x_{1,2}} = 0 \quad \Rightarrow$

$$\frac{\partial \varepsilon_T}{\partial t} = \underbrace{-\frac{2}{\sqrt{Bo}} \overline{\frac{\partial T'}{\partial x_j} \frac{\partial u_3'}{\partial x_j}} \frac{\partial \overline{T}}{\partial x_3}}_{P^1_{\varepsilon_T}} \underbrace{- \frac{2}{\sqrt{Bo}} \overline{u_3' \frac{\partial T'}{\partial x_3}} \frac{\partial^2 \overline{T}}{\partial x_3 \partial x_3}}_{P^2_{\varepsilon_T}}$$

$$\underbrace{-\frac{2}{\sqrt{Bo}} \overline{\frac{\partial T'}{\partial x_l} \frac{\partial T'}{\partial x_3}} \frac{\partial \overline{u}_l}{\partial x_3}}_{P^3_{\varepsilon_T}} \underbrace{- \frac{2}{\sqrt{Bo}} \overline{\frac{\partial T'}{\partial x_j} \frac{\partial u_l'}{\partial x_j} \frac{\partial T'}{\partial x_l}}}_{P^4_{\varepsilon_T}}$$

$$\underbrace{-\frac{2}{Bo} \overline{\left(\frac{\partial^2 T'}{\partial x_j \partial x_l} \right)^2}}_{\gamma_{\varepsilon_T}} \underbrace{- \frac{\partial}{\partial x_3} \left(\overline{\varepsilon_T' u_3'} - \frac{1}{\sqrt{Bo}} \frac{\partial \varepsilon_T}{\partial x_3} \right)}_{D_{\varepsilon_T} = D_{\varepsilon_T, t} + D_{\varepsilon_T, m}}$$

$P^1_{\varepsilon_T}, P^2_{\varepsilon_T}$ = generation by mean temperature field

$P^3_{\varepsilon_T}$ = generation by mean velocity field

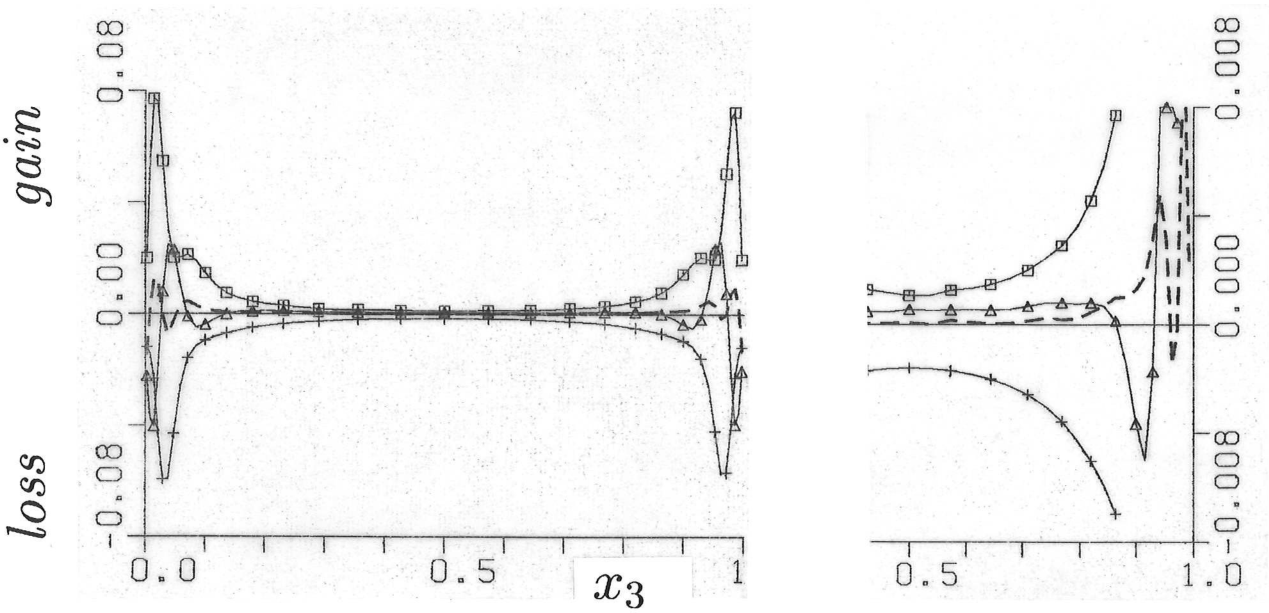
$P^4_{\varepsilon_T}$ = generation by fine scale turbulence interaction

