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The Balanced Oscillator Experiment

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Abstract. The "Balanced Oscillator Experiment" is a new type of oscillator experiment to measure transfer functions of nuclear fast reactors.

The technique consists in injecting in a reactor at the same time sinusoidal signals of reactivity and coolant flow of the same frequency and related each other in such a way that the coolant temperatures remain constant.

In addition to the Doppler reactivity coefficient, this new method allows to measure the fuel thermal conductivity and the heat transfer coefficient between fuel and coolant.

Numerical examples are included with reference to the Southwest Experimental Fast Oxide Reactor (Sefor). (Bibl. 3.)

1. Introduction

The "Balanced Oscillator Experiment" consists in injecting in a fast reactor at the same time sinusoidal signals of reactivity and coolant flow of the same frequency and related each other in such a way that the coolant temperatures remain constant.

During the experiment the inlet coolant temperature must be kept constant by other means which are discussed later in para 6.

In this way the Doppler effect is the only reactivity temperature effect which is present during the experiment.

Fig. 1 shows the schematic reactor flow diagram.

Fig. 2 shows a schematic diagram of all the signals. The input signals to reactor are:

(i) reactivity signal \( \Delta K = \Delta K_{m} \sin \omega t \), \( k \)  
(ii) coolant flow signal \( \Delta \mu = \Delta \mu_{m} \sin (\omega t + z) \).  

The amplitude ratio, \( \alpha = \Delta \mu_{m}/\Delta K_{m} \), and the phase shift \( z \) of the two input signals must be chosen at any frequency in such a way that the outlet coolant temperature, \( \Theta_{out} \), remains constant. That is:

\[ \Delta \Theta_{out} = 0. \]  

The outlet coolant temperature is measured by a thermocouple (Fig. 2). \( \Delta \mu_{m}/\Delta k_{m} \) and \( \alpha \) are obviously function of \( \omega \).

The power signal \( \Delta P \) is measured by means of a flux detector (Fig. 2). The "Balanced Transfer Function Analyzer" (Fig. 2) allows us to evaluate the two transfer functions

\[ \alpha = \frac{\Delta \mu_{m}/\Delta K_{m}}{\Delta \mu/\Delta \mu_{m}} \]  

and

\[ \alpha = \frac{\Delta P/\Delta \mu}{\Delta P/\Delta \mu_{m}} \]  

where "*" indicates Laplace transform

subscript "o" indicates steady state condition

\( \beta = \) fraction of delayed neutrons.

2. Physical Fundamentals

In this paragraph we intend to find out which conditions should the amplitude ratio \( \alpha = \Delta \mu_{m}/\Delta k_{m} \) and the phase shift \( z \) satisfy in order to keep the coolant temperatures constant.
The reactor can be considered as divided in “n” cooling channels each including a fuel rod and its associated coolant. The heat balance equation of the coolant in an average channel is the following:

\[
\frac{2\pi R h}{c} \frac{T_0 - \theta}{\mu n} = \frac{\partial \theta}{\partial z} + \frac{1}{v} \frac{\partial \theta}{\partial t}
\]  

(1)

where:

- \( h \) = heat transfer coeff. between fuel surface and coolant (including the cladding)
- \( R \) = radius of fuel rod
- \( c \) = specific heat capacity of the coolant
- \( T_0 \) = fuel surface temperature
- \( \theta \) = coolant temperature
- \( z \) = axial coordinate
- \( v \) = coolant speed
- \( t \) = time
- \( n \) = number of cooling channels.

Eq. (1) can give \( \partial \theta / \partial t = 0 \) only if:

\[
\frac{T_0 - \theta}{\mu} = T_0 - \theta_0 = \text{function of “z” only}
\]  

(2)

where subscript “0” indicates initial steady state conditions. Taking into account (2), eq. (1) becomes:

\[
\frac{2\pi R h}{c} \frac{T_0 - \theta}{\mu n} = \frac{\partial \theta}{\partial z} + \frac{1}{v} \frac{\partial \theta}{\partial t}.
\]  

(3)

If we introduce the change of coolant temperature \( \Delta \theta \):

\[
\Delta \theta = \theta - \theta_0.
\]  

(4)

eq. (3) becomes:

\[
\frac{2\pi R h}{c} \frac{T_0 - \theta}{\mu n} = \frac{d \theta}{dz} + \frac{1}{v} \frac{\partial \theta}{\partial t}.
\]  

