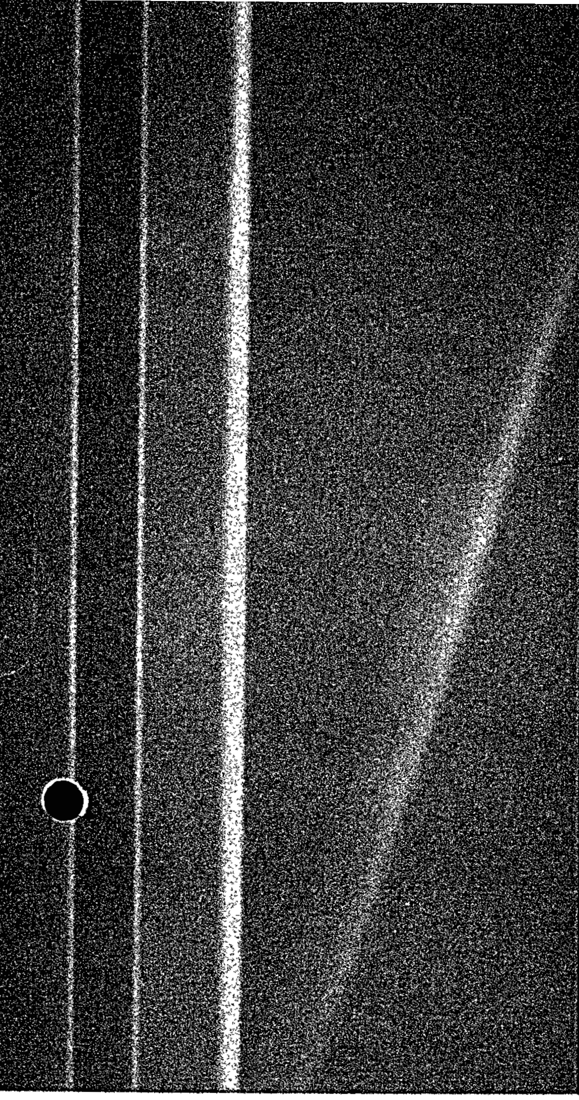

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Inelastic Scattering of Neutrons by U^{238} below 1 Mev

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The neutron excitation cross sections for the individual energy levels of U^{238} below 1 Mev have been calculated for neutron energies up to 1 Mev with the statistical theory of Hauser and Feshbach.

INTRODUCTION

For the calculation of the performance of fast breeder reactors which use U^{238} as the fertile material, it is necessary to know the reaction cross sections of U^{238} with fast neutrons fairly precisely. Particularly important are the partial inelastic scattering cross sections, which play the chief role in the energy loss process. Also important is the (n, γ) cross section, which influences the breeding gain through absorption. At present, there are five experimental measurements of the inelastic cross section known (*1–5*), but these, quite unfortunately, are not at all consistent with one another. Therefore, we shall compare these measurements with theoretical calculations in order to try to decide among the different results.

For this purpose the cross sections must be calculated rather exactly. The results of the simple optical model (*6*) are generally too inaccurate for this purpose. Consequently, we employ in this work the formulas of the statistical model of nuclear reactions (*7, 8*), as we would have if we had used the optical model, but determine the necessary physical parameters which appear in these formulas (e.g., transmission coefficients), not from pure theory, but rather from the most credible part of the experimental results. Then with the help of a few simple theoretical assumptions, we calculate the remaining reaction cross sections and make a meaningful comparison with experiment.

This work is divided into three parts. In the first, we discuss briefly the experimental results up to 1 Mev. In the second, we describe the theoretical basis for the calculations, and in the third, we con pare the calculations with the experiments, ai recommend what in our opinion are the best curre results up to 1 Mev.

I. DISCUSSION OF THE EXPERIMENTAL RESUL

The inelastic scattering cross section has be measured by three different methods. The fir method consists of a transmission experiment wi spherical shells. Measurements of this kind in whi proportional counters were employed as detecto have been published by Allen (*1*), Batchelor (*2* and Beyster *et al.* (*3*). As a result of the use of pr portional counters, neutrons which excited low lyi energy levels were only partly resolved from ela tically scattered neutrons. This represents a serio disadvantage of this method. Moreover, in most these measurements the spherical shells were thick that the neutrons were multiply scattere The order of magnitude of this last effect can estimated and a correction for it made, but this h been omitted by some authors. The results of tl three considered measurements are tabulated Table I, together with some brief remarks.

