A key parameter to characterize Taylor flow in narrow circular and rectangular channels

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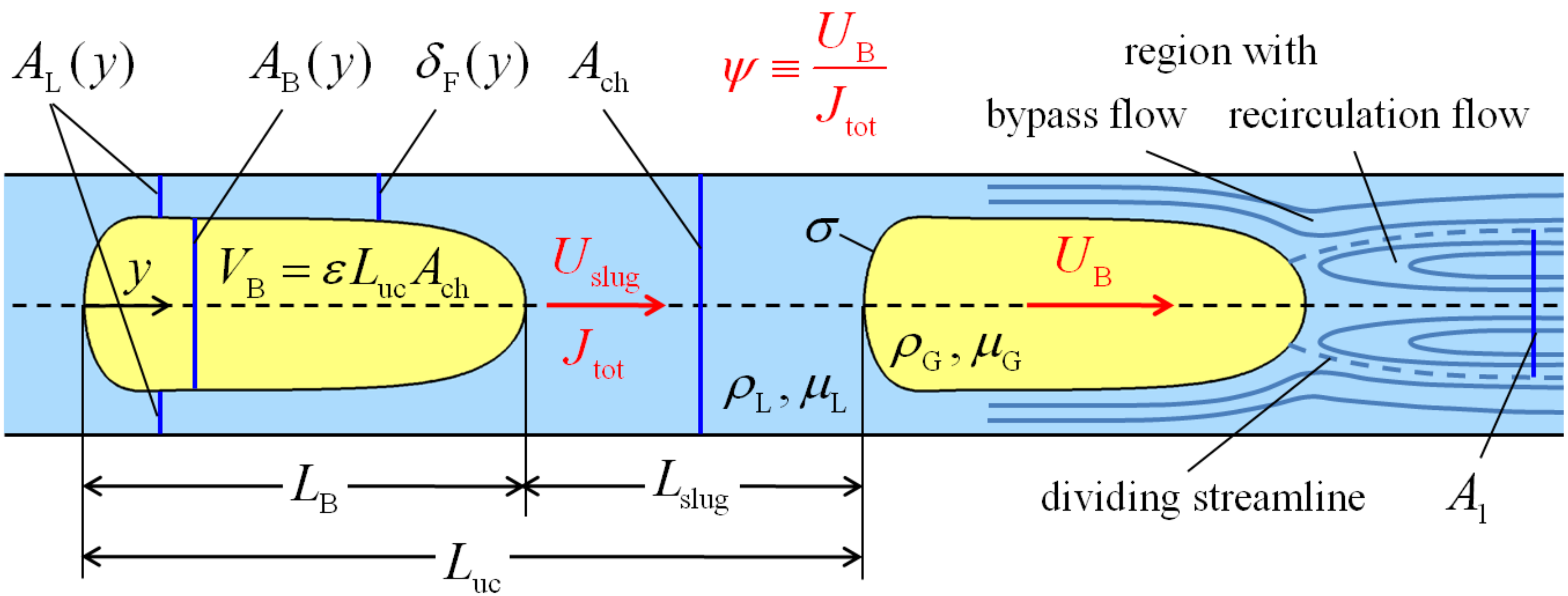

Outline

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  - Relevance of Taylor flow
  - Objective
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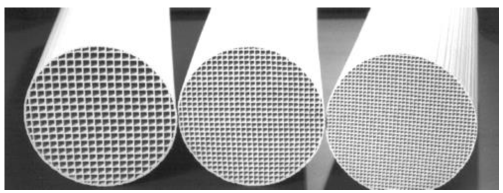
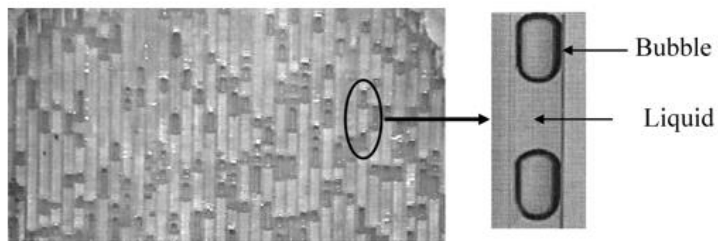
- The key parameter $\psi \equiv U_B/J_{tot}$
  - Relations by definition
  - Bubble diameter and liquid film thickness
  - Recirculation flow and bypass flow
  - Correlations for ψ
- Conclusions