(5)

It is:

\[
\frac{2\pi R h}{c} \frac{T_0 - \theta}{\mu n} = \frac{d \theta_0}{dz}.
\]  

(6)

Eq. (5) becomes therefore:

\[
\frac{d \theta_0}{dz} + \frac{1}{v} \frac{\partial \theta_0}{\partial t} = 0.
\]  

(7)

Eq. (7) associated to the boundary condition that the inlet coolant temperature, \( \theta_i \), is constant (\( \Delta \theta = 0 \)), gives:

\[
\Delta \theta = 0.
\]  

(8)

We have therefore shown that condition (2) is necessary and sufficient in order to keep the coolant temperatures constant with time.

Taking into account (4) and (8), condition (2) may be written as follows:

\[
\frac{\mu}{\mu_0} = \frac{T_0 - \theta}{T_0 - \theta_0} = \frac{T_0 - \theta_0}{T_0 - \theta_0}.
\]  

(9)

Introducing

\[
\Delta \mu = \mu - \mu_0
\]  

(10)

and

\[
\Delta T_0 = T_0 - T_0
\]  

(11)

eq. (9) becomes:

\[
\frac{\Delta \mu}{\mu_0} = \frac{\Delta T_0}{T_0 - \theta_0}.
\]  

(12)

The Laplace transform of eq. (12) is:

\[
\frac{\Delta \mu^*}{\mu_0} = \frac{\Delta T_0^*}{T_0 - \theta_0}.
\]  

(13)

It is:

\[
T_{eo} - \theta_0 = \frac{R}{2k} \frac{P_0}{V_f} M(z)
\]  

(14)

where:

- \( P_0 \) = reactor power at steady state
- \( V_f \) = volume of fuel in reactor = \( n \pi R^2 H \) (\( H \) being the height of the fuel rod)
- \( M(z) \) = normalized function expressing power distribution along the axis of a fuel rod

\[
\left| \frac{1}{H} \int_0^H M(z) dz = 1 \right|.
\]

Since the coolant temperatures are constant, it is [according to Ref. 2, para 2, eq. (20)]:

\[
\Delta T_0^* = \frac{R}{2k} \frac{1}{V_f} \int F(s) \cdot \Delta P^*(s) dt
\]  

(15)

where

\[
\Delta P^*(s) = \text{Laplace transform of the power change}
\]

and

\[
F(s) \cdot \Delta P = \text{normalized transfer function between fuel surface temperature and power}
\]

\[
F_0 = \text{fuel surface temperature}
\]

\[
F = \text{fuel specific heat capacity} \times \text{fuel thermal conductivity} \times \text{radius}^2.
\]

Putting (14) and (15) in (13), we have:

\[
\mu^* = \frac{1}{\mu} \frac{\Delta T_0^*}{T_0 - \theta_0} = F(s) \cdot \Delta P^*(s)
\]  

(16)

which is independent on the axial coordinate “z”.

In the time domain eq. (16) becomes:

\[
\frac{\Delta \mu}{\mu_0} = \frac{\Delta T_0}{T_0 - \theta_0} = \frac{1}{P_0} \int_0^t f(x) \Delta P(t - x) dx
\]  

(17)

where

\[
f(t) = L^{-1} [F(s) \cdot \Delta P]
\]  

(18)

and \( L^{-1} \) indicates antitransformation.

The demonstration given in this paragraph starts from the assumption (2) where \( \mu \) can be dependent on “z” which is physically impossible. At the end [eqs. (16) and (17)] we find out that \( \mu \) is function only of the time.

In Appendix 1 a more refined demonstration is given: starting from condition (17) where \( \mu \) is only a time dependent function, it is shown that the coolant temperatures remain constant.

Eq. (16) allows us to find which conditions should be the amplitude ratio \( A \Delta H_n \Delta K_m \) and the phase shift “x” satisfy, in order to keep the coolant temperatures constant.

