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KFK-153

FORMAL REPORT

KFK-153
EURFNR -67F

UNITED STATES – EURATOM FAST REACTOR EXCHANGE PROGRAM

Nicht entleihbar

Original report number KFK-153

Title THEORY OF DELAYED SUPERCRITICAL EXCURSIONS TO DETERMININE DOPPLER COEFFICIENTS OF FAST REACTORS.

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Date of original report issuance June 1963

Reporting period covered

THIS REPORT MAY BE GIVEN UNLIMITED DISTRIBUTION

USAEC Division of Technical Information Extension, Oak Ridge, Tennessee

KERNFORSCHUNGSZENTRUM KARLSRUHE

Juni 1963 KFK 153

Institut für Neutronenphysik und Reaktortechnik

Theory of Delayed Supercritical Excursions to Determine Doppler-Coefficients of Fast Reactors

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by

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Abstract:

The determination of the prompt power coefficient of a fast reactor by means of special delayed superprompt critical excursions has been investigated. If the reactivity is inserted as a fast ramp limited below prompt critical, the changes of the neutron flux after the prompt jump result from the competition of the increase of feed-back and delayed neutrons. The height of the prompt jump and the "slope" of the flux after the prompt jump permits to determine the prompt power coefficient. In the special case in which the competition is complete, the prompt power coefficient is directly connected to the height of the prompt jump. The accuracy of the method is discussed extensively and improvements of the accuracy are investigated. The achievable high accuracy can be used to test finer details of the Doppler coefficient theory as e.g. the dependence on temperature and temperature profile and on different fuel compositions in fairly small zones.

I. Introduction

The very short generation time of the neutrons in fast reactors is an advantage, in limiting power excursions, if there is an inherent prompt negative temperature coefficient of a sufficient magnitude. The inherent prompt temperature coefficient in ceramic fueled reactors is mainly caused by Doppler-broadened resonances. The calculation of its reactivity effect can be done only with a very limited accuracy because the nuclear resonance data especially for the fissile material are fairly uncertain in the main range of interest in fast reactors. A sufficient improvement of these nuclear data from the theoretical or experimental side is a long term undertaking. Therefore an extensive direct measurement of the Doppler coefficient and its dependency on fuel composition and temperature is improtant for the design of fast reactors.

The "measurement" needs theoretical relations between the temperature coefficient (Doppler-coefficient) and directly measureable quantities. In this paper the theory of special delayed supercritical excursions has been investigated, to find such relations. The very short generation- (or life)-time of the neutrons allows to simplify the mathematical treatment so much, that a simple relation connects the Doppler-coefficient to a special flux value (equ.(21)). The accuracy of this measurement can be increased, if the measured flux "shape" during the excursion is used (equ.(23)).

II. Simplified mathematical treatment

The time dependent neutron flux during a power excursion is to be calculated from the reactor dynamic differential equation

(1) $$l\,\dot{\phi}(t) = \beta\left[\Delta k(t) - 1 - R(t)\right]\phi(t) + Q(t)$$

where

l is the mean neutron life-time,

$\phi(t)$ is proportional to the space and energy integrated neutron flux and in the following measured in units of 1 MW;

$\Delta k(t)$ is the excess multiplication factor in units of β, describing a fast insertion of $\Delta k < 1$ (β is the fraction of delayed neutrons);

Q(t) is proportional to the source of delayed neutrons [the constant of proportionality $\frac{1}{\bar{\Sigma}_a}$ is not relevant for the following consideration ($\bar{\Sigma}_a$ is the mean macroscopic absorption cross section)]

R(t) describes the prompt reactivity feed-back.

Neglecting the heat transfer to the coolant in a first short time interval after the beginning of the excursion, the feed-back term splits up in the prompt power coefficient (γ in $\frac{\beta}{MWs}$) and the integrated additional power (in MWs):

(2) $$R(t) = \gamma \int_0^t \left[\phi(t') - \phi_o\right] dt'$$

which is used in the following. ϕ_o denotes the initial power (at t=o).

Instead of (2) an expression containing an exponential function describing the heat transfer could also be used.

