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Technik und Umwelt
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# The influence of gas-liquid density ratio on shape and rise velocity of an ellipsoidal bubble: a numerical study by 3D volume-of-fluid (VOF) computations

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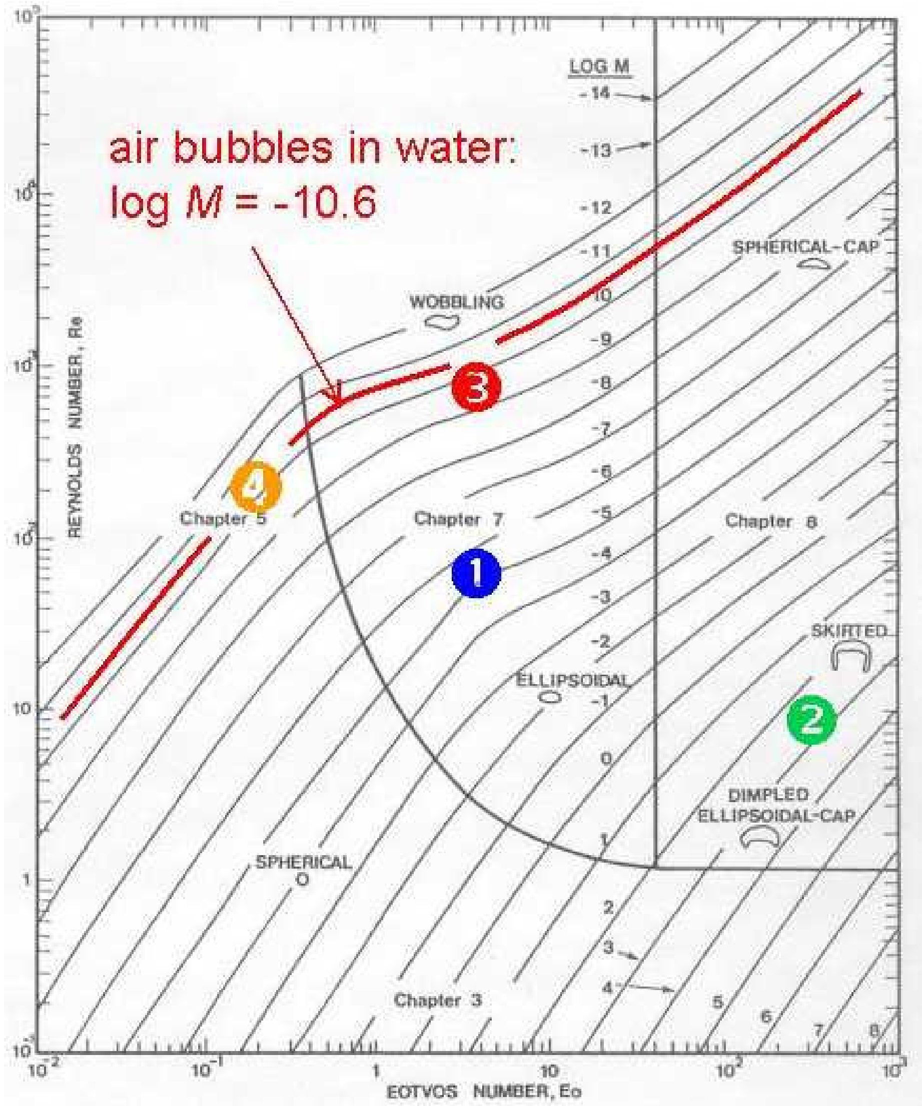
# Motivation




- VOF simulations* (❶❷❸❹) performed for density ratio 0.5 yield a similar bubble shape as experiments with density ratio ≈ 0.001
- <u>Here:</u> detailed investigation of bubble rise velocity and shape for different density ratios (ellipsoidal bubble ❶)

* Sabisch FZKA Report # 6478 (2000)

# Similitude analysis

- Single bubble rising steadily in liquid of infinite extent
- Relevant phys. quantities: $F\left(\rho_l^*, \rho_g^*, \mu_l^*, \mu_g^*, \sigma^*, g^*, d_V^*, U_T^*\right) = 0$
- Non-dimensional bubble terminal rise velocity:

$$Re_B = f\left(M, E\ddot{o}_B, \Gamma_\rho, \Gamma_\mu\right)$$

- Bubble Reynolds number $Re_B \equiv \rho_l^* d_V^* U_T^* / \mu_l^*$

  Morton number $M \equiv \left(\rho_l^* - \rho_g^*\right) g^* \mu_l^{*4} / \left(\rho_l^{*2} \sigma^{*3}\right)$