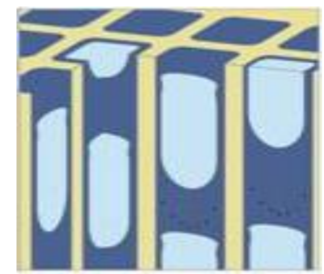
Relevance of Taylor flow

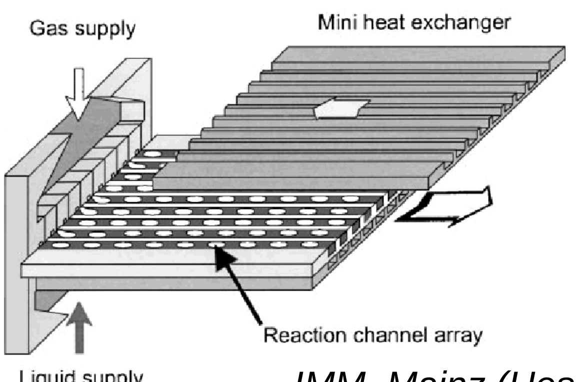
- Taylor flow is a special kind of slug flow in small channels, where the liquid plugs which separate the elongated bullet-shaped bubbles (Taylor bubbles) are free from gas entrainment

Delft group (Moulijn)

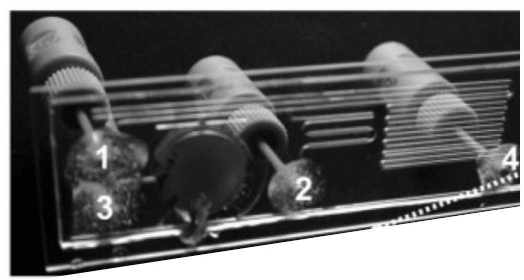


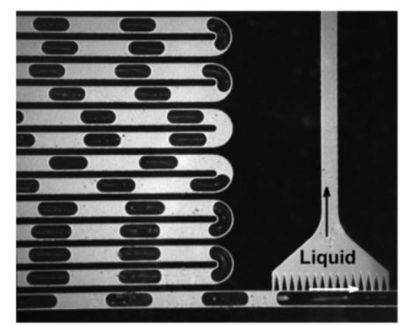






IMM, Mainz (Hessel)





MIT (Jensen)

Advantages of Taylor flow

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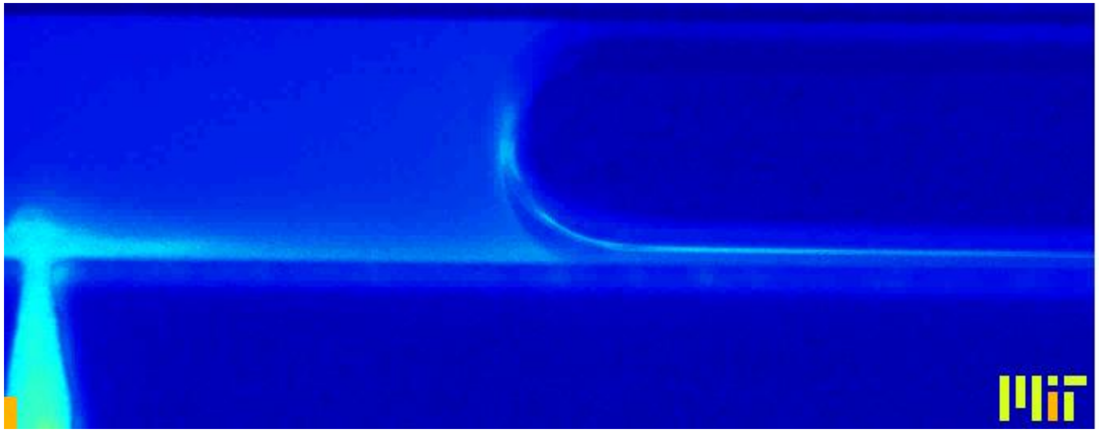
- Large specific interfacial area
  - efficient heat and mass transfer
- Segmentation of the liquid
  - reduced axial dispersion
- Good mixing in gas and liquid due to recirculation



