Modelling turbulent dissipation rate for Rayleigh-Bénard convection

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June 6, 1997

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Contents

- Equations governing the second moments:

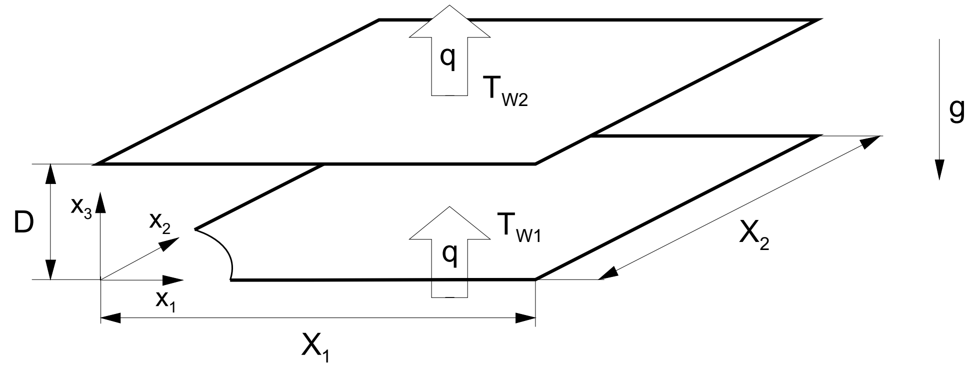
$$\begin{aligned}\frac{\partial \overline{u_i u_j}}{\partial t} &+ U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k} + \overline{u_i u_k}\frac{\partial U_j}{\partial x_k} \\ &+ \beta g_j \overline{\theta u_i} + \beta g_i \overline{\theta u_j} + \frac{\partial \overline{u_i u_j u_k}}{\partial x_k} \\ &+ \frac{1}{\rho}[\overline{u_j \frac{\partial p}{\partial x_i}} + \overline{u_i \frac{\partial p}{\partial x_j}}] + 2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} \\ &- \nu \Delta_x \overline{u_i u_j} = 0\end{aligned}$$

- Dissipation correlations in $\overline{u_i u_j}$ equation:

$$\epsilon_{ij} = \nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}$$

- Modelling of ϵ_{ij} is very important for reliable flow prediction
- Requirement of special care for the flows dominated by buoyancy
- Objective: Turbulence closure for dissipation rate
- Tools: -DNS data for Rayleigh-Bénard convection
  -Two-point correlation technique
  -Invariant theory

DNS data for Rayleigh-Bénard convection



- The simulation were performed with the TURBIT code(Grötzbach, 1987).
- Parameters of the simulation data

Pr	Ra	Gr	Re_t	Pe_t
0.006	6,000	10^6	497	3
0.006	24,000	4×10^6	2240	13
0.7	381,000	5.4×10^5	109	76
0.7	630,000	8.9×10^5	154	107

- Using two-point correlation technique (Chou 1945, Kolovandin & Vatutin 1972), the dissipation tensor ϵ_{ij} can be decomposed into:

$$\epsilon_{ij} = \nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}} = \frac{1}{4}\nu \Delta_x \overline{u_i u_j} - \nu(\Delta_\xi \overline{u_i u_j'})_0$$

The dissipation rate ϵ:

$$\epsilon = \nu \overline{\frac{\partial u_s}{\partial x_k}\frac{\partial u_s}{\partial x_k}} = \underbrace{\frac{1}{4}\nu \Delta_x \overline{u_s u_s}}_{inhomogeneous} \overbrace{\underbrace{- \nu(\Delta_\xi \overline{u_s u_s'})_0}_{homogeneous}}^{\epsilon_h},$$

