Consistent modelling of fluctuating temperature-gradient-velocity-gradient correlations for natural convection

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Background and Motivation

- natural convection in nuclear safety research

  - in-vessel cooling of decay-heated molten core
  - sump cooling of core melt by water layer

  ⇒ explore flow features by engineering CFD codes

- statistical turbulence models for natural convection

  - typically low Reynolds and Peclet numbers
  - no universal law of the wall for natural convection

    ⇒ <u>low Reynolds number model required</u>
  - complex geometrical configurations

    ⇒ <u>avoid wall-distance in model</u>

Objectives

develop simple low Reynolds number models for closure terms important in buoyant flows

- buoyancy production in dissipation equation

$$P_{\varepsilon b} = -2\nu\beta g_i \underbrace{\overline{\frac{\partial\theta}{\partial x_l}\frac{\partial u_i}{\partial x_l}}}_{\Upsilon_i}$$

- molecular dissipation term in $\overline{\theta u_i}$-equation

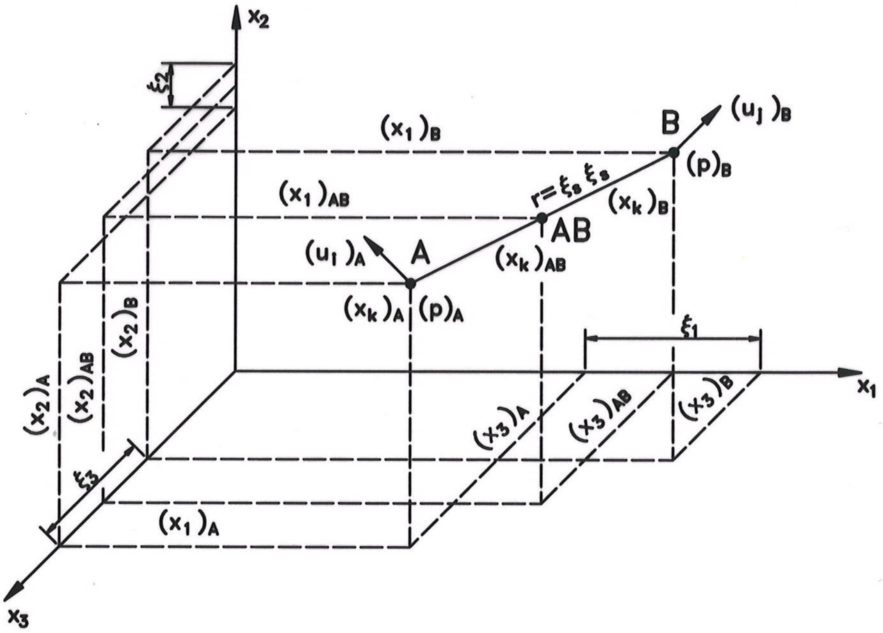
$$\varepsilon_{\theta i} = (\nu + \kappa)\underbrace{\overline{\frac{\partial\theta}{\partial x_l}\frac{\partial u_i}{\partial x_l}}}_{\Upsilon_i}$$

present approach for model development[1]:
- apply two-point correlation technique
- a priori testing by DNS data for natural convection

[1] Jovanovic, Ye, Durst: Statistical interpretation of the turbulent dissipation rate in wall-bounded flow, *J. Fluid Mech.* 293:321-347

Two-point correlation technique[2]

- Local coordinate system: $\xi_k = (x_k)_B - (x_k)_A$



- Partial differential operators at points A and B:

$$\left(\frac{\partial}{\partial x_k}\right)_A = \frac{1}{2}\left(\frac{\partial}{\partial x_k}\right)_{AB} - \frac{\partial}{\partial \xi_k}$$