It is:

\[
D(j\omega) = \frac{\Delta P^*(j\omega) \cdot \Delta \mu}{\mu_0} \frac{1}{F(s) \cdot \Delta P(j\omega)}
\]  

(19)

Taking into account (16) and (19) we have:

\[
\Delta T_0^*(j\omega) = \frac{\Delta \mu}{\mu_0} \cdot \frac{\Delta P^*(j\omega)}{\Delta \mu(j\omega)} \cdot \frac{\mu_0}{\mu(j\omega)} \cdot D(j\omega) F(s) \cdot \Delta P(j\omega)
\]  

(20)
and therefore:

\[ \frac{\Delta \mu_m}{\Delta \mu_n} = \frac{\mu_m}{\beta} |D(j\omega)| \cdot |F(j\omega t)| \]  (21)

\[ \alpha = \varphi_0(j\omega) + \varphi_s(j\omega t) \]  (22)

\varphi_0 and \varphi_s being the phases respectively of the functions \( D(j\omega) \) and \( F(j\omega t) \).

3. Results obtained from the new oscillator experiment

We have already said in para 1 that the “Balanced Transfer Function Analyser” allows us to evaluate the two transfer functions:

\[ E_s(j\omega t) = \frac{\Delta P_s(j\omega)|\mu_s}{\Delta P_s(j\omega)|P_2} \]  (1)

and

\[ D(j\omega) = \frac{\Delta P_s(j\omega)|\mu_s}{\Delta P_s(j\omega)|\beta} \]  (2)

As we have already shown in para 2 [eq. (16)], \( E_s(j\omega t) \) is the transfer function between fuel surface temperature and power.

3.1. Determination of the parameters \( t_r \) and \( \gamma \) from the transfer function \( E_s(j\omega t) = \frac{\Delta P_s(j\omega)|\mu_s}{\Delta P_s(j\omega)|P_2} \)

The author has found the following theoretical expression for \( E_s(\sigma) \) with \( \sigma = \omega t_r \) (Ref. 2):

\[ E_s(\sigma) = \frac{1}{1 + \gamma |Z(\sigma)|} \]  (3)

where

\[ t_r = \text{radial time scale} = \frac{v^2}{\lambda} R^2 = \frac{\text{fuel density} \times \text{fuel specific heat capacity}}{\text{fuel thermal conductivity} \times (\text{radius})^2} \]  (4)

\[ \gamma = \frac{\lambda}{2hR} = \frac{\text{fuel thermal conductivity}}{2 \times \text{heat transfer coefficient} \times \text{radius}} \]  (4')

\[ Z(\sigma) = -\frac{J_0((\omega - \sigma)\beta)}{\sigma J_1((\omega - \sigma))} \]  (5)

\( J_0 \) and \( J_1 \) being Bessel functions of the first kind.

Putting \( \sigma = \omega t_r \), we can write:

\[ Z(j\omega) = X(\nu) + j Y(\nu) \]  (6)

With \( X(\nu) \) and \( Y(\nu) \) respectively real and imaginary part of \( Z(j\nu) \). Introducing (6) in (3), we can write:

\[ \frac{1}{E_s(j\omega t_r)} = -\frac{\cos \varphi_s}{M_s} - j \frac{\sin \varphi_s}{M_s} \]  (8)

By comparing (7) with (8) we have:

\[ \frac{\cos \varphi_s}{M_s} = -\nu Y(\nu) \]  (9)

and

\[ -\frac{\sin \varphi_s}{M_s} = \nu X(\nu) + \gamma t_r \omega \]  (10)

The functions \(-\nu Y(\nu)\) and \(\nu X(\nu)\) have been calculated and are given respectively in Figs. 2 and 4.

For a chosen value of \( \nu \) we can evaluate \( \cos \varphi_s/M_s \) experimentally. Using eq. (8), and Fig. 3 we get \( \nu \) and, since \( \nu = \omega t_r \) or \( t_r \) in determined.

In eq. (10) the term \(-\sin \varphi_s/M_s \) on the left side is evaluated experimentally for the chosen \( \omega \). \( \nu X(\nu) \) is also known because \( \nu \) is known, and we can therefore determine \( \gamma \) and \( t_r \).

The functions \(-\nu Y(\nu)\) and \(\nu X(\nu)\) have been programmed on the IBM 7070 computer, so that \( t_r \) and \( \gamma \) can be more precisely determined by means of numerical methods instead of using the graphs of Figs. 3 and 4. From \( t_r \) and \( \gamma \), it is possible to evaluate the thermal conductivity, \( \lambda \), and the heat transfer coefficient, \( h \), if the density, \( \sigma \), the specific heat capacity, \( c \), and radius, \( R \), of the fuel rod have been previously determined.

