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THE INFLUENCE OF MULTIPLE SCATTERING ON THE MEASUREMENT OF THE CIRCULAR POLARIZATION OF γ-RAYS

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The circular polarization of γ-rays is usually measured by Compton forward scattering from comparatively thick (about 1 cm) magnetizied iron cylinders. Therefore, multiple scattering was determined by measuring the counting rate as a function of the iron thickness and was indeed found large (up to 35%). Nevertheless, it was shown that multiple scattering decreases the polarization efficiency by only a few percent for the photon energy range between 600 and 1220 keV.

1. Introduction

Following the discovery of parity non-conservation in β-decay the measurement of the circular polarization of γ-rays became of increasing importance during the last years as it proved to be a very powerful tool for nuclear spectroscopy. Recently some discrepancies between the results of different laboratories were found and, therefore, a search for possible systematic errors seemed appropriate.

For photon energies between several hundred keV and about 1.5 MeV the most suitable and frequently used method for the measurement of the circular polarization of γ-rays is the Compton forward scattering from magnetized iron[1]). This method gives a comparatively high polarization efficiency combined with high counting rates. In order to reduce the still long counting times iron cylinders up to 15 mm thick are usually used. As the half value thicknesses for the absorption of γ-rays are much smaller it is to be expected that multiple scattering might not be negligible compared to Compton single scattering. A calculation of the intensity of the multiple scattering and its influence on the polarization efficiency is very difficult. A rather crude estimate[1]) showed that in spite of the large intensity of double scattering the polarization efficiency should not be changed much. This surprising result stems from the peculiar behaviour of the Compton cross section.

[1]) H. Schopper, Nuclear Instr. and Meth. 3 (1958) 158.

As the polarization efficiency can be calculated quite reliably for single Compton scattering but not for multiple scattering an experimental investigation of the latter seemed necessary. As multiple scattering increases more rapidly with the thickness of the scatterer than single scattering the two effects were separated by observing the scattered intensity and the polarization effect as a function of the iron thickness†. Monoenergetic γ-rays of radioactive sources (Hg^{203}, Cs^{137} and Co^{60}) were used to study the scattered intensity. The polarization effect was investigated with circularly polarized bremsstrahlung produced by longitudinally polarized β-particles (Sr^{90} + Y^{90}).

2. Experimental Apparatus

The experimental set-up is shown in fig. 1. It is conventional with the exception of the shape of the iron yoke for the magnetic return flux and the position of the magnetizing coils. As thin iron cylinders were to be used the coil could not be wound around the scattering cylinder in order to avoid the scattering from the copper. With the yoke shape and the lead shieldings shown in fig. 1 only a small γ-background was observed when the scattering cylinder was removed (cf. fig. 2).

† If there were no absorption by photoelectric effects the intensity for single scattering would be proportional to the iron thickness, whereas double scattering would go with its square as long as the incident beam is not attenuated appreciably. Because of the superposition of absorption and scattering things are more complicated.

The scattering cylinders could easily be exchanged. Four cylinders were used with thicknesses of 1.5, 3.2, 6.2 and 14.2 mm, respectively.

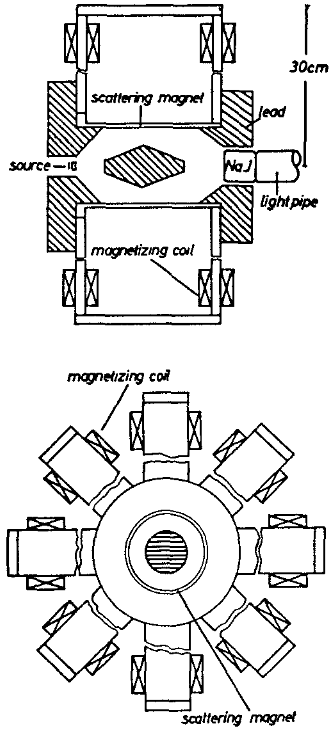

Fig. 1. Experimental arrangement for the measurement of the circular polarization of bremsstrahlung. (a) side view; (b) front view.