γ_{ε_T} = destruction by fine scale turbulence interaction

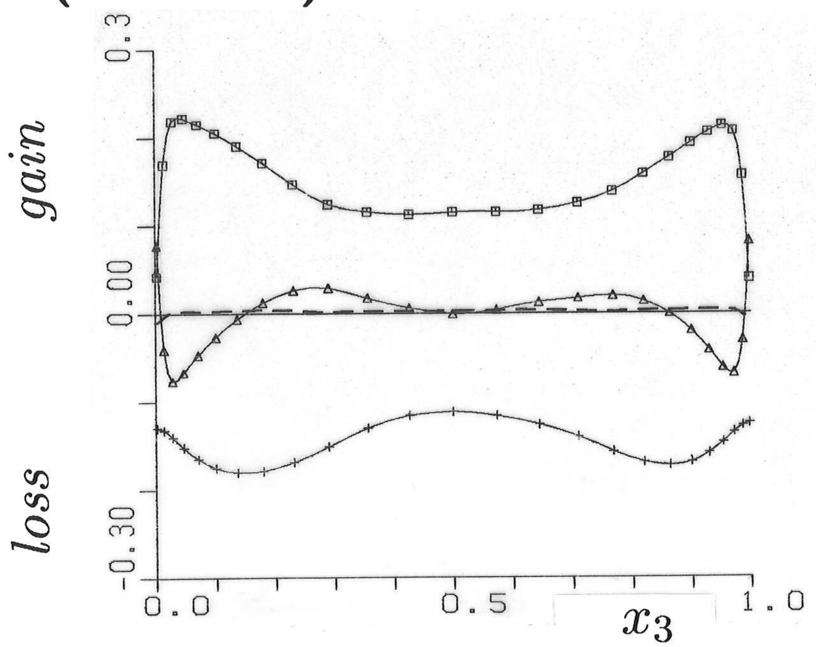
D_{ε_T} = turbulent and molecular diffusion

Budget of ε_T

- **air** ($Bo = 447,300$)



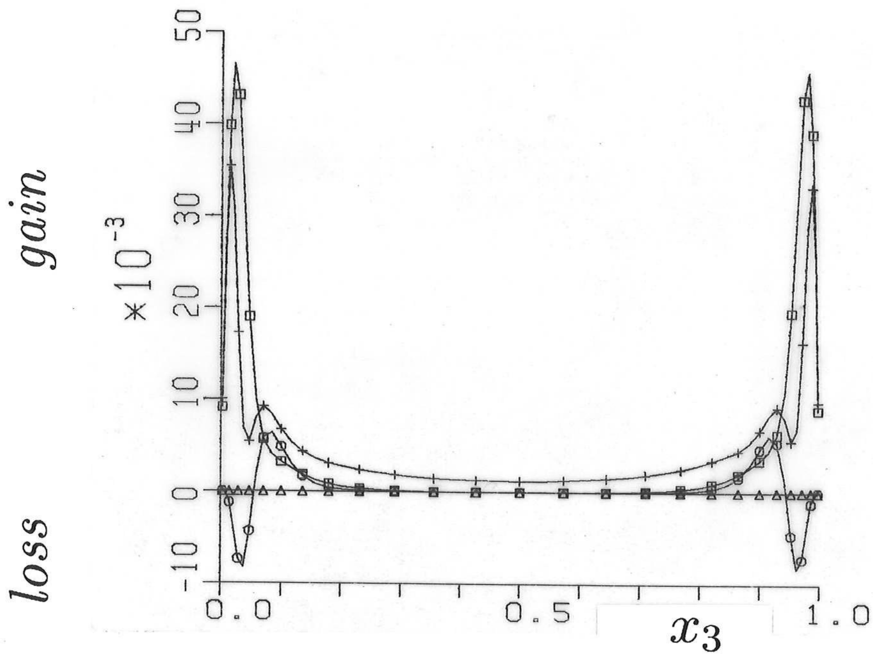
- **sodium** ($Bo = 144$)