where $\xi_k = (x_k)_B - (x_k)_A$

- Dynamic equation for ϵ_h

$$\begin{aligned}
\frac{\partial \epsilon_h}{\partial t} + U_k \frac{\partial \epsilon_h}{\partial x_k} &= \nu[(\Delta_\xi \overline{u_k u_s'})_0 + (\Delta_\xi \overline{u_s u_k'})_0]\frac{\partial U_s}{\partial x_k} \\
&+ \frac{\nu}{4}\left[2\overline{u_s u_k}\Delta_x \frac{\partial U_s}{\partial x_k} + (\Delta_x U_k)\frac{\partial}{\partial x_k}\overline{u_s u_s}\right] \\
&+ \nu\left[\left(\frac{\partial}{\partial \xi_l}\overline{u_s u_k'}\right)_0 - \left(\frac{\partial}{\partial \xi_l}\overline{u_k u_s'}\right)_0\right]\frac{\partial^2 U_s}{\partial x_l \partial x_k} \\
&+ 2\nu\left(\frac{\partial^2}{\partial \xi_l \partial \xi_k}\overline{u_s u_s'}\right)_0 \frac{\partial U_k}{\partial x_l} + \nu\beta g_s[(\Delta_\xi \overline{\theta u_s'})_0 + (\Delta_\xi \overline{u_s \theta'})_0] \\
&+ \frac{\nu}{2}\frac{\partial}{\partial x_k}[(\Delta_\xi \overline{u_s u_k u_s'})_0 + (\Delta_\xi \overline{u_s u_s' u_k'})_0] \\
&+ \nu[\Delta_\xi \frac{\partial}{\partial \xi_k}(\overline{u_s u_s' u_k'} - \overline{u_s u_k u_s'})]_0 \\
&+ \frac{\nu}{2\rho}\frac{\partial}{\partial x_s}[(\Delta_\xi \overline{p u_s'})_0 + (\Delta_\xi \overline{u_s p'})_0] \\
&- \frac{\nu}{\rho}[\Delta_\xi \frac{\partial}{\partial \xi_s}(\overline{p u_s'} - \overline{u_s p'})]_0 + \frac{1}{2}\nu \Delta_x \epsilon_h - 2\nu^2(\Delta_\xi \Delta_\xi \overline{u_s u_s'})_0
\end{aligned}$$

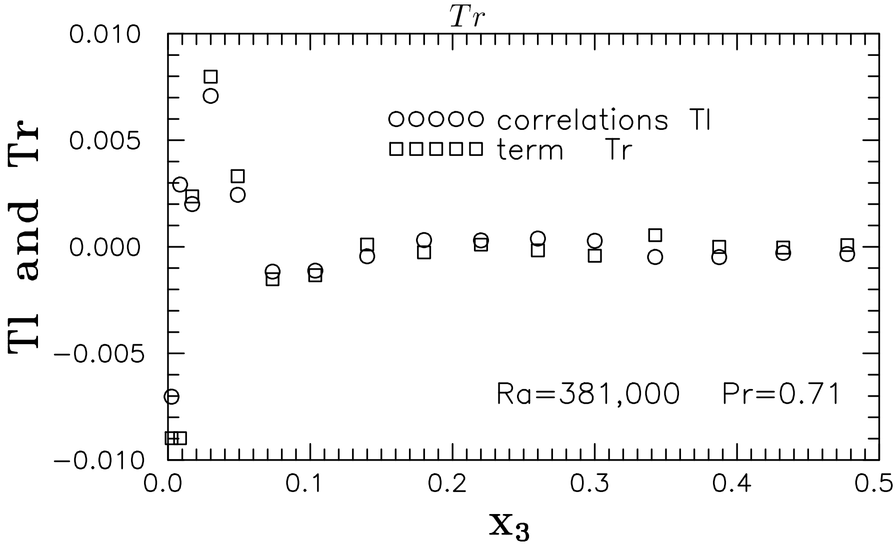
- The properties of homogeneous turbulence for two-point correlations:

$$\begin{array}{rcl} \overline{u_s u_k'} = \overline{u_k u_s'} & & (\frac{\partial}{\partial \xi}\overline{u_s u_k'})_0 - (\frac{\partial}{\partial \xi}\overline{u_k u_s'})_0 = 0 \\ \overline{u_s u_s' u_k'} = -\overline{u_s u_k u_s'} & \Rightarrow & (\Delta_\xi \overline{u_s u_s' u_k'})_0 + (\Delta_\xi \overline{u_s u_k u_s'})_0 = 0 \\ \overline{\gamma u_s'} = -\overline{u_s \gamma'} & & (\Delta_\xi \overline{\gamma u_s'})_0 + (\Delta_\xi \overline{u_s \gamma'})_0 = 0 \end{array}$$

- Testing the assumption of the local homogeneity for two-point velocity correlations of third rank

If $\quad (\Delta_\xi \overline{u_s u_k u_s'})_0 + (\Delta_\xi \overline{u_s u_s' u_k'})_0 \simeq 0 \quad \rightsquigarrow$