The pulses from the multiplier were fed into an amplifier and analyzed with a single channel analyser. For the polarization measurements the counting rates were printed every 10 min. and the magnetization of the iron cylinder reversed automatically. As the change of the counting rate on reversing the magnetization amounted to only a few percent several millions of counts had to be taken to obtain a sufficient statistical accuracy. The running time for a single point was about 12 days. In order to suppress the influence of the magnetic stray field on the multiplier a light guide 60 cm long was used and the multiplier was surrounded by μ-metal and 4 iron shields. The change of the counting rate due to the stray field was less than 0.05 %.

For the intensity measurements the monoenergetic γ-lines of 280 keV (Hg^{203}), 660 keV (Cs^{137}) and 1.17 + 1.33 MeV (Co^{60}) were used. In fig. 2 the pulse height spectrum produced by the scattered radiation is shown for Cs^{137}. The photo-

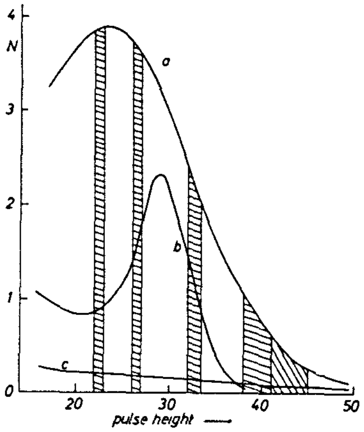

Fig. 2. Pulse height distribution of photons scattered from a cylinder 0.62 cm thick. (a) bremsstrahlung from Sr^{90} + Y^{90}; (b) Cs^{137}; (c) background for bremsstrahlung (scattering cylinder removed). The shaded areas indicate the pulse heights selected for the measurements at various k.

electric peak is rather broad as the scattering angle ranges from 59° to 64°.

The polarization measurements were performed with a 40 mC Sr^{90} source on a thick silver backing. The electrons were stopped in a lead absorber where they produced circularly polarized bremsstrahlung[2,3]. This absorber was made 1 cm thick in order to filter out the bremsstrahlung quanta with energies below 300 keV as these have only small polarizations. The depolarization of quanta with higher energies by this lead filter should be small and does not impair this investigation as we are interested in a relative measurement only. The pulse height spectrum for the bremsstrahlung is also shown in fig. 2.

3. Calculation of the Intensity and the Polarization Effect for Single Scattering

The effect of multiple scattering can be detected by comparing the measured results with those calculated for single scattering. The intensity and polarization efficiency for monoenergetic γ-rays

[2]) M. Goldhaber, A. W. Sunyar and L. Grodzins, Physical Review **106** (1957) 826.

[3]) S. Galster and H. Schopper, Physical Review Letters **1** (1958) 330.
S. Galster, Zeitschrift für Physik **161** (1961) 46.

can be calculated in a straight forward though laborious and lengthy way.[1]) The counting rate as a function of the γ-energy k and the thickness d of the iron is given for a cylindrical geometry by

$$N(k, d) = \int \left\langle \frac{\bar{F}\bar{\varepsilon}}{r_1^2 r_2^2} \bar{r}\, d\sigma(k) \right\rangle \cdot$$

$$\left\{ \frac{1 - \exp\left[-(\sigma_1 + g\sigma_2) n_0\, d \sin^{-1} \psi\right]}{\sigma_1 + g\sigma_2} \right\} \sin \psi\, dz \quad (3.1)$$

where

$\bar{F}$ = effective area of the scintillation crystal

$\bar{\varepsilon}$ = detection efficiency for photons entering the scintillation crystal

σ_1, σ_2 = total absorption coefficient/electron for incident and scattered photons

n_0 = number of electrons/cm^3

$g = s_2/s_1$

The quantities r_1, r_2, s_1, s_2, d, $\bar{r}$ and ψ are defined in fig. 3. The Compton cross section $d\sigma$ can be split up in a polarization independent part $d\sigma_0$ and a part sensitive to the circular polarization $d\sigma_c$.

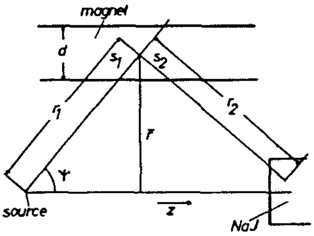

Fig. 3. Definition of geometrical quantities.

$$d\sigma(k) = d\sigma_0(k) \pm fP_c\, d\sigma_c(k) \quad (3.2)$$

where P_c is the degree of polarization of the photons and f is the fraction of polarized electrons per atom. The + (−) sign corresponds to the case in which electron spins and γ-momentum are parallel (antiparallel).