  Bubble Eötvös number $E\ddot{o}_B \equiv \left(\rho_l^* - \rho_g^*\right) g^* d_V^{*2} / \sigma^*$

  Gas-liquid density ratio $\Gamma_\rho \equiv \rho_g^* / \rho_l^*$

  Gas-liquid viscosity ratio $\Gamma_\mu \equiv \mu_g^* / \mu_l^*$

(dimensional variables are marked by supersript *)

# Non-dimensional governing equations of our VOF method

$$\mathbf{x} = \frac{\mathbf{x}^*}{L_{ref}^*},\ \mathbf{u}_k = \frac{\mathbf{u}_k^*}{U_{ref}^*},\ t = \frac{t^* U_{ref}^*}{L_{ref}^*},\ \rho_k = \frac{\rho_k^*}{\rho_l^*},\ \mu_k = \frac{\mu_k^*}{\mu_l^*},\ P = \frac{p^* + p_0^* - \rho_l^* \mathbf{g}^* \cdot \mathbf{x}^*}{\rho_l^* U_{ref}^{*\,2}} \quad (k \in l, g)$$

$$\frac{\partial}{\partial t}\rho_m \mathbf{u_m} + \nabla\cdot\rho_m \mathbf{u_m}\mathbf{u_m} = -\nabla P + \frac{1}{Re_{ref}}\nabla\cdot\left[\mu_\mathrm{m}\left(\nabla\mathbf{u_m} + \nabla\mathbf{u_m}^\mathrm{T}\right)\right] - (1-f)\frac{E\ddot{o}_{ref}}{We_{ref}}\frac{\mathbf{g}^*}{g^*} + \frac{a_{int}\kappa\mathbf{n}}{We_{ref}}$$

$$\nabla\cdot\mathbf{u_m} = 0$$

$$\frac{\partial f}{\partial t} + \nabla\cdot f\,\mathbf{u_m} = 0$$

$$\left(f \equiv \alpha_l, 0 \le f \le 1\right) \quad \mathbf{u_m} \equiv \frac{1}{U_{ref}^*}\frac{f\rho_l^*\mathbf{u}_l^* + (1-f)\rho_g^*\mathbf{u}_g^*}{f\rho_l^* + (1-f)\rho_g^*}$$

$$\rho_m \equiv \frac{f\rho_l^* + (1-f)\rho_g^*}{\rho_l^*} = f + (1-f)\Gamma_\rho, \quad \mu_m \equiv \frac{f\mu_l^* + (1-f)\mu_g^*}{\mu_l^*} = f + (1-f)\Gamma_\mu$$

$$Re_{ref} \equiv \frac{\rho_l^* L_{ref}^* U_{ref}^*}{\mu_l^*},\ E\ddot{o}_{ref} \equiv \frac{(\rho_l^* - \rho_g^*)g^* L_{ref}^{*\,2}}{\sigma^*},\ We_{ref} \equiv \frac{\rho_l^* L_{ref}^* U_{ref}^{*\,2}}{\sigma^*},\ M = \frac{E\ddot{o}_{ref}We_{ref}^2}{Re_{ref}^4} = \frac{E\ddot{o}_B We_B^2}{Re_B^4}$$

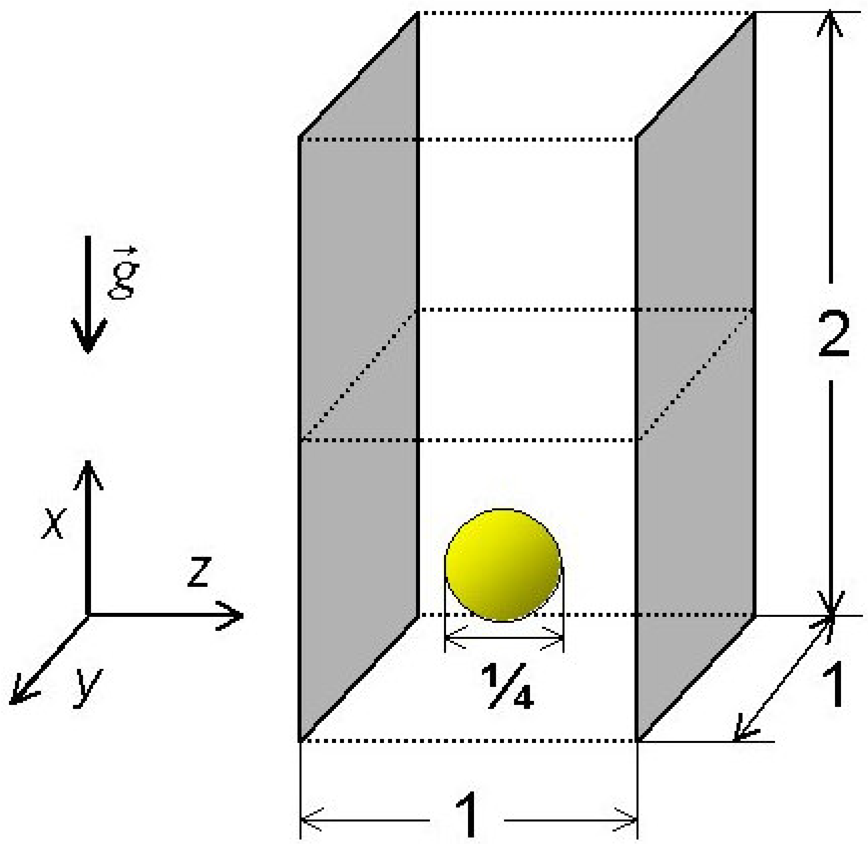
(dimensional variables are marked by superscript *)

