KFK-260

KERNFORSCHUNGSZENTRUM

KARLSRUHE

Februar 1966

KFK 260

Institut für Angewandte Reaktorphysik

Experiments to Evaluate Reactivity Coefficients with Reactor Operating at Steady State Conditions (Static Experiments)

L. Caldarola, W. Häfele, W. Schikarski

GESELLSCHAFT FUR KERNFORSCHUNG M.B.H.

KARLSRUHE

KERNFORSCHUNGSZENTRUM KARLSRUHE

Februar 1966 K F K 260

Institut für Angewandte Reaktorphysik

Experiments to evaluate reactivity coefficients with reactor operating at steady state conditions (Static Experiments)+)

von

L. Caldarola++)
W. Häfele+++)
W. Schikarski+++)

Gesellschaft für Kernforschung m.b.H., Karlsruhe

+) Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung m.b.H., Karlsruhe

++) Euratom, Brussels, delegated to the Karlsruhe Fast Breeder Project, Institut für Angewandte Reaktorphysik

+++) Kernforschungszentrum Karlsruhe - Institut für Angewandte Reaktorphysik

Abstract

The experiments to evaluate reactivity coefficients with reactor operating at steady state conditions are described.

Three reactivity coefficients are defined

(i) the global power coefficient, G, referred to the reactor power

(ii) the global coolant temperature coefficient, γ, referred to the average coolant temperature

(iii) the secondary radial expansion temperature coefficient, γ_r, referred to the coolant temperature rise in the reactor.

Each coefficient is measured by a separate experiment in which the reactivity effects due to the other two are made equal to zero.

The error analysis is developed with numerical examples referred to the Southwest Experimental Fast Oxide Reactor (SEFOR).

With the aim of minimizing the total error, attention is given to the choice of the best power interval for the measurement of the global power coefficient.

The control phylosophy of the reactor system during the experiments is also studied. (auth.)

Contents

Corrigendum

page 1	line 19 from above: read "mainly"
page 2	line 7 from below: after "parameters T_{eff}", read komma instead of period.
page 3	line 11 from above: after "primary coolant", read komma instead of period.
page 8	formula (8): read $E_r = \sqrt{\varepsilon_o^2 + \varepsilon_i^2 + \varepsilon_\rho^2 + \varepsilon_\Phi^2} = \frac{A}{\Delta P}$ = total random error
page 16	formula (3): read $\bar{\xi} = \left\lvert \frac{\gamma}{\gamma_r} \frac{\Delta\bar{\Theta}}{\Delta\Theta_r} \right\rvert$
page 23	formula (11): read $\Gamma_f = \frac{dk_f}{dT_{eff}} \approx \frac{V_f}{V_{reactor}} \alpha_o$

1. Introduction

One of the most important safety features of fast power reactors arises from negative reactivity temperature and power coefficients. Therefore the experimental evaluation of reactivity coefficients and its dependence on the various reactor parameters are essential. Besides the possibilities to measure experimentally reactivity coefficients by dynamic methods, the determination can be performed by the so called "Static Experiments".

With the "Static Experiments", the power and the temperature coefficients are determined by means of reactivity, power and temperature measurements carried out at steady state conditions. The experiments described in this paper will be carried out on the SEFOR (Southwest Experimental Fast Oxide Reactor - Bibl.1) and refers in some particular points to this reactor. However, applicability to other reactors is possible. The authors (Bibl.6) have shown that Doppler reactivity effects in a core can be analyzed in terms of P/nH (Power/number of fuel rods x rod height) and that the radius of the fuel rod does not enter directly. Thus, one can examine the Doppler effect in a high power reactor with a large number of small radius fuel rods by doing experiments in a low power reactor like SEFOR with a small number of large radius fuel rods.

The reactivity changes with reactor temperatures are manily due to the following causes:

a) Changes of fuel temperatures, T

These generate a change in reactivity due to the Doppler effect and to the fuel expansion. This second effect is less important because of the small expansion coefficient of oxide materials and because of the special construction of the SEFOR fuel element.

b) Changes of cladding temperatures, T_c

These generate variations in the neutron leakage, which in turn produce a change in reactivity. In the SEFOR case the reactivity effect due to the cladding expansion may be considered as dependent on the coolant temperatures, Θ, since cladding and coolant temperatures differ only slightly.

c) Changes of coolant temperatures, Θ

These generate variations in the neutron leakage and in the neutron moderation and absorption of Sodium, which in turn produces a change in reactivity.

d) Changes of reactor structure temperatures

These changes generate mainly variations in the radial dimensions of the reactor and therefore in the neutron leakage and in the Sodium quantity present in the core. The temperature of reactor core structures depends on the primary coolant reactor inlet and outlet temperatures (Θ_i and Θ_o). In the following pages we shall consider this reactivity effect due to core expansion as dependent on the average coolant temperature $\bar{\Theta}$ and the primary coolant temperature rise, Θ_r.