Another way to obtain the inelastic scatteri cross section is to subtract the integrated measur differential scattering cross section from the tot measured cross section. Also in this method tl difficulty of insufficient resolution of elastical scattered neutrons and neutrons which excite lo lying states appears, particularly in the measur ment of the differential elastic scattering cross se tion. In Table I are values obtained by this meth by Allen (*1*) and Walt and Barschall (*4*).

The third method with which the partial inelast cross sections can be measured is the time-of-flig

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TABLE I

SUMMARY OF THE EXPERIMENTAL DATA ON INELASTIC SCATTERING IN U^{238} UP TO 1 MEV

E (kev)	$\sigma(44)$ (mb)	$\sigma(146)$ (mb)	$\sigma(300)$ (mb)	$\sigma(730)$ (mb)	$\sigma_{\text{in tot}}$ (mb)	Refs.
150	440 ± 200^{a}				440 ± 200^{g}	^{k}
250	$>190 \pm 100^{a, b}$	290 ± 150^{a}			480 ± 180^{g}	
500	$>180 \pm 100^{a, b}$	490 ± 250^{a}			670 ± 270^{g}	
1000					$1400 \pm 200^{h, i}$	
500					$>800 \pm 200^{a, b}$	^{o}
1000					$>1100 \pm 200^{a, i}$	
1000					1350 ± 200^{a}	^{l}
550	$1430 \pm 200^{c, d}$	290 ± 60^{c}			1720 ± 210^{g}	^{m}
1000	$>900 \pm 250^{b, c, e}$	$600 \pm 190^{c, e}$	$90 \pm 50^{c, e}$	$520 \pm 80^{c, e}$	2100 ± 330^{g}	
1000					$1200 \pm 700^{h, i, j}$	^{n}

[a] Sphere transmission value, correction for radiative capture made with an assumed value of $\sigma(n, \gamma)$.
[b] Lower limit due to bad resolution.
[c] Time-of-flight value.
[d] Differential inelastic scattering cross section measured.
[e] 4π times the differential inelastic scattering cross section at 90°.
[f] 145°/90° scattering ratio = 1.00 ± 0.2.
[g] Sum, not a directly measured value.
[h] Value obtained from measured total and differential elastic scattering cross sections and an assumed value of $\sigma(n, \gamma)$.
[i] Without excitation of the levels at 44 and 146 kev.
[j] Error estimate by L. Cranberg (LA-2177).
[k] See reference *1*. Includes results of R. C. Allen, R. B. Walton, R. B. Perkins, R. A. Olson, and R. F. Taschek, *Phys Rev*. **104**, 731 (1956) and R. C. Allen, *ibid.* **105**, 1796 (1957).
[l] See reference *3*.
[m] See reference *5*.
[n] See reference *4*.
[o] See reference *2*.

method. In this method resolution and multiple scattering difficulties also enter, but not as strongly as in other methods. In Table I are given the time-of-flight results of Cranberg and Levin (*5*).

When one examines the entries in Table I, one sees that they sharply contradict one another. The exact reasons for these contradictions are not known, but Cranberg has suggested in a new work (*9*) that they originate in the resolution and multiple scattering difficulties already mentioned.

In any case, we believe that the time-of-flight values are the most reliable. Particularly, we assume that the two time-of-flight values at 550 kev are essentially correct. For, firstly, the resolution at 550 kev was rather good, as can be seen from the published time spectra; and, secondly, the extent of multiple scattering was not large. For this latter Cranberg has tried to correct with the help of Monte Carlo calculations. (Furthermore, Cranberg *et al.* measured the angular distribution at 550 kev, whereas at 1 Mev they only measured the differential scattering cross section at 90°). These two 550 kev values will then be used in what follows to determine the other inelastic cross sections.

II. THEORETICAL BASIS OF THE CALCULATIONS

In the calculations we employ the well-known method of Hauser and Feshbach (*7*), augmented however, in the following respects:

1. The strength functions will not be determined from the optical model but will be taken as free parameters. (However, we will use the optical model as a guide in what follows.) In order to calculate the cross sections, we must consider neutrons of orbital angular momenta, l, up to $l = 4$. To each angular momentum belongs a strength function, so that we have, in all, five free parameters. These parameters can be determined for low energies as follows: For $l = 0$ one can obtain the strength function from the resonance data of Rosen (*10*). For the special case of U^{238} we can set the strength functions belonging to $l = 2$ and $l = 4$ equal to that belonging to $l = 0$.