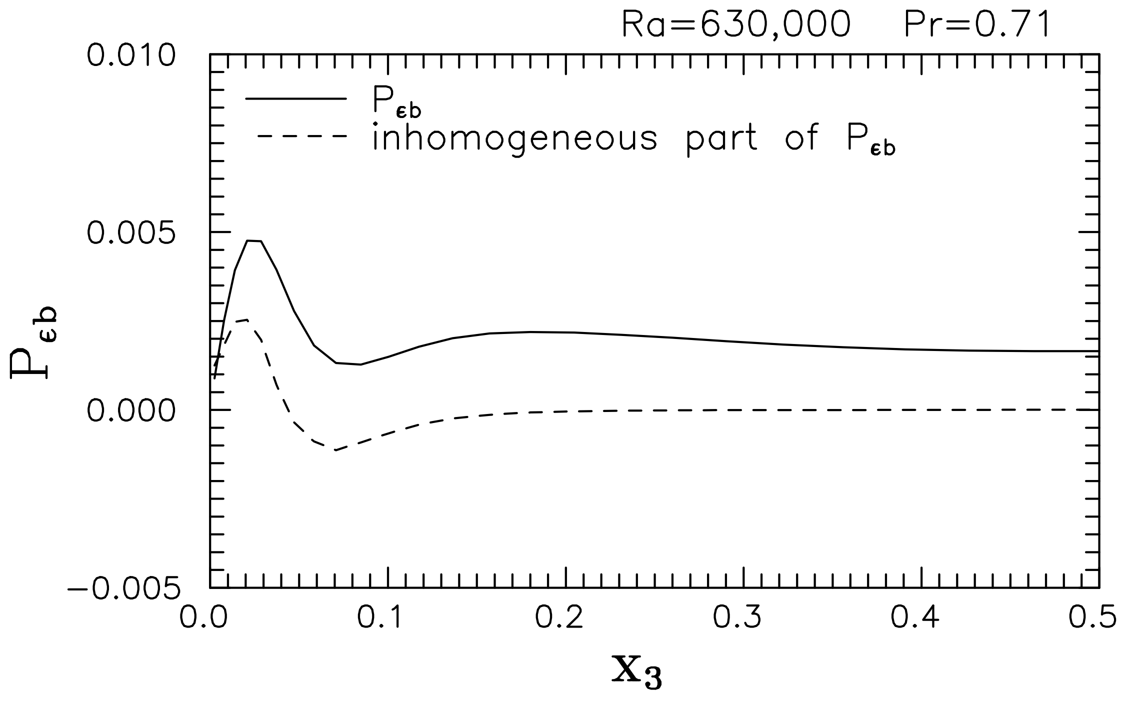
$$\begin{aligned} \underbrace{\nu \frac{\partial}{\partial x_k} \overline{\frac{\partial u_i}{\partial x_l} \frac{\partial u_k u_i}{\partial x_l}}}_{Tl} &= \frac{1}{4}\nu \frac{\partial}{\partial x_k} \Delta_x \overline{u_i u_k u_i} \\ &- \frac{1}{2}\nu \frac{\partial}{\partial x_k} [(\Delta_\xi \overline{u_i u_k u_i'})_0 + (\Delta_\xi \overline{u_i u_k' u_i'})_0] \\ &\simeq \underbrace{\frac{1}{4}\nu \frac{\partial}{\partial x_k} \Delta_x \overline{u_i u_k u_i}}_{Tr} \end{aligned}$$



- Testing the assumption of the local homogeneity for two-point temperature-velocity correlations

If $\quad (\Delta_\xi \overline{\theta u'_s})_0 + (\Delta_\xi \overline{u_s \theta'})_0 \simeq 0 \quad \rightsquigarrow$

