



3D volume-of-fluid simulation of a wobbling bubble in a gas-liquid system of low Morton number

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Background and motivation

- Bubbly two-phase flow
 - enhancement of turbulence by bubbles (bubble-induced turbulence = BIT, pseudo-turbulence)
 - phenomenon is not fully understood
 - no general models available for engineering CFD codes
- Goal: “direct” simulations with volume of fluid (VOF) method
 - achieve low Morton numbers to realize “wobbling” bubbles
 - extend basic understanding of BIT
 - use simulation results to develop improved models for BIT

Non-dimensional volume-averaged equations*

$$\nabla \cdot \mathbf{u}_m = 0$$

$$\frac{\partial}{\partial t} \rho_m \mathbf{u}_m + \nabla \cdot (\rho_m \mathbf{u}_m \mathbf{u}_m) = -\nabla p + \frac{1}{\text{Re}_{\text{ref}}} \nabla \cdot [\mu_m (\nabla \mathbf{u}_m + \nabla \mathbf{u}_m^T)]$$

$$- (1-f) \frac{\text{Eo}_{\text{ref}}}{\text{We}_{\text{ref}}} \frac{\mathbf{g}}{g} + \frac{a_{\text{int}} \kappa \vec{n}}{\text{We}_{\text{ref}}}$$

$$\rho_m \equiv \frac{f \rho_1^* + (1-f) \rho_2^*}{\rho_{\text{ref}}^*} \quad \mu_m \equiv \frac{f \mu_1^* + (1-f) \mu_2^*}{\mu_{\text{ref}}^*} \quad f \equiv \alpha_1$$

$$\mathbf{u}_m \equiv \frac{f \rho_1^* \vec{u}_1^* + (1-f) \rho_2^* \vec{u}_2^*}{\rho_m^* U_{\text{ref}}^*} \quad \mathbf{u}_r \equiv \frac{\vec{u}_2^* - \vec{u}_1^*}{U_{\text{ref}}^*} \quad \text{here : } \mathbf{u}_r = \mathbf{0}$$

*see presentation # 245 in session DK this afternoon

3

Volume-of-fluid interface tracking

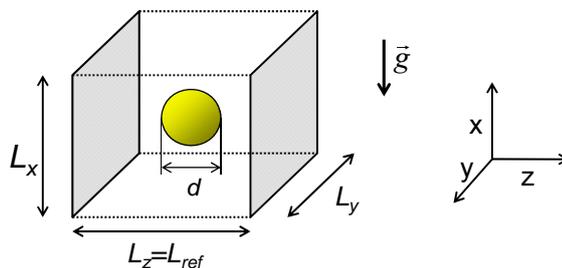
$$\frac{\partial f}{\partial t} + \nabla \cdot (f \mathbf{u}_m) = 0$$

- New algorithm for interface reconstruction:
EPIRA = **E**xact **P**lane **I**nterface **R**econstruction **A**lgorithm
 always reconstructs a 3D plane interface correctly,
 regardless of its orientation (see paper)
- Advection step by naive unsplit method

Computer code TURBIT-VOF

- Finite volume method on regular staggered grid
- Explicit Runge-Kutta time integration scheme (3rd O.)
- Central difference scheme (CDS) for diffusive terms (2nd O.)
- CDS or W-ENO scheme (3rd O.) for convective terms
- Surface tension:
 - use of normal vector of VOF reconstruction
 - use of a_{int} instead of delta function
- Conjugate gradient solver for Pressure-Poisson-equation

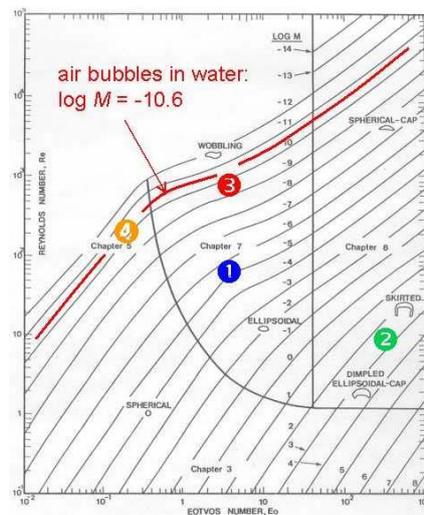
Computational domain and boundary conditions



- x-, y-direction: periodic boundary conditions
- z-direction: rigid walls
- present simulations: domain = 1 x 1 x 1 (64 x 64 x 64 cells)
 $d / L_{ref} = 0.25$ (=16 cells per bubble diam.)

