Invariance of the velocity field induced by a bubble rising steadily through liquid under variation of the gas-liquid density ratio

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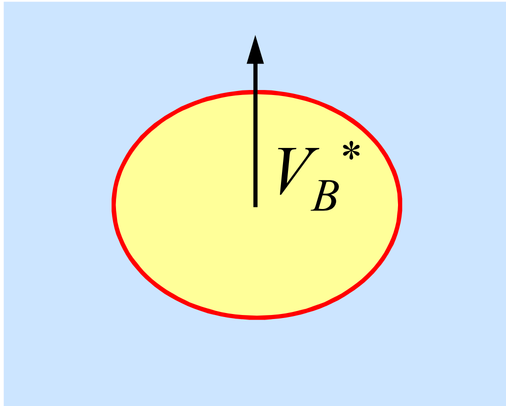
Motivation

- Buoyancy driven motion of single bubble in “infinite” liquid
  - Gas-liquid density ratio (Γ_ρ) is one of 5 dimensionless groups
- Direct Numerical Simulation (DNS) of two-phase flow
  - Numerical difficulties associated with low values of Γ_ρ
  - Usually a density ratio $\Gamma_\rho \approx 1/50$ is used instead of $1/1000$
- Objective
  - How is influence of gas-liquid density ratio on
    - bubble rise velocity
    - bubble shape
    - local motion in liquid and gas phase
  - Can results for $\Gamma_\rho = O(0.1)$ be transferred to $\Gamma_\rho = O(0.001)$?
  - <u>Here:</u> Investigation by 3D Volume-of-fluid computations

Dimensional equations in steady frame of reference



$$\nabla^* \cdot \mathbf{u}_c^* = 0, \quad \frac{\partial^* \rho_c^* \mathbf{u}_c^*}{\partial^* t^*} + \nabla^* \cdot \rho_c^* \mathbf{u}_c^* \mathbf{u}_c^* = -\nabla^* p_c^* + \mu_c^* \nabla^{*2} \mathbf{u}_c^* + \rho_c^* \mathbf{g}^*, \quad \mathbf{x}^* \in \Omega_c^*\left(t^*\right)$$

$$\nabla^* \cdot \mathbf{u}_d^* = 0, \quad \frac{\partial^* \rho_d^* \mathbf{u}_d^*}{\partial^* t^*} + \nabla^* \cdot \rho_d^* \mathbf{u}_d^* \mathbf{u}_d^* = -\nabla^* p_d^* + \mu_d^* \nabla^{*2} \mathbf{u}_d^* + \rho_d^* \mathbf{g}^*, \quad \mathbf{x}^* \in \Omega_d^*\left(t^*\right)$$

$$\left.\begin{array}{c} \mathbf{u}_c^* = \mathbf{u}_d^* = \mathbf{u}_i^* \\ \left(p_c^* - p_d^* + \kappa^* \sigma^*\right)\mathbf{n}_i = \left[\mu_c^*(\nabla^* \mathbf{u}_c^* + \nabla^* \mathbf{u}_c^{*\mathrm{T}}) - \mu_d^*(\nabla^* \mathbf{u}_d^* + \nabla^* \mathbf{u}_d^{*\mathrm{T}})\right] \cdot \mathbf{n}_i \end{array}\right\} \mathbf{x}_i^* \in S_i^*(t^*)$$

(dimensional variables are marked by superscript *)

Normalization of variables

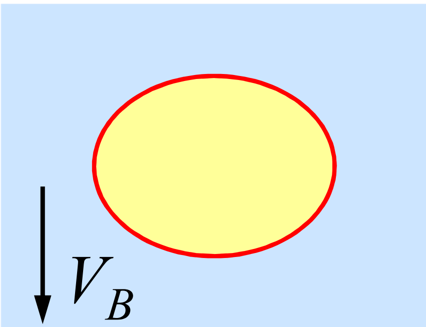
- **Reference scales**
  - length: sphere-equivalent diameter d_B^*
  - velocity: bubble rise velocity V_B^*
  - density: liquid phase density ρ_c^*
  - viscosity: liquid phase viscosity μ_c^*
- **Non-dimensional quantities**
  - $\mathbf{x} = \mathbf{x}^* / d_B^*$, $\mathbf{u}_c = \mathbf{u}_c^* / V_B^*$, $\mathbf{u}_d = \mathbf{u}_d^* / V_B^*$, etc.
  - $\rho_c = 1$, $\rho_d = \rho_d^* / \rho_c^* \equiv \Gamma_\rho$, $\mu_c = 1$, $\mu_d = \mu_d^* / \mu_c^* \equiv \Gamma_\mu$
  - $p_c \equiv \dfrac{p_c^* - \rho_c^* \mathbf{g}^* \cdot \mathbf{x}^*}{\rho_c^* V_B^{*2}}, \quad p_d \equiv \dfrac{p_d^* - \rho_c^* \mathbf{g}^* \cdot \mathbf{x}^*}{\rho_c^* V_B^{*2}}$

Non-dimensional equations in <u>moving</u> frame of reference

$$\nabla' \cdot \mathbf{w}_c = 0, \quad \frac{\partial \mathbf{w}_c}{\partial t} + \nabla' \cdot \mathbf{w}_c \mathbf{w}_c = -\nabla' p_c + \frac{1}{Re_B} \nabla'^2 \mathbf{w}_c - \frac{\mathrm{d}\mathbf{V}_B}{\mathrm{d}t}, \quad \mathbf{z} \in \Omega'_c(t)$$

$$\nabla' \cdot \mathbf{w}_d = 0, \quad \Gamma_\rho \left(\frac{\partial \mathbf{w}_d}{\partial t} + \nabla' \cdot \mathbf{w}_d \mathbf{w}_d \right) = -\nabla' p_d + \frac{\Gamma_\mu}{Re_B} \nabla'^2 \mathbf{w}_d - \sqrt{\frac{E\ddot{o}_B^3}{MoRe_B^4}} \mathbf{n}_g - \Gamma_\rho \frac{\mathrm{d}\mathbf{V}_B}{\mathrm{d}t}, \quad \mathbf{z} \in \Omega'_d(t)$$

$$\left.\begin{array}{c} \mathbf{w}_c = \mathbf{w}_d = \mathbf{w}_i \\ \left(p_c - p_d + \kappa \sqrt{\dfrac{E\ddot{o}_B}{MoRe_B^4}} \right) \mathbf{n}_i = \dfrac{1}{Re_B} \left[(\nabla' \mathbf{w}_c + \nabla' \mathbf{w}_c^{\mathrm{T}}) - \Gamma_\mu (\nabla' \mathbf{w}_d + \nabla' \mathbf{w}_d^{\mathrm{T}}) \right] \cdot \mathbf{n}_i \end{array} \right\} \mathbf{z}_i \in S_i'(t)$$



$$Re_B \equiv \frac{\rho_c^* d_B^* V_B^*}{\mu_c^*}, \; E\ddot{o}_B \equiv \frac{g^* \left(\rho_c^* - \rho_d^* \right) d_B^{*2}}{\sigma^*}, \; Mo \equiv \frac{(\rho_c^* - \rho_d^*) g^* \mu_c^{*4}}{\rho_c^{*2} \sigma^{*3}}$$

Steady non-dimensional equations in moving frame of reference

$$\nabla' \cdot \mathbf{w}_c = 0, \qquad \nabla' \cdot \mathbf{w}_c \mathbf{w}_c = -\nabla' p_c + \frac{1}{Re_B} \nabla'^2 \mathbf{w}_c , \qquad \mathbf{z} \in \Omega'_c$$

$$\nabla' \cdot \mathbf{w}_d = 0, \quad \Gamma_\rho \nabla' \cdot \mathbf{w}_d \mathbf{w}_d = -\nabla' p_d + \frac{\Gamma_\mu}{Re_B} \nabla'^2 \mathbf{w}_d - \sqrt{\frac{E\ddot{o}_B^3}{MoRe_B^4}} \mathbf{n}_g , \qquad \mathbf{z} \in \Omega'_d$$

$$\left. \begin{aligned} & \mathbf{w}_c = \mathbf{w}_d = \mathbf{w}_i \\ & \left(p_c - p_d + \kappa \sqrt{\frac{E\ddot{o}_B}{MoRe_B^4}} \right) \mathbf{n}_i = \frac{1}{Re_B} \left[(\nabla' \mathbf{w}_c + \nabla' \mathbf{w}_c^{\mathrm{T}}) - \Gamma_\mu (\nabla' \mathbf{w}_d + \nabla' \mathbf{w}_d^{\mathrm{T}}) \right] \cdot \mathbf{n}_i \end{aligned} \right\} \mathbf{z}_i \in S'_i$$