3.2. Determination of the Doppler power coefficient and of the parameter \( \sigma t_r \), from the reactor transfer function

\[ D(j\omega) = \frac{\Delta P_s(j\omega)|\mu_s}{\Delta k^0|j\omega\beta} \]  (11)

Fig. 5 shows a schematic block diagram of the reactor transfer functions defined as:

\[ D_0(\sigma) = \frac{\Delta P_s(\sigma)|P_2}{\Delta k^0(\sigma)|\beta} \] = zero power transfer function,
\[ N(s) = \frac{A_k T_{ij}(s) \beta}{\Delta P^*(s) R} = \text{feedback transfer function}, \quad (12) \]

\[ D(s) = \frac{\Delta P^*(s) R}{A_k T_{ij}(s) \beta} = \frac{D_{ij}(s)}{1 + D_{ij}(s) \cdot N(s)} \quad (13) \]

where:

\[ A_k = \Delta k_i + \Delta k_j. \]

Since the coolant temperatures are constant, the reactivity feedback \((\Delta k_i)\) will depend only on the fuel temperatures. It is:

\[ A_k = \gamma_i A_{T_{eff}} \quad (15) \]

where:

\[ \gamma_i = \text{fuel temperature coefficient (mainly Doppler)} \]

\[ T_{eff} = \text{effective fuel temperature}. \]

The effective fuel temperature is defined by:

\[ A_{T_{eff}} = \int_{all \, rods} \Phi \frac{\Phi' \cdot A_{T_{av}}}{\Phi \cdot \Phi' \cdot dV} \quad (16) \]

\( \Phi \) and \( \Phi' \) being respectively flux and adjoint flux, \( V \) volume and \( T_{av} \) the average temperature of a section of a fuel rod. Since the coolant temperatures are constant, \( A_{T_{av}} \) will depend only on \( \Delta P \). It is [according to Ref. 2 para 2 eq. (22a)]:

\[ A_{T_{av}} = \frac{R}{2k} \left( 1 + \frac{1}{8 \gamma_i} \right) F_{av}(\delta \cdot t_r) M(\gamma_i) \frac{1}{\gamma_i} \Delta P^*(s) \quad (17) \]

Putting (16) and (17) in (15), and taking into account that \( V_f = n \pi R^2 H \), we get:

\[ A_k = G \cdot F_{av}(\delta \cdot t_r) \frac{1}{nH} \Delta P^*(s) \quad (18) \]

where:

\[ G = \text{Doppler power coefficient} = \frac{1}{\gamma_i} \frac{1}{2 \pi R H} \left( 1 + \frac{1}{8 \gamma_i} \right), \]

\[ A_{T_{av}} = \int_{all \, rods} \frac{\Phi \cdot \Phi' \cdot dV}{\Phi \cdot \Phi' \cdot dV} \quad (19) \]

Taking into account (18), eq. (12) becomes:

\[ N(s) = \frac{P_c}{nH \cdot \beta} \frac{1}{G} \cdot F_{av}(\delta \cdot t_r) \quad (21) \]

The author has shown in Ref. 2 that the function \( F_{av}(\delta \cdot t_r) \) with very good approximation is given by:

\[ F_{av}(\delta \cdot t_r) \approx 1 + \eta \cdot \sigma_1 \quad (22) \]

\( \sigma_1 \) being the first root of the Bessel functions equation:

\[ J_0(2\gamma) = 0. \]

Putting (22) in (21), we get:

\[ N(s) \approx \frac{P_c}{nH \cdot \beta} \frac{1}{G} \frac{1}{1 + \eta \cdot \sigma_1} \quad (24) \]

From eq. (13) we have:

\[ N(s) = \frac{1}{D(s)} - \frac{1}{D_{ij}(s)} \quad (25) \]

From (24) and (25) we obtain (putting \( s = jw \)):

\[ \frac{P_c}{nH \cdot \beta} \frac{1}{1 + j\omega \tau_i / \alpha_1} \approx \frac{1}{D(jw)} \frac{1}{D_{ij}(jw)}. \]

If \( \varphi_D \) and \( \varphi_R \) are the phases respectively of \( D(jw) \) and \( D_{ij}(jw) \), from (26) we have:

\[ \frac{P_c}{nH \cdot \beta} \frac{1}{1 + j\omega \tau_i / \alpha_1} \approx \frac{1}{D(jw)} \frac{1}{D_{ij}(jw)} \quad (27) \]

and

\[ \sigma_1 = \omega \cot \varphi_D \quad (28) \]