For, the curve of the $l = 0$ strength function as a function of mass number has a minimum near $A = 238$ (*11*). In this case the strength functions belonging to partial waves of even l are in antiresonance, from which one can conclude, that the even l strength functions are all virtually equal (at least for not too large l). This particular problem has been dealt with by the author in somewhat greater detail in another work (*12*), and the conclusion successfully compared with the simple optical model of Feshbach *et al.* (*6*) in a special case.

Since the strength functions belonging to odd l are nearly in resonance at $A = 238$, they are not equal to one another. These two remaining parameters then are those which shall be determined from the experimental data.

2. The energy dependence of the strength functions is largely determined by two things: (*i*) by the "giant" resonances of a single nucleon (*6*, *13*), and (*ii*) by the interference of the many particle resonances of the compound nucleus at high energies. We approximate these two energy dependences in the following way:

For an ordinary central interaction between the nucleus and neutron the following equation holds (*14*):

$$T_l = \frac{4s_l/M_l}{[(N_l - \Delta_l)/M_l]^2 + [1 + (s_l/M_l)]^2} \quad (1)$$

where

T_l = the transmission coefficient of the nucleus for neutrons with angular momentum l;

N_l = the real part of the logarithmic derivative, $(1/\psi)(d/dr)(r\psi)$, of the radial wave function of the neutron on the nuclear surface;

$-M_l$ = the corresponding imaginary part; and

s_l and Δ_l = the penetration factor and level shift factor, respectively, defined by the following recursion relation:

$$s_0 = x, \qquad \Delta_0 = 0 \quad (2a)$$

$$s_l = \frac{x^2 s_{l-1}}{(l - \Delta_{l-1})^2 + s_{l-1}^2} \quad (2b)$$

$$\Delta_l = \frac{x^2(l - \Delta_{l-1})}{(l - \Delta_{l-1})^2 + s_{l-1}^2} - l \quad (2c)$$

where x is the nuclear radius, R, times the wave number, k, of the incident neutrons.

According to Blatt and Weisskopf (*14*) the dependence of $N_l - iM_l$ on energy is described by the function $-KR \tan [Z(E) + iq]$, where $Z(E)$ is a monotone increasing function of E, and KR and q are positive numbers. When q is small the transmission coefficients will exhibit characteristic giant resonance behavior. This happens near $Z(E) = n\pi$ at which point $N_l = 0$ and $M_l \approx qKR$, its minimum value. Actually, the peak value of the transmission coefficient lies virtually at the point at which $N_l = \Delta_l$, which is the familiar level shift phenomenon. When $Z(E) = (n + \frac{1}{2})\pi$, $N_l = 0$ and $M_l = KR/q$, its maximum value. These latter conditions characterize partial waves in antiresonance; for them it follows from (1) that

$$T_l = 4s_l/M_l \quad (3)$$

since in most cases of interest $M_l \gg s_l$, Δ_l at antiresonance. If we assume that M_l varies slowly with energy, (3) then implies that the transmission coefficients of partial waves which are not in resonance are proportional to s_l.

For orbital angular momenta which are in resonance, we neglect the expression $[(N_l - \Delta_l)/M_l]^2$ in (1) and obtain for the transmission coefficients

$$T_l = \frac{4s_l/M_l}{[1 + (s_l/M_l)]^2} \quad (4)$$

Again we assume that M_l varies only slightly with energy, although in this case this supposition cannot be very accurate.

In case the resonances of the compound nucleus are well separated, the equations

$$T_l = 2\pi\Gamma_{nl}/D \quad (5a)$$

$$\Gamma_{nl} = 2s_l\gamma_{nl}^2 \quad (5b)$$

also hold, where

Γ_{nl} = neutron width for neutrons of angular momentum l;

γ_{nl} = the corresponding reduced width; and

D = the average spacing between adjacent resonances.

With the help of these relations one can relate the ratio γ_{nl}^2/D, the so-called strength function, with the transmission coefficient as follows:

$$T_l = 4\pi s_l(\gamma_{nl}^2/D) \quad (6)^1$$

Equation (6) applies, however, only when $T_l \ll 1$, which is the condition for good separation of the compound nucleus' resonances. To fulfill this condition, let us consider very low bombarding energies, for which the s_l, and therefore T_l, are very small.

[1] It is worth noting here that in general D depends on J, the spin of the relevant levels of the compound nucleus. When there is only a pure central interaction between the neutron and nucleus, γ_{nl}^2 must have the same J dependence as D, and the ratio γ_{nl}^2/D be J independent.