# Numerical method

- In-house computer code TURBIT-VOF
- Finite volume method on rectangular staggered grid
- Spatial discretization by central differences (2nd O.)
- Explicit Runge-Kutta time integration scheme (3rd O.)
- Conjugate gradient solver for Pressure-Poisson-eq.
- Piecewise-linear interface calculation by EPIRA algorithm (a 3D plane interface is always reconstructed correctly, regardless of its orientation)

# Computational set-up

- Domain: 2 x 1 x 1
- Grid: 128 x 64 x 64
- Bubble diameter: 0.25 ( = 16 mesh cells)
- Gas holdup: ≈ 0.4%
- Boundary conditions
  - walls at *z* = 0 and *z* = 1
  - periodic in *x* and *y*
- Liquid & gas initially at rest



# Input parameters for code

1. Fixed values for $Eö_B$, $M$, $\Gamma_\mu$:
   $Eö_B = 3.06$, $M = 3.09 \cdot 10^{-6}$, $\Gamma_\mu = 1$

($Eö_B$, $M$) = bubble in surface tension dominated regime

2. Fixed values for reference quantities:

$$L^*_{ref} = 4\,\mathrm{m}, \quad U^*_{ref} = 1\,\mathrm{ms}^{-1}, \quad g^* = 9.81\,\mathrm{ms}^{-2}$$

3. Density ratio $\Gamma_\rho$ to be varied $\Rightarrow$ successively compute

$$Eö_{ref} = \left(\frac{L^*_{ref}}{d^*_V}\right)^2 Eö_B, \quad We_{ref} = \frac{Eö_{ref}}{1-\Gamma_\rho}\frac{U^{*\,2}_{ref}}{g^* L^*_{ref}}, \quad Re_{ref} = \left(\frac{Eö_{ref} We^2_{ref}}{M}\right)^{0.25}$$

Note: we do not give explicit values for $\rho^*_l, \rho^*_g, \mu^*_l, \mu^*_g, \sigma^*$!

# Simulation parameters

$$E\ddot{o}_B = 3.06,\ \ M = 3.09 \cdot 10^{-6},\ \Gamma_\mu = 1$$

| Run | $\Gamma_\rho$ | **$1 / \Gamma_\rho$** | $E\ddot{o}_{ref}$ | $We_{ref}$ | $Re_{ref}$ | $\Delta t$ | $N_t$ |
|---|---|---|---|---|---|---|---|
| R2 | 0.5 | **2** | 49.05 | 2.5 | 100.00 | 0.0005 | 1,100 |
| R5 | 0.2 | **5** | 49.05 | 1.563 | 78.90 | 0.0003 | 1,800 |
| R10 | 0.1 | **10** | 49.05 | 1.389 | 74.39 | 0.00015 | 3,200 |
| R50 | 0.02 | **50** | 49.05 | 1.276 | 71.28 | 0.00003 | 13,000 |
| | 0 | **∞** | 49.05 | 1.25 | 70.57 | | |