The neutron flux in excursions following a fast Δk-insertion limited below β increases very rapidly to realise the new source multiplication. This "prompt jump" occurs with a time constant

(3) $$t^{*} = \frac{l}{\beta(1 - \Delta k)} \quad (= 10^{-3}\text{sec for } \frac{l}{\beta} = 10^{-4} \text{ sec and } \Delta k = 0{,}9)$$

After the prompt jump the delayed neutron source Q(t) as well as the reactivity feed-back increase, caused by the increased neutron flux. The time constants of the changes of Q and R are of the order of 1 second and the time constant of the further neutron flux alteration must be of the same order of magnitude. Therefore $\dot{\emptyset}$ is small so that $\dot{\emptyset}$ times l, the very short neutron life-time, can be dropped, and the competition of the Q and R increase determines the further flux shape. In this approximation the prompt jump is contracted in a step. The flux after the prompt jump is the solution of the following system of algebraic and differential equations:

$$(4) \qquad 0 = \beta\left[\Delta k - 1 - r(\tau)\right] \varphi(\tau) + q(\tau)$$

with

$$(5) \qquad q(\tau) = \sum_i \lambda_i c_i(\tau)$$

and

$$(6) \qquad \dot{c}_i(\tau) = -\lambda_i c_i(\tau) + \beta_i \varphi(\tau)$$

where τ denotes the time after the (suitable defined) end of the prompt jump. φ, q, r and c_i are the "approximative" quantities for $\emptyset$, Q, R and C_i. c_i, λ_i and β_i are the concentrations, decay-constants and the contributions of the delayed neutron precursors.

Because $\varphi(\tau)$ is a slowly varying function we solve the system (4) to (6) in terms of a Taylor expansion:

$$(7) \qquad \varphi(\tau) = \sum_{v=0}^{\infty} \varphi_v \tau^v$$

$$(8) \qquad c_i(\tau) = \sum_{v=0}^{\infty} c_{iv} \tau^v$$

$$(9) \qquad q(\tau) = \sum_{v=0}^{\infty} q_v \tau^v$$

$$(10) \qquad r(\tau) = \sum_{v=1}^{\infty} r_v \tau^v$$

Insertion of (7) and (8) into (6) leads to a recursion-formula for the calculation of the expansion-coefficients of the concentrations of the delayed neutron precurcors:

(11) $c_{iv} = \frac{1}{v} (\beta_i \varphi_{v-1} - \lambda_i c_{iv-1})$; $v = 1,2,3 \ldots$

with

(12) $c_{io} = \frac{\beta_i}{\lambda_i} \phi_o$

ϕ_o denotes the constant neutron flux before the excursion, whereas $\varphi(o)=\varphi_o$ in equation (7) means the neutron flux after the prompt jump.
The calculation of the first few coefficients c_{iv} yields after insertion into equation (5) the expansion-coefficients of the delayed neutron source:

(13.0) $q_o = \beta \phi_o$

(13.1) $q_1 = \beta(\varphi_o - \phi_o) \overline{\lambda}$

(13.2) $q_2 = \frac{\beta}{2} (\varphi_1 \overline{\lambda} - \varphi_o \Delta k \overline{\lambda^2})$

where

(14) $\overline{\lambda^m} = \frac{1}{\beta} \sum_i \beta_i \lambda_i^m$

Equation (2) connects the expansion-coefficients of the reactivity feedback and the neutron flux:

(15) $r(\tau) = \gamma(\varphi_o - \phi_o)\tau + \gamma \sum_{v=1}^{\infty} \frac{\tau^{v+1}}{v+1} \varphi_v$

resulting

(16.1) $r_1 = \gamma(\varphi_o - \phi_o)$

(16.2) $r_v = \gamma \frac{\varphi_{v-1}}{v}$; $v = 2,3 \ldots$

r_2 and higher terms would be influenced if we would include heat transfer to the coolant.
After expressing all the coefficients in terms of the coefficients φ_v insertion into the "reduced" dynamic equation allows the calculation of the flux coefficients:

$$ß\,(1-\Delta k + r_1\tau + \ldots)\,(\varphi_o + \varphi_1\tau + \ldots) = q_o + q_1\tau + \ldots$$