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In this case from (6) and (3) or (4), respectively, follows

$$\left(\frac{\gamma_{nl}^2}{D}\right)_0 = \frac{1}{\pi M_l} \quad \text{(for resonance and antiresonance)} \quad (7)$$

where the index zero denotes the value of the strength function corresponding to zero energy. We now assume that Eq. (7) also holds for higher energy. Then, one can employ Eq. (6) also at high energy *if* one defines

$$\frac{\gamma_{nl}^2}{D} = (\gamma_{nl}^2/D)_0 \quad \text{(for antiresonance)} \quad (8a)$$

$$\frac{\gamma_{nl}^2}{D} = \frac{(\gamma_{nl}^2/D)_0}{[1 + \pi s_l(\gamma_{nl}^2/D)_0]^2} \quad \text{(for resonance)} \quad (8b)$$

Equation (8a) is partly a tautology since (3) and (6) only apply under the conditions that $T_l \ll 1$. In our calculations (8a) sufficed for those angular momenta which were in antiresonance, because for these T_l was never greater than 1. When, however, values of γ_{n1}^2/D and γ_{n3}^2/D were obtained from the experimental data at 550 kev, and from them values of T_1 and T_3 calculated with (8a), occasionally these latter values were greater than unity. This, of course, cannot be since it implies the production of neutrons. Thus (8b) must be employed at all energies for $l = 1$ and $l = 3$. This problem has been discussed in detail by Thomas (*8*), and has also been mentioned by Lane and Lynn (*15*).

3. In the formula of Hauser and Feshbach (7) expressions of the following form appear:

$$\frac{\pi}{k^2}\frac{2J+1}{2(2I+1)} \cdot \frac{T_l\,\epsilon(IlJ)\cdot T_{l'}\,\epsilon(I'l'J)}{\sum_{l''I''} T_{l''}\,\epsilon(I''l''J)} = \sigma_J(Il;I'l') \quad (9)$$

where I and I' are, respectively, the spins of the target and residual nuclei and $\epsilon(IlJ)$, which can only be equal to 0, 1, or 2, is the number of channel spins with which neutrons of orbital angular momentum l and a target or residual nucleus of spin I can combine to form a compound nucleus of spin J. The sum in (9) contains all possible modes of decay of the compound nucleus (with spin J). The expression (9) is actually one of the partial cross sections for a transition from an initial state with spin I to a final state of spin I' through intermediate compound states of spin J.

In the case that the resonances of the compound nucleus are well separated, (5a) holds. Then, however, $\sigma_J(Il;I'l')$ is proportional to

$$\frac{\Gamma_{nl}\,\epsilon(IlJ)\Gamma_{nl'}\epsilon(I'l'J)}{\sum_{l''I''}\Gamma_{nl''}\,\epsilon(I''l''J)} \quad (10)$$

According to Porter and Thomas (*16*), the Γ_{nl} are not constant, but rather are statistically distributed. They have determined this distribution and it is, as a matter of fact, a chi-squared distribution of one degree of freedom (*17*). Consequently we must average the ratio (10) over this distribution and employ this average in (9).

The author has dealt with this problem in another work (*12, 18*) as follows: Let us set

$$R = \left\langle\frac{\Gamma_a\,\Gamma_b}{\Gamma_a + \Gamma_b + \Gamma_c + \cdots}\right\rangle \cdot \frac{\langle\Gamma_a\rangle + \langle\Gamma_b\rangle + \langle\Gamma_c\rangle + \cdots}{\langle\Gamma_a\rangle\langle\Gamma_b\rangle} \quad (11)$$

In general, the calculation of R entails an N-fold integration where N is the number of partial widths involved. When the partial widths are distributed in member distributions of the chi-squared family, however, these N integrations can be reduced to just one. This single integration can be easily evaluated numerically.

In (9) the products $\Gamma_{nl}\epsilon(IlJ)$ appear in the following way: $\epsilon(IlJ)\Gamma_{nl}$ is the sum of the Γ_{nl} for all the different channel spins which are consistent with I, l, and J. Now the widths are not, by supposition, dependent on channel spin. In this case, when they are constant for all compound levels of the same J, we interpret the sum simply as a number equal to $\epsilon(IlJ)\Gamma_{nl}$. If the widths, however, are statistical variables we must interpret this sum as a statistical variable with the average $\epsilon(IlJ)\langle\Gamma_{nl}\rangle$ which is distributed in a chi-squared distribution of $\epsilon(IlJ)$ degrees of freedom. A slightly more detailed discussion of this point can be found in references (*12*) and (*18*).