In order to calculate the scattered intensity for unpolarized ($P_c = 0$) γ-quanta of energy k as a function of the iron thickness d the integration over the magnet geometry was carried out by numerical methods. The results for three energies are displayed in fig. 4. The intensity increases first linearly with the thickness but very soon levels off because the intensity of the primary and scattered beam inside the iron drops exponentially.

For circularly polarized γ-rays the relative change in counting rate on reversing the magnetization is given by

$$E(k, d) = \frac{N^- - N^+}{(N^- + N^+)/2} = 2fP_c < d\sigma_c^-/d\sigma_0 >$$

$$= P_c E_0(k, d) \quad (3.3)$$

Here N^- and N^+ are the counting rates for the two directions of magnetization, respectively. The brackets indicate the averaging over the magnet geometry. An expression for $\langle d\sigma_c^-/d\sigma_0 \rangle$ for large iron thickness has been given previously[1]). A generalisation of this formula for a finite iron thickness is given in the appendix. E_0 gives the relative change of counting rate for completely polarized photons.

As bremsstrahlung produced by β-particles was used as a convenient source for circularly polarized γ-rays formula (3.3) derived for monoenergetic

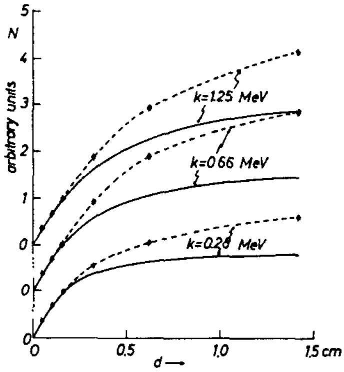

Fig. 4. Intensity of scattered photons as a function of iron thickness for 3 photon energies. Full lines calculated for single scattering, broken lines: experimental values.

photons cannot be used immediately. The relations are complicated by the fact that even monoenergetic γ-rays produce a broad pulse height distribution at the multiplier output consisting of a photopeak and a Compton distribution. Hence a certain pulse height selected by the single channel analyser cannot be associated uniquely with a definite γ-energy. Therefore, the polarization effect given by (3.3) has to be averaged over a bremsstrahlung spectrum taking into account that photons with different energies carry various degrees of polarization[3]):

$$\bar{E}(H, d) = \frac{\int P_c(k)E_0(k, d)I(k)G(k, H)\,dk}{\int P_c(k)I(k)G(k, H)\,dk} \tag{3.4}$$

where the symbols have the following meaning:

H, pulse height intervall selected by the single channel analyser.

$I(k)$, bremsstrahlung intensity as a function of the photon energy k. The theoretical spectrum[4]) has been used folded with the β-spectrum and taking into account the lead filter.

$G(k, H)$ gives the pulse height distribution at the multiplier output for monoenergetic γ-rays scattered from the iron. This spectrum was determined experimentally for the energies 280, 660 and (1170 + 1330) keV and interpolated for other energies.

$P_c(k)$ the circular polarization of the bremsstrahlung was calculated with formulae given by several authors[5]) folded with the β-spectrum which agree fairly well with experimental results[3]).

An average photon energy $\bar{k}$ can be associated with a certain analyser setting H in the following way

$$\bar{k}(H) = \frac{\int kI(k)G(kH)\,dk}{\int I(k)G(kH)\,dk} \tag{3.5}$$

The evaluation of $\langle d\sigma_c^-/d\sigma_0\rangle$ was performed with a computer ZUSE. The results for $E(k, d)$ as a function of the iron thickness are shown in fig. 6.

4. Experimental Results and Discussion

The experimental results for the scattering intensity and the polarization effect shall now be compared with the values calculated for single scattering.

The numbers of scattered quanta as a function of the iron thickness are plotted in fig. 4. The experimental curves are normalized to the calculated ones at the smallest thickness of 0.5 mm as it is thought that the probability for multiple scattering is quite small for such a thin scatterer.