- Example: dissipation rate ε

$$\varepsilon = \nu \overline{\frac{\partial u_s}{\partial x_l}\frac{\partial u_s}{\partial x_l}} = \underbrace{\frac{1}{4}\nu \Delta_x \overline{u_s u_s}}_{inhomogeneous} \underbrace{- \nu(\Delta_\xi \overline{u_s u_s'})_0}_{homogeneous} = \frac{1}{2}\nu \Delta_x k + \varepsilon_h$$

[2]Chou (1945), Kolovandin & Vatutin (1972)

Development of model for Υ_i

$$\Upsilon_i = \overline{\frac{\partial\theta}{\partial x_l}\frac{\partial u_i}{\partial x_l}} = \underbrace{\frac{1}{4}\Delta_x\overline{\theta u_i}}_{\Upsilon_{i,inh}} \underbrace{-\frac{1}{2}[(\Delta_\xi\overline{\theta u_i'})_0 + (\Delta_\xi\overline{u_i\theta'})_0]}_{\Upsilon_{i,hom}}$$

- inhomogeneous part $\Upsilon_{i,inh}$ needs no approximation
- to model homogeneous part $\Upsilon_{i,hom}$ recall model of Shikazono&Kasagi[3] for total correlation Υ_i :

$$\frac{\overline{\frac{\partial\theta}{\partial x_l}\frac{\partial u_i}{\partial x_l}}}{\sqrt{\overline{\left(\frac{\partial\theta}{\partial x_l}\right)^2}}\sqrt{\overline{\left(\frac{\partial u_{(i)}}{\partial x_l}\right)^2}}} \simeq \mathcal{C}\frac{\overline{\theta u_i}}{\sqrt{\overline{\theta^2}}\sqrt{\overline{u_{(i)}^2}}}$$

$$\frac{\overline{u_{(i)}^2}}{\overline{\left(\frac{\partial u_{(i)}}{\partial x_l}\right)^2}} \simeq \frac{2k}{\frac{\varepsilon}{\nu}}, \qquad R = \frac{\overline{\theta^2}}{2\varepsilon_\theta}\frac{\varepsilon}{k}$$

$$\Rightarrow \quad \boxed{\Upsilon_i \simeq \frac{\mathcal{C}}{2\sqrt{\nu\kappa R}}\frac{\varepsilon}{k}\overline{\theta u_i}}$$

[3] Int. J. Heat Mass Transfer 39 (1996) 2977

Model for $\Upsilon_{i,hom}$

model only homogeneous part $\Upsilon_{i,hom}$ of Υ_i

$\Rightarrow$ replace ε by $\varepsilon_h = \varepsilon - \frac{1}{2}\nu\Delta_x k$

and ε_θ by $\varepsilon_{\theta h} = \varepsilon_\theta - \frac{1}{4}\kappa\Delta_x\overline{\theta^2}$

$\Rightarrow$ 'basic' model

$$\boxed{\overline{\frac{\partial\theta}{\partial x_l}\frac{\partial u_i}{\partial x_l}} = \frac{1}{4}\Delta_x\overline{\theta u_i} + \frac{\mathcal{C}}{2\sqrt{\nu\kappa R_h}}\frac{\varepsilon_h}{k}\overline{\theta u_i}}$$

respectively

$$\varepsilon^*_{\theta i,b} = \frac{1}{4}(\nu+\kappa)\frac{\partial\overline{\theta u_i}}{\partial x_l \partial x_l} + \frac{1}{2}\mathcal{C}\left(1+\frac{1}{Pr}\right)\sqrt{\frac{Pr}{R_h}\frac{\varepsilon_h}{k}}\,\overline{\theta u_i}$$

$$P^*_{\varepsilon b,b} = -\frac{1}{2}\nu\beta g_i\frac{\partial\overline{\theta u_i}}{\partial x_l \partial x_l} - \beta g_i\mathcal{C}\sqrt{\frac{Pr}{R_h}\frac{\varepsilon_h}{k}}\,\overline{\theta u_i}$$

where

$$R_h = \frac{\overline{\theta^2}}{2\varepsilon_{\theta h}}\frac{\varepsilon_h}{k}$$