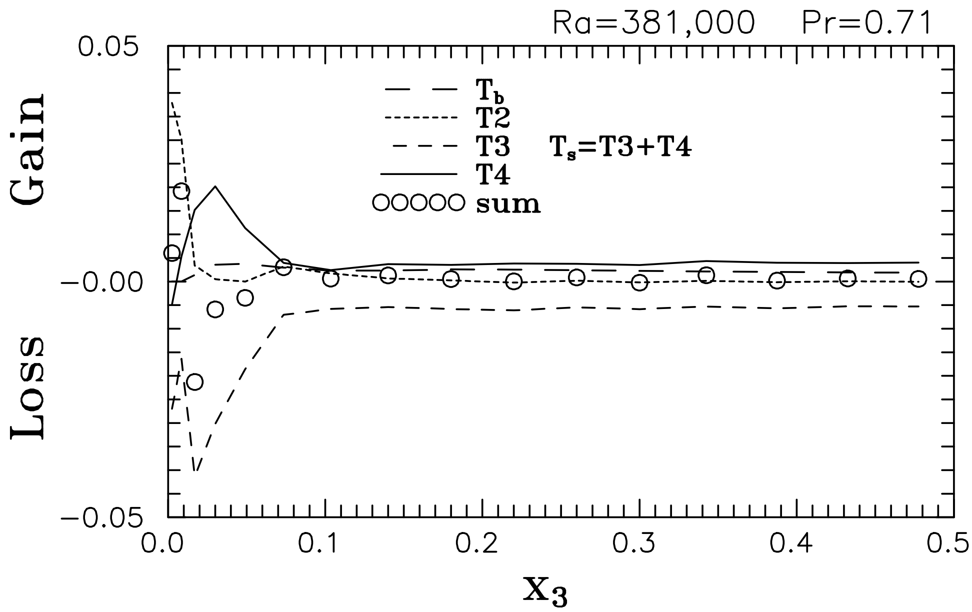
$$
\begin{aligned}
P_{\epsilon b} &= -2\nu\beta g_i \overline{\frac{\partial \theta}{\partial x_l}\frac{\partial u_i}{\partial x_l}} \\
&= -\frac{1}{2}\nu\beta g_i \Delta_x \overline{\theta u_i} + \nu\beta g_i[(\Delta_\xi \overline{\theta u'_i})_0 + (\Delta_\xi \overline{u_i \theta'})_0] \\
&\simeq \underbrace{-\frac{1}{2}\nu\beta g_i \Delta_x \overline{\theta u_i}}_{\text{inhomogeneous part}}
\end{aligned}
$$



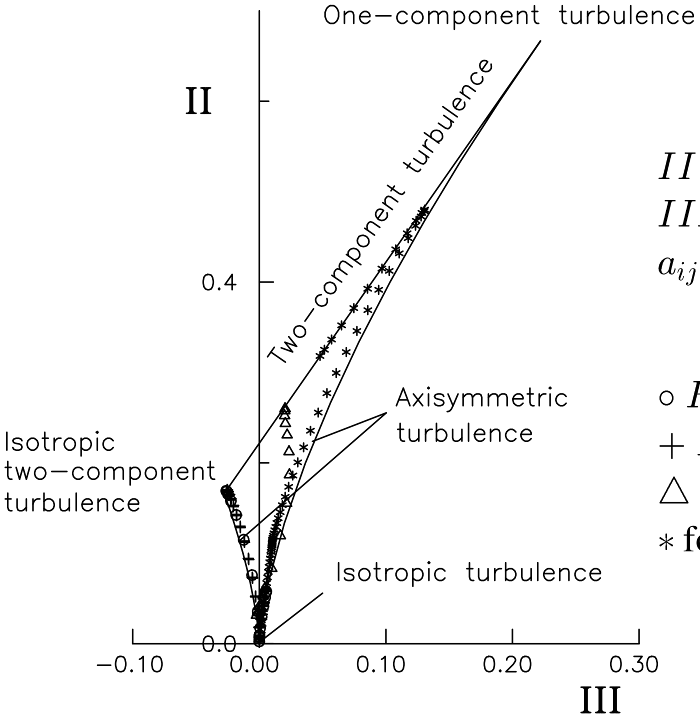
- For Rayleigh-Bénard convection the approximate equation for ϵ_h:

$$\frac{\partial \epsilon_h}{\partial t} \simeq \underbrace{\nu \beta g_s [(\Delta_\xi \overline{\theta u'_s})_0 + (\Delta_\xi \overline{u_s \theta'})_0]}_{T_b} + \frac{1}{2}\nu \Delta_x \epsilon_h$$
$$\underbrace{-2\nu^2 (\Delta_\xi \Delta_\xi \overline{u_s u'_s})_0 - 2\nu \left(\Delta_\xi \frac{\partial}{\partial \xi_k} \overline{u_s u_k u'_s}\right)_0}_{T_s}$$

- Budget of the equation above



Anisotropy invariant map of Reynolds stress



$II = a_{ij}a_{ji}$
$III = a_{ij}a_{ik}a_{jk}$
$a_{ij} = 2\overline{u_i u_j}/k - 1/3\delta_{ij}$

$\circ$ $Ra = 381,000$, $Pr = 0.71$
$+$ $Ra = 630,000$, $Pr = 0.71$
$\triangle$ $Ra = 6000$, $Pr = 0.006$
$*$ forced convection (Kim et al)