Gas-liquid density ratio is without influence for

- limit $\Gamma_\rho \rightarrow 0$
- no internal circulation ($\mathbf{w}_d = 0$)

Momentum theorem

- Integration of disperse phase momentum Eq. over Ω'_d and application of Gauss-Ostrogradskii divergence theorem

$$\iint_{S_i'} \mathbf{n}_\mathrm{i} \cdot \left\{ -\Gamma_\rho \mathbf{w}_d \mathbf{w}_d - \mathbf{I} p_d + \frac{\Gamma_\mu}{Re_B} \left[\nabla' \mathbf{w}_d + \nabla' \mathbf{w}_d{}^\mathrm{T} \right] \right\} \mathrm{d}S = \mathcal{V}_B \sqrt{\frac{E\ddot{o}_B^3}{MoRe_B^4}} \mathbf{n}_g$$

- At interface: $\Gamma_\rho \mathbf{n}_i' \cdot \mathbf{w}_d \mathbf{w}_d = \Gamma_\rho \mathbf{n}_i' \cdot \mathbf{w}_i \mathbf{w}_i = \left(\mathbf{n}_i' \cdot \mathbf{w}_i \right) \mathbf{w}_i = W_{i\perp} \mathbf{w}_i = 0$

$$\Rightarrow \boxed{\iint_{S_i'} \mathbf{n}_\mathrm{i} \cdot \left\{ -\mathbf{I} p_d + \frac{\Gamma_\mu}{Re_B} \left[\nabla' \mathbf{w}_d + \nabla' \mathbf{w}_d{}^\mathrm{T} \right] \right\} \mathrm{d}S = \mathcal{V}_B \sqrt{\frac{E\ddot{o}_B^3}{MoRe_B^4}} \mathbf{n}_g}$$

- If the bubble shape is independent of Γ_ρ then it is likely that this is also true for the bubble-driven liquid motion

TURBIT-VOF: Governing equations

$$\mathbf{x}=\frac{\mathbf{x}^*}{L_{ref}^*},\quad \mathbf{u}_k=\frac{\mathbf{u}_k^*}{U_{ref}^*},\quad t=\frac{t^*U_{ref}^*}{L_{ref}^*},\quad \rho_k=\frac{\rho_k^*}{\rho_c^*},\quad \mu_k=\frac{\mu_k^*}{\mu_c^*},\quad P_k=\frac{p_k^*+p_0^*-\rho_c^*\mathbf{g}^*\cdot\mathbf{x}^*}{\rho_c^*U_{ref}^{*\,2}}\quad (k\in c,d)$$

$$\frac{\partial}{\partial t}\rho_m\mathbf{u}_m+\nabla\cdot\rho_m\mathbf{u}_m\mathbf{u}_m=-\nabla P+\frac{1}{Re_{ref}}\nabla\cdot\left[\mu_m\left(\nabla\mathbf{u}_m+\nabla\mathbf{u}_m^{\mathrm{T}}\right)\right]-(1-f)\frac{E\ddot{o}_{ref}}{We_{ref}}\frac{\mathbf{g}^*}{g^*}+\frac{a_{int}\kappa\mathbf{n}_i}{We_{ref}}$$

$$\nabla\cdot\mathbf{u}_m=0$$

$$\frac{\partial f}{\partial t}+\nabla\cdot f\mathbf{u}_m=0$$

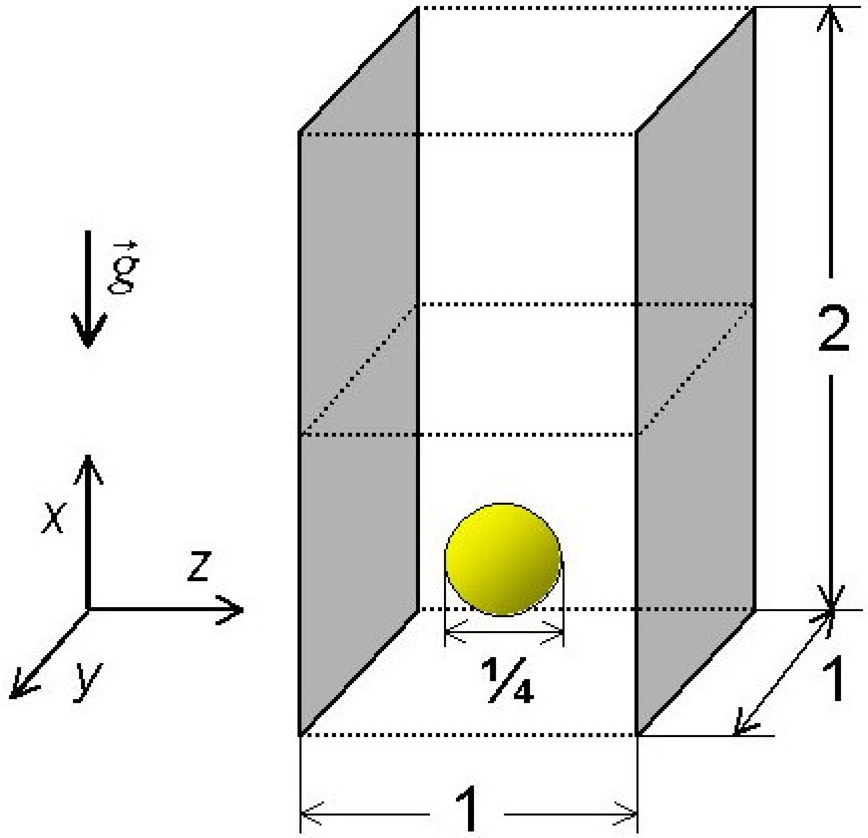
$$\left(f\equiv\alpha_c,\ 0\le f\le 1\right)\quad \mathbf{u}_m\equiv\frac{1}{U_{ref}^*}\frac{f\rho_c^*\mathbf{u}_c^*+(1-f)\rho_d^*\mathbf{u}_d^*}{f\rho_c^*+(1-f)\rho_d^*}$$

$$\rho_m\equiv\frac{f\rho_c^*+\left(1-f\right)\rho_d^*}{\rho_c^*}=f+\left(1-f\right)\Gamma_\rho,\quad \mu_m\equiv\frac{f\mu_c^*+\left(1-f\right)\mu_d^*}{\mu_c^*}=f+\left(1-f\right)\Gamma_\mu$$

