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Experimental Determination of the Photofraction of a Cylindrical 4″ x 6″ Na I (Tl) Scintillation Detector

Monte-Carlo Calculation of Photofractions and Intrinsic Efficiencies of Cylindrical Na I (Tl) Scintillation Detectors

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EXPERIMENTAL DETERMINATION OF THE PHOTOFRACTION OF A CYLINDRICAL 4″ × 6″ NaI(Tl) SCINTILLATION DETECTOR

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The photofraction of a NaI(Tl) crystal of 4″ diameter × 6″ length is determined for gamma-rays between 0.32 and 2.76 MeV. The method used eliminates all perturbative effects except Compton scattering in the source which is corrected for analytically.

1. Introduction

Measurement of the photofraction of NaI(Tl) scintillation detectors[1–5]) in most cases yield results considerably lower than the theoretical values. As the experiments can never exactly represent the simple conditions assumed in the calculations, and as few of the measurements try to remove or correct for these effects, it appeared interesting to set up a simple arrangement allowing the determination of the photofraction under elimination of background, backscatter from the crystal housing and from the wall of the laboratory, counting rate dependence of the amplification factor and deadtime effects in the pulse height analyzer. Compton scattering in the source was allowed for by analytical corrections. The measurements were performed with uncollimated radiation and with a collimator of the aperture $\theta = \frac{1}{2}\theta_{max}$, θ_{max} being the total angle subtended by the crystal. Compared to the method of Ricci[6]) the present arrangement has the advantage of using activities smaller by a factor of 10^3, thus requiring no shielding and hence reducing scattering a priori to a minimum.

2. Experimental Procedure

The scintillation detector was a 4″ diam. × 6″ Harshaw NaI(Tl) crystal mounted on an RCA 7046 photomultiplier tube. The pulses of the tenth dynode were fed to a 256 channel pulse-height analyzer. The zero level was determined by means of a weak source of Cs^{137} or Au^{198}. The conversion lines of these nuclides (32 and 69 keV) are well suited for calibration purposes.

The activities of the sources had to be small enough to avoid the relatively strong counting rate dependence of the amplification factor of the photomultiplier[7]) but high enough to maintain good statistics when subtracting background radiation. The optimum activities were between 2 and 7 μC, depending upon the different γ-energies of the nuclides. The source material was enclosed in small plexiglas cylinders.

Background and radiation scattered from the walls of the laboratory were subtracted from the original spectra. The direct radiation was kept aloof by a lead cone (see fig. 1a). This lead cone, however, prevented not only the direct radiation of the source from reaching the crystal but also a certain amount of background and backscatter. To compensate this effect, the "subtracting time" was increased by an appropriate factor.

When subtracting background and backscatter radiation from the spectrum, the influence of gammas scattered in the crystal container is not

[1]) S. H. Vegors, L. L. Marsden and R. L Heath, IDO-16370 (1958).

[2]) W E. Kreger, Phys Rev. **96** (1954) 1554.

[3]) E R Rathburn and C. E Crouthamel, Applied Gamma-Ray Spectrometry (C E. Crouthamel ed); (Pergamon Press, Oxford 1960).

[4]) W. E. Kreger and R M. Brown, Nucl. Instr. and Meth. **11** (1961) 290

[5]) H M. Childers, Rev. Sci. Instr. **30** (1959) 810.

[6]) R. A. Ricci, Physica **24** (1958) 289.

[7]) C. Weitkamp, Diplomarbeit Karlsruhe 1962 (unpublished).

taken into account. To eliminate this effect, the following simple extrapolation method was used: A tube of thickness equivalent to the thickness of the crystal container, reflector material etc. was placed over the front part of the detector in order to double the number of gammas scattered into the crystal, and a second tube to treble it. Each measurement was then performed without any tube, with one tube, and with two tubes over the crystal housing; the three photofraction results were linearly extrapolated to container thickness zero. In the case of collimated radiation the collimator kept direct radiation nearly completely off the crystal housing so that no such correction was necessary.