From these observations we can conclude that it is possible to consider the reactivity effects as dependent mainly on the three following parameters

(i) An effective value of fuel temperature, T_{eff} (Appendix 1)

(ii) The average primary coolant temperature, $\bar{\Theta} = (\Theta_o+\Theta_i)/2$

(iii) The primary coolant temperature rise, $\Theta_r = \Theta_o-\Theta_i$.

In order to adjust these three parameters we have three independent variables, namely the reactivity, Δk, the primary coolant flow μ, and the secondary coolant flow, ν. Θ_i depends mainly on ν, whereas μ and Δk influence strongly both T_{eff} and the Sodium outlet temperature Θ_o. $\bar{\Theta}$ depends directly on Θ_o and Θ_i and it is therefore theoretically possible to adjust ν, μ and Δk in such a way that only one of the three parameters T_{eff}. $\bar{\Theta}$ and Θ_r changes whereas the two others are kept constant. This should give a possibility to measure the various temperature coefficients separately.

The outlet temperature Θ_o is a "true" reactor variable because it exists physically behind the mixing plenum. It is possible therefore to measure it in a definite procedure. The same is true for the inlet temperature Θ_i. But it is difficult to measure T_{eff}. T_{eff} does not exist as such: it is a mathematical quantity. On the other hand it is possible to control its constancy. For that only the constancy of the average coolant temperature and of the power is necessary (App.2). This can be measured. The real inherent value of a static test in SEFOR is the direct access to the universal power coefficient, G. From this one gets the advice to measure as a third variable the reactor power. The relative changes of the reactor power can be measured by ionization chamber techniques, which are fairly accurate. We therefore have the following scheme of measuring two temperature and one power coefficients.

2. Physical Fundamentals

The reactor system is illustrated schematically in Fig.1. The heat produced in the core is removed by the primary coolant. Sodium, and is then transferred to the secondary Sodium circuit. From here the heat is rejected to the atmosphere by open circuit forced air cooling.

In the following investigation these symbols have been used:

Δk = total reactivity change due to temperature variations in reactor

Δk_f = reactivity change due to variations of the effective fuel temperature

Δk_{Na} = reactivity change due to variations of the average Sodium temperature

Δk_c = reactivity change due to variations of the average cladding temperature

Δk_{st} = reactivity change due to the expansion of the reactor structures

T_{eff} = effective fuel temperature defined in Appendixes 1 and 2

Θ_i = primary coolant reactor inlet temperature

Θ_o = primary coolant reactor outlet temperature

$T_{c\ eff}$ = effective cladding temperature

$\bar{\Theta}$ = primary coolant average temperature = $\frac{\Theta_o + \Theta_i}{2}$

P = power generated by reactor

n = number of fuel rods in reactor

H = height of the fuel rod

Γ_f = differential reactivity temperature coefficient due to the fuel, referred to the effective fuel temperature

Γ_c = differential reactivity temperature coefficient due to the cladding, referred to the effective cladding temperature

Γ_{Na} = differential reactivity temperature coefficient due to the coolant, referred to the average coolant temperature

Γ_{st} = main differential reactivity temperature coefficient due to the expansion of the reactor structures, referred to the average coolant temperature

Γ_r = secondary differential reactivity temperature coefficient due to the expansion of the reactor structures, referred to the coolant temperature rise in the reactor