Putting (16), (17), and (18) into account (24), (25), (27), and (28), we obtain:

\[ \frac{P_c}{nH \cdot \beta} = \frac{1}{D(jw)} \frac{1}{D_{ij}(jw)} \quad (29) \]

and

\[ \frac{D(jw)}{D_{ij}(jw)} = \frac{1}{\sin \varphi_D} \quad (30) \]

Taking into account (29), (30) and (31), eq. (27) becomes:

\[ \lim_{\omega \to \infty} \frac{1}{D(jw)} = \frac{P_c}{nH \cdot \beta} \quad (32) \]

For \( \omega \) very small (in Sefer smaller than \( 3 \cdot 10^{-2} \text{ sec}^{-1} \)), the power transfer function \( D(j\omega) \) tends to the asymptotic value \( nH \cdot \beta / PG \). Since \( P_c/nH \beta \) and \( \beta \) are known, eq. (32) allows to determine the Doppler power coefficient \( G \). We can conclude that \( G \) can be determined by measuring the transfer function \( D(j\omega) \) only.

Eq. (33) allows to determine the parameter \( \sigma_1 / \tau_i \). From the point of view of the accuracy, it is convenient to carry out this evaluation when \( \varphi_D \approx 1 \) that is when \( \omega \approx \tau_i / \alpha_1 \).

The determination of \( \sigma_1 / \tau_i \) implies in the most general cases the measurement of both the transfer functions \( D(j\omega) \) and \( D_{ij}(j\omega) \). In some cases the conditions (29) and (30) are already satisfied in the frequency region under consideration and \( \sigma_1 / \tau_i \) is not more simply given by

\[ \sigma_1 / \tau_i = \omega \cot \varphi_D \quad (33) \]

This happens when:

\[ \sigma_1 / \tau_i \ll \lambda_\text{hor} \quad (34) \]

where

\[ \lambda_\text{hor} \equiv \frac{1}{\sum_{i=1}^{\omega} \beta_i} \quad (35) \]

\( \beta_i \) and \( \lambda_i \) being respectively fraction and decay constant associated to the \( i-th \) group of delayed neutrons. The determination of \( \sigma_1 / \tau_i \) can be used as a countercheck of the results obtained in para 3.1. Since \( \tau_i \) and \( \gamma \) have already been determined (para 3.1) and \( \sigma_1 \) is function of \( \gamma \) (Fig. 6), the ratio \( \sigma_1 / \tau_i \) can be also theoretically calculated and compared with that obtained experimentally.
4. Numerical examples for Sefor reactor

Figs. 7 and 8 show respectively amplitude ($M_\omega$) and phase ($\varphi_\omega$) of the function $F_\omega(j\omega)$ as it is expected to be in Sefor.

Taking for example $\omega = 0.0625$ rad/sec, we have:

\[ M_\omega = 0.4537 \quad \text{from Fig. 7,} \]
\[ \varphi_\omega = -51.75^\circ \quad \text{from Fig. 8.} \]

From (1) and (2), we have:

\[ \cos \varphi_\omega = \frac{0.4225}{M_\omega} = 1.375 = -vY(v), \]
\[ \sin \varphi_\omega = \frac{0.7826}{M_\omega} = 1.37 = vX(v) + \gamma t / \omega. \]

From Fig. 3, we see that:

\[ -vY(v) = 1.375 \text{ when } v = 10. \]

We can therefore determine $t_r$:

\[ t_r = \frac{v}{\omega} = \frac{10}{0.0625} = 160 \text{ secs.} \]

From Fig. 4, when $v = 10$, we have

\[ vX(v) \equiv 1.03. \]

From eq. (4) we get:

\[ \gamma t_r = \frac{1.73 - 1.03}{0.0625} = 11.2 \text{ secs} \]

and therefore we determine $\gamma$

\[ \gamma = \frac{11.2}{160} = 0.07. \]

From Fig. 6 for $\gamma = 0.07$ we get:

\[ \sigma_1 = 4.4068 \]

and

\[ \sigma_1 = \frac{4.4068}{160} = 0.0276 \text{ rad/sec.} \]

Figs. 9 and 10 show respectively amplitude and phase of the power transfer function $D(j\omega)$ in the low frequency region as it is expected to be in the case of Sefor.