The timer unit used allowed direct deadtime correction

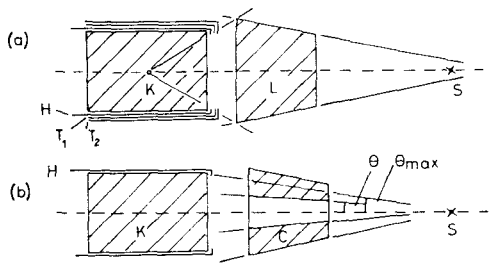

Fig. 1. Geometry of the arrangement.
S source K NaI(Tl) crystal
L lead cone H crystal housing
C collimator T_1, T_2 aluminum tubes.

3. Treatment of Experimental Data

The influence of photons scattered in the source material was not negligible in all cases, especially for low gamma energies. To calculate the correction of the measured photofractions, the following simplifying assumptions were made:

1) Photons subject to a Compton effect within the source can no longer cause a pulse contributing to the full energy peak of the spectrum.

2) A mean path, $\bar{l}$, of the gammas in the source may be defined so that the ratio of the number of scattered photons to the number of photons not scattered is

$$B = (1 - e^{-\Sigma_s \bar{l}})e^{-\Sigma_c \bar{l}} = e^{\Sigma_c \bar{l}} - 1$$

(Σ_c = macroscopic Compton cross section).

3) The sources are infinitely long cylinders of radius R. Then $\bar{l}$ is given by (see fig. 2)

$$\bar{l} = \frac{\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi}\int_{r=0}^{R}\int_{\alpha=0}^{2\pi} l(r,\alpha,\vartheta)\,d\alpha\, r\,dr\,d\varphi\sin\vartheta\,d\vartheta}{\int_0^{\pi}\sin\vartheta\,d\vartheta\int_0^{2\pi}d\varphi\int_0^{R}r\,dr\int_0^{2\pi}d\alpha} = \tfrac{4}{3}R.$$

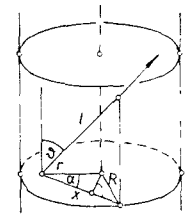

Fig. 2. Explication of parameters involved in the calculation of the mean path $\bar{l}$ of photons in the source.

As the scattered gammas have a smaller energy than the unscattered, the mean efficiency T_2 of scattered photons is larger than the efficiency T_1 of the unscattered photons. Calling $P^* = P + \Delta P$ the "true" photofraction, one has

$$P = \frac{T_1 P^* e^{-\Sigma_c \bar{l}}}{T_1 e^{-\Sigma_c \bar{l}} + T_2(1 - e^{-\Sigma_c \bar{l}})}$$

and, hence, the correction

$$\Delta P = P \cdot T_2/T_1 \cdot B .$$

For collimated radiation another correction should be made taking into account the influence of gammas scattered by the collimator. A quantitative calculation of this effect is difficult because of the strong dependence on the dimensions of the source and on its adjustment. But it can be seen qualitatively that the correction will be small for the lower energies, where primary photons striking the collimator as well as secondary gammas are very likely to be absorbed within the lead, whilst for higher energies the effect may be rather significant.

4. Results

Fig. 3 gives the corrected results, P^*, for uncollimated and for collimated radiation as a

function of the gamma-ray energy E_γ. Agreement with the calculated photofractions[8]) is very good except for collimated radiation of energy > 1 MeV by reducing the mass of the source material or by using several sources of various thicknesses and extrapolating to source thickness zero. A vertical

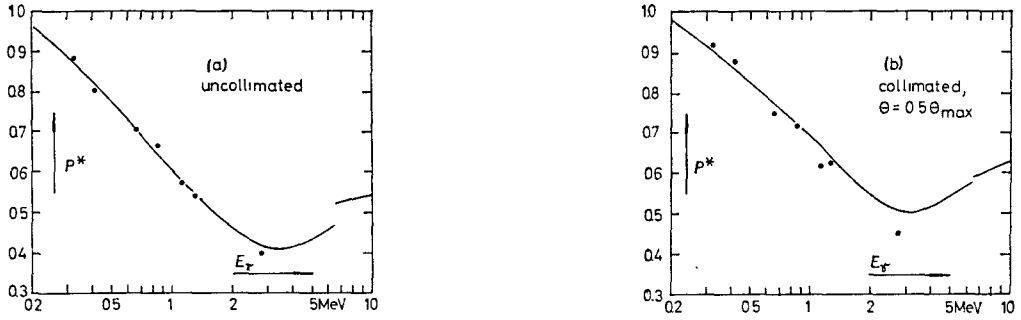

Fig. 3. Comparison of experimental (. . .) and theoretical (—) photofractions of a $4''\phi \times 6''$ NaI(Tl) crystal for uncollimated (a) and collimated radiation (b) as a function of the gamma energy E_γ.