(17.o) $$\varphi_o = \frac{q_o}{ß(1-\Delta k)} \text{ (prompt jump)}$$

(17.1) $$\varphi_1 = \frac{\varphi_o - \phi_o}{1-\Delta k}(\bar{\lambda} - \gamma\varphi_o)$$

(17.2) $$\varphi_2 = \frac{1}{1-\Delta k}\left[\frac{\varphi_1}{2}(\bar{\lambda} - \gamma\varphi_o) - \varphi_o\Delta k\left(\gamma\varphi_1 + \frac{1}{2}\overline{\lambda^2}\right)\right]$$

(17.o) connects the prompt jump with Δk. Together with (13.o) results

(18) $$\varphi_o = \frac{\phi_o}{1-\Delta k}$$

(17.1) shows that the flux rise after the prompt jump consists of a positive term caused by the increasing delayed neutron source which is proportional to $\varphi_o - \phi_o$ and a negative feed-back part proportional to $(\varphi_o - \phi_o)\cdot\varphi_o$. The choice of φ_o permits to change the degree of compensation. φ_o and φ_1 can be determinded by flux measurement during the excursion. Then (17.1) gives the prompt power coefficient as

(19) $$\gamma = \frac{\bar{\lambda}}{\varphi_o} - \frac{\varphi_1\ \phi_o}{\varphi_o^2(\varphi_o - \phi_o)}$$

If one realizes experimentally the excursion for which

(20) $$\varphi_1 = 0$$

(this special φ_o we denote by φ_o^*), then (19) simplifies to

(21) $$\gamma = \frac{\bar{\lambda}}{\varphi_o^*}$$

(Note: it is not necessary to know Δk!)

Example:

$$\bar{\lambda} = 0.40 \text{ sec}^{-1} \text{ for } U^{235}$$

$$\varphi_o^{*} = 200 \text{ MW (e.q.)}$$

then

$$\gamma = \frac{0.40 \text{ sec}^{-1}}{200 \text{ MW}} = 2.0 \cdot 10^{-3} \frac{\$}{\text{MWs}}$$

Using power distribution and the heat capacity of the fuel one can evaluate the Doppler-coefficient in $\frac{\$}{^{o}C}$.

III. Discussion of the accuracy.

As a more detailed example we can use the calculated excursion shapes plotted in figure 1. Each curve belongs to a step excursion with Δk-value (in $) as designed on the right.

The parameters used in calculating figs. 1 to 4:

$\phi_o = 10$ MW

$\frac{\beta}{l} = 10^{4} \text{sec}^{-1}$ (a reasonable value for a fast reactor with a ceramic fuel)

The $\frac{\beta_i}{\beta}$ - and λ_i-values are reasonable values for U^{235} [1].

Tabelle 1

i	$\frac{\beta_i}{\beta}$	λ_i
1	0.033	0.012
2	0.2191	0.030
3	0.1959	0.111
4	0.3951	0.303
5	0.1152	1.136
6	0.0417	3.013

In figure 1 are plotted flux shapes of such excursions for which the alteration of the flux after the prompt jump is small. As can be seen from figure 1 the flux of the .943 excursion clearly increases and that of the .947 excursion clearly decreases after the prompt jump. For the .945 excursion the first order alteration of the flux seems to vanish. This flux

$$\varphi_o^* = 18{,}22\ \phi_o = 182{,}2\ \text{MW}$$

which gives $$\gamma = \frac{0.40}{182.2}\ \frac{\$}{\text{MWs}} = 2.2\ \ .\ 10^{-3}\ \frac{\$}{\text{MWs}}$$

The error limits are smaller than 18,2 $\pm$ 0.35 that means smaller than 2%.

The inaccuracy coming from the flux measurement relative to ϕ_o should be smaller at these flux levels.