Channel cross-section:
400 µm × 280 µm

Movie from von Günther et al.
Langmuir ***21*** *(2005) 1547-1555*

Objective

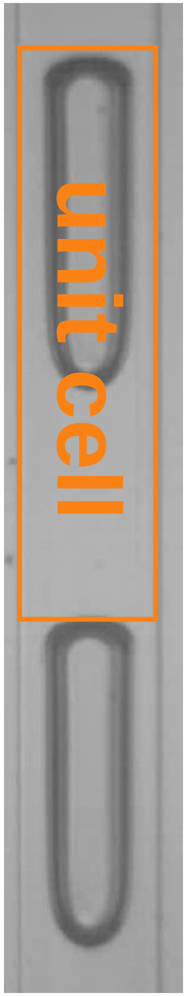
- Many features of Taylor flow such as the liquid film thickness depend on the capillary number Ca_{B}

$$Ca_{\mathrm{B}} \equiv \frac{\mu_{\mathrm{L}} U_{\mathrm{B}}}{\sigma} \qquad \text{where } U_{\mathrm{B}} = \text{bubble velocity}$$

  - The capillary number is not known a priory for given gas and liquid flow rates, but adjusts accordingly
- <u>Objective:</u>
  - propose the use of correlations for global and local quantities in Taylor flow in terms of “a priori” known flow parameters
  - propose a unified consistent approach, which relates many global and local features of Taylor flow to a single functional dependence

Similitude analysis for a unit cell

- Physical properties (6)
  - Gas and liquid density (ρ_G, ρ_L) and viscosity (μ_G, μ_L)
  - Coefficient of surface tension (σ), gravity constant (g)
- Flow specific quantities (3)
  - Gas and liquid flow rates (Q_G,Q_L), pressure drop (Δp_{uc})
- Geometrical quantities for a rectangular channel (5)
  - Angle of channel orientation (φ) with respect to **g**
  - Channel height (H) and width (B)
  - Length of unit cell (L_{uc})
  - Bubble volume in the unit cell (V_B)
- Basic dimensions: kg, m, s (3)
- Pi-theorem: there are 14 – 3 = **11** independent non-dimensional groups



Pi-theorem

- A priori unknown groups

$$\Pi_1 \equiv \frac{L_{\mathrm{uc}}}{D_{\mathrm{h}}} \equiv \Lambda \qquad \Pi_2 \equiv \frac{V_{\mathrm{B}}}{A_{\mathrm{ch}} L_{\mathrm{uc}}} \equiv \varepsilon \qquad \Pi_3 \equiv \frac{\Delta p_{\mathrm{uc}}}{\rho_{\mathrm{L}}} \left(\frac{A_{\mathrm{ch}}}{Q_{\mathrm{G}} + Q_{\mathrm{L}}} \right)^2 = \frac{\Delta p_{\mathrm{uc}}}{\rho_{\mathrm{L}} J_{\mathrm{tot}}^2} \equiv Eu_{\mathrm{uc}}$$

- A priori known groups

$$\Pi_4 \equiv \frac{\rho_{\mathrm{G}}}{\rho_{\mathrm{L}}} \equiv \rho' \qquad \Pi_5 \equiv \frac{\mu_{\mathrm{G}}}{\mu_{\mathrm{L}}} \equiv \mu' \qquad \Pi_6 \equiv \frac{Q_{\mathrm{G}}}{Q_{\mathrm{G}} + Q_{\mathrm{L}}} \equiv \beta \qquad \Pi_7 \equiv \frac{\mu_{\mathrm{L}}}{\sigma} \frac{Q_{\mathrm{G}} + Q_{\mathrm{L}}}{A_{\mathrm{ch}}} = \frac{\mu_{\mathrm{L}} J_{\mathrm{tot}}}{\sigma} \equiv Ca_{\mathrm{J}}$$

$$\Pi_8 \equiv \frac{\sigma \rho_{\mathrm{L}} D_{\mathrm{h}}}{\mu_{\mathrm{L}}^2} \equiv La \qquad \Pi_9 \equiv \frac{g(\rho_{\mathrm{L}} - \rho_{\mathrm{G}}) D_{\mathrm{h}}^2}{\sigma} \equiv E\ddot{o} \qquad \Pi_{10} \equiv \frac{H}{B} = \chi \qquad \Pi_{11} \equiv \varphi$$