where the collimator influence, as discussed above, should not be neglected.

The accuracy of the method might be improved position of the detector would allow to diminish considerably the mass of the crystal container.

[8]) C. Weitkamp, Nucl. Instr. and Meth. **23** (1963) 13.

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MONTE-CARLO CALCULATION OF PHOTOFRACTIONS AND INTRINSIC EFFICIENCIES OF CYLINDRICAL NaI(Tl) SCINTILLATION DETECTORS

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This paper describes a Monte-Carlo calculation of efficiencies and photofractions of NaI(Tl) detectors for gamma-rays between 0.2 and 10.0 MeV. The crystals considered are right circular cylinders ranging in size from 3″ diameter × 2″ length to 5″ diameter × 6″ length. The results may be used to determine the optimum crystal dimensions for given crystal volume and gamma-ray energy. The influence of collimators of different apertures is investigated. Except two characteristic cases, however, results are given for uncollimated radiation only.

1. Introduction

As the photofraction of a gamma-ray scintillation detector is an important parameter in nuclear spectroscopy, many authors have performed calculations of the photofractions for given source and crystal geometry. While the approximations used in older publications[1–5]) are somewhat coarse (pair production is often considered as complete absorption of the incident photon), more recent papers give a rigorous treatment of the annihilation radiation[6]) and sometimes take into account even small effects like bremsstrahlung and electron escape energy losses from the crystal[7,8]). None of the papers mentioned, however, allows the photofraction optimization of the dimensions of scintillation detectors as used in capture gamma coincidence and angular correlation work. Either the variety of crystals considered is too small or too widely spaced, or the calculations do not take into account the whole energy range. Moreover, most of the programs existing, especially those calculating the entire response spectrum, take rather a long time, even when run on a fast computer.

A rapid Monte-Carlo program therefore has been developed to calculate the photofraction and the efficiency of right cylindrical NaI(Tl) detectors for γ-ray energies from 0.2 to 10 MeV. Pair production was treated thoroughly, but no corrections were made for bremsstrahlung and escape of electrons from the crystal. It can be seen easily that, even for the smallest crystal and the highest energy, the error introduced by those effects is below the statistical fluctuations. The calculations were carried out for point sources located on the crystal axis at various distances; the influence of collimators of different apertures was investigated by dividing the crystal face into ten concentric rings. The program was set up for the simultaneous consideration of four crystals of the same diameter but different length, and 6000 Monte-Carlo histories were computed for each set of energy and geometry parameters.

2. Method of Calculation

2.1. PHOTON HISTORIES

The principle of the Monte-Carlo method consists in simulating the history of a great number, N, of individual photons of energy E_γ on their way through the crystal. Physical quantities that obey

[1]) D. Maeder, R. Müller and V. Wintersteiger, Helv. Phys. Acta **27** (1954) 3.

[2]) J. G. Campbell and A. J. F. Boyle, Australian J. Phys. **6** (1953) 171.

[3]) M. J. Berger and J. Doggett, Rev. Sci. Instr. **27** (1956) 269.

[4]) M. J. Berger and J. Doggett, J. Res. Nat. Bur. Stand. **56** (1956) 355.

[5]) M. H. Wächter, W. H. Ellett and G. L. Brownell, Rev. Sci. Instr. **28** (1957) 717.

[6]) W. F. Miller and W. J. Snow, Rev. Sci. Instr. **31** (1960) 905.

[7]) C. O. Zerby, TID-7594, Paper 8 (1960).