P_{ε_T} ($\square$), γ_{ε_T} ($+$), D_{ε_T} ($\triangle$), balance difference (- - -)

Generation of ε_T

- **air** ($Bo = 447,300$)



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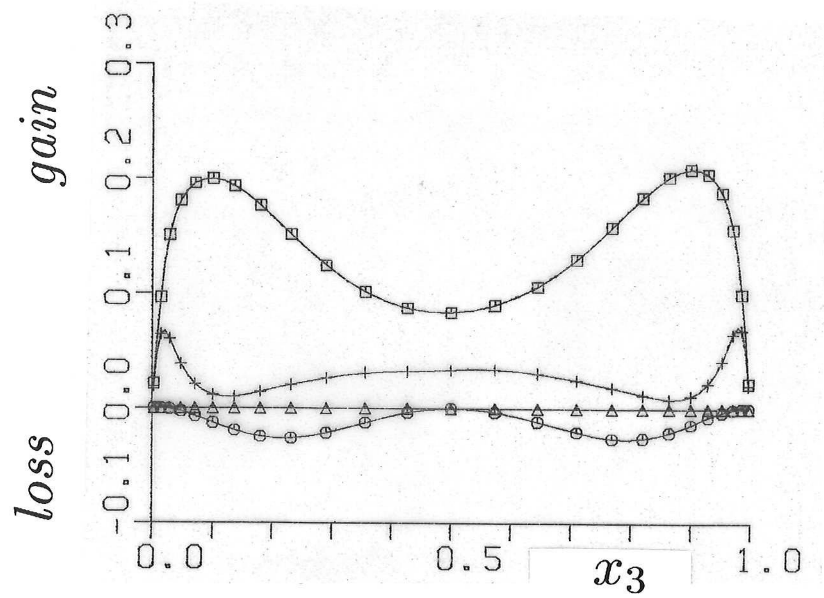


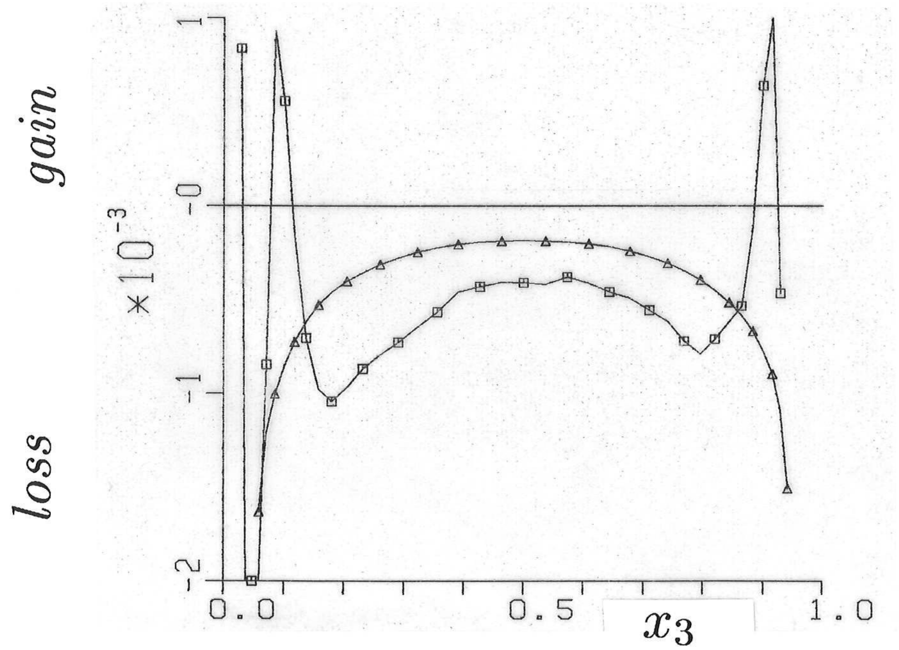
$P^1_{\varepsilon_T}(\square)$, $P^2_{\varepsilon_T}(\circ)$, $P^3_{\varepsilon_T}(\triangle)$, $P^4_{\varepsilon_T}(+)$