- Functional relationships

$$\varepsilon = F_{\varepsilon}(Eu_{\mathrm{uc}}, \Lambda, \rho', \mu', \beta, Ca_{\mathrm{J}}, La, E\ddot{o}, \chi, \varphi)$$

$$Eu_{\mathrm{uc}} = F_{Eu}(\varepsilon, \Lambda, \rho', \mu', \beta, Ca_{\mathrm{J}}, La, E\ddot{o}, \chi, \varphi)$$

unknown parameter

known parameter

Further non-dimensional groups

- Some dependent non-dimensional groups

$$Re_J \equiv \frac{\rho_L D_h J_{tot}}{\mu_L} = LaCa_J$$

$$We_J \equiv \frac{\rho_L D_h J_{tot}^2}{\sigma} = Ca_J Re_J = LaCa_J^2$$

$$Fr_J \equiv \frac{J_{tot}}{\sqrt{gD_h \frac{\rho_L - \rho_G}{\rho_L}}} = Ca_J \sqrt{\frac{La}{E\ddot{o}}}$$

$$Ca_B \equiv \boxed{\frac{U_B}{J_{tot}}} Ca_J = \boxed{\frac{\beta}{\varepsilon}} Ca_J$$

- The key parameter ψ

$$\boxed{\psi \equiv \frac{\beta}{\varepsilon} = \frac{U_B}{J_{tot}}}$$

Nomenclature ψ was introduced by Thulasidas et al.(1995)

$$\psi = \frac{\beta}{F_\varepsilon} \equiv F_\psi(Eu_{uc}, \Lambda, \rho', \mu', \beta, Ca_J, La, E\ddot{o}, \chi, \varphi)$$

Quantities and their relation to ψ

- Bubble velocity $$U_\mathrm{B} = \psi J_\mathrm{tot}$$
- Mean liquid velocity $$U_\mathrm{L} = \frac{1-\beta}{1-\beta/\psi} J_\mathrm{tot}$$
- Gas hold-up in unit cell $$\varepsilon = \frac{\beta}{\psi}$$
- Relative bubble velocity $$W \equiv \frac{U_\mathrm{B} - J_\mathrm{tot}}{U_\mathrm{B}} = 1 - \frac{1}{\psi}$$ $$Z \equiv \frac{U_\mathrm{B} - J_\mathrm{tot}}{J_\mathrm{tot}} = \psi - 1$$
- Capillary number $$Ca_\mathrm{B} = \psi Ca_\mathrm{J}$$
- Bubble Reynolds number $$Re_\mathrm{B} = \psi Re_\mathrm{J} = \psi La Ca_\mathrm{J}$$

Liquid mass balance

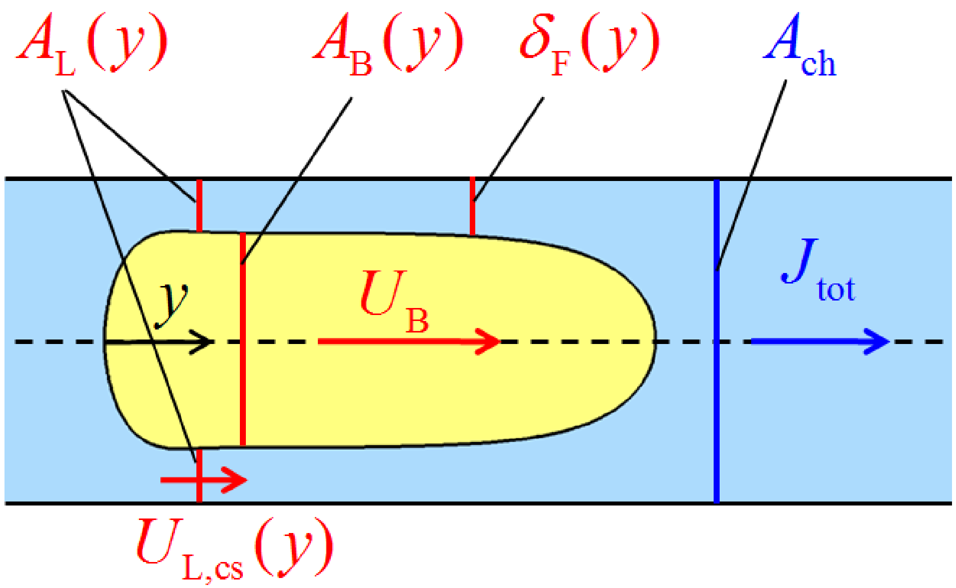
- A mass balance for the liquid phase in a frame of reference moving with the bubble at an arbitrary axial position *y* yields