It is quite necessary to apply the correction represented by (11), since it can occasionally influence the terms (9) by as much as a factor of 2 (*12, 18*). In case Γ_{nl} is not $\ll D$, (8a) no longer holds, but rather (8b). When the second term in the denominator in (8b) is not too large, we can neglect its statistical variation and retain the correction factor (11).

4. In the calculation of the inelastic cross section the J and energy dependence of D plays no role (*12*). Since, however, these dependences are important in the calculation of the radiative capture cross section, we assume, following Lang and Le Couteur (*19*),

that D_J is proportional to $(2J + 1)^{-1}$ and that it varies with energy according to the formula published by these authors. Finally we assume that Γ_γ, the radiative width, is energy-independent and is the same for all resonances.

5. For this calculation one must know the spin and parity of each level which is excited. In U^{238}, besides the ground state, there are four excited states known below 1 Mev, lying, respectively, at 44, 146, 300, and ~ 730 kev (*1–5*). For even-even nuclei the ground state is 0+; the first excited state at 44 kev has been found by Coulomb excitation and is clearly determined as a 2+ level (*20*). The energies of the first three excited states are nearly in the ratio 3:10:21, which characterizes a rotational band (*21*). Therefore, we can be relatively sure that the spins and parities of these levels are, respectively, 2+, 4+, and 6+.

For the level at 730 kev, there are no such simple considerations available with which one can determine its spin and parity. Nevertheless the possibilities are few in number. For nuclei, two possible kinds of levels are known, ordinary "nucleonic" levels and collective levels. For heavy, even-even nuclei only spins and parities of 0+ or 2+ have been observed for the first excited "nucleonic" level (*22*). The most probable collective motions are either the β or γ

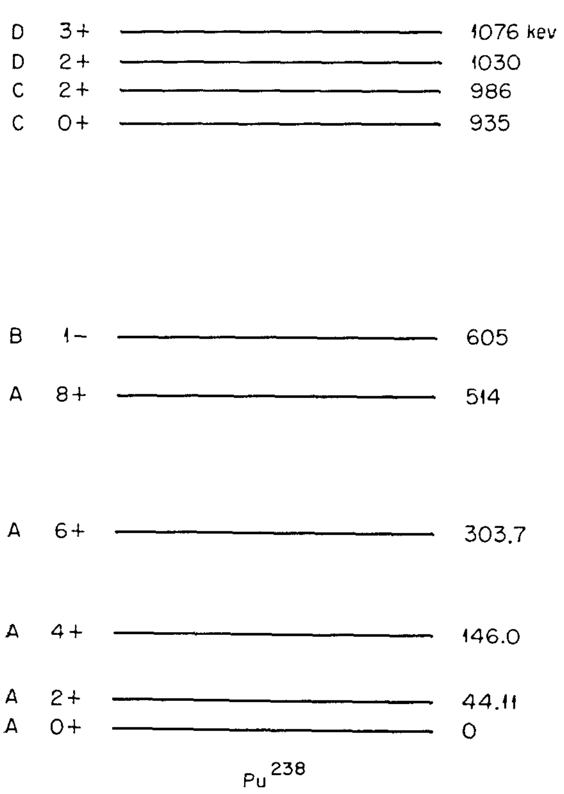

FIG. 1 The level scheme of Pu^{238} up to about 1 Mev

quadrupole vibrations of Bohr and Mottelson or octupole vibrations (*22, 23*). The first two tions imply the respective choices 0+ or 2+ for spin and parity. In the case of octupole vibrat one obtains levels of negative parity, of which levels have already been observed at low excita energies in heavy even-even nuclei (*23*). Thus expect a spin and parity for the state at 730 k most probably of either 0+, 2+ or 1−.

We can test these ideas on the well-known lev scheme of Pu^{238} (*24*), which is given in Fig. 1. The the determination of spins and parities is relatively sure. There are four groups of different levels. The first, marked A, is the ground-state rotational band. The second, marked B, consists of a single 1− level. The third, marked C, is a 0+ level, upon which a further rotational level is built. Presumably this 0+ level is a β vibration. The fourth group, marked *D*, is a 2+ level, also upon which a further rotational level is built. This 2+ level is presumably a γ vibration.