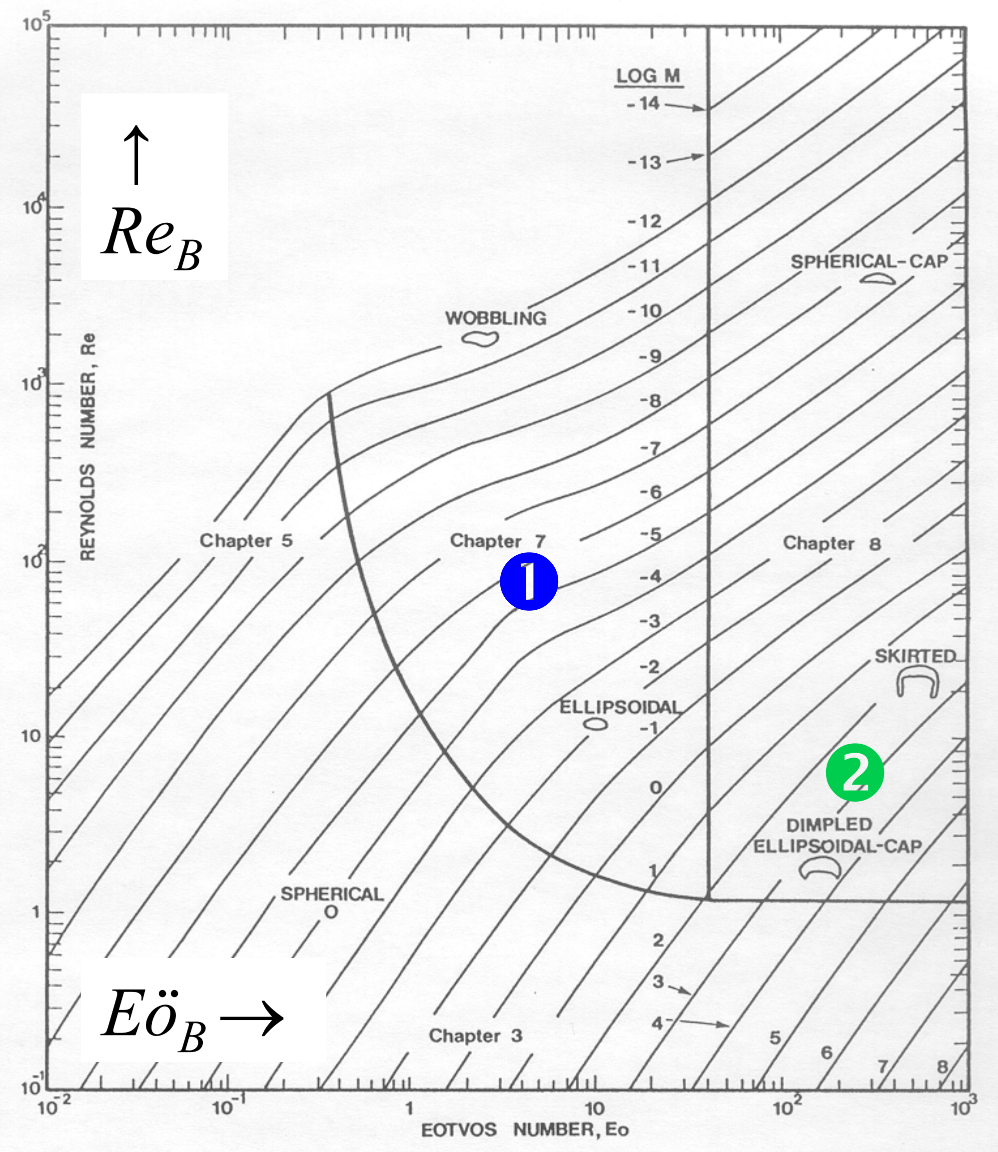
$$Re_{ref}\equiv\frac{\rho_c^*L_{ref}^*U_{ref}^*}{\mu_c^*},\quad E\ddot{o}_{ref}\equiv\frac{\left(\rho_c^*-\rho_d^*\right)g^*L_{ref}^{*\,2}}{\sigma^*},\quad We_{ref}\equiv\frac{\rho_c^*L_{ref}^*U_{ref}^{*\,2}}{\sigma^*}=\sqrt{\frac{MoRe_{ref}^4}{E\ddot{o}_{ref}}}$$

Computational set-up

- Domain: $2 \times 1 \times 1$
- Grid: $128 \times 64 \times 64$
- Bubble diameter: 0.25 ($= 16$ mesh cells)
- Gas holdup: $\approx 0.4\%$
- Boundary conditions
  - walls at $z = 0$ and $z = 1$
  - periodic in x and y
- Liquid & gas initially at rest