Model for the sink term T_s

$$T_s = -\psi\frac{\epsilon_h^2}{k}, \;\; \psi = (1-F)\psi_{2C} + F\psi_{axi}, \;\; F = J^2/L,$$

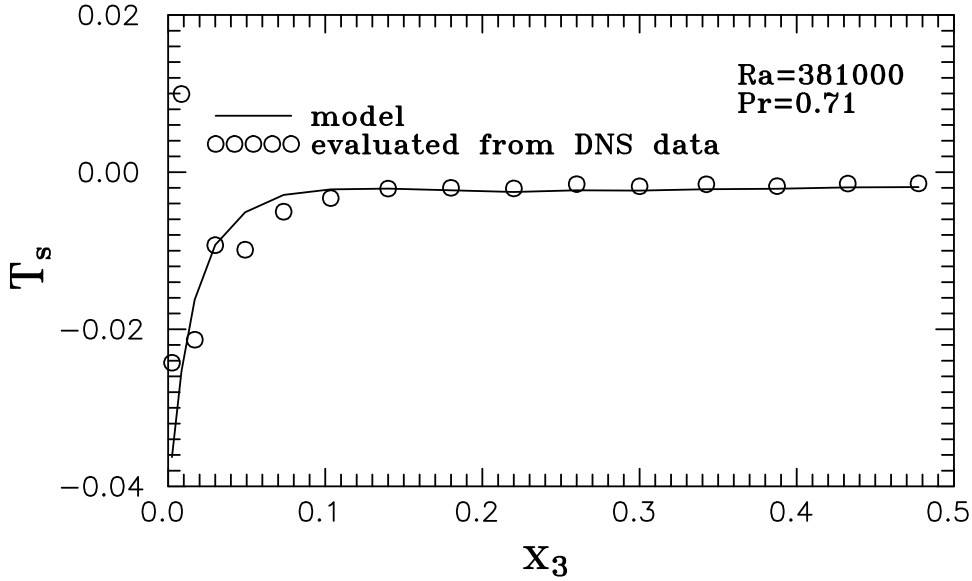
$$J = 1 - 9(1/2II - III), \;\; L = 1 - 9[\frac{3}{4}(\frac{4}{3}III)^{2/3} - III],$$

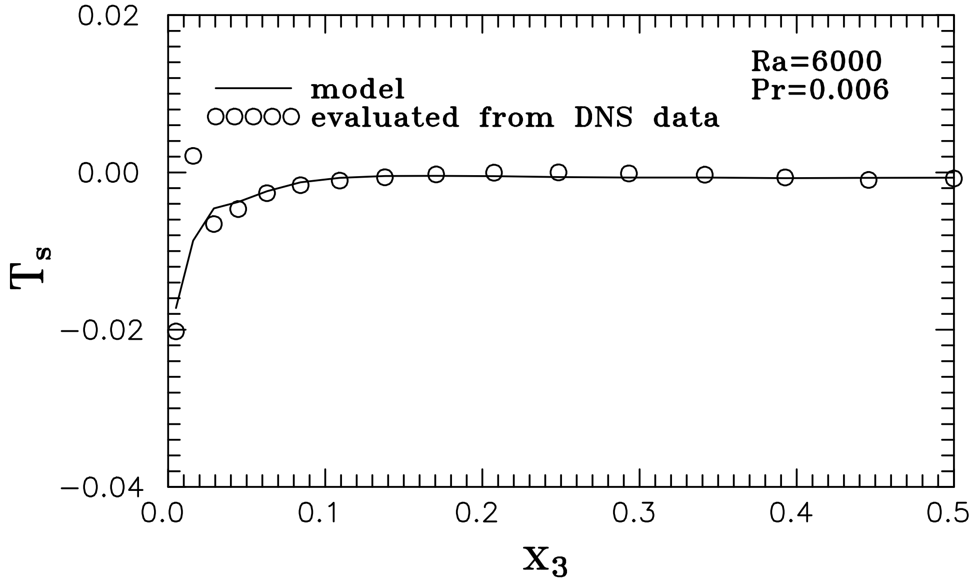
$$\psi_{2C} = [0.02 + 0.03\exp(-Re_t)]\sqrt{20Re_t},$$