If these levels are really collective, then one expects that two nuclei which differ only in the exchange of a pair of protons for a pair of neutrons will have quite similar level schemes. With this presupposition we can compare the level schemes of U^{238} and Pu^{238}. From this comparison it follows that the level at 730 kev is probably a 1− level, while the group of poorly resolved levels in U^{238} which lie at about 1 Mev probably are β and γ vibrations.

Upon the 1− level a rotational band can be built. Since this level is one of $K = 0$ [in the sense of the collective model (*21*)], the next level of the rotational band is a 3− level. If the moment of inertia is the same for this band as for the ground-state band, then the 3− level must lie at 805 kev. The next level of this band, the 5− level, can be neglected.[2] In our calculations both the 1− and 3− levels were included although experimentally only one level was observed. (It is clear that levels at 730 and 800 kev would quite probably not be resolved in a sphere transmission experiment with 1 Mev neutrons. This explanation is also a possibility in the time-of-flight method, in which no level at 800 kev was observed.)

Calculations have also been performed in which the two levels at 730 and 805 kev were chosen to have spins and parities of 0+ and 2+, respectively, thus characterizing a β vibration. In this case the 2+ level does not lie at 805 kev, but rather at 775 kev, this shift, however, introduces only a small perturbation. One also expects here a 4+ level at 880 kev,

[2] This neglect is discussed further in the next section

which has been neglected.[2] Further calculations have been performed in which the levels at 730 and 805 kev were taken as 2+ and 3+ levels, respectively, thus characterizing a γ vibration. Again the 805-kev level really appears at 775 kev, while a further 4+ level is expected at 835 kev. This last level is again neglected.[2]

III. DESCRIPTION OF THE CALCULATIONS AND COMPARISON WITH EXPERIMENT

The author has already published a calculation done by the above described method at 550 kev (*12*, *18*), the purpose of which was the determination of the strength functions for $l = 1$ and $l = 3$ from the experimental results given by Cranberg at this energy. The author used for $l = 0$ and $l = 2$ ($l = 4$ is negligible at 550 kev) the value of the strength function $\gamma_{n0}^2/D = 0.036$. This value came from the data published in BNL–325 (*25*). The results of these earlier calculations were

$$\frac{\gamma_{n1}^2}{D} = 0.040; \qquad \frac{\gamma_{n3}^2}{D} = 0.120 \qquad \text{at 550 kev} \quad (12)$$

Not long ago Rosen (*10*) measured a new value of the $l = 0$ strength function, namely, $\gamma_{n0}^2/D = 0.025$, and as a result the values for $l = 1$ and $l = 3$ have changed. Fortunately, it is not necessary to repeat the earlier calculations. For, at 550 kev all the levels which can be excited have the same parity. Therefore, the sum which gives the cross section splits into two groups of terms, all having the same form as in (9), but where, however, one group contains only even l-values while the other group contains only odd l-values. Since we have taken γ_{n0}^2/D equal to γ_{2n}^2/D the terms of the first group are directly proportional to γ_{n0}^2/D. (This result is quite independent of the statistical considerations previously mentioned). Therefore, we can correct these terms for the changed value of the $l = 0$ strength function. The treatment of the second group if terms, which refer to odd l-values, is similar but somewhat more complicated and is further discussed in reference 12. With the help of the assumption

$$\frac{\gamma_{n0}^2}{D} = \frac{\gamma_{n2}^2}{D} = \frac{\gamma_{n4}^2}{D} = 0.025 \qquad (13)$$

we obtain finally

$$\frac{\gamma_{n1}^2}{D} = 0.050; \qquad \frac{\gamma_{n3}^2}{D} = 0.135 \qquad \text{at 550 kev} \quad (14)$$

In addition we have taken $\Gamma_\gamma = 0.025$ ev and $D_{J=1/2}(E = 0) = 18$ ev (*10*).