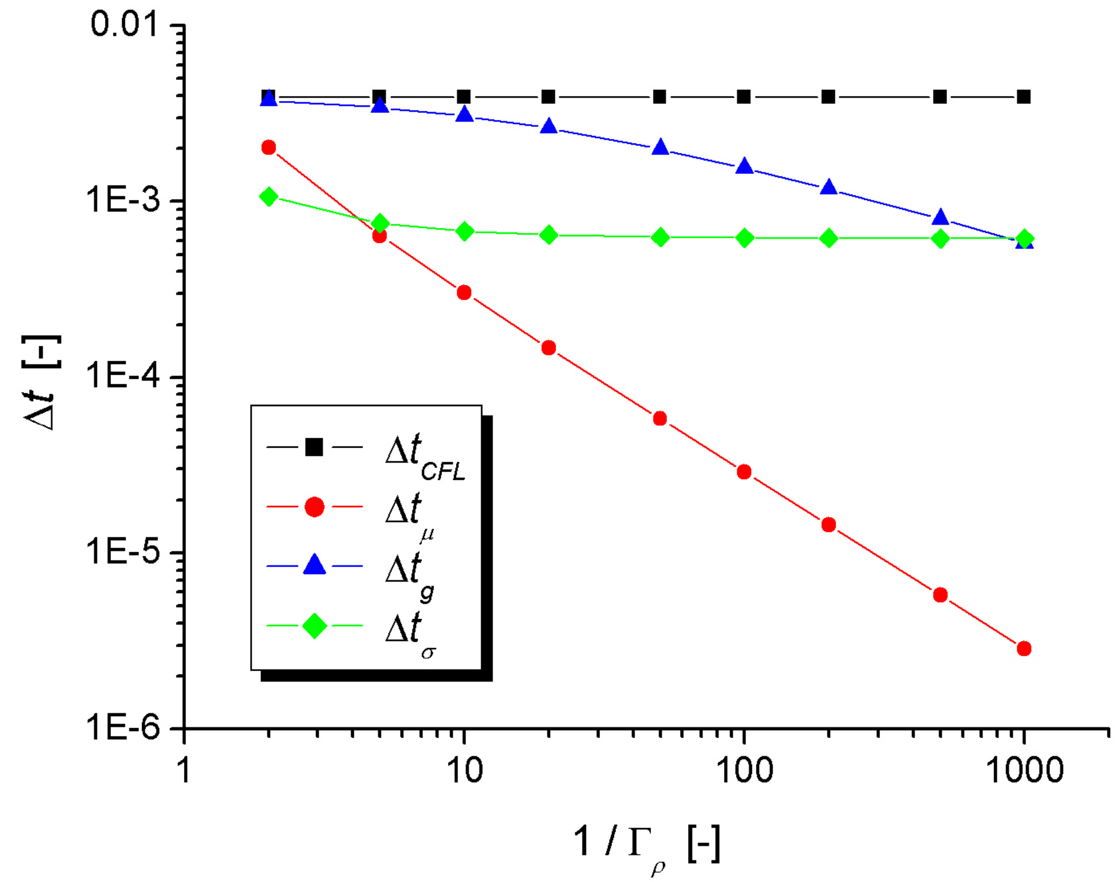
# Time step restrictions

$$\Delta t_{CFL} = \frac{\Delta x}{\left|\mathbf{u}^n_{max}\right|}$$

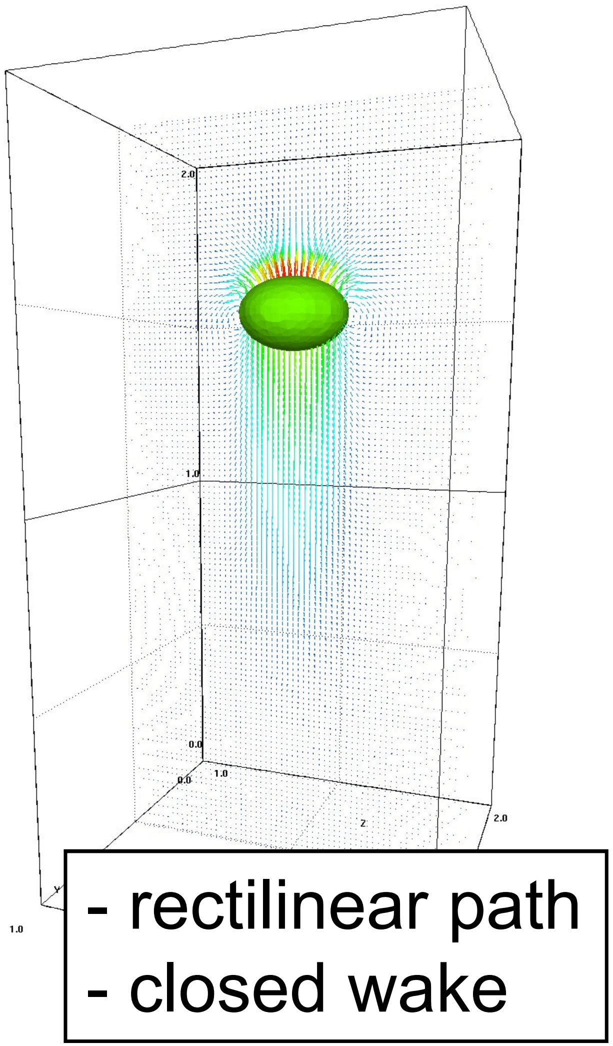
$$\Delta t_{\mu} = \frac{1}{6}\Delta x^2\, Re_{ref}\, \min\left(1, \frac{\Gamma_\rho}{\Gamma_\mu}\right)$$