Analysis of near-wall behaviour

$$\theta = a_\theta + b_\theta x_3 + c_\theta x_3^2 + d_\theta x_3^3 + \cdots$$

$$u_i = a_i + b_i x_3 + c_i x_3^2 + d_i x_3^3 + \cdots$$

- no slip boundary conditions $\rightarrow a_i = 0$
- incompressible fluid $\rightarrow b_3 = 0$
- isothermal walls $\rightarrow a_\theta = 0$

• near-wall behaviour of analytical term $\Upsilon_{3,hom}$

$$\Upsilon_{3,hom} = \Upsilon_3 - \Upsilon_{3,inh} = (2b_\theta c_3 x_3 + \cdots) - (\frac{6}{4} b_\theta c_3 x_3 + \cdots)$$

$$= \boxed{\frac{1}{2} b_\theta c_3 x_3} + \cdots$$

• near-wall behaviour of model for $\Upsilon_{3,hom}$

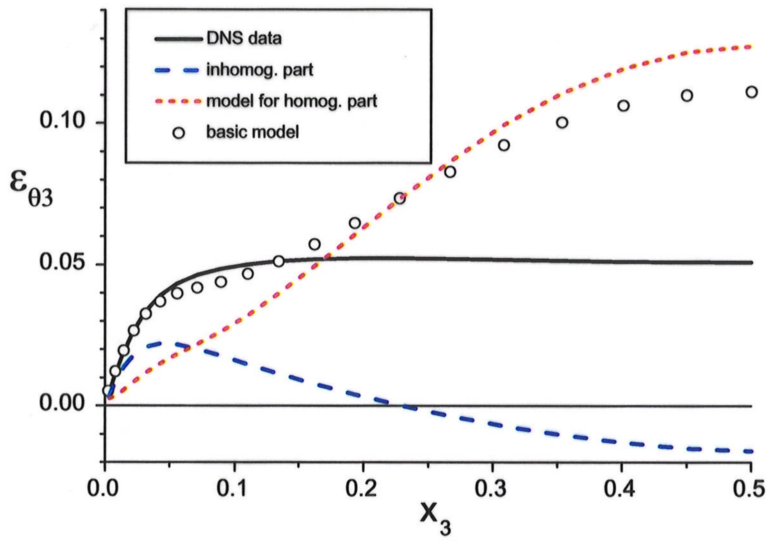
$$\Upsilon^*_{3,hom} = \frac{\mathcal{C}}{2\sqrt{\nu\kappa(\frac{\nu}{\kappa} + \cdots)}} \frac{\frac{\nu}{2}(b_1^2 + b_2^2) + \cdots}{\frac{1}{2}(b_1^2 + b_2^2)x_3^2 + \cdots} \cdot (b_\theta c_3 x_3^3 + \cdots)$$

$$= \boxed{\frac{\mathcal{C}}{2} b_\theta c_3 x_3} + \cdots$$

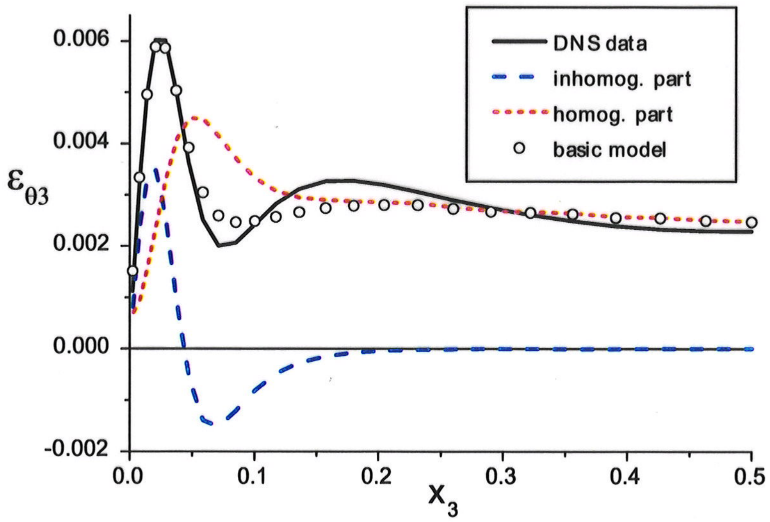
$\Rightarrow$ correct wall-limiting behaviour requires $\mathcal{C} = 1 + \cdots$