The fact that in practice a reactivity step can be realized only as a limited fast ramp has only a small influence on the results, because the flux shape shortly after the prompt jump is nearly the same in both (step-and limited ramp) cases (the main flux rise comes after the reactivity input is finished) (comp. figure 2). The temperature dependence of the Doppler-coefficient is neglected above. As can be seen from figure 1 the described measurement needs a time of about 50 msec after the prompt jump. The temperature increases of the fuel at the power φ_o^* in this time interval is of the order of magnitude of 10°C. Therefore the method described above measures the Doppler-coefficient at a temperature T in a small interval ΔT ($\approx$10°C). A roughly known temperature dependence can be taken into account as a perturbation to improve the accuracy of the measurement.
As mentioned above the transfer of the "additional" heat to the coolant can easily be taken into account, equs.(19) and (21), however, remain unchanged. Another possible source of error comes from higher flux modes. If e.g. the limited ramp Δk is produced by ejecting an absorbing rod, the higher modes describing the flux depression near the initially inserted rod will decrease during the excursion. Their influence, however, should be small if $\Delta k < 1$ \$ and if Δk = constant during this part of the excursion which is used for the measurement.

An inherent uncertainty, however, is that of $\bar{\lambda}$, which is of the order of magnitude of some percent.

The discussion of the different sources of errors shows that there are two inherent ones:

1. that of the method itself (neglecting the structure of the prompt jump)
2. the uncertainty of $\bar{\lambda}$.

In the following chapter improvements of these two uncertainties are investigated.

IV. Improvement of the accuracy.

In the method described in section II the derivative term $\dot{\emptyset}1$ is neglected which has two consequences:

1. The prompt jump is contracted to a flux step, which has according to the smallness of the mean neutron life-time in itself a very small influence on the result.
2. During this "contracted" prompt jump the increase of feed-back and of the delayed neutron source is neglected.

This second consequence affects slightly the height of the prompt jump and gives small corrections to the formula for the "tangent" after the prompt jump derived in section II. This correction term can be calculated from the "shape" of the prompt jump which must be limited for this correction to suitable time after the beginning of the excursion.

In a first order approximation the shape of the prompt jump can be calculated using the initial values of feed-back and delayed neutron source:

(22) $R(t) = 0$ and $Q(t) = Q_o$, during the prompt jump.

$\Delta k(t)$ can be simplified as a limited ramp. From this flux shape the feed-back and the increase of $Q(t)$ "during" the prompt jump can be estimated and be used in calculating the correction terms mentioned above.

The order of magnitude of this corrections on the height of the prompt jump can be seen from figure 1, where the respective correct values are indicated on the ordinate. A further improvement of this correction term is unnecessary but it could be reached using a measured correct shape of the prompt jump instead of the first order approximation.

But if one wants to use a measured flux shape at all it is better to turn to a method in which the dynamic equation is inversed still more directly than in the method described in sect.II. The connection of these two methods is discussed below.

By inversion of the differential equation (1) we define a function

$\tilde{\gamma}(t,\Delta k)$, where k is a constant parameter:

(23) $$\tilde{\gamma}(t,\Delta k) = \frac{Q(t)-\beta(1-\Delta k)\phi(t) - l\dot{\phi}}{\beta\phi(t)\int_0^t (\phi(t')-\phi_o)dt'} \quad ^{x)}$$

with

(24) $$Q(t) = \sum_i \lambda_i C_i(t) = \sum_i \lambda_i \, e^{-\lambda_i t}\left(C_i(o) + \beta_i \int_0^t e^{\lambda_i t'} \phi(t')dt'\right)$$

λ_i and β_i in (24) are known quantities. We do not want to consider effects coming from uncertainties in λ_i and β_i for the moment. Then $\tilde{\gamma}$ $(t,\Delta k)$ can be calculated using the measured flux $\phi(t)$. It turns out that $\tilde{\gamma}$ is strongly time-dependent as long as for the "parameter" Δk is not inserted very precisely that value which has produced the excursion shape $\phi(t)$. For the "correct" value of Δk the function $\tilde{\gamma}$ is constant and equals the prompt power coefficient:

(25) $$\tilde{\gamma}(t,\Delta k_{correct}) = \gamma = \text{const.}$$

for $t > \bar{t}$, where $\bar{t}$ is the time at which the total reactivity is inserted.