Model for generation/destruction terms [1]

$$P^1_{\varepsilon_T} + P^3_{\varepsilon_T} + P^4_{\varepsilon_T} + \gamma_{\varepsilon_T} = \underbrace{C_{P1}\frac{\varepsilon_T}{\overline{T'^2}}P_T}_{M_1} + \underbrace{C_{P2}\frac{\varepsilon_T}{k}P_k}_{M_2} \underbrace{-C_{D1}\frac{\varepsilon_T^2}{\overline{T'^2}}}_{M_3} \underbrace{-C_{D2}\frac{\varepsilon\varepsilon_T}{k}}_{M_4}$$

where $P_T = -\overline{u'_j T'} \cdot \frac{\partial \overline{T}}{\partial x_j}$, $P_k = -\overline{u'_i u'_j} \cdot \frac{\partial \overline{u_i}}{\partial x_j}$

- **results for air** [2]



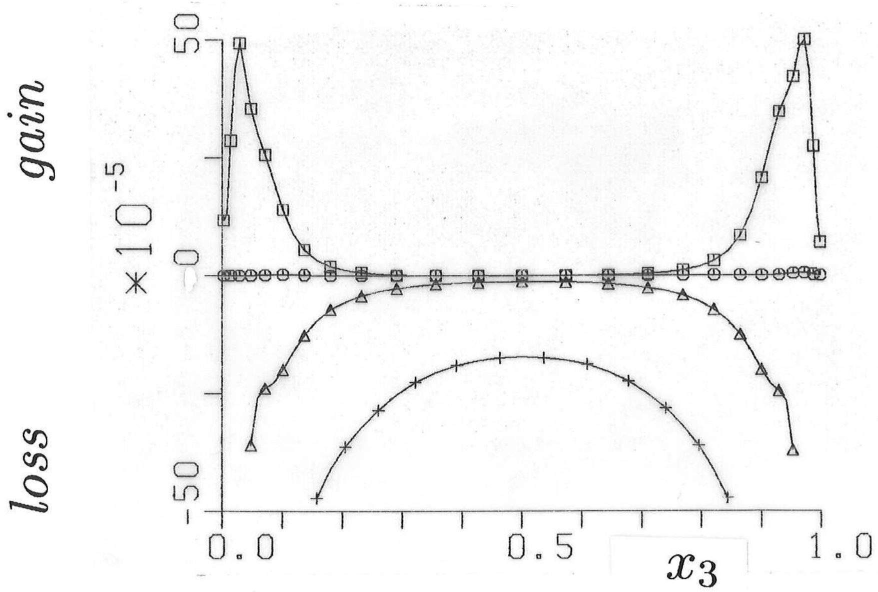
$P^1_{\varepsilon_T} + P^3_{\varepsilon_T} + P^4_{\varepsilon_T} + \gamma_{\varepsilon_T}(\square)$, $M_1 + M_2 + M_3 + M_4(\triangle)$

[1] Nagano & Kim, J. Heat Transfer, **110** (1988) 583

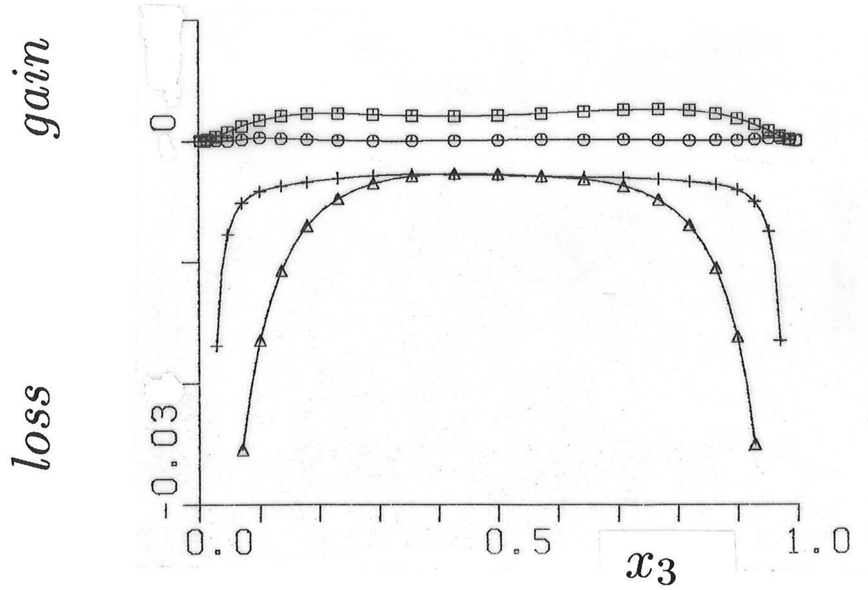
[2] $C_{P1} = 1.8, C_{P2} = 0.72, C_{D1} = 2.2, C_{D2} = 0.8$