[8]) W. F. Miller and W. J. Snow, ANL-6318 (1961).

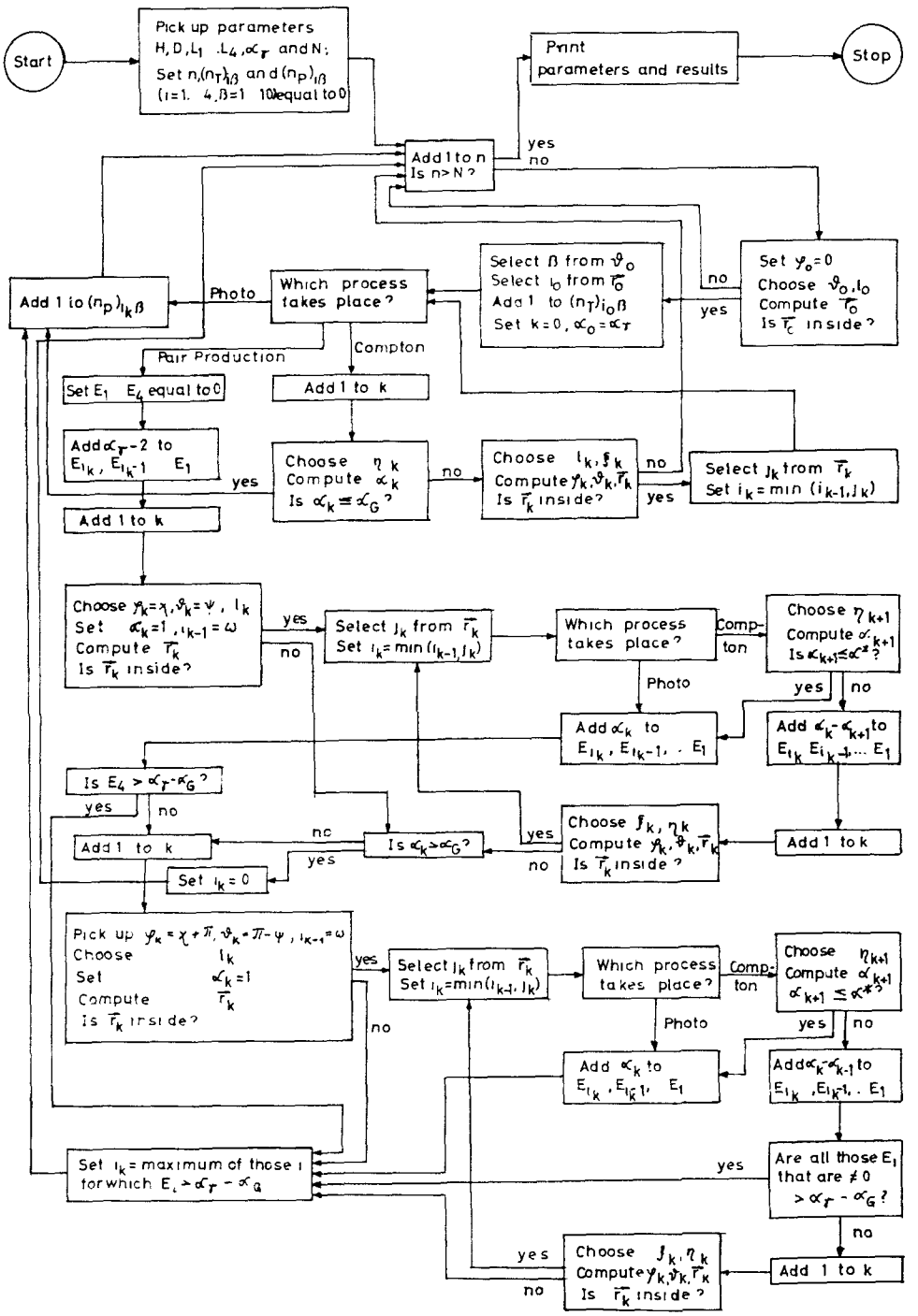

Fig. 1. Schematic flow diagram.

a statistical distribution are sampled according to the given distribution function.

