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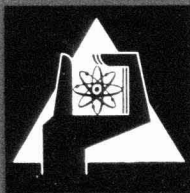
KFK 234

Institut für Strahlenbiologie

Comment on the Applicability of Compartment  
Theory to Interacting Biochemical Systems

P. F. Periti

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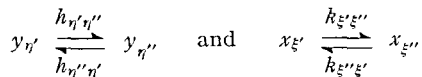
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Comment on the Applicability of ~~Zentralschere~~  
 Theory to Interacting Biochemical Systems

During the past two decades considerable progress has been made in biochemistry and in pharmacology due to development of the 'compartment theory'<sup>1-4</sup>. The mathematical discussion of the different phenomena which occur during metabolism of drugs or of biochemical compounds proved to be useful as a tool of analysis.

A fundamental property of living organisms is that there exist chemically defined compartments represented by compounds with 'catalytic' properties, i.e. compounds influencing the reactions taking place in another compartment-system<sup>5</sup>, as, for instance, in the case of a given hormone or enzyme which represents the end-compartment of one system of synthesis and simultaneously influences the kinetic reaction(s) between two or more compartments of another system.

If we express this fact in mathematical terms by means of the symbols used in the compartment theory<sup>6</sup>, we can designate the compartments of one system Y as  $y_1, y_2, \dots, y_n, \dots, y_m$  and the compartments of a second system X as  $x_1, x_2, \dots, x_\xi, \dots, x_n$ . The functions expressing the amount of metabolites in the x and in the y compartments are  $x = x(t)$  and  $y = y(t)$  respectively. The reactions schemes are then

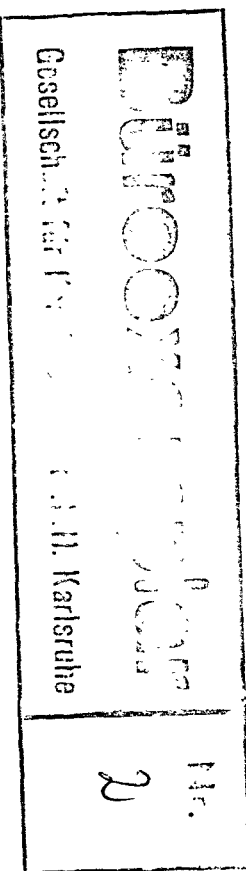


Such systems, as X and Y, can be solved with a system of ordinary linear differential equations with constant coefficients.

In the general case, i.e. if all the compartments of Y influence the kinetic coefficients  $k$  of the X system, we may write:

$$k_{\xi'\xi''} = f_{\xi'\xi''}(y_1, y_2, \dots, y_m)$$

where the function  $f$  indicates, in the simplest situation, linear functions of the  $y(t)$  functions. Such a case would



be for a set of enzymatically controlled reactions whose rates are proportional to the amounts of reacting enzymes.

The solutions of the system  $Y$  are represented by multi-exponential functions of the time. By substituting for the  $k$  the corresponding  $f$  functions, we obtain a system of  $n$  linear differential equations, but no longer a system with constant coefficients. This system, however, can be solved by the classical analytical methods.

If we consider two compartment-systems between which exist such relations as described above, we may write:

$$Y \rightrightarrows X$$

where the double arrow represents the dependence of the transfer rates of the system  $X$  upon the functions  $y(t)$ .

This relation can be extended straightforwardly to the general case of  $E$  compartment-systems which interact one with another by a well-known and defined set of functions. If the compartment system  $X_{\epsilon_0}$  of the system  $E$  possesses  $n_{\epsilon_0}$  compartments the total number of  $k$  functions of  $X_{\epsilon_0}$  will be  $n_{\epsilon_0} (n_{\epsilon_0} - 1)$ .

Because the total number of compartments of  $E$  is  $\sum_{\epsilon=1}^E n_{\epsilon}$ , every  $k$  of the  $X_{\epsilon_0}$  system will be expressed as a function of  $\left( \sum_{\epsilon=1}^E n_{\epsilon} \right) - n_{\epsilon_0}$  functions of the time. By substituting for every  $k$  of every compartment the corresponding expressions, one obtains a system of  $\sum_{\epsilon=1}^E n_{\epsilon}$  ordinary non-linear differential equations solved for the derivative.

Such an interpretation may prove to be useful in describing the interaction phenomena between several metabolic chains or between several endocrine glands; or, in general, in the description of every homeostatic mechanism.

*Riassunto.* Viene tratteggiata riassuntivamente l'impostazione matematica del problema dell'interazione fra più sistemi a compartimenti.

P. F. PERITI<sup>7</sup>

*Institut für Strahlenbiologie am Kernforschungszentrum  
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- <sup>7</sup> Present address: Istituto di Radiologia, Università degli Studi, Modena (Italy).