For $\omega < 3 \cdot 10^{-3}$ sec$^{-1}$, $|D(j\omega)|$ tends to the asymptotic value of 0.424 $\text{s}^{-1}$. From eq. (28) of para 3.2 putting $\omega = 0$, we get

\[ \frac{P_n \cdot \beta}{nH} \beta = 1 = 2.36 \text{ s.} \]

Since:

\[ P_n = 20 \text{ MW,} \]
\[ nH = 500 \text{ m,} \]
\[ \beta = 3.395 \cdot 10^{-3}, \]

$G$ can be determined

\[ G = \frac{2.36 \cdot 500 \cdot 3.395 \cdot 10^{-3}}{20} = 0.2 \text{ A k} \cdot \text{MW.} \]

For $\omega = 0.029$ sec$^{-1}$ (which is not too different from the theoretical value of $\sigma_1/\beta$, given by (11)), we have:

\[ |D_\alpha| = 4.24 \quad \text{(from Fig. 11),} \]
\[ |D| = 0.622 \quad \text{(from Fig. 9),} \]
\[ \varphi_\alpha = -55^\circ \quad \text{(from Fig. 12),} \]
\[ \varphi_D = +36^\circ \text{S} \quad \text{(from Fig. 10).} \]
Using eq. (28) of para 3.2, we have:
\[
\sigma_{t} = 0.629 \cdot 1.33 \cdot \frac{0.622}{1 - 4.24 - 0.8} \cdot \frac{0.622}{1 + 4.24 - 0.586} = 0.629 \cdot 1.33 \cdot 0.740 = 0.6286 \text{ (sec}^{-2}\text{)}.
\]
The value of \(\sigma_{t}/\gamma\) calculated by (21) differs slightly from that given by (11) because the reactivity feedback transfer function is only approximately expressed by one pole [eqs. (22) and (24) of para 3.2].

5. Comparison with the traditional oscillator experiment

With the traditional oscillator experiment only the reactivity signal is introduced in the reactor. The advantages of the “balanced oscillator experiment” in comparison with the traditional one, are mainly the following:

(i) Since the coolant temperatures are constant, it is possible to separate the Doppler temperature effect on reactivity from the other temperature effects. It is a real clean oscillator experiment.

(ii) The normalized transfer function \(F_{c}(j\omega_{t})\) between fuel surface temperature and power is determined by indirect measurements. The direct measurement of \(F_{c}(j\omega_{t})\) would imply the measurement of the fuel surface temperature, which is technically difficult and inaccurate. The measurement of \(F_{c}(j\omega_{t})\) allows to determine the parameters \(\gamma\) and \(\gamma_{t}\) and therefore the fuel conductivity, \(\lambda_{f}\), and the heat transfer coefficient, \(h\), can be calculated.

(iii) The normalized transfer function \(F_{av}(j\omega_{t})\) between average fuel temperature and power is determined and therefore the parameter \(\sigma_{av}/\gamma_{av}\) can be calculated.

With the traditional type of oscillator experiment the transfer function \(F_{c}(j\omega_{t})\) cannot be determined and therefore \(\gamma\) and \(\gamma_{t}\) cannot be evaluated.

In addition, since the coolant temperatures are not kept constant, the Doppler temperature effect on reactivity is not rigorously separated from the other temperature effects. The calculation of the Doppler power coefficients \(G_{t}\), and of the parameter, \(\sigma_{t}/\gamma_{t}\), from the power transfer function \(D(j\omega_{t})\) is therefore more complicated and it is dependent upon the knowledge of the other reactivity coefficients and their associated time constants.

6. Final Comments

The method of introducing in a system two or more sinusoidal signals related in such a way that a specific physical quantity easily measurable does not change, can be considered a very general method to measure transfer functions indirectly. This “balance technique” may have a wide application especially when it is difficult to carry out the direct measurement of a transfer function.

A simple and well known example of “balance technique” is the Wheatstone Bridge to measure electric impedances. In the Wheatstone Bridge the impedences are balanced in modulus and phase in such a way that no current passes through the diagonal. When this condition is fulfilled, the unknown impedance can be determined by a simple relationship with the other three known impedences.