Relevant non-dimensional groups

Group	type	value
$Re_B = \rho_1^* U_B^* d_B^* / \mu_1^*$	result	10 - 1000
$Eo_B = (\rho_1^* - \rho_2^*) g^* d_B^{*2} / \sigma^*$	input	3.07 - 243
$Mo = \frac{g^* \mu_1^{*4} (\rho_1^* - \rho_2^*)}{\rho_1^{*2} \sigma^{*3}} = \frac{EoWe^2}{Re^4}$	input	$2.5 \cdot 10^{-10}$ - 266
ρ_2^* / ρ_1^*	input	0.5
μ_2^* / μ_1^*	input	1



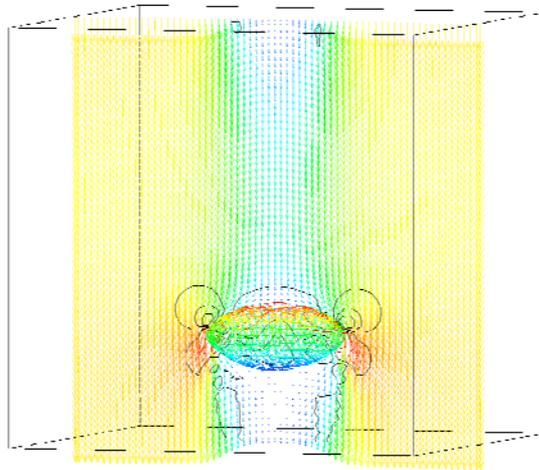
**Case ① :
ellipsoidal bubble**

($Eo_B=3.07$, $M=3.1 \cdot 10^{-6}$)

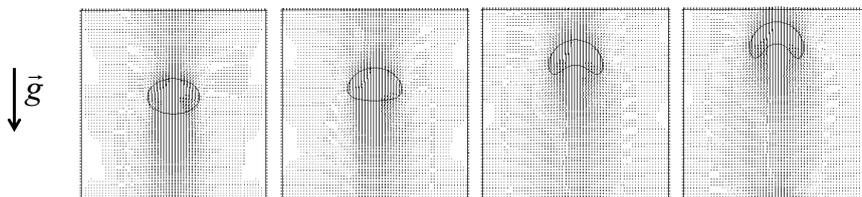
- rectilinear path
- steady shape

	Exp.	Sim.
axis-ratio	0,72	0,71
Re_B	50...70	61,5

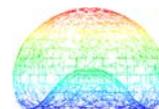
initial cond. for ② + ③



Case ② : spherical cap bubble ($Eo_B=243$, $M=266$)



- Rectilinear path
- Steady bubble shape:



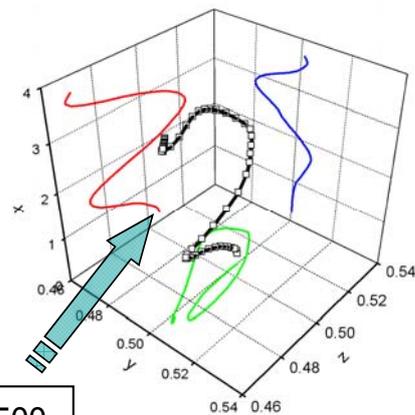
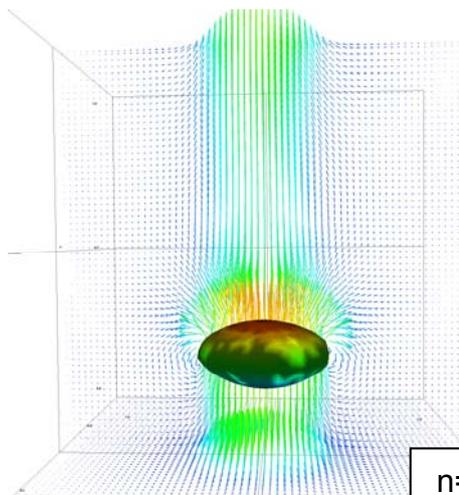
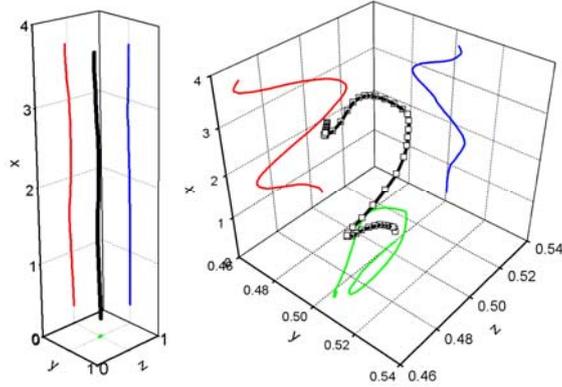
Exp. Bhaga&Weber