If n_T is the number of photons interacting at least once, thus causing a pulse at the detector output, and if n_P photons lose all† their energy within the crystal, the intrinsic efficiency†† is defined as $T = n_T/N$, and the photofraction is $P = n_P/n_T$. n_T and n_P, and therefore T and P are characterized by two indices $i(i = 1 \ldots 4)$ and $\beta(\beta = 1 \ldots 10)$ indicating the number of the crystal of the "set" and the collimator aperture, respectively.

The sampling of the photon histories may easily be understood from the schematic diagram of fig. 1 and need not be described in detail. Fig. 2 gives

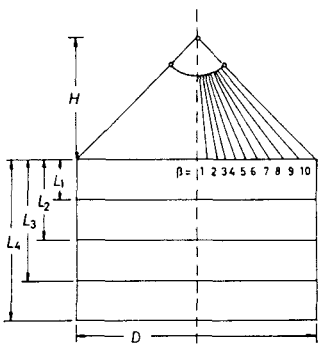

Fig. 2. Geometry parameters.

the meaning of the geometry parameters. n is the number of Monte-Carlo histories already considered, l is the free path of the photon between two successive interactions, φ, ϑ and r are the azimuthal and polar angles of the propagation direction and the endpoint of the gamma-ray in the crystal coordinate system, and η and ξ represent the scattering angle and azimuth involved in Compton scattering. α stands for the energy of the photon in units of m_0c^2; the mean free path of gamma-rays with energies < 50 keV being 3×10^{-2} cm or less, photons with $\alpha < \alpha^* \equiv 0.1$ are assumed to be completely absorbed. The subscript k indicates the number of the interaction undergone by the photon under consideration. In the case of a pair production the separation of the amounts of energy lost in the different crystals is somewhat more difficult than for Compton and photoelectric processes. Therefore four quantities $E_1 \ldots E_4$ are introduced "gathering" the fraction of energy dissipated by the gamma in the largest crystal only, in the two largest crystals, in the three largest ones and in all four crystals, respectively.

† Because of the finite resolution of the detector system a photon may contribute to the photopeak of the pulse height spectrum even when losing less energy within the crystal than E_γ, say $E_\gamma - E_G$. The choice of E_G is not critical. It appears reasonable to take $E_G = C.\sqrt{E_\gamma}$, the constant C being determined by setting $(E_G/E_\gamma)\,(E_G/E_\partial)_{E\partial\,=\,662\ \mathrm{keV}}\ 10\,\%$. In the higher energy region ($E_\gamma \geqslant 2$ MeV) E_G has been chosen from the resolution measurements of Koch and Foote[9]).

†† Often referred to as interaction ratio.

2.2. RANDOM SAMPLING

If $W(y)$ is the given probability distribution of the quantity $y(a \leqslant y \leqslant b)$, and $r(0 \leqslant r \leqslant 1)$ are so-called random numbers with the distribution function

$$F(r) = \begin{cases} 1 & 0 \leqslant r \leqslant 1 \\ 0 & \text{elsewhere,} \end{cases}$$

an individual value y may be obtained from a number r by the transformation

$$\frac{\int_a^y W(y')\,\mathrm{d}y'}{\int_a^b W(y')\,\mathrm{d}y'} = \frac{\int_0^r F(r')\,\mathrm{d}r'}{\int_0^1 F(r')\,\mathrm{d}r'} = r\,. \qquad (1)$$

The (pseudo) random numbers employed are of the form

$$r_n = 2^{-36} x_n$$

$$x_n = 5^{13} x_{n-1} \bmod 2^{36}\,, \quad \text{with}$$

$$x_0 = 1\,.$$

They may be very rapidly generated by the computers used (Z 22 and IBM 704). Rief[10]) tried their "randomness" by extensive tests.

The application of transformation (1) to the calculation of ξ_k, l_k, ϑ_0 etc. does not present any difficulties. The random sampling of the Compton scattering η_k is somewhat cumbersome. Integration of the Klein-Nishina formula yields the transcendental equation

$$f(v) \equiv v + a \log v + b/v + c/v^2 + d + er = 0\,, \qquad (2)$$

where

$$v = 1 + \alpha_k(1 - \cos \eta_k)$$

[9]) H. W. Koch and R. S. Foote, Nucleonics **12** No. 3 (1954) 51.