As can be seen the multiple scattering gets more important with increasing thickness as is to be expected. At the largest thickness it contributes more than 30 % to the scattered intensity. In fig. 5 the ratio of the measured intensity and the one calculated for single scattering is displayed as a function of the incident γ-energy. For a given thickness the intensity of multiple scattering increases rapidly, reaches a maximum

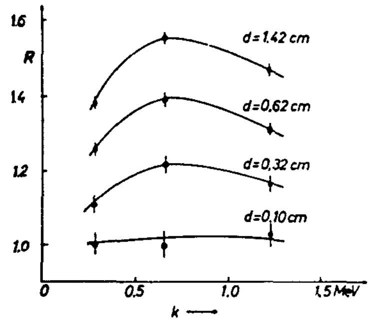

Fig. 5. Ratio R of measured intensity to intensity calculated for single scattering for 4 iron thicknesses d.

at about 600 keV and drops off slowly for higher photon energies. This behaviour can be understood qualitatively by taking into account that at the lowest energies the photoelectric effect predominates and suppresses double scattering. With increasing energy the photoelectric absorption drops quickly and multiple scattering becomes more important accordingly. At the highest energy, however, Compton scattering is directed more at forward angles and hence a double scattering process under a total angle of about 60° gets less probable. Thus multiple scattering decreases slowly.

The interesting question now is whether this comparatively large contribution of multiple scattering impairs the polarization measurements. Circularly polarized bremsstrahlung was scattered for this purpose from various iron cylinders. The measured relative changes in counting rates on reversing the magnetisation are displayed in fig. 6 as a function of the iron thickness. First it may be remarked that the absolute magnitude of the effect is in good agreement with previous

[4]) H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146** (1934) 83.
A. Busolati, Nuovo Cim. **13** (1959) 909.

[5]) K. W. McVoy, Phys. Rev. **106** (1957) 828 and **110** (1958) 1484;
C. Fronsdal and H. Überall, Phys. Rev. **111** (1958) 580;
H. Olsen and L. C. Maximon, Phys. Rev. **114** (1959) 887.

measurements[3]) if one takes into account that the magnetization in this experiment is somewhat lower ($f = 0.057$) and that there is some back

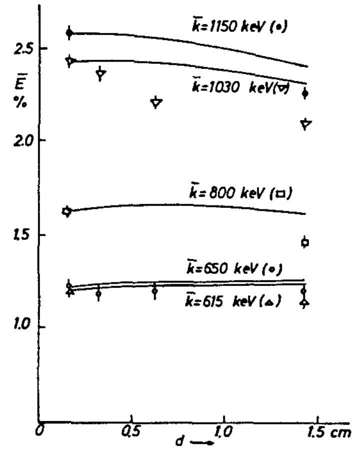

Fig. 6. Polarization efficiency E as a function of iron thickness d for five photon energies; full line: calculated for single scattering Points: experimental values.

scattering of electrons from the thick source backing. However, here we are little interested in the absolute v alue of E but rather in its change with the iron thickness. The experimental points were normalized to the calculated curves at $d = 1.5$ mm as at this thickness the contribution of multiple scattering is negligible (see fig. 4). A comparison of the measurements and the values evaluated for single scattering show that the polarization efficiency is only very little effected by multiple scattering. For $d = 14.2$ mm the deviations seem to be independent of energy within the experimental errors and the average decrease of E is $7\,\% \pm 3\,\%$ (fig. 7).

The surprising result that the polarization efficiency changes only by a few percent although the contribution of multiple scattering is more than 30% can be understood qualitatively. As it has been pointed out previously[1]) the polarization sensitivity of two consecutive scatterings with a total angle of about 60° is approximately the same as that of a single scattering. This peculiar behaviour of the Compton cross section results in a small difference between these two processes as far as the polarisation sensitivity is concerned. A detailed evaluation of the influence of multiple scattering seems very difficult, however, and can probably be achieved only by Monte Carlo calculations. The experimental results presented in this paper show that the polarization efficiency for Compton forward

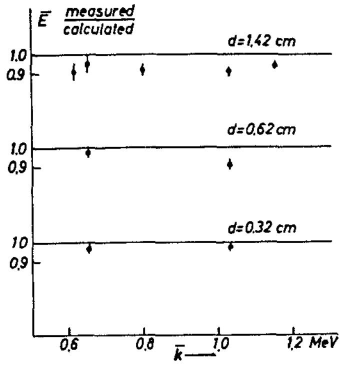

Fig. 7. Polarization efficiency E as a function of average photon energy k for three iron thicknesses d. E is normalized to the polarization efficiency for single scattering at $d = 0.15$ cm where plural scattering is negligible (cf fig. 4).

achieved scattering can be computed by taking into account single scattering only within an accuracy of a few percent. Multiple scattering can be corrected for easily by using the results given in fig. 7.