$$\psi_{axi} = 1.4 + L(\frac{7\sqrt{3}}{90}f_\epsilon - 1.4), \quad III > 0$$

$$\psi_{axi} = 1.2 + L(\frac{7\sqrt{3}}{90}f_\epsilon - 1.2), \quad III < 0$$

$$f_\epsilon = \frac{54\sqrt{3}}{7}[1 - 0.222\exp(-0.336\sqrt{Re_t})].$$





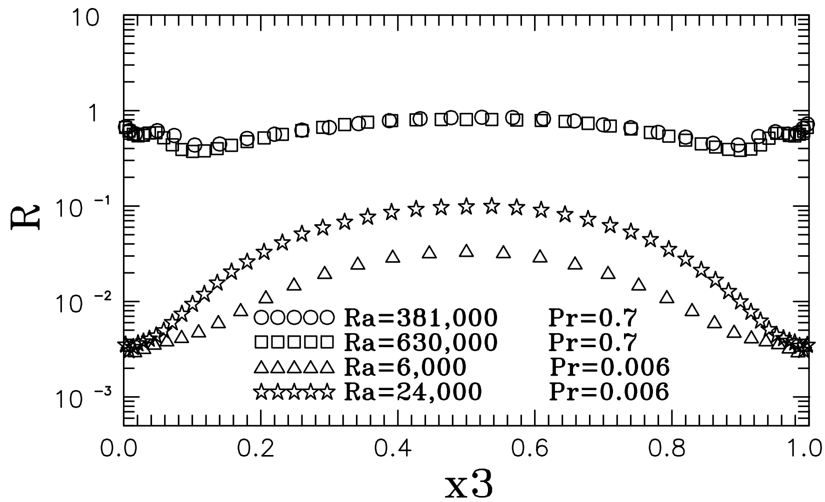
Vertical profile of the sink term T_s

- Model for the buoyant production term T_b in ϵ_h Eq.:

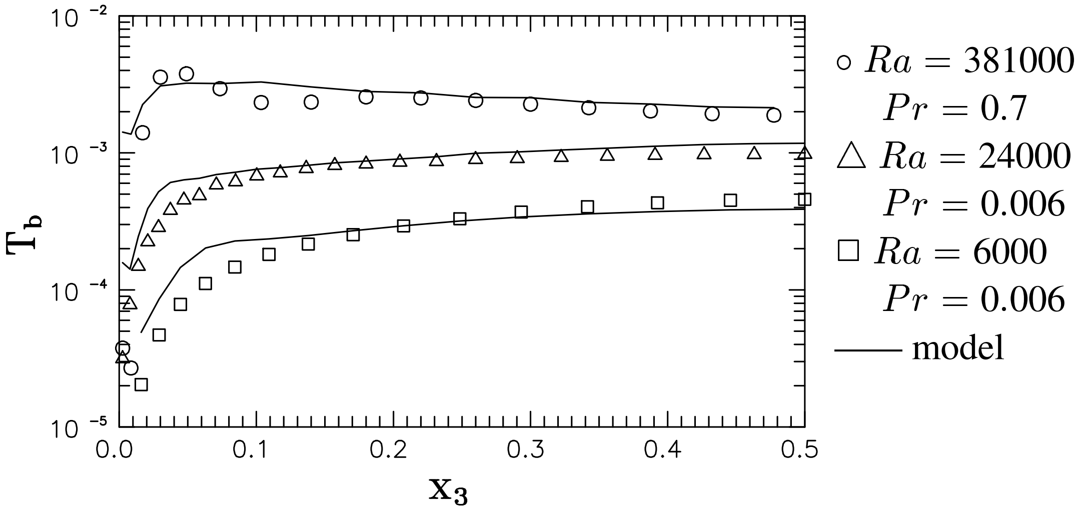
$$T_b \simeq C_{\epsilon 3}\frac{\epsilon_h}{k}G = \left(\frac{Pr}{R}\right)^{0.7}\frac{\epsilon_h}{k}G, \quad G = -\beta g_s \overline{u_s \theta},$$

where time scale ratio $R = \tau_\theta/\tau = (\overline{\theta^2}/2\epsilon_\theta)/(k/\epsilon)$ (ϵ_θ is the dissipation of $\overline{\theta^2}$, $\overline{\theta^2}$ is temp. variance).