Figure 3 and 4 show the sensitivity to the choice of Δk. An error in Δk of 10^{-3} ¢ ($\approx 3 \cdot 10^{-6}$) produces a strong time dependency. In figure 3 $\tilde{\gamma}$ is plotted for step excursions and in figure 4 for limited ramp (5¢/sec) excursions. This method therefore simultaneously gives γ and Δk very accurately

V. The influence of the uncertainties of λ_i and β_i.

The accuracy of the methods described in section II and IV is mainly limited by the uncertainty of $\bar{\lambda}$, the mean decay constant of the delayed neutron

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x) (23) gets indentical with (21), if $\phi(t) = \varphi_o^x$ = constant precursors. In (21) γ is directly proportional to $\bar{\lambda}$. In equation (23) the dependency on the delayed neutron constants is more complicated. Therefore we rewrite (23) to show that in a first order approximation the main uncertainty comes from $\bar{\lambda}$ also:

$$(26) \quad Q(t) = \beta \left\{ \phi_o + \sum_i \lambda_i b_i \int_0^t (\phi(t') - \phi_o)\, e^{-\lambda_i (t-t')} dt' \right\}$$

where (12) is used and b_i is

$$(27) \quad b_i = \frac{\beta_i}{\beta}$$

Because we are interested in the behavior in a small time interval after the beginning of the excursion, we can expand the exponential function in (26) and get

$$(28) \quad Q(t) = \beta \left\{ \phi_o + \bar{\lambda}\, \Delta\phi^o - \overline{\lambda^2} \Delta\phi^1 + - \ldots \right\}$$

with

$$(29) \quad \Delta\phi^n = \int_0^t (\phi(t') - \phi_o)(t-t')^n dt'$$

To find out the influence of the inaccuracy of the $\overline{\lambda^m}$ we split

$$(30) \quad \overline{\lambda^m} = \overline{\lambda^m_c} + \delta\overline{\lambda^m},$$

where $\overline{\lambda^m_c}$ designs the unknown "correct" values of decay constant moments. Then Q(t) splits into a correct part and into an error part:

$$(31) \quad Q(t) = Q_c(t) + \beta\, (\delta\bar{\lambda}\Delta\phi^o - \delta\overline{\lambda^2}\Delta\phi^1 + - \ldots .)$$

Insertion into (23) gives

$$(32) \quad \hat{\gamma}(t,\Delta k) = \frac{\frac{1}{\beta} Q_c(t) - (1-\Delta k)\phi - \frac{1}{\beta}\dot{\phi}}{\phi\,(t)\Delta\phi^o} + \frac{\delta\bar{\lambda}}{\phi} - \delta\overline{\lambda^2}\, \frac{\Delta\phi^1}{\phi\,\Delta\phi^o} + - \ldots$$

The first term in (32) equals the "correct" function $\tilde{\gamma}_c\ (t, \Delta k)$, which is constant for the "correct" value of Δk. The main error comes from the term $\delta\bar{\lambda}/\phi$ which gets evidently smaller for larger values of ϕ. The higher terms in (32) are small for a short time interval after the beginning of the excursion. The formula

$$(33) \quad \tilde{\gamma}(t, \Delta k_{corr.}) = \gamma + \frac{\delta\bar{\lambda}}{\phi}$$

permits to eliminate the error $\delta\bar{\lambda}$ if one uses quite different values of ϕ.

VI. Conclusions.

The methods described in this paper permit a very accurate determination of the Doppler-coefficient in a fast reactor. The temperature rise during the excursions considered is of the order of magnitude of 10^{6}C. The temperature at the beginning of this interval can be chosen freely in wide limits by varying initial power and coolant flow. Therefore the temperature dependency of the Doppler-coefficient can be determinded in this way.

The high accuracy of the methods may permit to determine the Doppler-coefficient of a big enough central zone charged with different fuel compositions.

Acknowledgements:

It is a pleasure to acknowledge the calculations of the flux shapes for several excursions performed by Dr. A. Fraude and D.Math. W. Höbel. Thus the feasibility of the method described could be tested extensively.