Γ_p = differential power reactivity coefficient

μ = primary circuit Sodium flow

ν = secondary circuit Sodium flow

The static change in reactivity, Δk, due to temperature effects in a nuclear reactor, which moves from steady state "1" to steady state "2", is the following:

$$\Delta k = \Delta k_f + \Delta k_c + \Delta k_{Na} + \Delta k_{st} \qquad (1)$$

where:

$$\Delta k_f = \gamma_f (T_{2eff} - T_{1eff}) = \gamma_f \, \Delta T_{eff} \qquad (2)$$

$$\Delta k_c = \gamma_c (T_{c2eff} - T_{c1eff}) = \gamma_c \, \Delta T_{ceff} \qquad (3)$$

$$\Delta k_{Na} = \gamma_{Na} (\bar{\theta}_2 - \bar{\theta}_1) = \gamma_{Na} \, \Delta \bar{\theta} \qquad (4)$$

$$\Delta k_{st} = \gamma_{st} (\bar{\theta}_2 - \bar{\theta}_1) + \gamma_r \, (\theta_{r2} - \theta_{r1}) = \gamma_{st} \, \Delta \bar{\theta} + \gamma_r \, \Delta \theta_r \qquad (5)$$

and:

$$\gamma_f = \frac{1}{\Delta T_{eff}} \int_1^2 \Gamma_f \cdot dT_{eff} \qquad (6)$$

$$\gamma_c = \frac{1}{\Delta T_{ceff}} \int_1^2 \Gamma_c \cdot dT_{ceff} \qquad (7)$$

$$\gamma_{Na} = \frac{1}{\Delta \bar{\theta}} \int_1^2 \Gamma_{Na} \cdot d\bar{\theta} \qquad (8)$$

$$\gamma_{st} = \frac{1}{\Delta\bar{\theta}} \int_1^2 \Gamma_{st} \cdot d\bar{\theta} \tag{9}$$

$$\gamma_r = \frac{1}{\Delta\theta_r} \int_1^2 \Gamma_r \, d\theta_r \tag{10}$$

In Appendix 2 we see that it is possible in the practical cases to express the effective fuel temperature, T_{eff}, for a given flux distribution, as function of the average coolant temperature $\bar{\Theta}$ and of the power P.

$$T_{eff} = \bar{\Theta} + A_f R_f \frac{P}{nH} \tag{11}$$

The product $A_f R_f$ is a function of P and a linear function of $\bar{\Theta}$. $A_f R_f$ is a mathematical quantity. It can be calculated only by knowing flux and temperature distributions in the reactor. The knowledge of $A_f R_f$ would mean therefore a tremendous number of measuring points in the reactor, which is not practical. We can conclude that $A_f R_f$ is unknown and T_{eff} therefore is also unknown. On the other hand we can control the constancy of T_{eff} by keeping $\bar{\Theta}$ and P constant. Since T_{eff} is practically not measurable, it seems convenient to divide its effect on reactivity in two parts: one referred to the average coolant temperature $\bar{\Theta}$ and the other to power P. We can write

$$\Delta k_f = G_f \Delta \left(\frac{P}{nH}\right) + \gamma_{fc} \Delta\bar{\Theta} \tag{12}$$

where G_f and γ_{fc} are defined respectively by eqs.(59) and (58) in Appendix 2.

The same considerations can be applied to the reactivity effect dependent on the effective cladding temperature T_{ceff}:

$$\Delta k_c = G_c \Delta \left(\frac{P}{nH}\right) + \gamma_c' \Delta\bar{\Theta} \tag{13}$$

In the SEFOR case cladding and coolant temperatures differ only slightly, $T_{ceff} \cong \bar{\Theta}$. It is therefore:

$$\gamma_c' \cong \gamma_c \tag{14}$$

Taking into account eqs. (4,5,12,13), eq. (1) can be written as follows

$$\Delta k = G\Delta \left(\frac{P}{nH}\right) + \gamma\Delta\bar{\theta} + \gamma_r \Delta\theta_r \qquad (15)$$

where:

$G = G_f + G_c$ = global power coefficient (in SEFOR $G \cong G_f$) (16)

$\gamma = \gamma_{Na} + \gamma_{fc} + \gamma_c' + \gamma_{st}$ = global coolant coefficient

γ_r = secondary radial expansion temperature coefficient due to change of coolant temperature rise in the reactor. (in SEFOR $\gamma_r \cong 0$) (18)

We shall measure the three coefficients G, γ and γ_r separately by means of three different static experiments.

I Measurement of the global power coefficient G

The coolant temperatures (inlet and outlet and therefore also the average) are held constant as power is changed. This is equivalent to put $\Delta\bar{\theta} = \Delta\theta_r = 0$ in eq. (15).

We get:

$$G = \frac{\Delta k}{\Delta(P/_{nH})} \qquad (19)$$

II Measurement of the global coolant temperature coefficient γ

Coolant temperature rise and power are held constant as average coolant temperature is changed. This is equivalent to put $\Delta P = \Delta\theta_r = 0$ in eq. (15).