To end our comments about the application of the “balanced oscillator experiment” on Sefor, we must say that the coolant flow signal may cause a noticeable disturbance in the inlet coolant temperature, \(\Theta_{t}\), through the primary heat exchanger. Since \(\Theta_{t}\) must be kept constant during the experiment, it is necessary to balance this effect. This may be obtained by introducing in the system a third signal \(\Delta \xi = \Delta \xi_{p} \sin (\omega_{t} + \delta)\) to the pump of the secondary coolant circuit (Fig. 1). \(\Delta \xi_{p}\) and \(\delta\) must of course chosen in such a way that no change occurs in \(\Theta_{t}\).

A better solution could be obtained by putting a by-pass value across the primary heat exchanger from the side of the primary coolant circuit (Fig. 1). This value should of course be operated in such a way that \(\Theta_{t}\) remains constant during the experiment.

In the analysis developed in this paper, the thermal capacity of the fuel cladding has been purposely neglected in order to show the essential parts of the new experiment. In the heat transfer coefficient “\(h\)” are included the heat transfer coefficients fuel to cladding, internal to external surface of the cladding and cladding to coolant. However, if the thermal capacity of the fuel cladding must be taken into account, the philosophy of the experiment is still valid, but the mathematical relationships will be slightly more complicated.

Appendix I

**Demonstration that the coolant temperatures, \(\Theta_{t}\), remain constant during the experiment \([4\Theta(z, t) = 0] \)**

The heat balance equation of the coolant in a cooling channel is the following:

\[
2\pi Rh \frac{T_{c} - \Theta}{\rho c_{p}} = \frac{\partial \Theta}{\partial t} + \frac{1}{c_{p} c_{t}} \partial \Theta \partial t
\]

(i)
where:

\[ R = \text{radius of fuel rod} \]
\[ h = \text{heat transfer coefficient between fuel and coolant (including the cladding)} \]
\[ c = \text{specific heat capacity of the coolant} \]
\[ \mu = \text{coolant flow} \]
\[ T_s = \text{surface fuel temperature} \]
\[ z = \text{axial coordinate} \]
\[ v = \text{cooler flow speed} \]
\[ t = \text{time} \]
\[ n = \text{number of cooling channels}. \]

We introduce:

\[ T_i = T_{i0} + \Delta T_{i}, \quad (2) \]
\[ \Theta = \Theta_{0} + \Delta \Theta, \quad (3) \]
\[ \mu = \mu_{0} + \Delta \mu, \quad (4) \]

where subscript \( _0 \) indicates initial steady state conditions and \( \Delta \) variation from steady state condition.

The fuel surface temperature, \( T_s \), may be expressed as function of the coolant temperature, \( \Theta \), and of the power, \( P \) [according to Ref. 2 para 2 eq. (20)]:

\[
\Delta T_s^* (s, t; z) = G_{i}(s, t) \Delta \Theta^* (s; z) + \frac{R}{2h} F_{i}(s, t) \frac{\Delta P^*(s)}{V_j} M(z) \quad (5)
\]

where:

\( \Delta T_s^* \) indicates Laplace transform
\( s = \text{complex variable of Laplace transformation} \)
\( V_j = \text{volume of fuel in reactor} = n \pi R^2 H \) (\( H \) being the height of the fuel rod)
\( M(z) = \text{normalized function expressing power distribution along the axis of a fuel rod} \)
\( t_r = \text{radial time scale} = \frac{0.67}{R} \]
\( \gamma = \frac{\lambda}{2hR} \)

Eq. (5) becomes:

\[
\Delta T_s^* (s; z) = \Delta \Theta^* (s; z) \left[ 1 - \gamma t_r s F_{i}(s, t) \right] + \frac{R}{2h} F_{i}(s, t) \frac{\Delta P^*(s)}{V_j} M(z) \quad (6)
\]

It is [Ref. 2 para 2 eq. (24)]:

\[
G_{i}(s, t) = 1 - \gamma t_r s F_{i}(s, t) \quad (7)
\]

Differentiating (15) in respect to \( z \) and putting \( z = 0 \), we get:

\[
\left( \frac{\partial \Delta \Theta}{\partial z} \right)_{z=0} = 0. \quad (18)
\]

By successive differentiations we get for each \( m \)

\[
\left( \frac{\partial^m \Delta \Theta}{\partial z^m} \right)_{z=0} = 0. \quad (20)
\]
We can conclude that the solution of eq. (15) with the boundary condition (16) is:

$$\Delta \theta(z; t) = 0.$$  \hspace{1cm} (21)

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References:


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