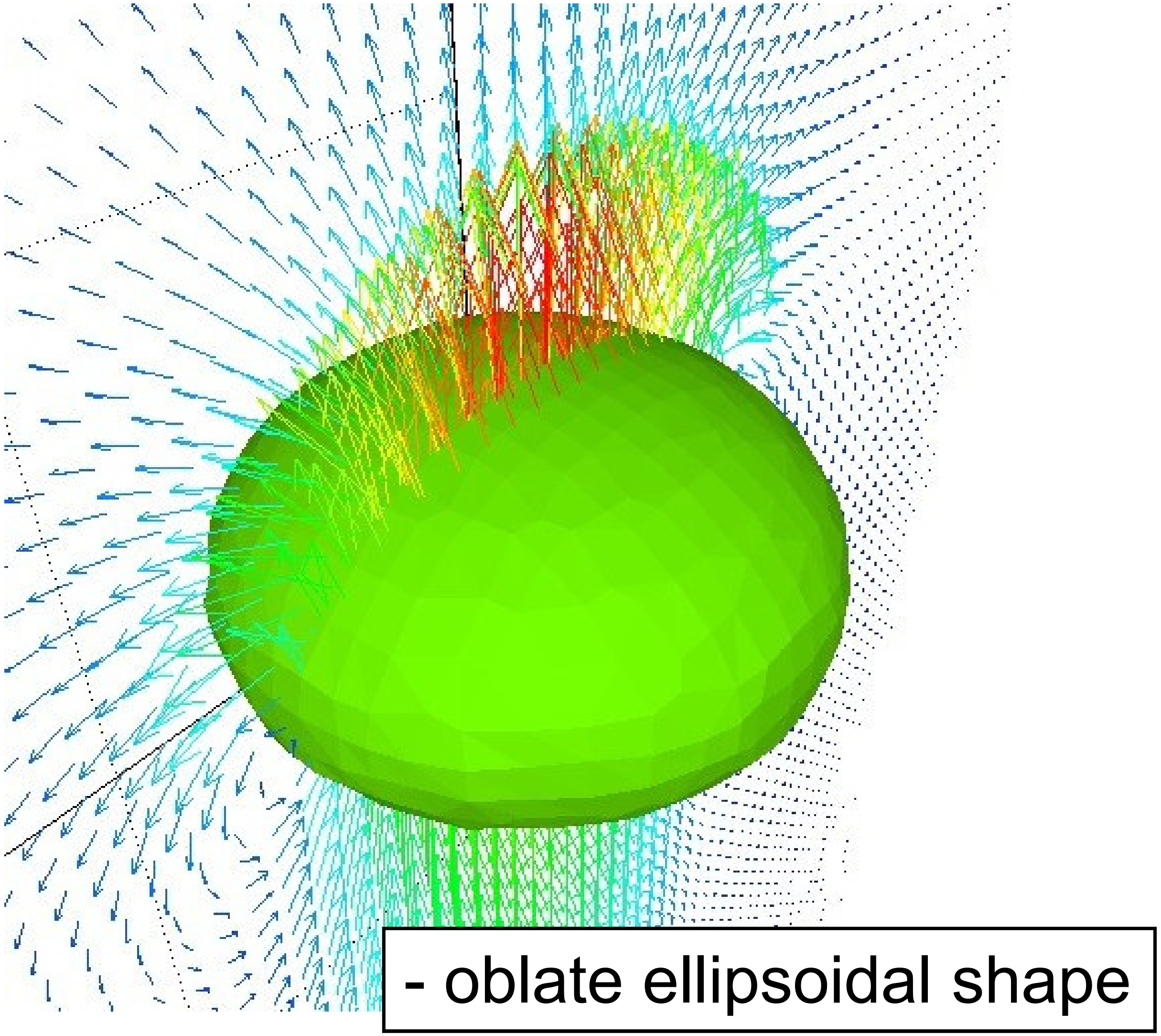
$$\Delta t_g = \frac{2\Delta x}{\left|\mathbf{u}^n_{max}\right| + \sqrt{\left|\mathbf{u}^n_{max}\right|^2 + \frac{4\Delta x}{\Gamma_\rho}\left|\frac{E\ddot{o}_{ref}}{We_{ref}}\right|}}$$

$$\Delta t_\sigma = \sqrt{\frac{\Delta x^3}{4\pi} We_{ref}\left(1+\Gamma_\rho\right)}$$