Acknowledgements

We should like to thank Prof. Wirtz for making available the ZUSE computer to us and to Mr. H. Sanitz for his help in programming. This work was supported by the Bundesministerium für Atomkernenergie.

Appendix

THE POLARIZATION EFFECT FOR SINGLE SCATTERING AND FINITE THICKNESS OF SCATTERER

The analysing efficiency for single scattering from magnetized iron with finite thickness was calculated for monoenergetic γ-rays in the following way. The total cross-sections in eq. (3.1) can be split up into a polarization dependent and a polarization independent part:

$$\begin{aligned} \sigma_1 &= \sigma_{t1} \pm fP_c\sigma_{c1} \\ \sigma_2 &= \sigma_{t2} \pm \varkappa fP_c\sigma_{c2} \end{aligned} \qquad \text{(A.1)}$$

The subscripts 1 and 2 refer to the total cross-section for quantum energies before and after the scattering and $\sigma_{t\,1,2}$ are the sums of the photoelectric and ordinary Compton cross section. $\varkappa$ is the ratio of the polarizations after and before the scattering[6]). Introducing the abbreviations

$$(\sigma_{t1} + g\sigma_{t2})\, n_0/\sin\psi = A_t$$
$$(\sigma_{c1} + \varkappa\, g\sigma_{c2})\, n_0/\sin\psi = A_c \tag{A.2}$$

Equation (3.1) can be written

$$N^{\pm} = \int \frac{\overline{F\varepsilon}}{r_1^2 r_2^2}\, \bar{r}(\mathrm{d}\sigma_0 \pm fP_c\, \mathrm{d}\sigma_c) \left\{ \frac{1 - \mathrm{e}^{-A_t d \pm fPA_c d}}{A_t \pm fPA_c}\, n_0 \right\} \mathrm{d}z \tag{A.3}$$

As f is a small number and the coefficients going with f are smaller than 1 ($A_c/A_t \leqslant 0.4$ and $A_c d \leqslant 10^{-2}$) we can expand (A.3) and retain only the first powers of f making an error of less than 1 %. For $\langle \mathrm{d}\sigma_c^-/\mathrm{d}\sigma_0 \rangle$ defined in (3.3) we then obtain

$$\langle \mathrm{d}\sigma_c^-/\mathrm{d}\sigma_0 \rangle = \frac{\int \left\langle \frac{\overline{F\varepsilon}}{r_1^2 r_2^2}\, \bar{r}\, \mathrm{d}\sigma_c^- \right\rangle (1 - \mathrm{e}^{-A_t d}) W A_t^{-1}\, \mathrm{d}z}{\int \left\langle \frac{\overline{F\varepsilon}}{r_1^2 r_2^2}\, \bar{r}\, \mathrm{d}\sigma_0 \right\rangle (1 - \mathrm{e}^{-A_t d}) A_t^{-1}\, \mathrm{d}z} \tag{A.4}$$

with

$$w_p = 1 - A_c \langle \mathrm{d}\sigma_0 \rangle / A_t \langle \mathrm{d}\sigma_c^- \rangle$$
$$W = w_p + (1 - w_p) A_t\, d (\mathrm{e}^{A_t d} - 1)^{-1}$$

For the special cases of very thick or very thin scatterers we obtain from (A.4)

$$\langle \mathrm{d}\sigma_c^-/\mathrm{d}\sigma_0 \rangle = \frac{\int \left\langle \frac{\overline{F\varepsilon}}{r_1^2 r_2^2}\, \bar{r}\, \mathrm{d}\sigma_c^- \right\rangle w_p A_t^{-1}\, \mathrm{d}z}{\int \left\langle \frac{\overline{F\varepsilon}}{r^2 r_2^2}\, \bar{r}\, \mathrm{d}\sigma_0 \right\rangle A_t^{-1}\, \mathrm{d}z} \quad \text{for } A_t d \to \infty \tag{A.5}$$

$$\langle \mathrm{d}\sigma_c^-/\mathrm{d}\sigma_0 \rangle = \frac{\int \left\langle \frac{\overline{F\varepsilon}}{r_1^2 r_2^2}\, \bar{r}\, \mathrm{d}\sigma_c^- \right\rangle \mathrm{d}z}{\int \left\langle \frac{\overline{F\varepsilon}}{r_1^2 r_2^2}\, \bar{r}\, \mathrm{d}\sigma_0 \right\rangle \mathrm{d}z} \quad \text{for } A_t d \to 0 \tag{A.6}$$

The term W in eq. (A.4) accounts for the polarization dependent absorption in iron. For the geometry shown in fig. 1 W was calculated as a function of z and is displayed in fig. 8. As can be seen W differs from 1 only by a few percent at most and, therefore, the polarization sensitive absorption in the iron is relatively unimportant.