I,10

Fig. 1

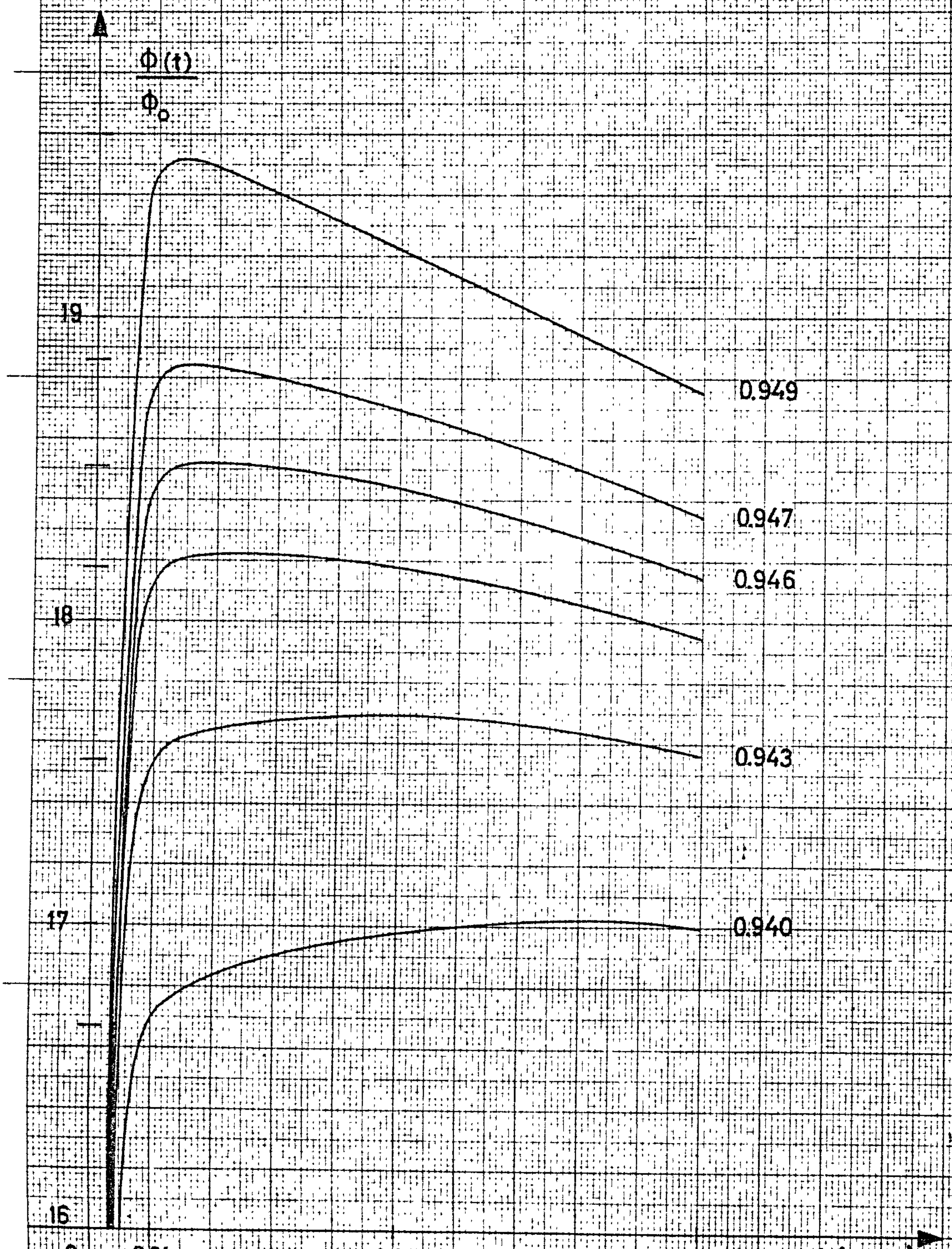


Fig. 3

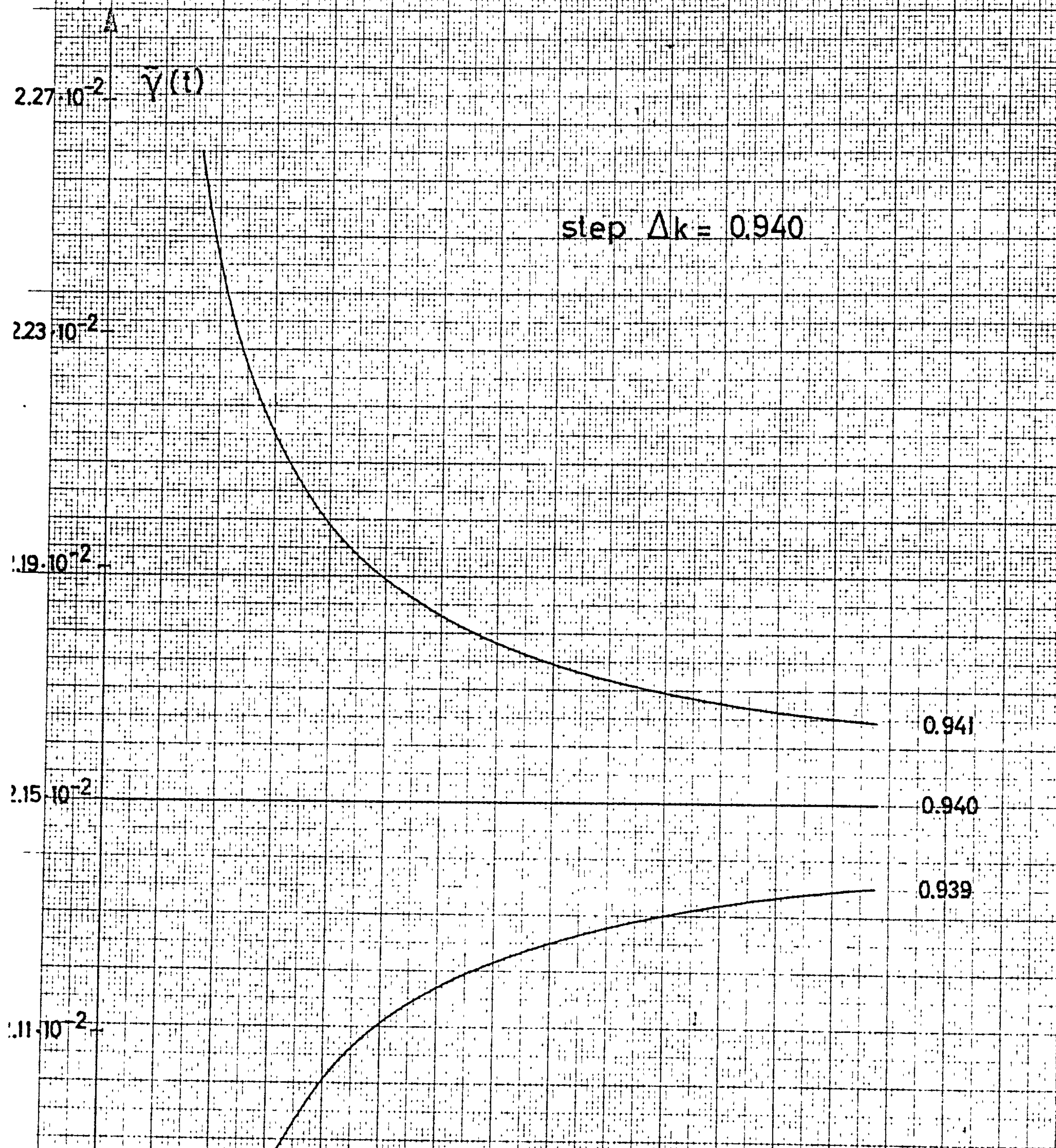
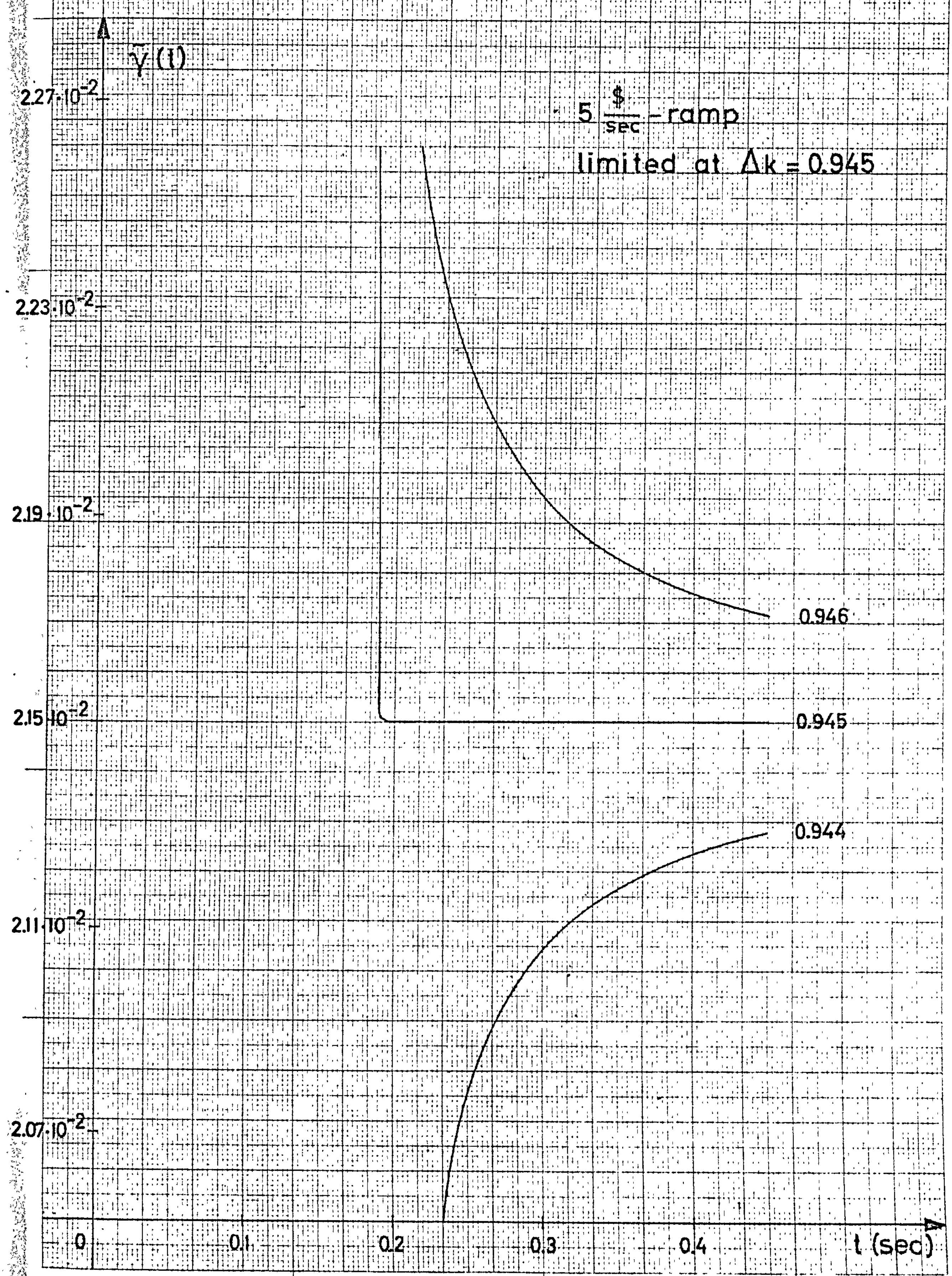

$\tilde{\gamma}(t)$
2.27·10⁻²
2.23·10⁻²
2.19·10⁻²
2.15·10⁻²
2.11·10⁻²
2.07·10⁻²
0
step Δk = 0.940
0.941
0.940
0.939
0.1
0.2
0.3
0.4
0.5 t(sec)

Fig. 4



$\bar{\gamma}(1)$
$2.27 \cdot 10^{-2}$
$2.23 \cdot 10^{-2}$
$2.19 \cdot 10^{-2}$
$2.15 \cdot 10^{-2}$
$2.11 \cdot 10^{-2}$
$2.07 \cdot 10^{-2}$
$5 \frac{\$}{sec}$ - ramp
limited at $\Delta k = 0.945$
0.946
0.945
0.944
0
0.1
0.2
0.3
0.4
t (sec)