$$(J_{\text{tot}} - U_{\text{B}})A_{\text{ch}} = \left[U_{\text{L,cs}}(y) - U_{\text{B}}\right]A_{\text{L}}(y)$$

so that

$$\frac{A_{\text{L}}(y)}{A_{\text{ch}}} = \frac{\psi - 1}{\psi - U_{\text{L,cs}}(y)/J_{\text{tot}}}$$

$$\frac{A_{\text{B}}(y)}{A_{\text{ch}}} = \frac{1 - U_{\text{L,cs}}(y)/J_{\text{tot}}}{\psi - U_{\text{L,cs}}(y)/J_{\text{tot}}}$$



Bubble diameter / film thickness

- For a stagnant liquid film the mass balance yields

$$\frac{A_{\mathrm{B}}}{A_{\mathrm{ch}}} = \frac{1}{\psi}$$

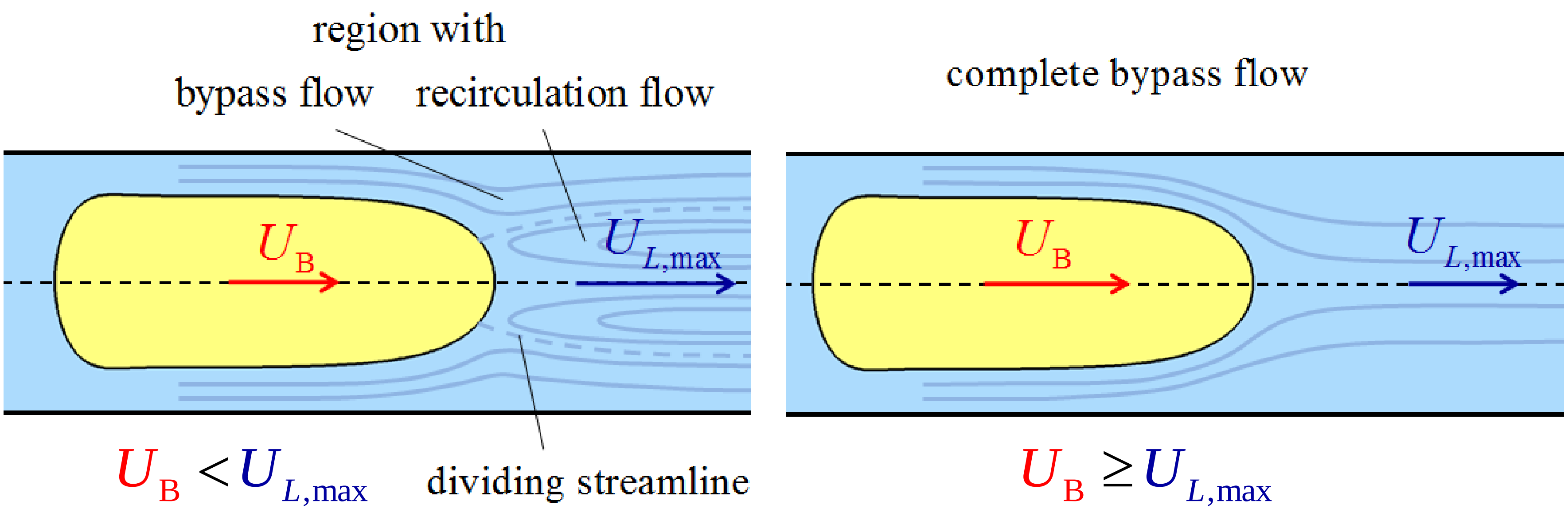
- For a circular channel with diameter D it follows for the bubble diameter D_{B} and the liquid film thickness δ_{F}

$$\frac{D_{\mathrm{B}}}{D} = \frac{1}{\sqrt{\psi}} \qquad \frac{\delta_{\mathrm{F}}}{D} = \frac{1}{2}\left(1 - \frac{1}{\sqrt{\psi}}\right)$$