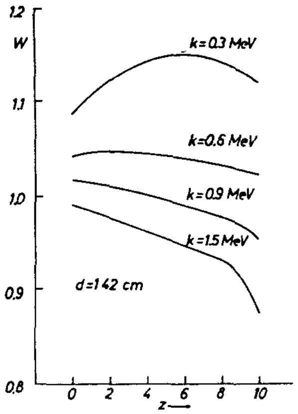

Fig. 8. The quantity W as defined by eq. (A.4) is displayed as a function of the coordinate z for 4 photon energies and d = 1.42 cm.

The brackets ⟨ ⟩ indicate an average over the iron thickness and we shall discuss the validity of this approximation now. As the same geometrical factors occur in the denominator and numerator of (A.4) and as the functions under the integral vary only slowly with z these weight functions are of

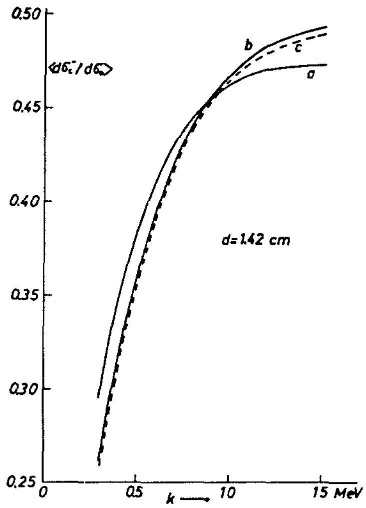

Fig. 9. $\langle \mathrm{d}\sigma_c^-/\mathrm{d}\sigma_0 \rangle$ as a function of the photon energy for d = 1.42 cm. (a) calculated according to eq. (A.4); (b) with $W = 1$; (c) $\langle \mathrm{d}\sigma_c^-/\mathrm{d}\sigma_0 \rangle = \int \langle \mathrm{d}\sigma_c^- \rangle \mathrm{d}z / \int \langle \mathrm{d}\sigma_0 \rangle \mathrm{d}z$.

6) F. W. Lipps and H. A. Tolhoek, Physica **20** (1954) 395.

little influence. In fact, if one puts them equal to one the result is hardly changed (see fig. 9, curve c) and, therefore, using the averages $\langle d\sigma_c^- \rangle$ and $\langle d\sigma_0 \rangle$ should introduce only a negligible error.

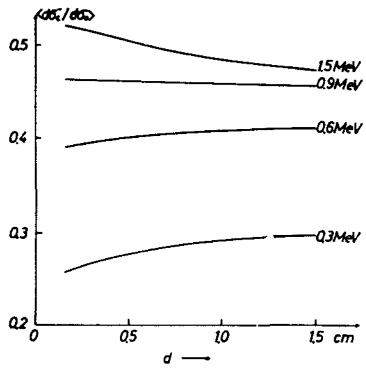

Fig. 10. $\langle d\sigma_c^-/d\sigma_0 \rangle$ as a function of the iron thickness d for 4 photon energies.

The final result for $\langle d\sigma_c^-/d\sigma_0 \rangle$ is shown in fig. 10 for 4 quantum energies. The change of $\langle d\sigma_c^-/d\sigma_0 \rangle$ with the thickness d is almost entirely due to W being a function of d. The different behaviour for high and low photon energies is accounted for by the fact that σ_c has a maximum at 0.3 MeV, changes sign at 0.66 MeV and has a minimum at about 6 MeV.

From fig. 10 it may be taken that for $d \geqslant 1.3$ cm the polarization effect has reached an almost constant value and therefore, putting $d \to \infty$ is a good approximation. It must be emphasized, however, that an iron thickness of about 1.5 cm is not sufficient to prevent scattering from a copper coil wound around the iron cylinder (see fig. 4 and compare[3])).