Bubble parameters



❶ „medium“ Morton number

ellipsoidal bubble

DNS for $\Gamma_\mu = 1$,

$Mo = 3.09 \cdot 10^{-6}$, $Eö_B = 3.06$

$\Gamma_\rho = 0.5, 0.2, 0.1, 0.02$

❷ „high“ Morton number

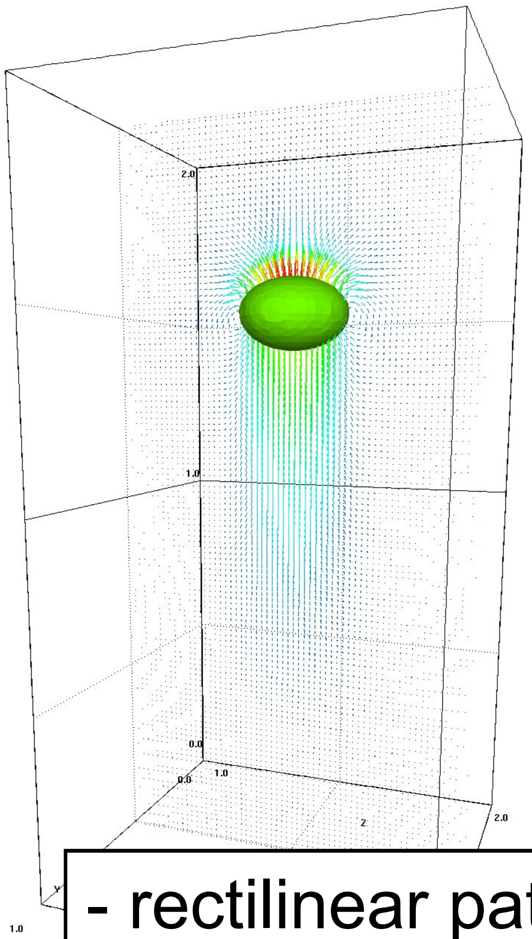
ellipsoidal cap bubble

DNS for $\Gamma_\mu = 1$,

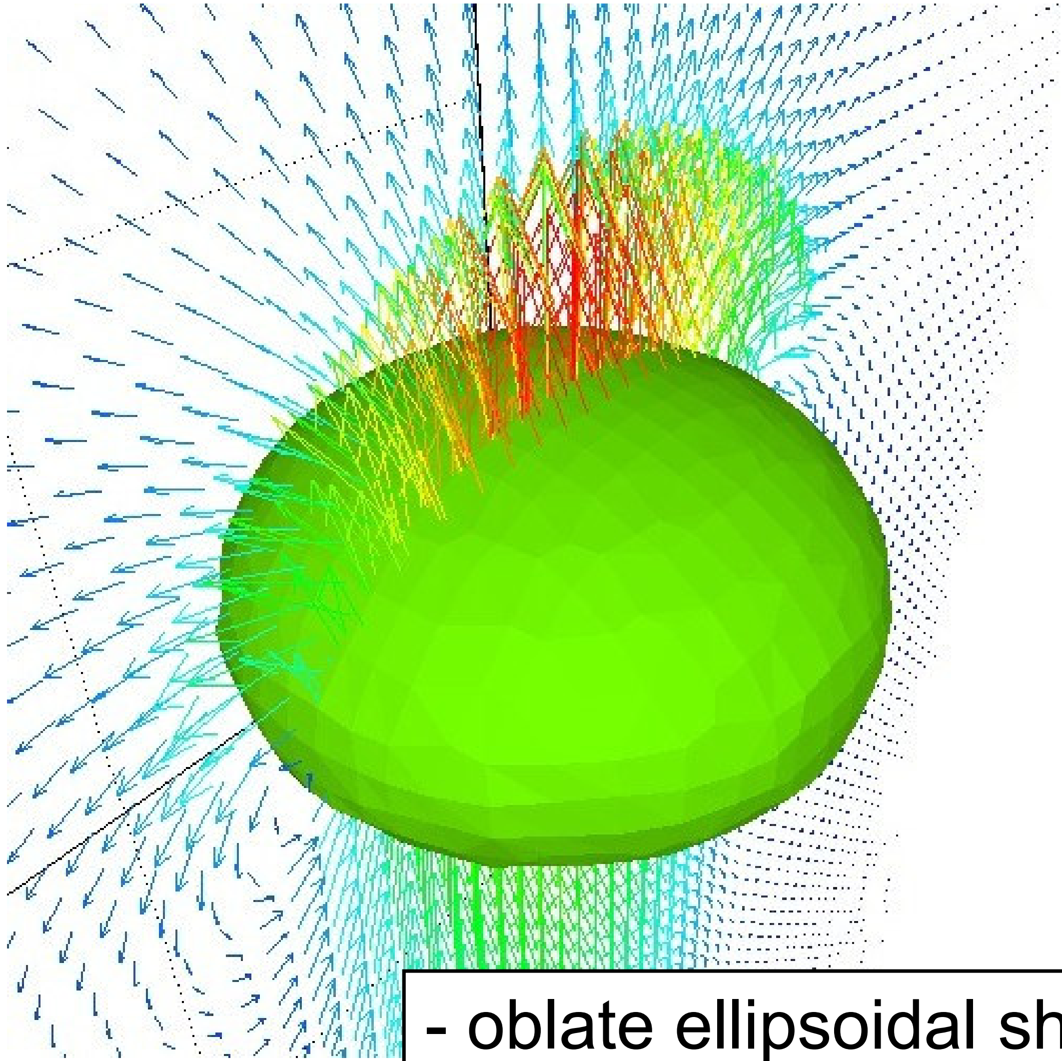
$Mo = 266$, $Eö_B = 243$

$\Gamma_\rho = 0.5, 0.2, 0.1$

Flow visualizations case ❶

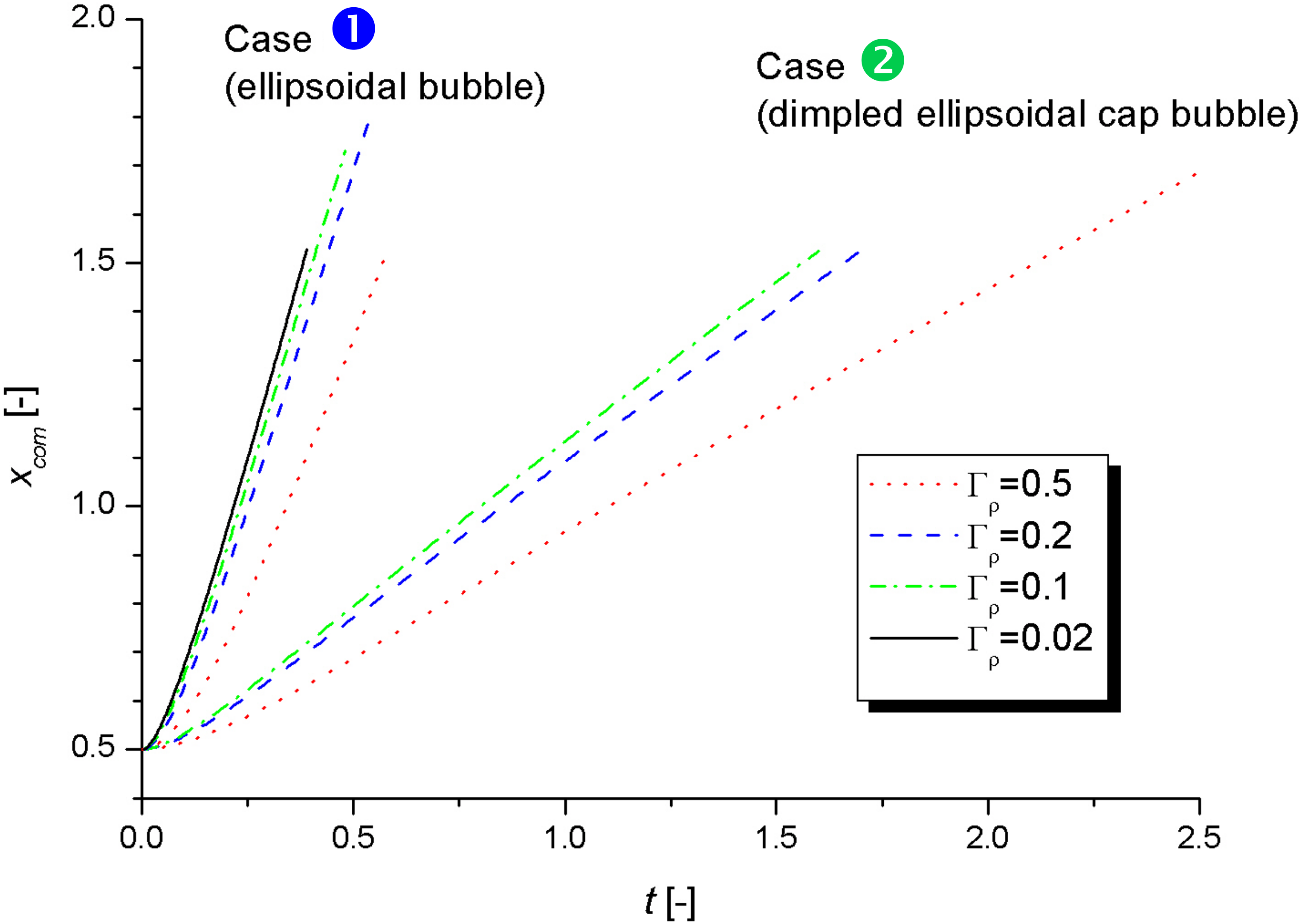


- rectilinear path
- closed wake

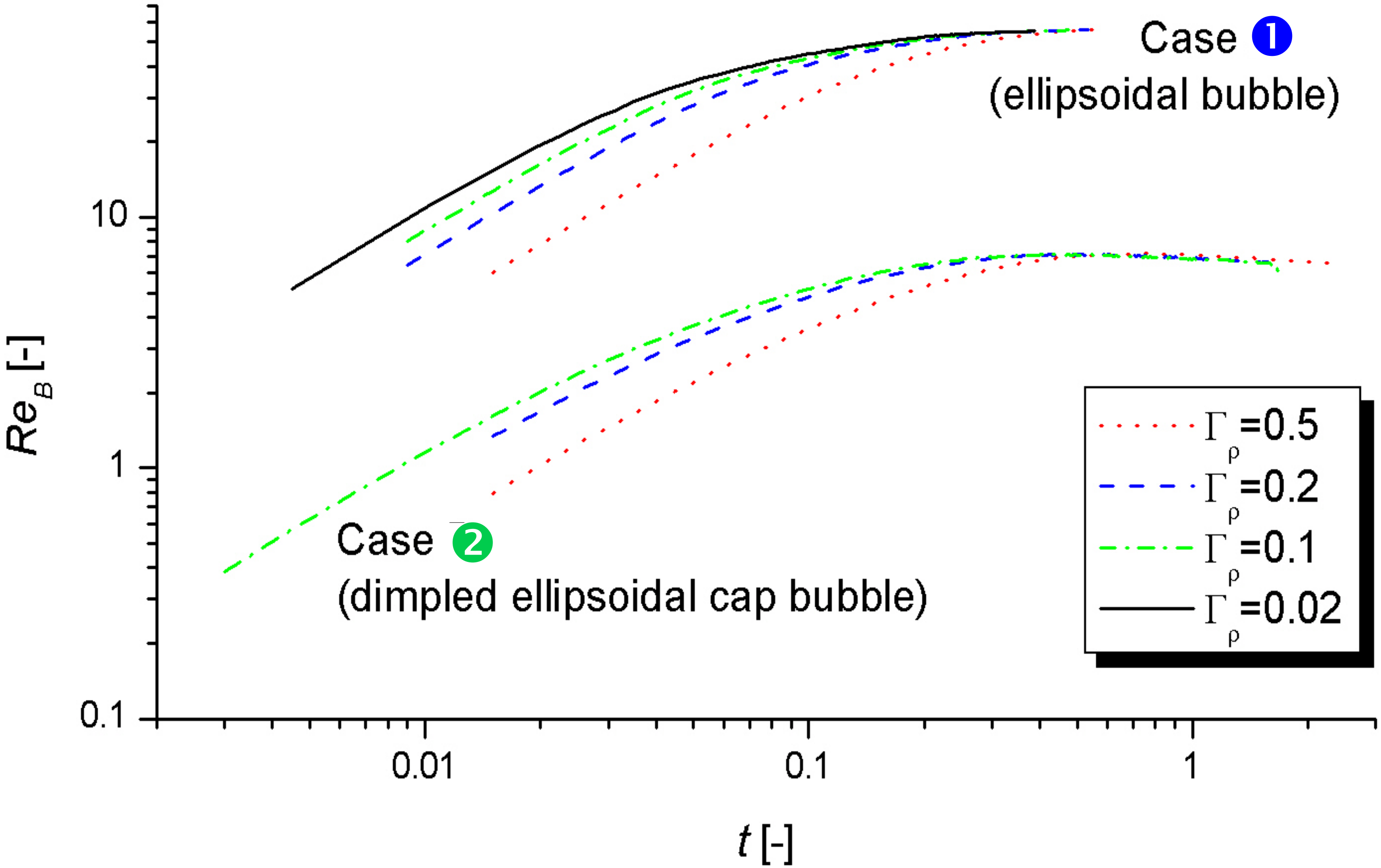


- oblate ellipsoidal shape

Time history of vertical bubble position



Time history of bubble Reynolds number



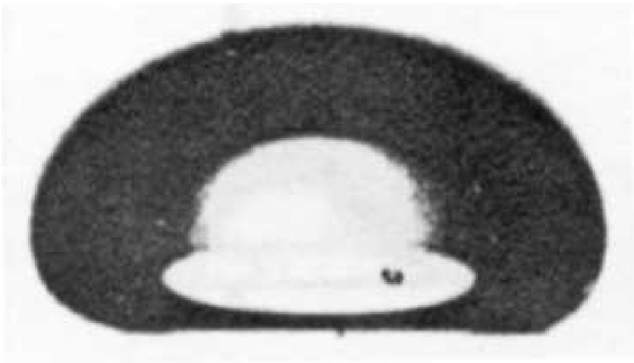
Comparison of bubble shape (case ❷)