[10]) H. Rief, private communication.

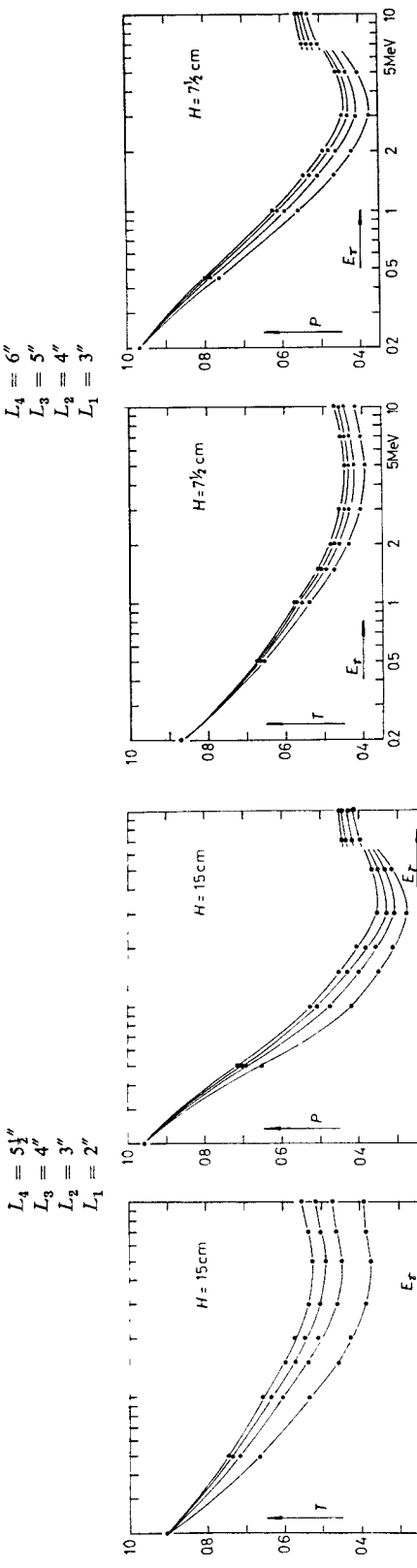

Fig. 3. Intrinsic efficiency T and photofraction P for uncollimated radiation of energy E_γ. Parameter is the crystal length L. Crystal diameter $D = 3''$, source distance $H = 15$ and 30 cm. The arrangement of each set of curves corresponds to the arrangement of the L values on top of the figure.

Fig. 4. Intrinsic efficiency T and photofracion P for uncollimated radiation of energy E_γ. Parameter is the crystal length L. Crystal diameter $D = 4''$, source distance $H = 7\frac{1}{2}$, 15 and 30 cm. The arrangement of each set of curves corresponds to the arrangement of the L values on top of the figure.

and a, b, c, d and e are functions of the energy parameter α_k of the incident photon. Starting with the solution $v = 1$, (2) may easily be solved by the method of Newton to any accuracy wanted.

2.3. CROSS SECTIONS

Using the tables of Grodstein[11]), Crouthamel[12]) has calculated the cross sections of 0.1 per cent thallium-activated sodium iodide for photoelectric effect, Compton effect and pair production. These data were used in the present calculation and, if necessary, interpolated quadratically. Rayleigh and Thomson scattering of gammas were not taken into account.

3. Results

The results of the calculation for uncollimated radiation are given in figs. 3, 4 and 5. It should be noted that the minimum of the photofraction does not coincide with the minimum of the intrinsic efficiency and of the total cross section of NaI located at 5 MeV but is shifted to about 3 MeV. This may be explained by the fact that, in this energy range, small Compton scattering angles and therefore small energy losses of the gammas are very frequent; thus, absorption of secondary gammas is not likely, and little energy will be transmitted to the crystal. Pair production, on the other hand, yields secondary radiation of relatively low energy which may easily be absorbed within the crystal, and therefore increases the photofraction in the energy region where its cross section becomes relevant. The discontinuity of the P curves shown in figs. 3, 4 and 5 results from the energy dependence of the resolution. The first escape peak was taken to be separated from the full energy peak up to 6.6 MeV only.