# Flow visualizations

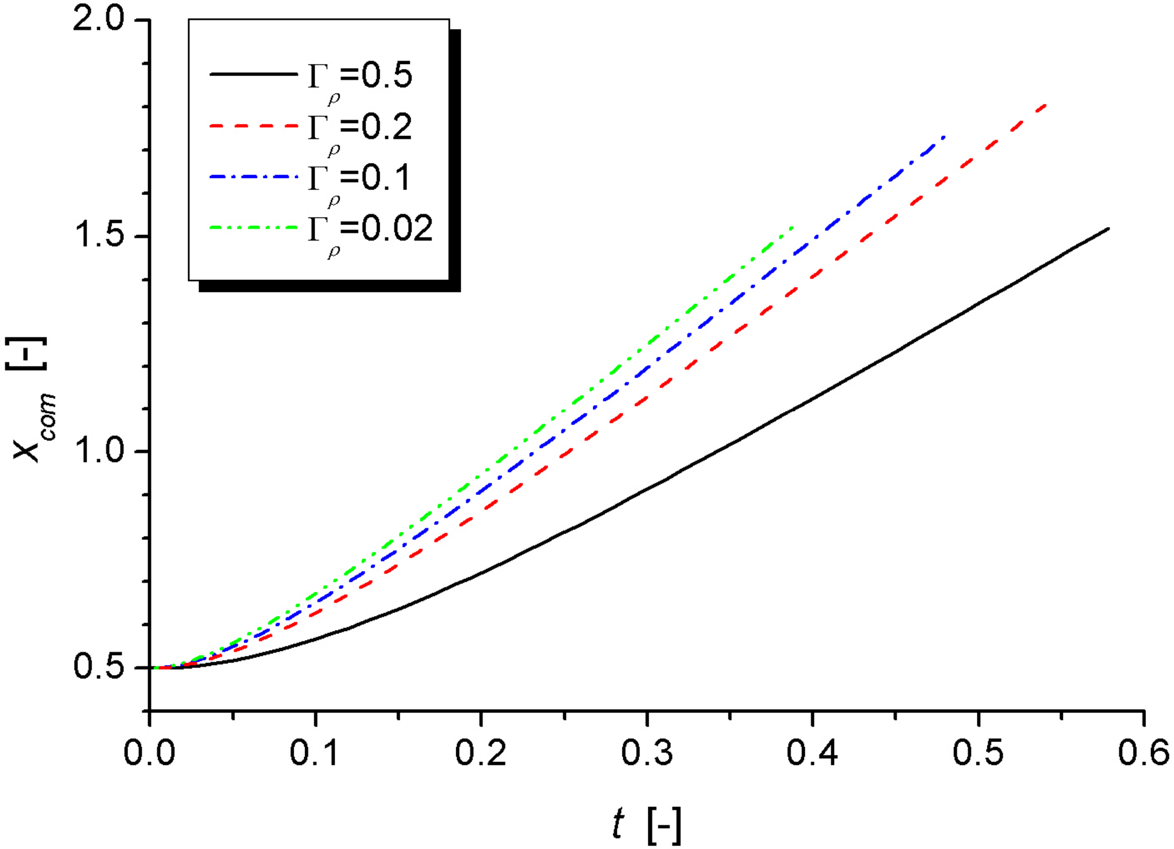


# Bubble vertical position

**Two distinct phases:**

- Initial phase:
  bubble accelerates from rest up to terminal velocity
- Subsequent phase:
  bubble rises steadily with terminal velocity



# Acceleration of bubble

- Balance between unsteady
  + inertial and buoyancy term

$$\rho_m \frac{D\mathbf{u}_m}{Dt} = -(1-f)\frac{E\ddot{o}_{ref}}{We_{ref}}\frac{\mathbf{g}^*}{g^*}$$