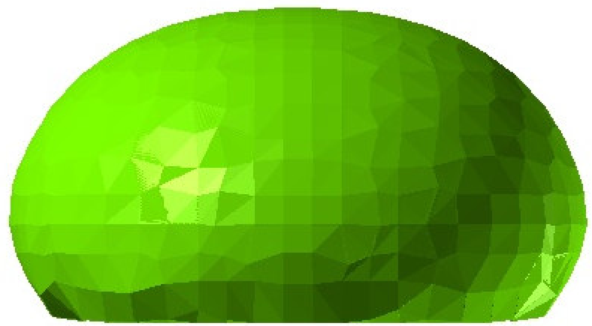
Experiment Bhaga & Weber*

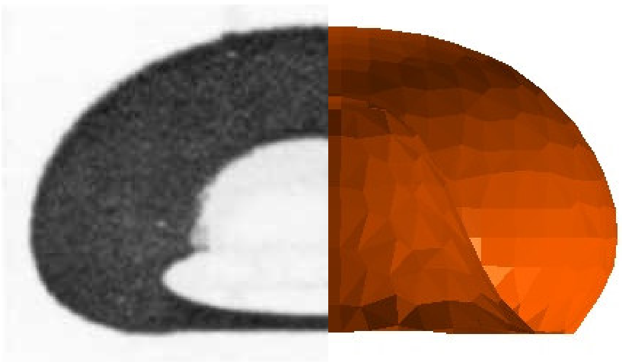
$(\Gamma_\rho \approx 0.0008, \Gamma_\mu \approx 10^{-5})$

TURBIT-VOF

$(\Gamma_\rho = 0.5, \Gamma_\mu = 1)$

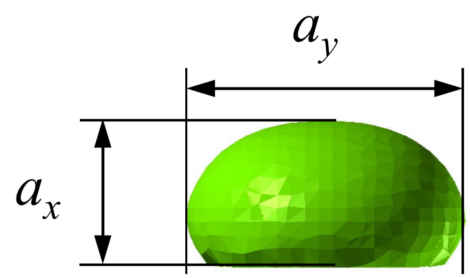






* D. Bhaga, M.E. Weber, J. Fluid Mech. 105:61-85, 1981

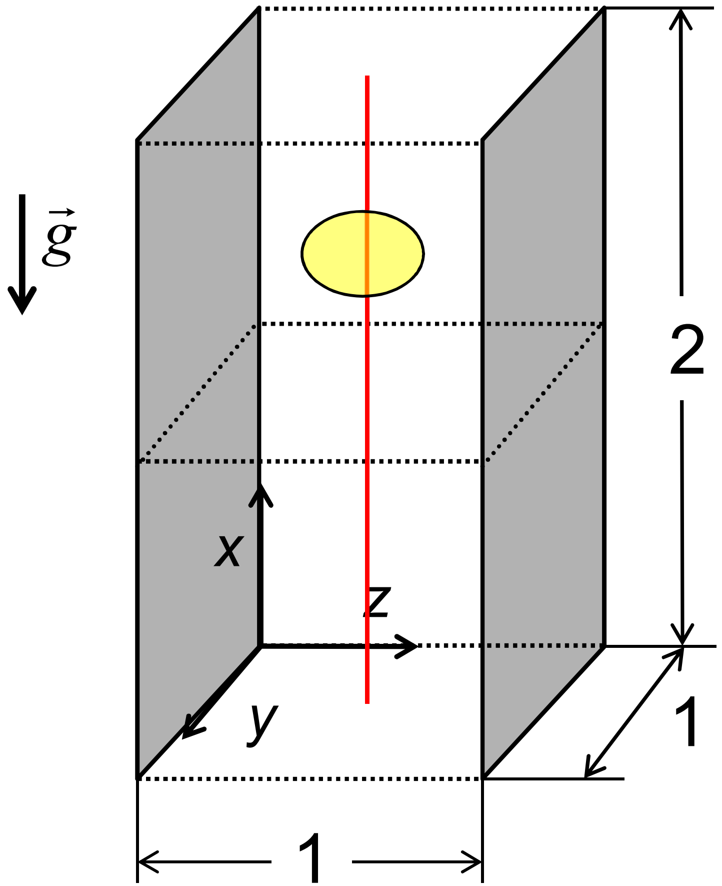
Influence of density ratio on bubble shape

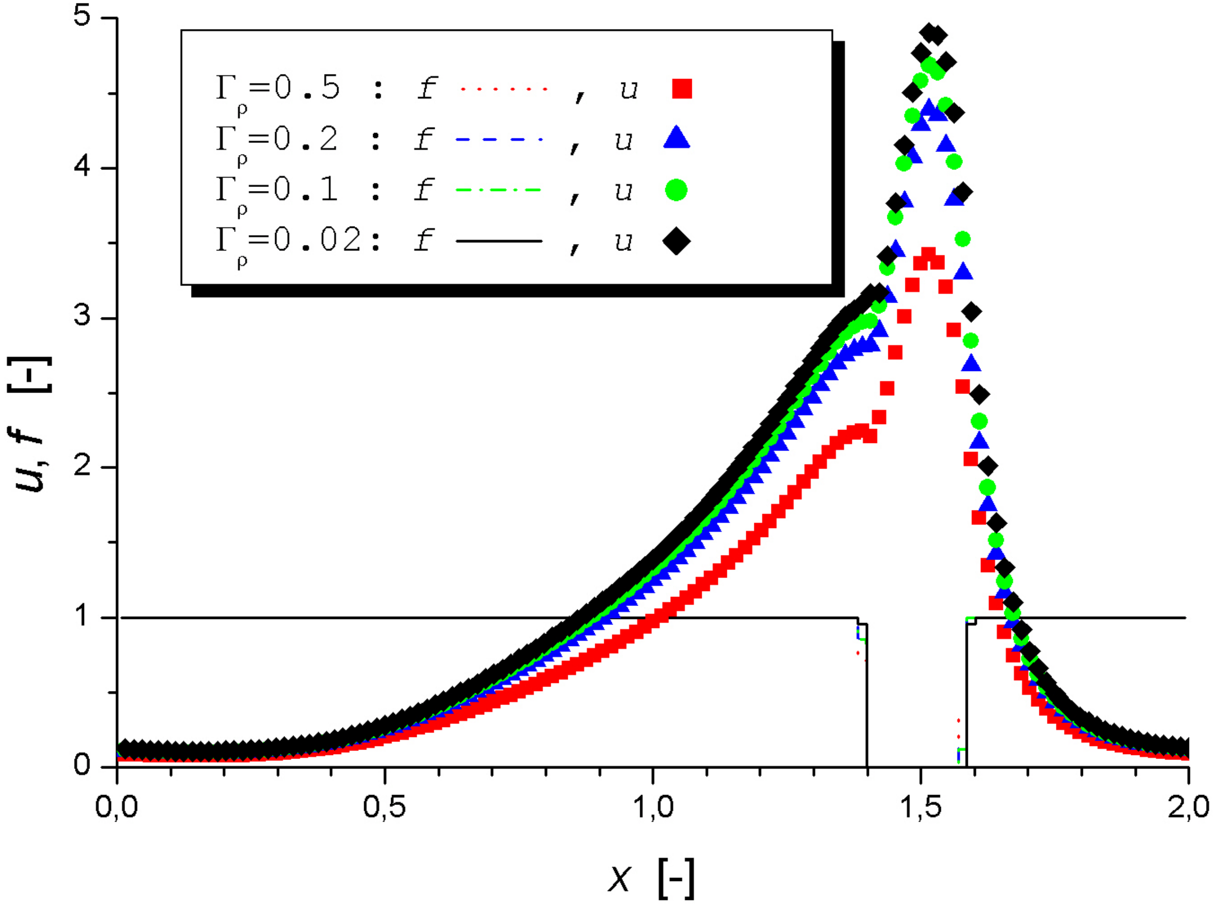


	Case ❶			Case ❷		
Γ_ρ	a_x / a_y	a_x / a_z	a_y / a_z	a_x / a_y	a_x / a_z	a_y / a_z
0.5	0.648	0.659	1.017	0.538	0.556	1.033
0.2	0.652	0.665	1.021	0.528	0.544	1.030
0.1	0.658	0.669	1.016	0.528	0.543	1.029
0.02	0.658	0.666	0.998	-	-	-