Table II gives the results of the new calculations at 1 Mev, as well as the values from reference 12, and the corresponding calculated radiative capture cross sections. It is interesting to note that the excitation cross section of the first three levels at 1 Mev is nearly independent of the choice of spins and parities of the upper levels, so that we cannot distinguish between the three possibilities considered on this ground at all. In distinction to the first measured partial excitation cross section, which represents a lower limit, the other measured partial cross sections at this energy can be directly compared with the calculated values. The agreement is quite good. Against 600 ± 190 mb measured for the excitation cross section of the 146 kev level we obtain in the three cases considered, respectively, 720, 725, and 660 mb. Against 90 ± 50 mb measured for the 300 kev level we obtain, respectively, 80, 80, and 70 mb. The three calculated excitation cross sections of the 44 kev level at 1 Mev are, respectively, 1380, 1390, and 1320 mb, although the measured value amounts to only 900 mb. We see in this difference the influence of insufficient resolution of the elastically scattered neutrons from those which excite the 44 kev state. We consider the calculated value more reliable than the measured one in this case, and in any case, consider our calculated values more trustworthy than the published experimental values in Table I due to authors other than Cranberg *et al.*

TABLE II

CALCULATED NEUTRON EXCITATION CROSS SECTIONS AND RADIATIVE CAPTURE CROSS SECTIONS

E_{kev}	$\sigma(44)_{mb}$	$\sigma(146)_{mb}$	$\sigma(300)_{mb}$	$\sigma(730)_{mb}$	$\sigma(805)_{mb}$	$\sigma_{tot\ mb}$	$\sigma_{n\gamma mb}$
150	810	—	—	—	—	810	180
550	1430[a]	290[a]	—	—	—	1720	140
1000	1380	720	80	395 (1^-)[b]	245 (3^-)	2820	140
1000	1390	725	80	190 (0^+)	400 (2^+)	2785	150
1000	1320	660	70	480 (2^+)	290 (3^+)	2820	140

[a] Normalization.

[b] Choice of spins and parities.

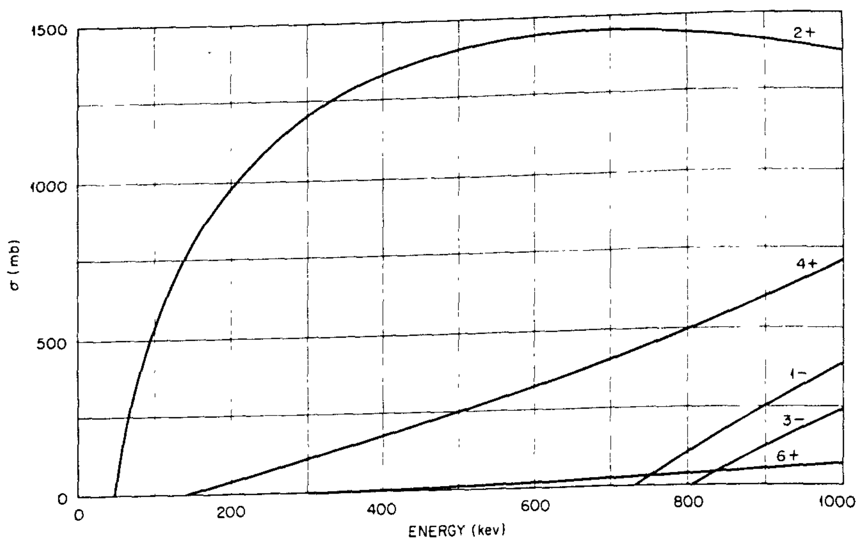

FIG. 2. The calculated partial neutron excitation cross sections assuming the level at 730 kev is a 1− level

Finally, the calculated capture cross sections agree well with the data published in BNL-325 (*25*).

Against 520 ± 80 mb which presumably represents the sum of the excitation cross sections of the remaining levels at 730 kev and above, the calculations give, respectively, 640, 690, and 895 mb. The last two values contain estimated contributions to the sum from the hitherto neglected 4+ states of 100 and 125 mb, respectively. The corresponding contribution to the first value from the 5− level has been estimated to be of the order of magnitude of 10 mb, in the case where this level lies as low as 835 kev.[3] In the third case the calculated value is certainly too large. Therefore, we discard the choice 2+ for the spin and parity of the 730 kev state. Between the other two possibilities we still cannot decide.

The accuracy of the calculated values of the excitation cross sections depends strongly on three different things. First, we have employed approximate values of the strength functions and transmission coefficients. We have, however, chosen these quantities so that the calculations necessarily give the experimental values at 550 kev, so that only small errors are expected from this source. Secondly, we have presupposed the spin-independence of the strength function, although it is known that the interaction between neutrons and nuclei is spin dependent. Again, however, we do not expect large errors from this for the same reason as before. Thirdly, there is the possibility that some of the levels may be excited by direct interaction of the neutrons with the nonspherical surface of the nucleus.