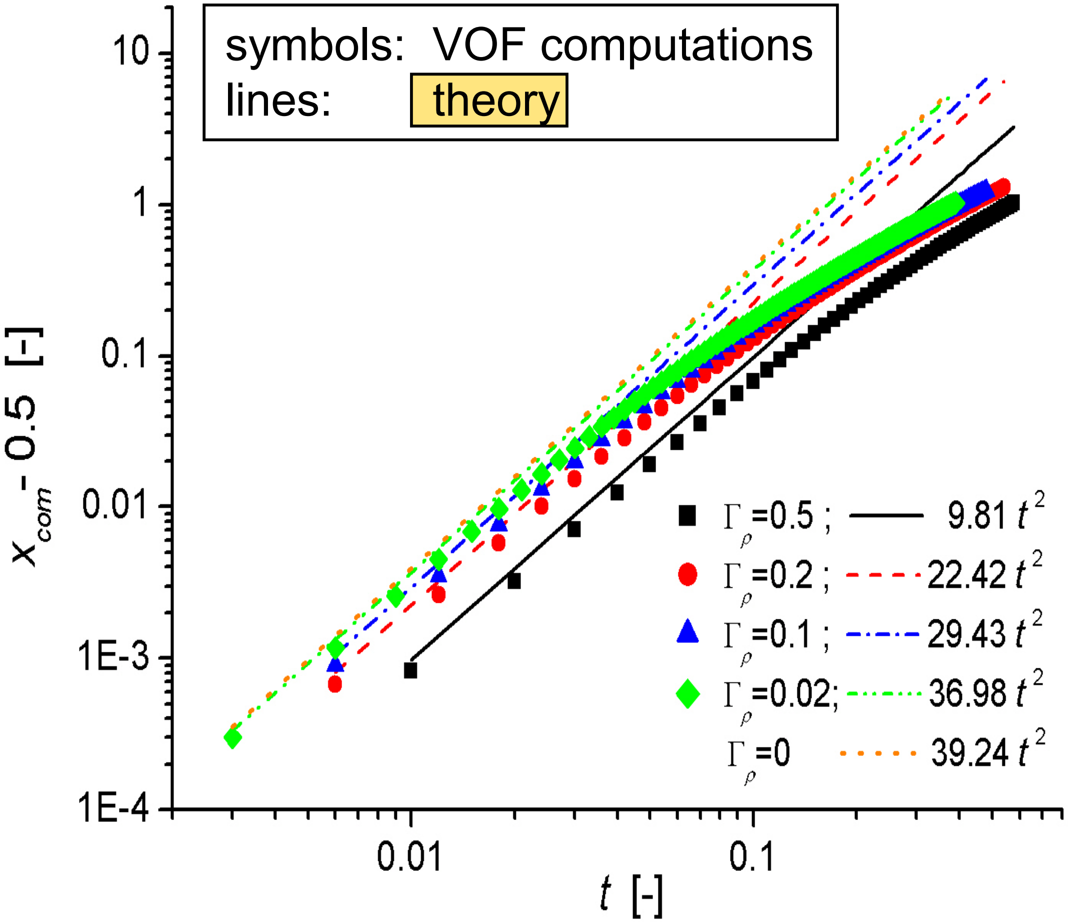
- Approx. for gas phase ($f = 0$):

$$u_{m,x} \approx U_B = Dx_{com}/Dt$$

$$\rho_m \approx \left(\rho_g^* + 0.5\rho_l^*\right)/\rho_l^*$$

**added mass !**

$$\Rightarrow \quad x_{com}(t) - 0.5 = \frac{1-\Gamma_\rho}{1+2\Gamma_\rho}\frac{g^* L_{ref}^*}{U_{ref}^{*\,2}} t^2$$