Local vertical velocity (case ❶)

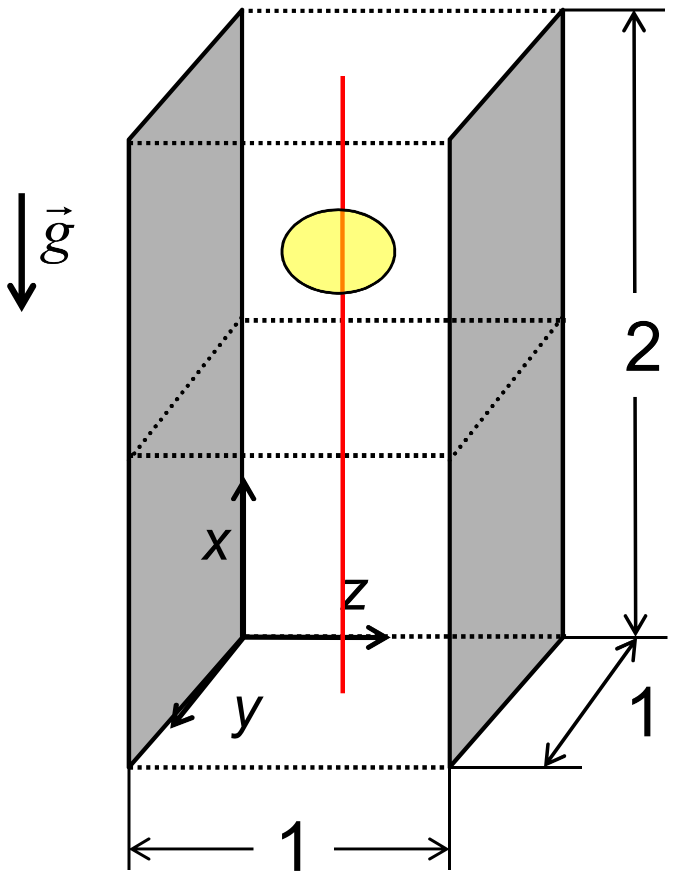
Comparison of profiles for instant t_C with $x_{com}(t_C) = 1.5$

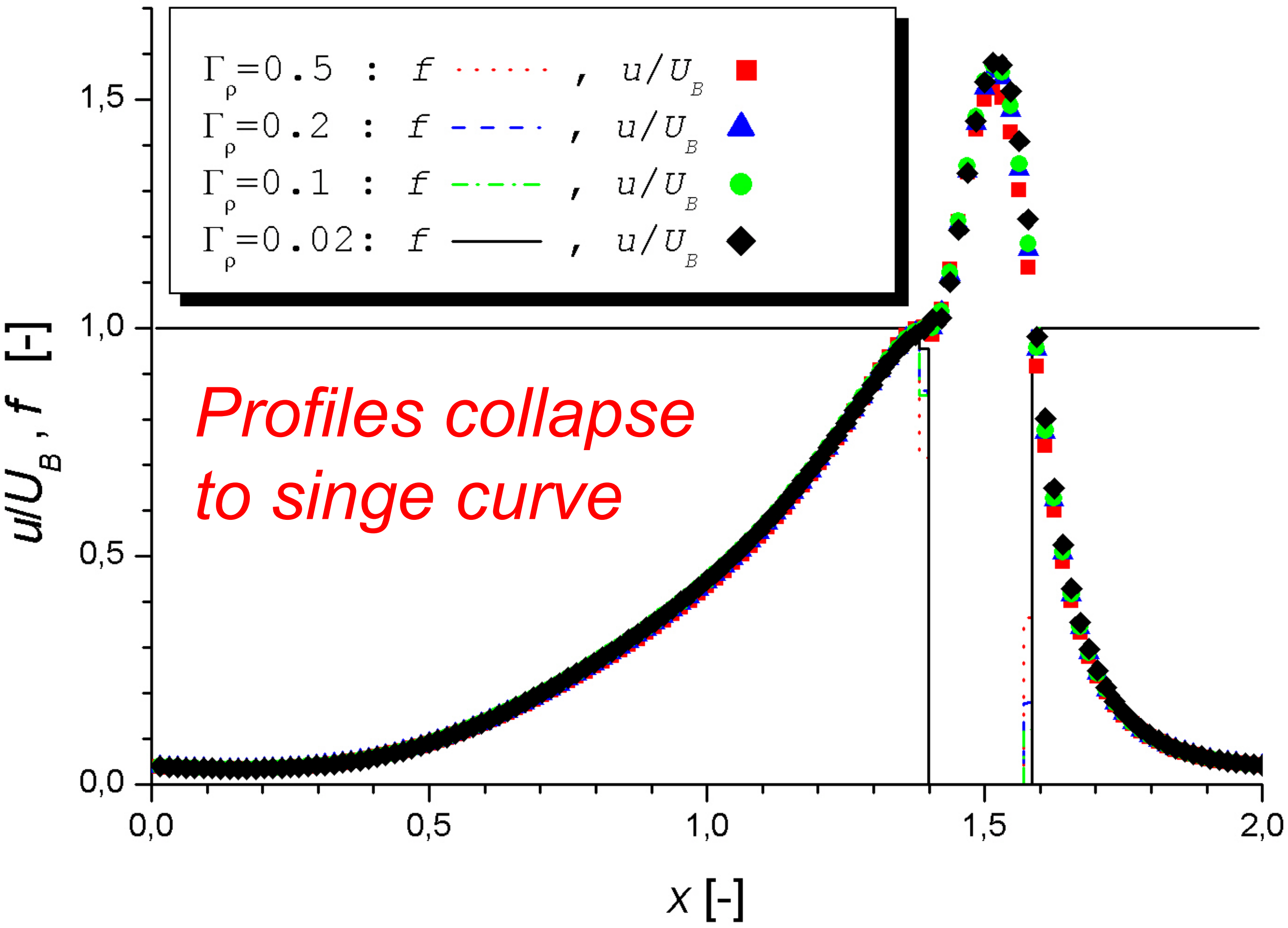




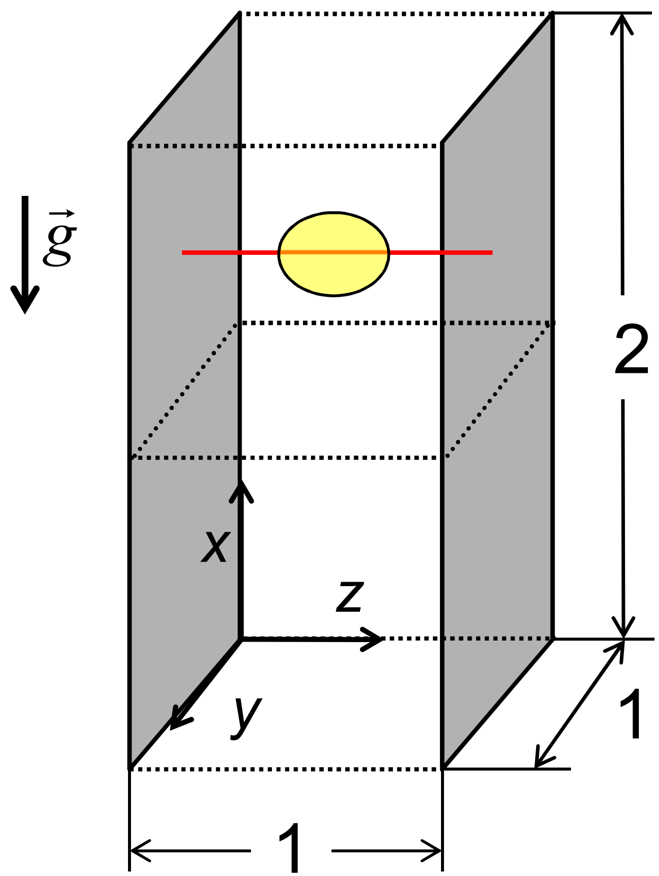
Local vertical velocity (case ❶)