Comparison of 'basic' model with DNS data for Rayleigh Bénard convection

- RBC in sodium ($Pr = 0.006, Ra = 12,000$)

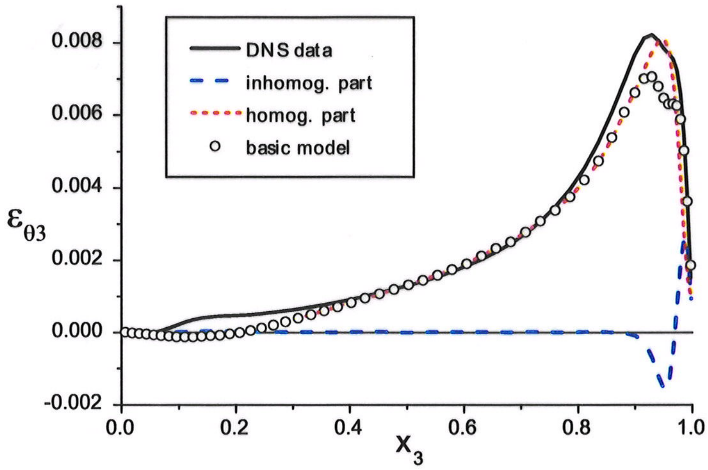


- RBC in air ($Pr = 0.71, Ra = 630,000$)

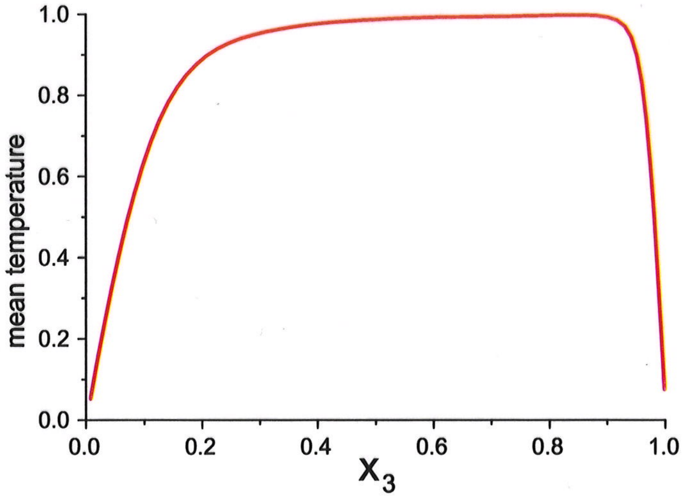


Comparison of 'basic' model with DNS data for internally heated convection

- $Pr = 7, Ra_I = 10^8$



- vertical profile of mean temperature Θ (DNS data)