- For a rectangular channel with an axisymmetric bubble and a stagnant liquid film it follows

$$\frac{D_{\mathrm{B}}}{B} = 2\sqrt{\frac{\chi}{\pi\psi}} \qquad \frac{D_{\mathrm{B}}}{H} = \frac{2}{\sqrt{\pi\chi\psi}} \qquad \frac{D_{\mathrm{B}}}{D_{\mathrm{h}}} = \frac{1+\chi}{\sqrt{\pi\chi\psi}}$$

Recirculation flow and bypass flow



Fully developed laminar flow: $U_{L,\max} = C U_{L,\mathrm{mean}} \quad (C_{\bigcirc} = 2;\ C_{\square} = 2.096)$

In Taylor flow it is: $U_{L,\mathrm{mean}} = J_{\mathrm{tot}}$

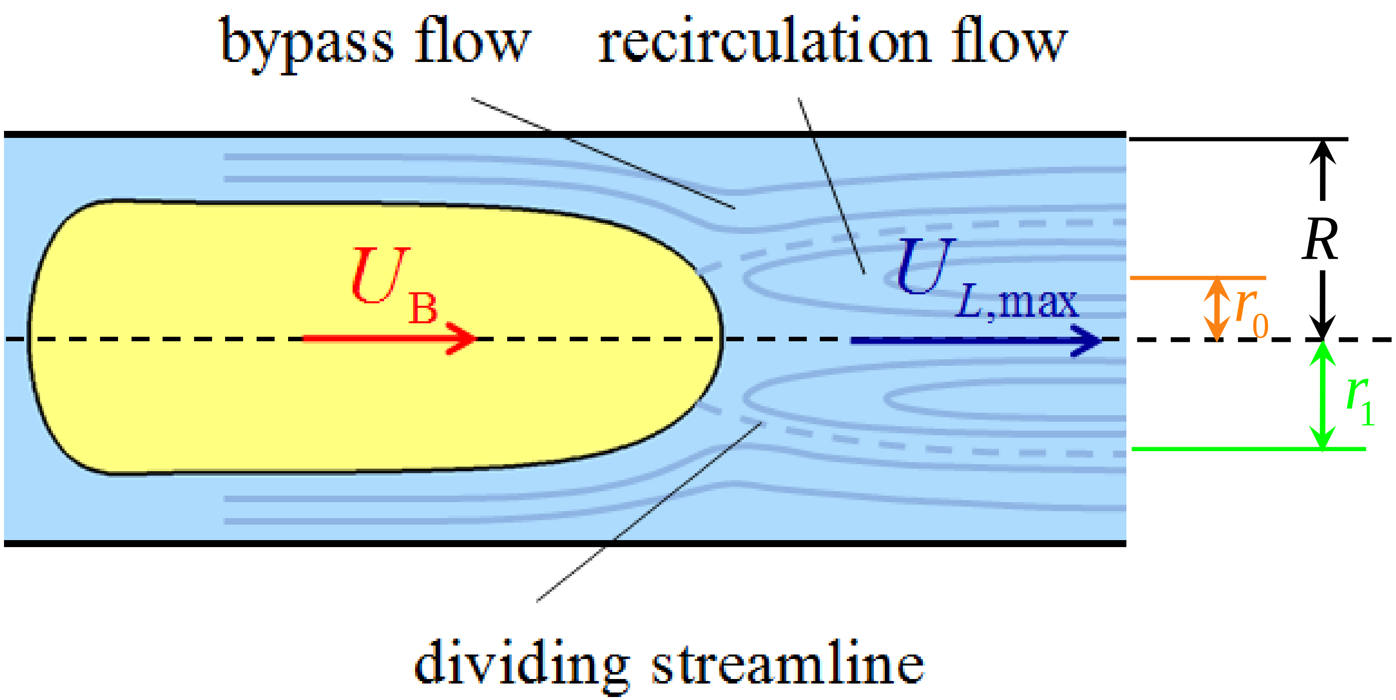
Condition for recirculation flow is	Condition for bypass flow is
$\psi = U_{\mathrm{B}} / U_{L,\mathrm{mean}} < C$	$\psi = U_{\mathrm{B}} / U_{L,\mathrm{mean}} \geq C$