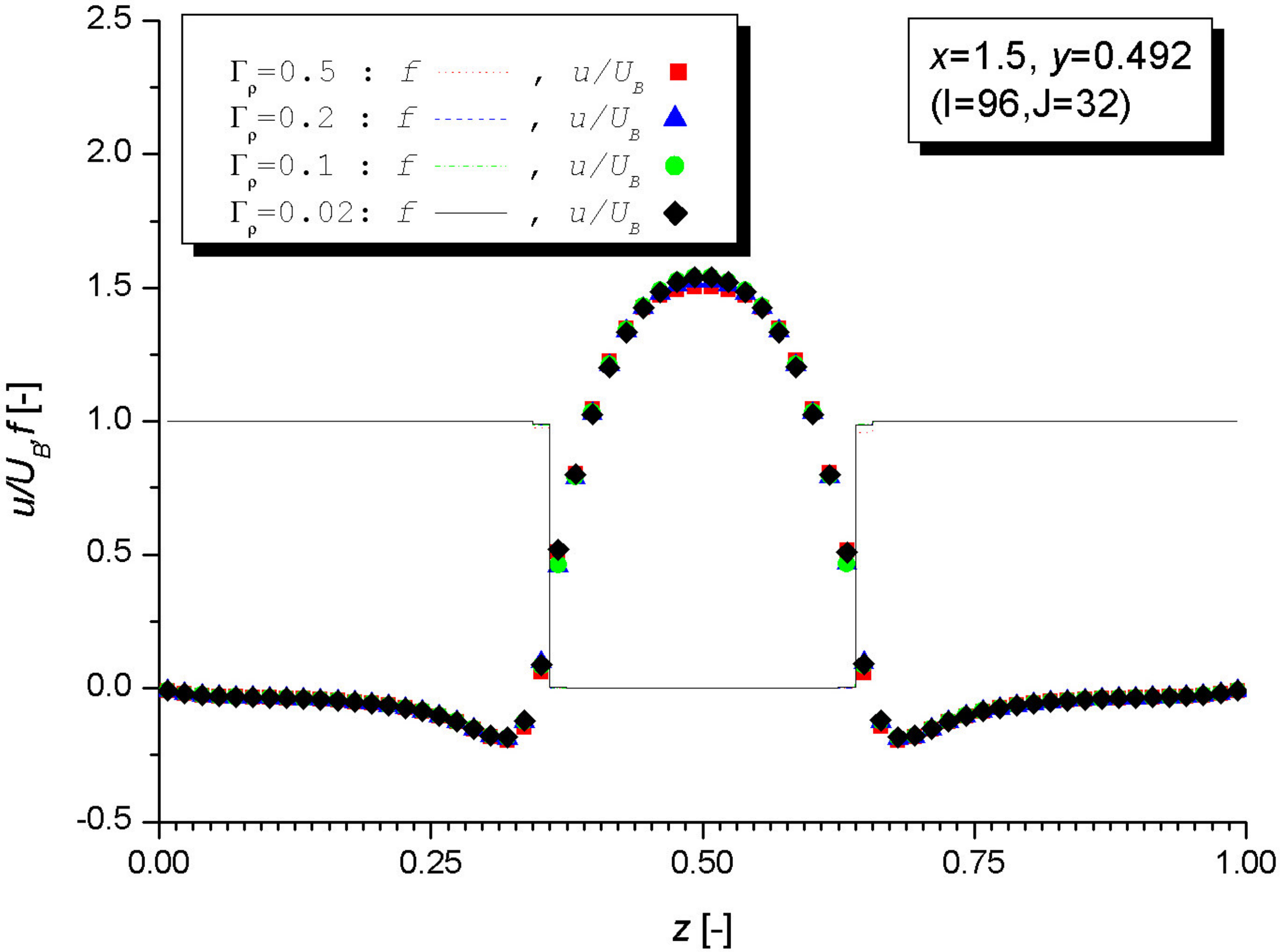
Normalization by respective bubble rise velocity U_B



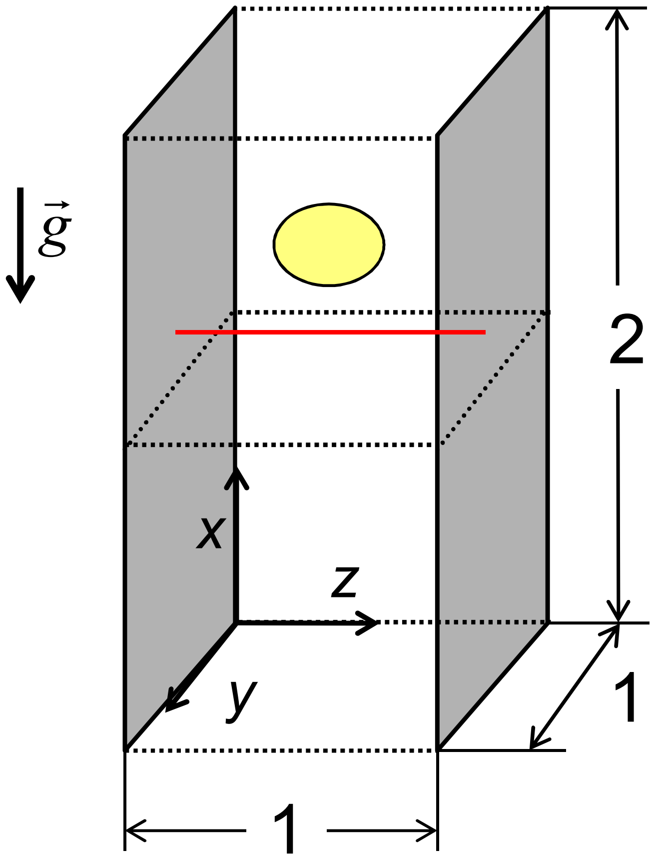


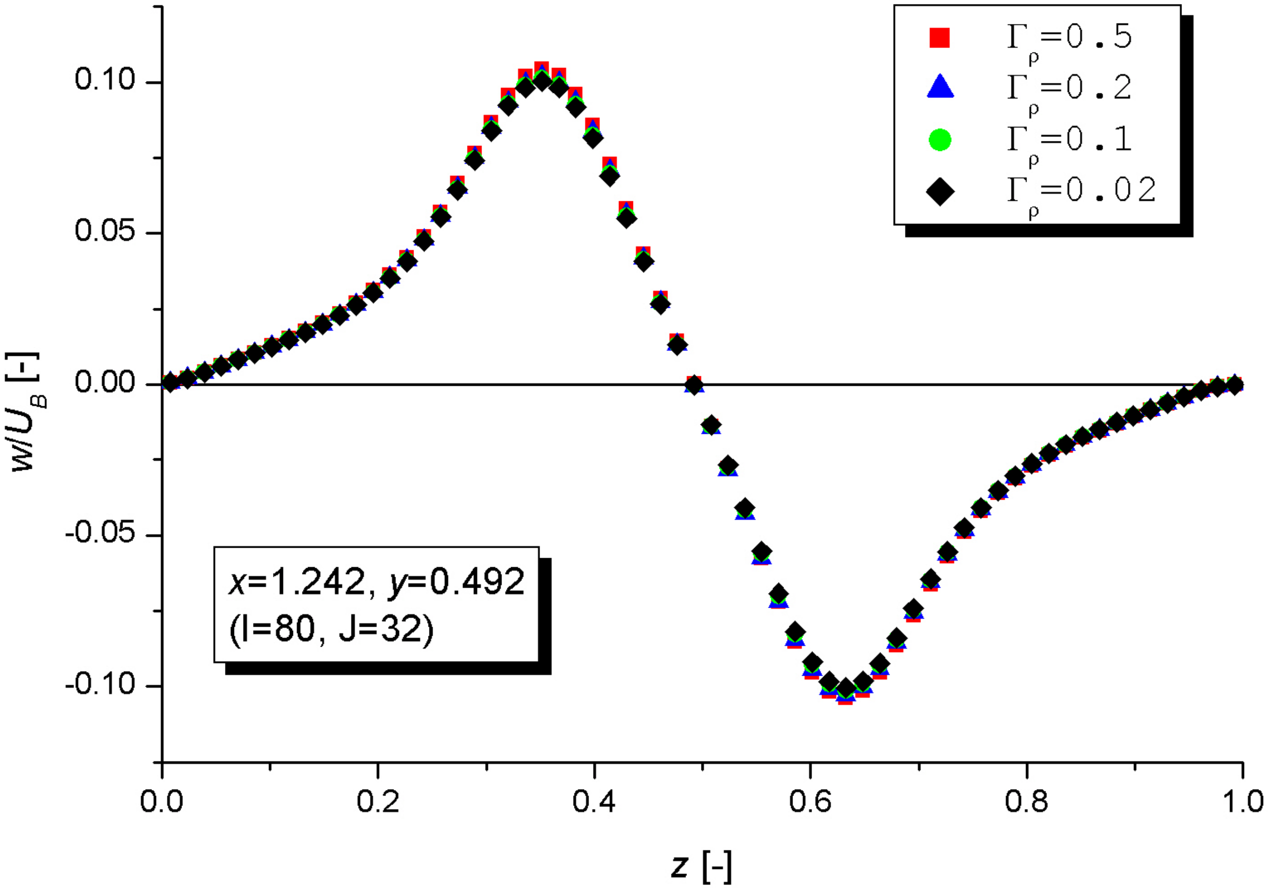
Local vertical velocity (case ❶)