Efficiency and photofraction are both increasing with increasing size of the crystal. But while the length dependence of T and P is about the same, the photofraction varies rather markedly with the diameter of the crystal, whereas the efficiency does not (cf. fig. 6). To choose the optimum length-to-

[11]) G. W. Grodstein, Nat. Bur. Stand. Circular 583 (1957).
[12]) C. E. Crouthamel ed., Applied Gamma-Ray Spectrometry (Pergamon Press, Oxford, 1960).

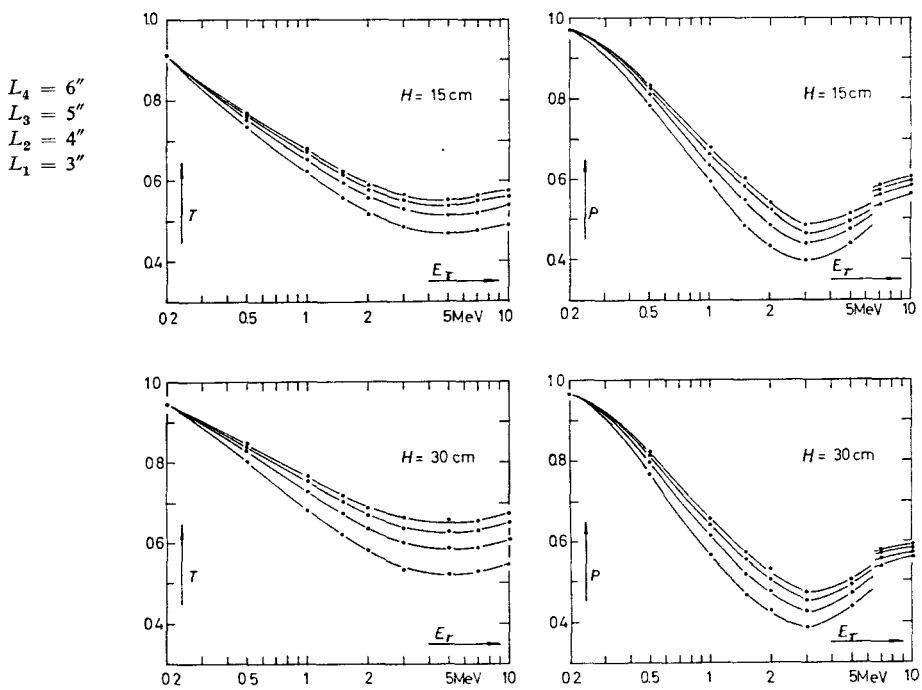

Fig. 5. Intrinsic efficiency T and photofraction P for uncollimated radiation of energy E_γ. The parameter is the crystal length L. Crystal diameter $D = 5''$, source distance $H = 15$ and 30 cm. The arrangement of each set of curves corresponds to the arrangement of the L values left of the figure.

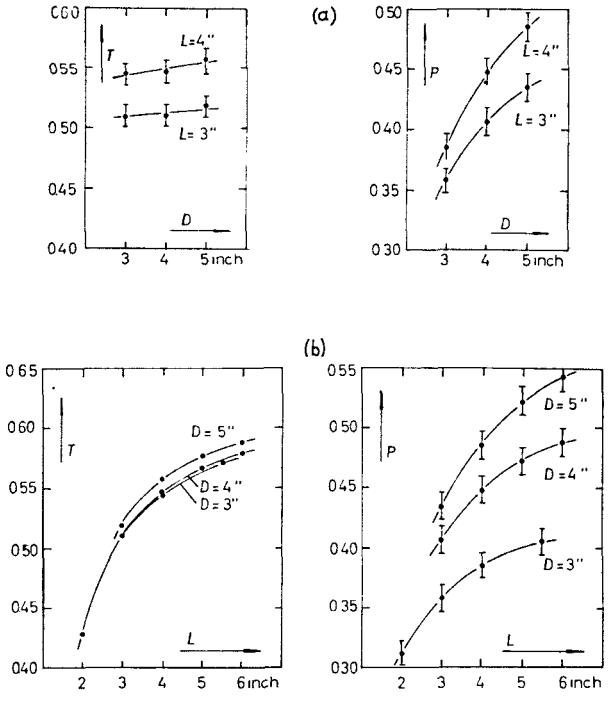

Fig. 6. Intrinsic efficiency T and photofraction P vs. crystal diameter (a) and crystal length (b). $E_\gamma = 2$ MeV, $H = 15$ cm.