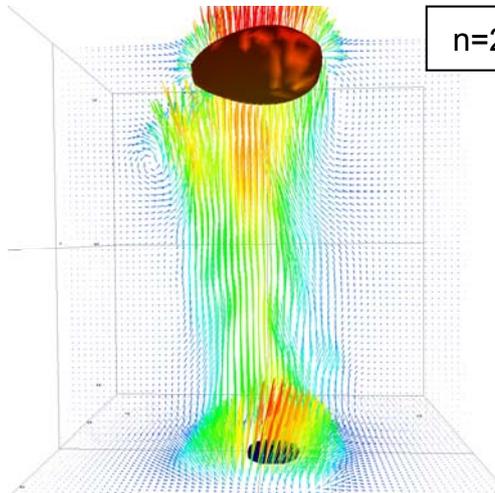
Simulation



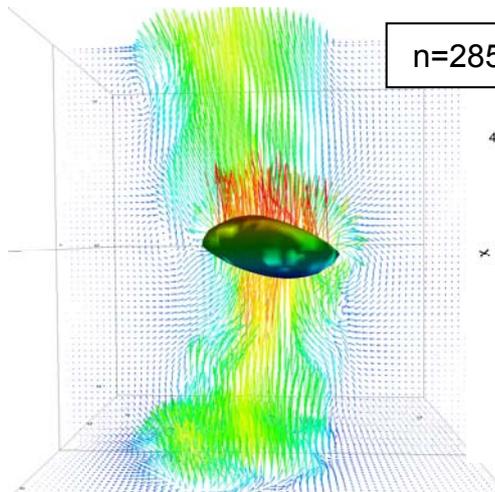
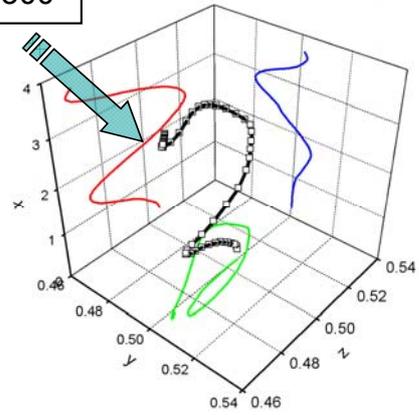
Case 3: “wobbling” bubble ($E_{0B}=3.07$, $M=2.5 \cdot 10^{-10}$)

- stepwise decrease of Morton number
- computation of 2050 time steps
- rectilinear path of case 1 becomes unstable
- bubble shape becomes unsteady and irregular

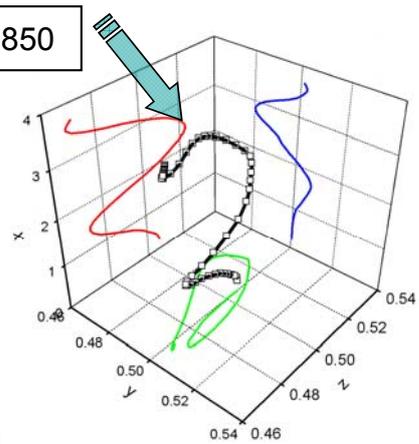


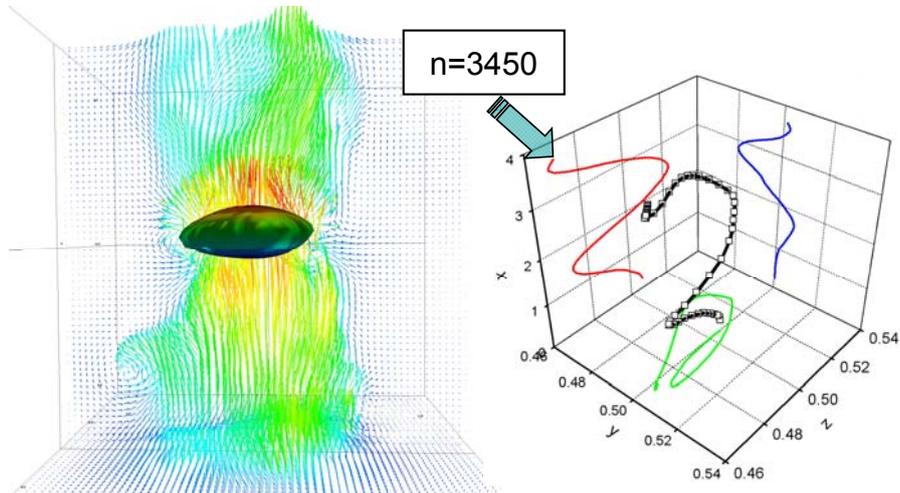


n=2500



n=2850





Conclusions

- present results for case ① and ② with $\rho_2/\rho_1 = 0.5$ compare well with experimental results for air-water systems
- bubble ③ behaves as to be expected from CGW diagram
- bubble shape, wake, and path
 - depend mainly on Eötvös and Morton number
 - density ratio and viscosity ratio are of minor importance
- \Rightarrow use $O(0.1)$ density ratio for efficient VOF simulations for bubble swarms in g/l systems of low Morton number
- \Rightarrow use results to develop models for bubble induced turbulence