Profiles of model terms $M_{1,2,3,4}$

- **air** ($Bo = 447,300$)



- **sodium** ($Bo = 144$)



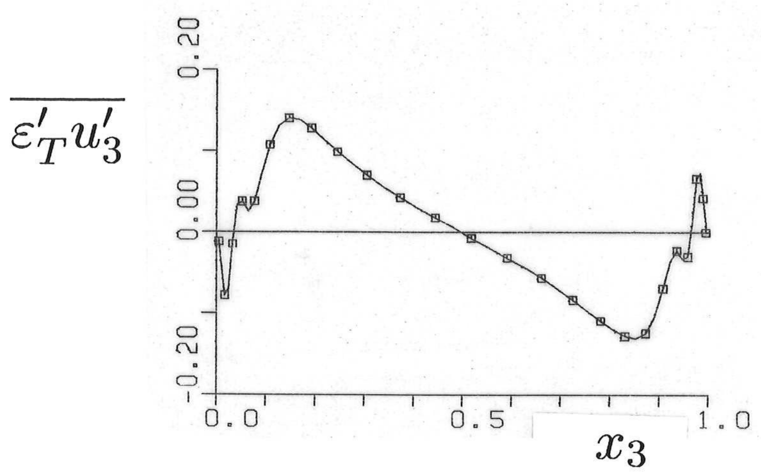
$M_1(\square), M_2(\circ), M_3(\triangle), M_4(+)$

Models for turbulent diffusion of ε_T

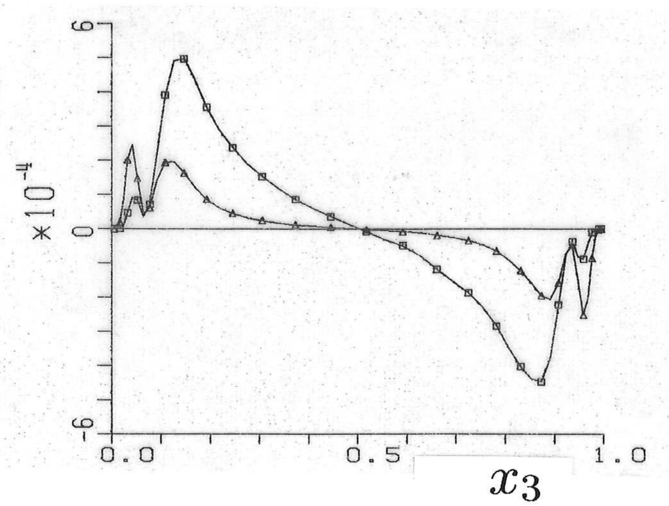
$$\overline{\varepsilon'_T u'_i} = -C_S \frac{k}{\varepsilon} \overline{u'_i u'_j} \frac{\partial \varepsilon_T}{\partial x_j}, \quad C_S = 0.22 \quad (1)$$

$$\overline{\varepsilon'_T u'_i} = -C_{DD} \frac{k^2}{\varepsilon} \frac{\partial \varepsilon_T}{\partial x_i}, \quad C_{DD} = 0.03 \quad (2)$$

- Evaluated profile of $\overline{\varepsilon'_T u'_3}$ for air



- Profile of $\overline{\varepsilon'_T u'_3}$ predicted by model 1 (□) and 2 (△)



Conclusions

- Direct Numerical Simulation data of turbulent natural convection
  - air $Gr = 887,324, \quad Bo = 447,300$
  - sodium $Gr = 4,000,000, Bo = 144$
- Budget of ε_T
  - almost local equilibrium $P_{\varepsilon_T} \approx \gamma_{\varepsilon_T}$
  - high Boussinesq number
    $\Rightarrow$ generation/destruction due to fine scale turbulence interaction are dominant terms
  - $P^2_{\varepsilon_T}$ changes sign
- Modelling of ε_T-equation
  - model for generation/destruction terms performs very well at high Bo
  - gradient diffusion model is inadequate