(sketches in moving frame of reference after Taylor 1961)

Recirculation area and recirculation time

- Relations for a circular pipe of radius R

  Thulasidas, Abraham, Cerro, Chem. Eng. Sci. **52** (1997) 2947



$$\frac{r_0}{R} = \sqrt{\frac{2-\psi}{2}}$$

$$\frac{r_1}{R} = \sqrt{2-\psi}$$

$$\tau = \left(\frac{1}{\psi} - \frac{1}{2}\right)^{-1}$$

For dependence of A_0/A_{ch}, A_1/A_{ch} and τ on ψ and χ in rectangular channels see Kececi, Wörner, Onea, Soyhan, Catalysis Today **147S** (2009) S125

Collection of literature data

- Thulasidas, Abraham, Cerro (1995) (Experiments, circular and square channel)
- Liu, Vandu, Krishna (2005) (Experiments, circular and square channel)

$$\boxed{\psi = \frac{1}{1-0.61 Ca_J^{0.33}}} \quad \text{for} \quad 0.0002 \le Ca_J \le 0.39$$

Many relations between ψ and Ca_J are implicit in ψ

- Fairbrother and Stubbs (1935) (Experiments, circular tube)

$$\frac{U_B - J_{tot}}{U_B} = Ca_B^{0.5} \quad \text{for} \quad 7.5\times 10^{-3} \le Ca_B \le 0.014 \quad \Rightarrow \quad 1-\frac{1}{\psi} = \psi^{0.5} Ca_J^{0.5}$$

- Bretherton (1961) (Theoretical, circular tube)

$$\frac{U_B - J_{tot}}{U_B} = 2.68 Ca_B^{2/3} \quad \text{for} \quad Ca_B \le 0.001 \quad \Rightarrow \quad 1-\frac{1}{\psi} = 2.68 \psi^{2/3} Ca_J^{2/3}$$