- Distribution of the time scale ratio R



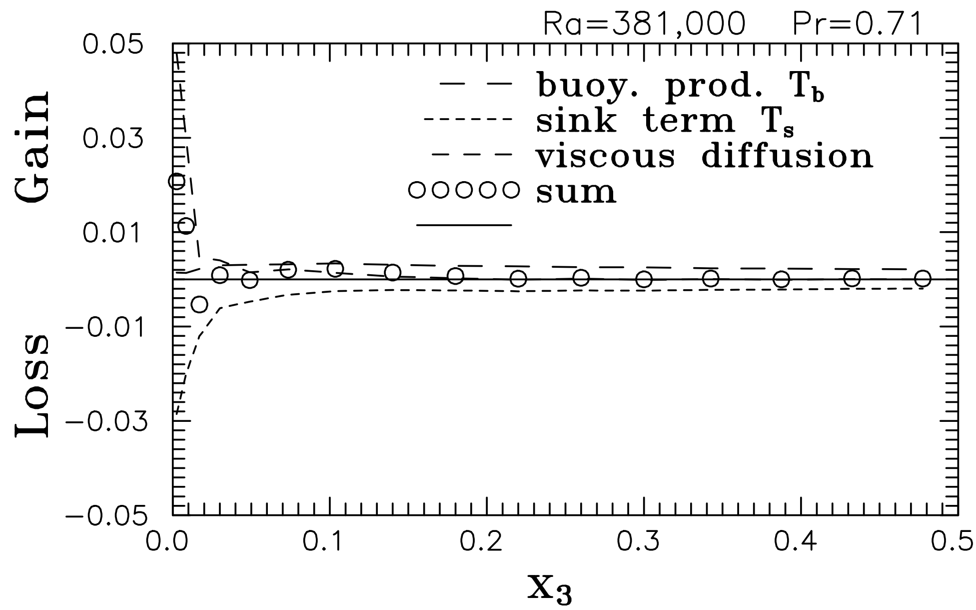
- Distribution of the buoyant production



- The modeled equation for ϵ_h:

$$\frac{\partial \epsilon_h}{\partial t} \simeq \left(\frac{Pr}{R}\right)^{0.7} \frac{\epsilon_h}{k} G - \psi \frac{\epsilon_h^2}{k} + \frac{1}{2} \nu \Delta_x \epsilon_h$$

- Terms in the modeled ϵ_h equation



- The closure of the equation for turbulent dissipation rate was investigated by means of the DNS data of Rayleigh-Bénard convection.
- The assumption of local homogeneity was tested:

  Valid: for the derivatives of two-point velocity correlations of third rank

  Except: for the derivatives of the two-point velocity/temp. correlations
- The derived closure for the sink term in the equation for the homogeneous part of the dissipation rate shows a good agreement with the DNS data.
- A new model for the buoyant production term in the equation for ϵ_h was proposed, which can account for the influence of Prandtl number and turbulence level.
- The model for buoyant production term can be also used for the modelling dissipation term $\epsilon_{i\theta}$ in the transport equation for the heat flux $\overline{u_i\theta}$

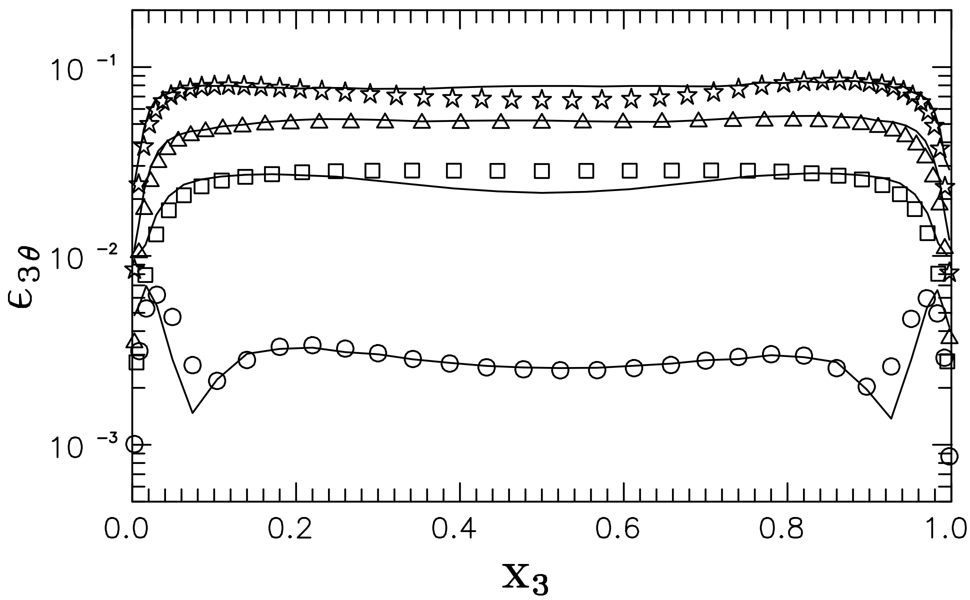
- Buoyant production term in ϵ equation

$$P_{\epsilon b} = -2\nu\beta g_i \overline{\frac{\partial\theta}{\partial x_l}\frac{\partial u_i}{\partial x_l}}$$
$$\simeq -\frac{1}{2}\nu\beta g_i \Delta_x \overline{\theta u_i} + \left(\frac{Pr}{R}\right)^{0.7} \frac{\epsilon_h}{k} G$$