diameter ratio for a given crystal volume and given energy of the incident gammas, it is useful to plot P against the crystal volume. Fig. 7 shows such a

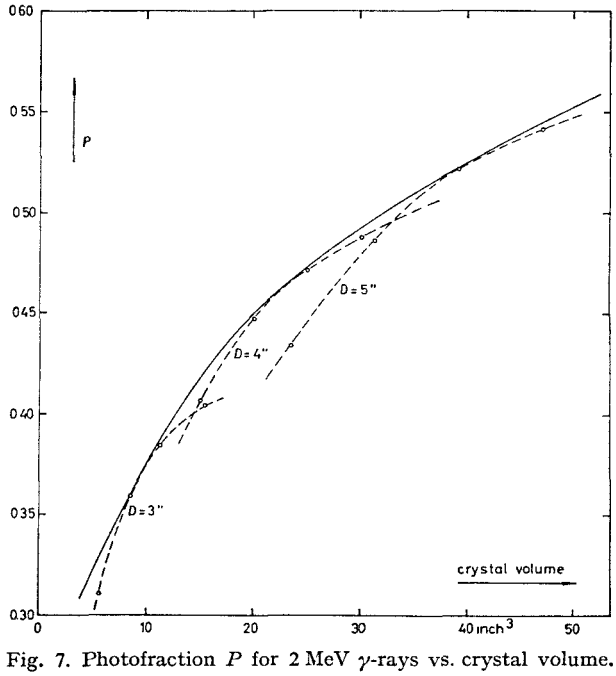

Fig. 7. Photofraction P for 2 MeV γ-rays vs. crystal volume. $E_\gamma = 2$ MeV, $H = 15$ cm.

plot for $E_\gamma = 2$ MeV and $H = 15$ cm. The dotted lines correspond to one set of crystals each, i.e. to one D value, the various points result from the variation of L. For every crystal volume there is one optimum diameter. It may be obtained by determining the (dotted) curve that touches the envelope (solid line) at the given volume. The corresponding D value is the crystal diameter wanted.

The collimator influence results in an improvement of the photofraction (see fig. 8a). For crystals of large diameter and small length it may happen, however, that at energies of about 3 MeV the photofraction as a function of the collimator aperture goes through a maximum (fig. 8b). The explanation is again the dominating forward direction of Compton scattering at higher energies where the probability of multiple processes increases rapidly with increasing "effective" thickness of the crystal.

Only few of the results of the present calculation

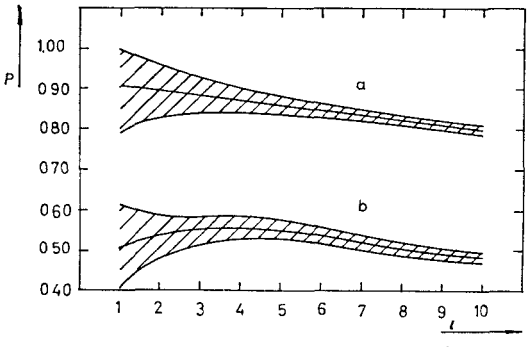

Fig. 8. Photofraction P vs. collimator aperture $\theta = i \times 0.1\theta_{max}$. Crystal dimensions, source distance and gamma-ray energy are $4''\phi \times 6''$, $7\frac{1}{2}$ cm and 500 keV (a) and $5''\phi \times 4''$, 15 cm and 2 MeV (b). The hatched areas indicate statistical errors.

can be compared with those from preceding publications. In all these cases agreement is very good.

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