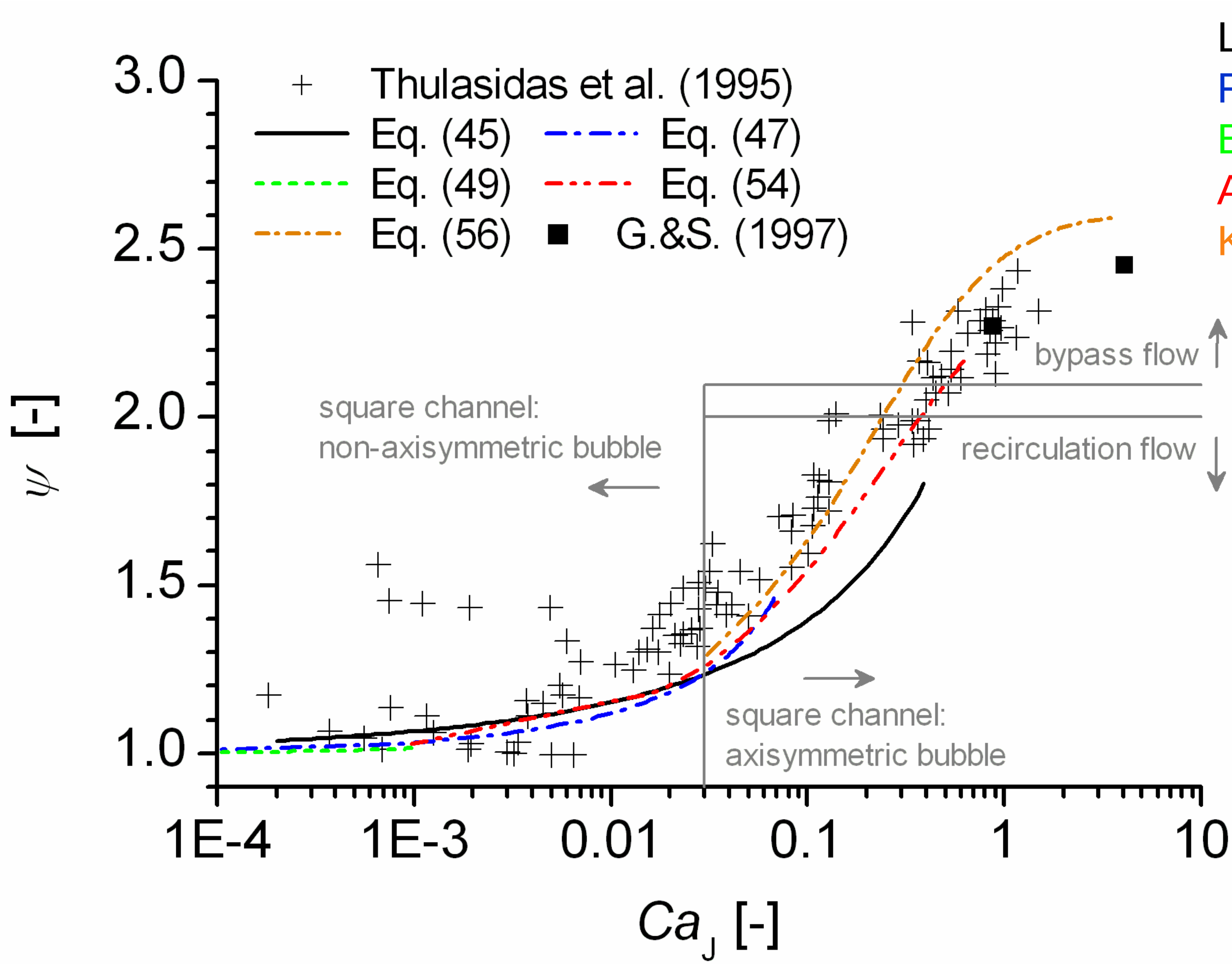
- Aussilious and Quere (2000) (Theoretical, circular pipe)

$$\frac{\delta_F}{D} = \frac{0.66 Ca_B^{2/3}}{1+3.33 Ca_B^{2/3}} \quad \text{for} \quad 0.001 \le Ca_B \le 1.4 \quad \Rightarrow \quad \psi \approx \left(\frac{1+3.33\psi^{2/3} Ca_J^{2/3}}{1+2\psi^{2/3} Ca_J^{2/3}} \right)^2$$

- Kreutzer, Kapteijn, Moulijn, Heiszwolf (2005) (Square channel)

$$\frac{D_{B,sq}}{D_h} = 0.7 + 0.5 \exp\left(-2.25 Ca_B^{0.445}\right) \quad \text{for} \quad Ca_B > 0.04 \quad \Rightarrow \quad \psi = \frac{4}{\pi}\left[0.7 + 0.5\exp\left(-2.25\psi^{0.445} Ca_J^{0.445}\right)\right]^{-2}$$

Comparison of literature data



Liu et al.
Fairbrother&Stubbs
Bretherton
Aussilious&Quere
Kreutzer et al.

Conclusions

- The ratio ψ between bubble velocity and total superficial velocity is a key parameter in Taylor flow
- Quantities that are related to ψ are
  - the mean liquid velocity, the mean relative velocity, the gas holdup
  - the local thickness of the liquid film (and the bubble diameter)
- ψ determines if complete bypass flow or recirculation flow occurs
  - The cross-sectional recirculation area and the non-dimensional recirculation time are unique functions of ψ (and in rectangular channels of the aspect ratio χ)
- Literature data show a clear trend for the dependence of ψ on Ca_J

$$\psi = F_\psi(Eu_{uc}, \Lambda, \rho', \mu', \beta, Ca_J, La, E\ddot{o}, \chi, \varphi) \quad \rightarrow \quad \boxed{\psi = F_\psi(\beta, Ca_J, La, E\ddot{o}, \chi, \varphi)}$$

- If this functional relation is known, then the hydrodynamics of Taylor flow is almost fully determined