This direct excitation was first treated by Brink (*26*) using first-order perturbation theory. In first order if the ground state is 0+, only 2+ levels can be excited through quadrupole deformation. The author has carried out this perturbation calculation (*12*) for the 44 kev level at 550 kev and finds a direct excitation cross section of about 100 mb. Chase *et al.* (*27*) have performed this calculation much more accurately without the use of perturbation theory and find essentially the same result. With octupole deformation only 3− levels can be excited in first order. The excitation cross section depends strongly on the magnitude of the deformation; we presume it is quite small in the case of octupole deformation. For these reasons we have neglected the direct excitation process.

In Fig. 2 are plotted the excitation cross sections for which the spin and parity of the 730 kev state have been chosen as 1−. In the case of the cross sections for the 44- and 146-kev levels we have sufficiently many points to draw a relatively good interpolation curve. In the case of the 300-kev level we have only two points, which we have joined by a reasonable curve. This curve is, of course, not very reliable, but the cross section is quite small, and the errors in it probably unimportant.

In the case of the 730-kev level (which is at the moment being considered as 1−) we have proceeded as follows: at 1 Mev about 90% of the cross section is contributed by those channels in which the emitted neutrons have $l = 0$ or 1. At lower energy this contribution is still larger. Therefore, we

[3] Moszkowski (*22*) has noticed that in Ra^{226} in the case of a rotational band of negative parity, the moment of inertia is about twice as large as in the case of positive parity. If this holds for U^{238} also, then the 5− state really appears at 835 kev.

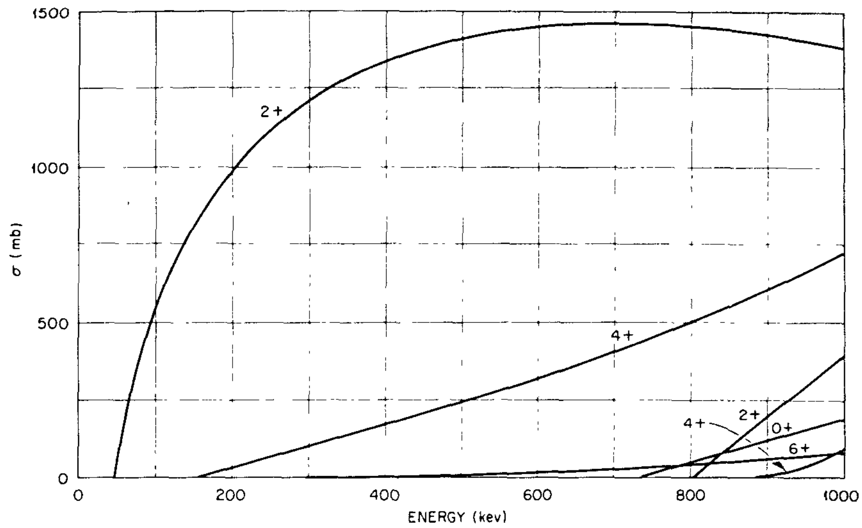

FIG. 3. The calculated partial neutron excitation cross sections assuming the level at 730 kev is a 0 + level

can approximate the energy dependence of the excitation cross section for the 730-kev level as

$$\sigma(E) = 114\left[\frac{s_0(x'')}{s_0(x')}\right] + 242\left[\frac{s_1(x'')}{s_1(x')}\right] \quad (15)$$

where x'' = the value of x corresponding to an energy $E - 730$ kev, x' = the vaue of x corresponding to an energy of 270 kev, and the numbers 114 and 242 are the respective contributions of the $l = 0$ and $l = 1$ exit channels. At 1 Mev we have employed the value from Table II [some 10% higher than the value given by (15)] and drawn a smooth curve through this point which becomes tangent to (15) at lower energies. In the case of the 805-kev level, which in this discussion is a 3− level, we have employed the same method. In both cases one obtains a nearly linear increase in the cross section. In Fig. 3, are plotted excitation cross sections obtained in the same way as above for the case that the spin and parity of the 730-kev level is 0+.

The calculated values are considered by the author to be more reliable than any measurements save those by Cranberg *et al.* For the excitation cross section of the level at 44 kev the calculated values are still, however, considered the more trustworthy. For the levels at 146 and 300 kev the calculated values and the measured ones agree within experimental error, so that a decision between them has little sense. For the levels between 730 kev and 1 Mev, the calculated values are just outside the quoted experimental error, so that a slight reduction may be applied to the calculated cross sections. In any case both the variation of these cross sections with energy, and the calculated ratio of excitation cross sections for the several states considered, is felt to be reliable.

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