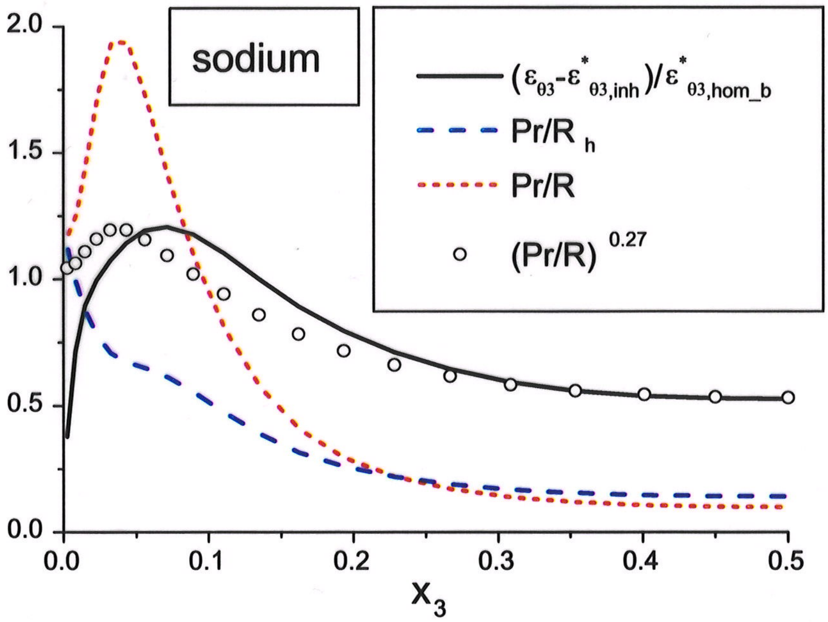


Modification of 'basic' model

- intention:
- improve performance of model in bulk region
- choose appropriate functional coefficient $\mathcal{C}$
- conserve correct wall-limiting behaviour of model

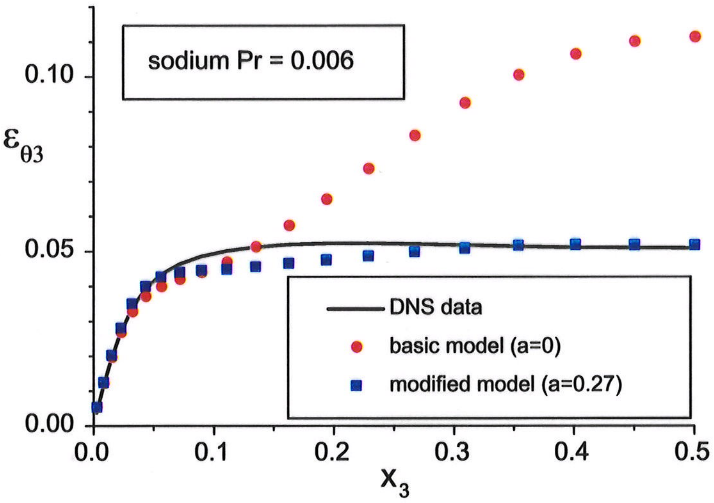
- key parameter for functional coefficient $\mathcal{C}$:
- turbulence time scale ratio R or R_h
- near-wall behaviour: $R = Pr + \cdots$, $R_h = Pr + \cdots$

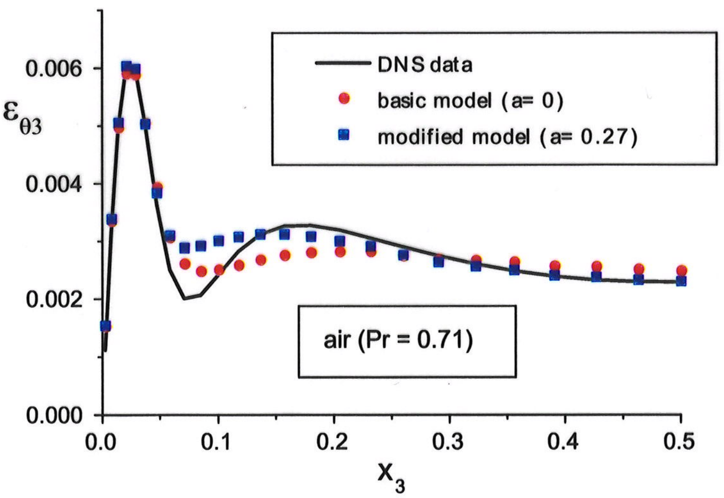
$\Rightarrow$ ansatz: $\mathcal{C} = \left(\frac{Pr}{R}\right)^a$ or $\mathcal{C} = \left(\frac{Pr}{R_h}\right)^a$, $a = a(Pr)$