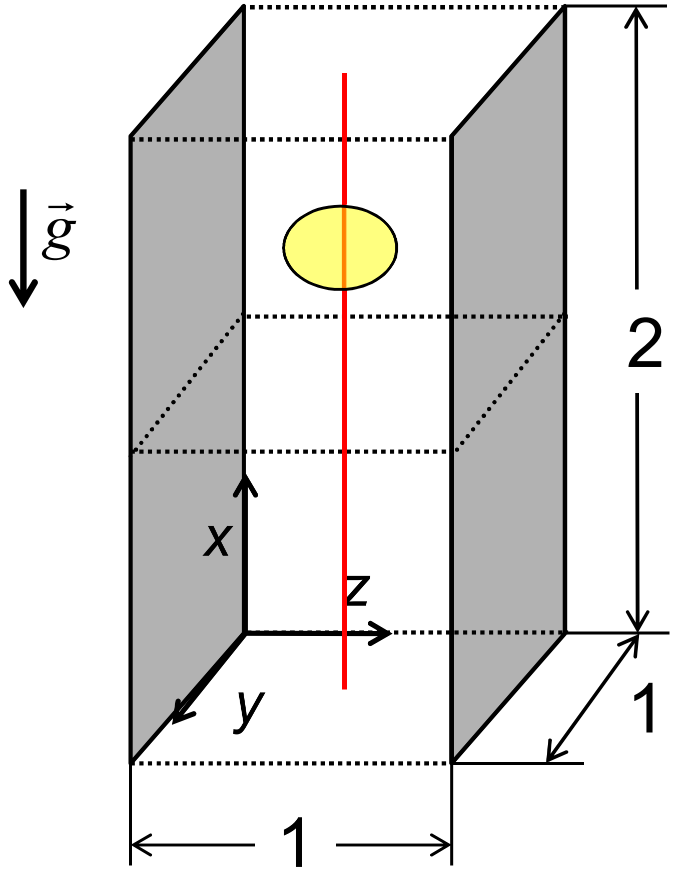


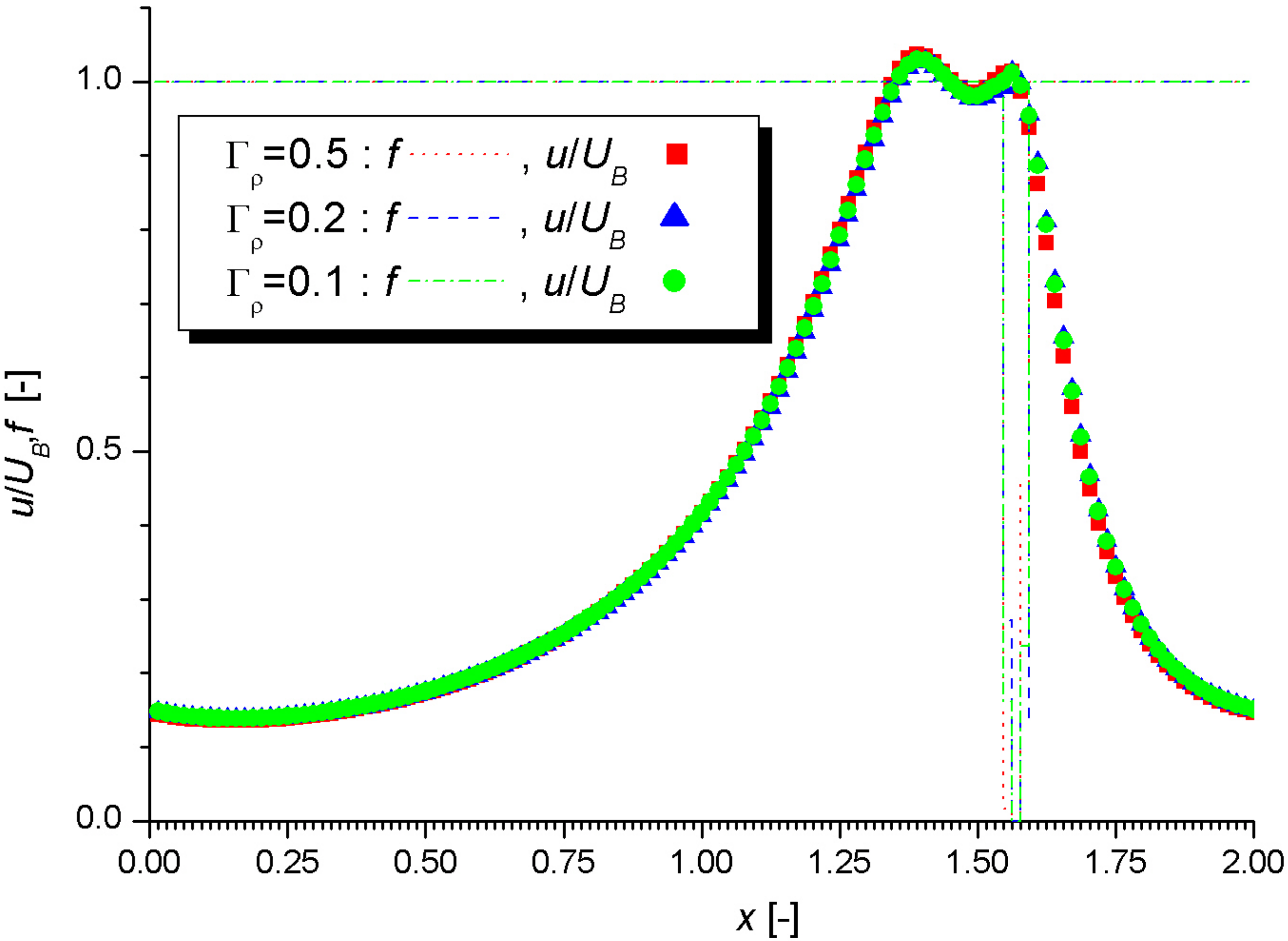
Local wall-normal velocity (case ❶)





Local vertical velocity (case ❷)





Conclusions

- DNS of ellipsoidal and ellipsoidal cap bubble for fixed values of $E\ddot{o}_B$ and Mo but different density ratios Γ_ρ
- Steady bubble: shape, bubble Reynolds number, and liquid velocity field are virtually independent of Γ_ρ

⇒ DNS results for steady bubbles obtained with $\Gamma_\rho = O(0.1)$ can be transferred to $\Gamma_\rho = O(0.001)$

- *Statistical models for bubble induced turbulence*
  - *formulate models in terms of Eötvös and Morton number*
  - *utilize DNS data for model development and testing*