- Dissipation term in heat flux $\overline{u_i\theta}$ equation

$$\epsilon_{i\theta} = (\nu+\alpha)\overline{\frac{\partial\theta}{\partial x_l}\frac{\partial u_i}{\partial x_l}}$$
$$\simeq \frac{1}{4}(\nu+\alpha)\Delta_x\overline{\theta u_i} + \frac{1}{2}(\nu+\alpha)\sqrt{Gr}\left(\frac{Pr}{R}\right)^{0.7}\frac{\epsilon_h}{k}G$$

- Vertical profiles of $\epsilon_{3\theta}$

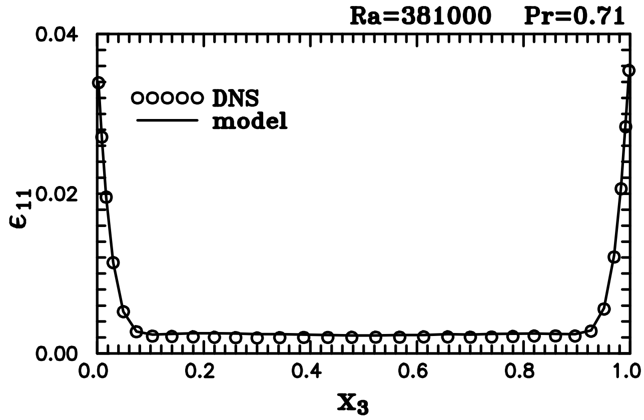


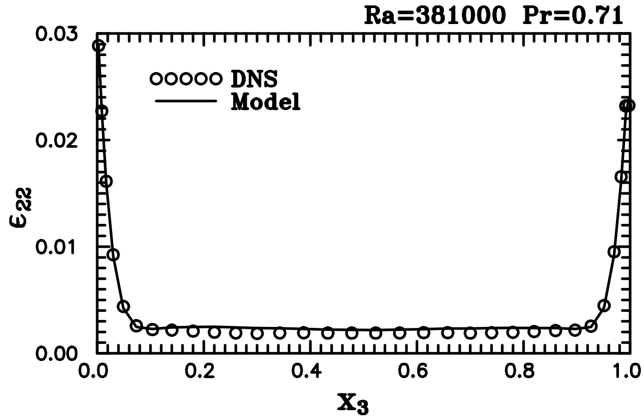
$Pr = 0.006$: $*$ $Ra = 24000$ $\quad$ $Pr = 0.71$: $\circ$ $Ra = 381000$

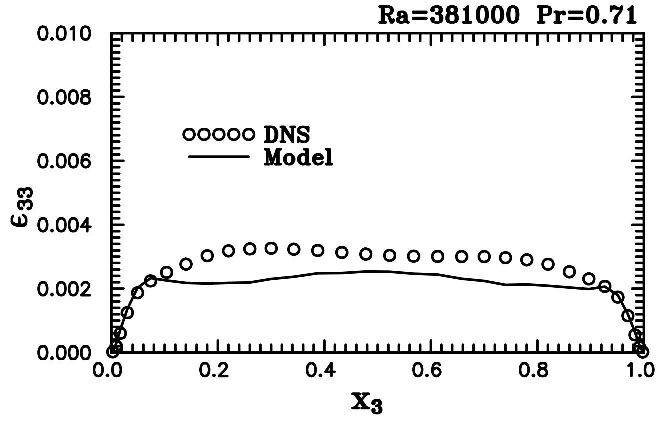
$\triangle$ $Ra = 12000$ $\quad$ —— model

$\square$ $Ra = 6000$

$$\epsilon_{ij} = \frac{1}{4}\nu\Delta_x\overline{u_iu_j} + (\epsilon_{ii} - \frac{1}{4}\nu\Delta_x q^2)\left\{\frac{1}{3}(1-A)\delta_{ij} + A\frac{\overline{u_iu_j}}{q^2}\right\}$$







Component of the dissipation tensor