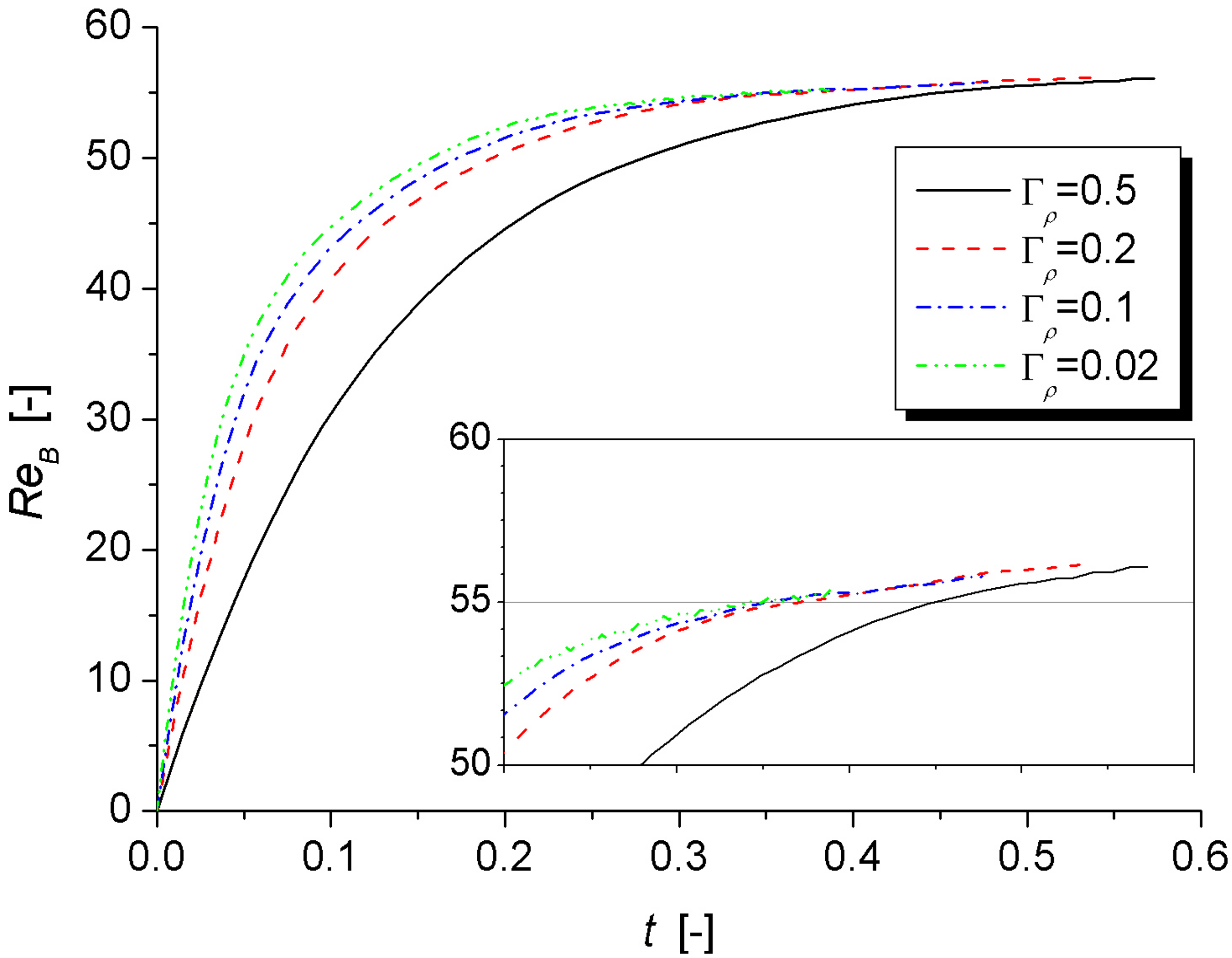
# Bubble Reynolds number

$$Re_B = \frac{d_V^*}{L_{ref}^*}\frac{U_B^*}{U_{ref}^*}Re_{ref}$$

$$= \frac{1}{4}U_B\, Re_{ref}$$

$$= \frac{1}{4}\frac{Dx_{com}(t)}{Dt}Re_{ref}$$

Terminal bubble Reynolds number is about 56 and does not vary with density ratio!



# Comparison with theory

- By analogy to two-phase wave theory Mendelson (1967) and Marrucci et al. (1970) proposed a relation for terminal rise velocity of a bubble in the surface tension or buoyancy dominated regime

$$U_T^* = \sqrt{\frac{2\sigma^*}{\rho_l^* d_V^*} + \frac{g^* d_V^* \left(\rho_l^* - \rho_g^*\right)}{2\rho_l^*}}$$

- Tomiyama et al. (1998 ) gave physical interpretation and noted

$$Re_B = \left(2 + \frac{1}{2} E\ddot{o}_B\right)^{0.5} \left(\frac{E\ddot{o}_B}{M}\right)^{0.25} \quad \Rightarrow \quad C_D \equiv \frac{4}{3} \frac{d_V^* g^* \Delta\rho^*}{\rho_l^* U_T^{*2}} = \frac{4}{3} \sqrt{\frac{E\ddot{o}_B^3}{M\, Re_B^4}} = \frac{8}{3} \frac{E\ddot{o}_B}{E\ddot{o}_B + 4}$$

- Here: $E\ddot{o}_B = 3.06,\ \ M = 3.09 \cdot 10^{-6} \ \Rightarrow \ Re_B = 59.3$ (VOF: $Re_B \approx 56$)

# Conclusions

- 3D VOF computations of ellipsoidal bubble for different density ratios but fixed Eötvös and Morton number
- Acceleration of bubble from rest to terminal velocity depends on density ratio (added mass force)
- Terminal bubble Reynolds number ($Re_B$) and bubble shape (see paper) do <u>not</u> depend on density ratio
- Good agreement of $Re_B$ with two-phase wave theory
- Results not yet directly verified by experiments
- Universal relation $Re_B = f\left(M, E\ddot{o}_B, \Gamma_\rho, \Gamma_\mu\right)$ ?

# Outlook

- Comparison of exterior and interior velocity field for runs with different density ratio
- Influence of density ratio on Reynolds number and shape for bubble in buoyancy dominated regime
- Experimental investigation of influence of $\Gamma_\rho$ requires
  - use of a gas-liquid and a liquid-liquid system with same Morton number but different density ratio
  - similarity of Eötvös number can be ensured by appropriate values of equivalent diameter of bubble/drop