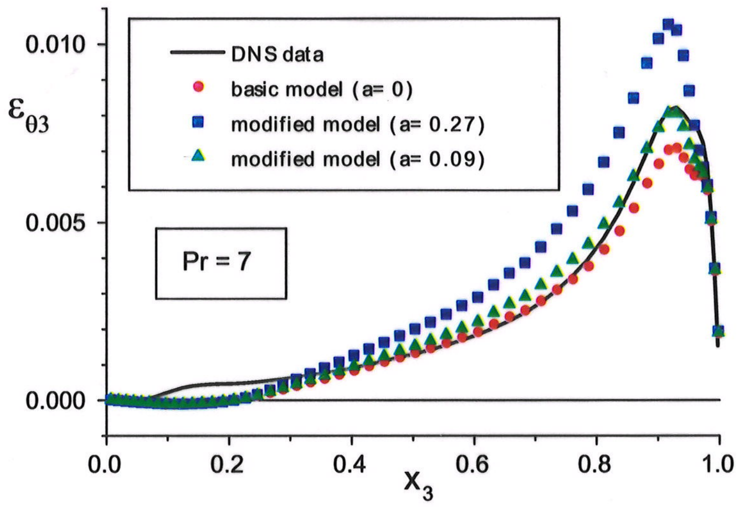
Performance of modified model for Rayleigh Bénard convection

$$\varepsilon^*_{\theta i} = \frac{1}{4}(\nu+\kappa)\frac{\partial\overline{\theta u_i}}{\partial x_l \partial x_l}+\frac{1}{2}\left(1+\frac{1}{Pr}\right)\left(\frac{Pr}{R}\right)^a\sqrt{\frac{Pr}{R_h}\frac{\varepsilon_h}{k}}\,\overline{\theta u_i}$$

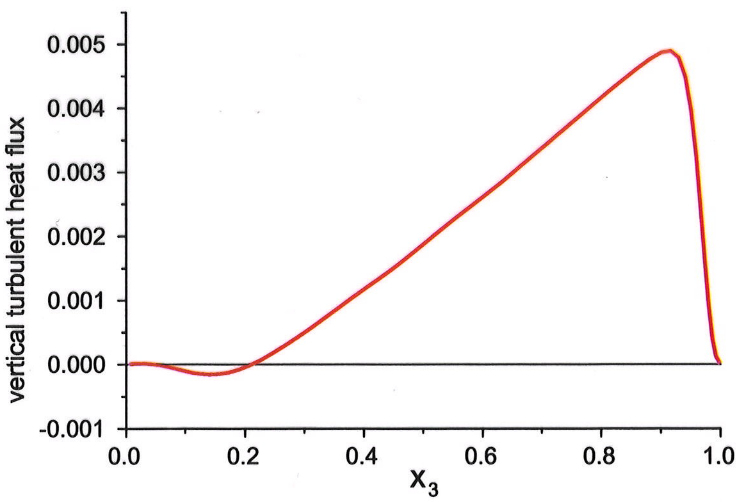




Performance of modified model for internally heated convection



- vertical turbulent heat flux $\overline{u_3\theta}$ (DNS data)



Conclusions

- development of model for $\overline{\frac{\partial \theta}{\partial x_l}\frac{\partial u_i}{\partial x_l}}$

- molecular dissipation in $\overline{u_i\theta}$-equation
- buoyancy production in dissipation equation

- features of model

- involves no wall-distance parameter
- obeys exact near wall behaviour
- consists of homogeneous and inhomogeneous part
- inhomogeneous part needs no approximation
- model for homogeneous part optimized by DNS data for natural convection with wide range of Pr

- performance in computer code FLUTAN

- $k-\varepsilon$ model + second moment closure for $\overline{u_i\theta}$
- improved results for flow along heated vertical wall[4]

- outlook

- comparison with DNS data for buoyant flows in vertical channels
- relationship $a = a(Pr)$ needs further investigation

[4] Experiments by Tsuji & Nagano