Simulation parameters

$$\rho_2 / \rho_1 = 1/2$$

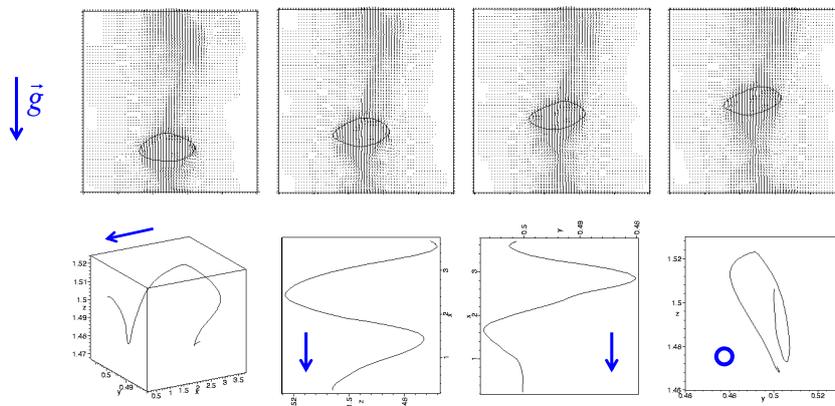
$$\mu_2 / \mu_1 = 1$$

- 1
- .
- .
- .
- 1
- 2
- 3

Case	Eo_b	Log M	Grid	Domain	d/L	Initial C.	Shape
1BLc	0,2	-10,6	64*64*64	1x1x1	1 / 4	1BLa	Spherical
1BLa	3,07	-5,51	64*64*64	1x1x1	1 / 4	$U=0$	Ellipsoidal
1BLaa	3,07	-5,51	64*64*128	1x1x2	1 / 8	$U=0$	Ellipsoidal
1Blab	3,07	-5,51	128*64*64	2x1x1	1 / 4	$U=0$	Ellipsoidal
1Blac	3,07	-5,51	128*128*128	1x1x1	1 / 4	$U=0$	Ellipsoidal
5BLa	3,07	-5,51	64*64*64	1x1x1	1 / 4	$U=0$	Ellipsoidal
1BLb	243	2,42	64*64*64	1x1x1	1 / 4	1BLa	Ellips.-cap
1BLd	3,07	-9,6	64*64*64	1x1x1	1 / 4	1BLa	„Wobbling“



Case 3: wobbling bubble



Normalisation and non-dimensional numbers

$$\mathbf{u} = \frac{\mathbf{u}^*}{U_{ref}^*}$$
$$\rho = \frac{\rho^*}{\rho_{ref}^*}, \quad \rho_{ref}^* = \rho_1^*$$
$$\mu = \frac{\mu^*}{\mu_{ref}^*}, \quad \mu_{ref}^* = \mu_1^*$$
$$\mathbf{x}^* = \frac{\mathbf{x}^*}{L_{ref}^*}, \quad t = \frac{t^* U_{ref}^*}{L_{ref}^*}$$
$$p^* = \frac{p^* + \rho_{ref}^* \mathbf{g}^* \cdot \mathbf{x}^*}{\rho_{ref}^* U_{ref}^{*2}}$$

$$Re_{ref} = \frac{\rho_{ref}^* U_{ref}^* L_{ref}^*}{\mu_{ref}^*}$$
$$Eo_{ref} = \frac{(\rho_1^* - \rho_2^*) g^* L_{ref}^{*2}}{\sigma^*}$$
$$We_{ref} = \frac{\rho_{ref}^* U_{ref}^{*2} L_{ref}^*}{\sigma^*}$$
$$Mo = \frac{g^* \mu_{ref}^{*4} (\rho_1^* - \rho_2^*)}{\rho_{ref}^{*2} \sigma^{*3}} = \frac{Eo_{ref} We_{ref}^2}{Re_{ref}^4}$$

Verification of TURBIT-VoF

- single phase flow (Taylor-Green vortices)
- bubble at rest (pressure jump due to surface tension)
- Rayleigh-Taylor instability (2D and 3D)
- gravity wave (comparison with analytical solution)
- capillary wave (comparison with analytical solution)



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