We get:

$$\gamma = \frac{\Delta k}{\Delta\bar{\theta}} \qquad (20)$$

III Measurement of the secondary radial expansion temperature coefficient γ_r

Power and average coolant temperature (and therefore the effective fuel temperature) are held constant as coolant temperature rise is changed. This is equivalent to put $\Delta P = \Delta\bar{\theta} = 0$ in eq. (15).

We get:

$$\gamma_r = \frac{\Delta k}{\Delta\theta_r} \qquad (21)$$

The average coolant temperature is held constant in this experiment by changing outlet and inlet coolant temperatures of the same amount in opposite directions ($\Delta\theta_o = - \Delta\theta_i$).

For completeness we must say that a fourth static experiment is also possible by means of which the over-all isothermal temperature coefficient, γ_{is}, of the reactor is measured. During this experiment the reactor is kept just critical without power production. Fuel and coolant temperatures are therefore all equal to the inlet coolant temperature. If this temperature is changed, there is a change also in reactivity:

$$\Delta k = (\gamma + \gamma_r)\, \Delta\theta_i = \gamma_{is}\, \Delta\theta_i \tag{22}$$

This fourth experiment can be used as a countercheck of the second and the third static experiments. Since this last is a very well known classical experiment, in the following pages only the first three static experiments will be analysed.

3. The measurement of the global power coefficient

3.1 Classification of the errors

As already stated in para 2, during the experiment, the coolant temperatures (inlet and outlet) are held constant as power is changed. The global power coefficient, G, is than given by:

$$G = \Delta k / \Delta \left(\frac{P}{nH}\right) \tag{1}$$

From eq. (15) of para 2 we get the following errors (according to Bibl.7):

$$\mathcal{E}_o = \left| \frac{nH(\frac{\gamma}{2} + \gamma_r)}{G} \, \frac{\Delta\theta_o}{\Delta P} \right| \tag{2}$$

is the random error due to the not exact constancy of the outlet coolant temperature, θ_o, during the experiment.

$$\mathcal{E}_i = \left| \frac{nH(\frac{\gamma}{2} - \gamma_r)}{G} \, \frac{\Delta\theta_i}{\Delta P} \right| \tag{3}$$

is the random error due to the not exact constancy of the inlet coolant temperature, θ_i, during the experiment.

$$\mathcal{E}_{\varsigma} = \left|\frac{\delta\Delta k}{\Delta k}\right| = \left|\frac{\sqrt{2}\,\delta k}{\Delta k}\right| = \sqrt{2}\left|\frac{nH}{G}\,\frac{\delta k}{\Delta P}\right| \qquad (4)$$

is the random error due to the measurement of the reactivity interval. "δk" is the error done in measuring each end of the interval.

$$\mathcal{E}_{p} = \left|\frac{\delta\Delta P}{\Delta P}\right| \qquad (5)$$

is the error due to the measurement of the power interval. This error may be divided in two parts: $\mathcal{E}_{\phi}$ is the random error due to the measurement of the current, I, produced by the neutron flux detector and $\mathcal{E}_K$ is the systematic error due to the determination of the calibration constant, K, of the neutron flux detector.

$$\mathcal{E}_{p} = \frac{\delta\Delta P}{\Delta P} = \left|\frac{\delta\Delta(KI)}{\Delta(KI)}\right| = \left|\frac{\sqrt{2}\,\delta I}{\Delta I}\right| + \left|\frac{\delta K}{K}\right| = \left|\frac{\sqrt{2}\,\delta P}{\Delta P}\right| + \left|\frac{\delta K}{K}\right| = \mathcal{E}_{\phi} + \mathcal{E}_{K} \qquad (6)$$

$\mathcal{E}_K$ has been defined supposing that the calibration curve of the detector and of all the measuring equipment is linear, which is true if the instrumentation is used in the proper range. The calibration constant K will be determined by means of the calorimetric method with reactor at full power (Appendix 3).

Taking into account the definition of the errors from (2) to (6) and their nature (systematic or random), we can write that the total experimental error, E, is given by:

$$E = \mathcal{E}_K + \sqrt{\mathcal{E}_o^2 + \mathcal{E}_i^2 + \mathcal{E}_{\varsigma}^2 + \mathcal{E}_{\phi}^2} = \mathcal{E}_K + E_r \qquad (7)$$

where:

$\mathcal{E}_K$ = total systematic error

$$E_r = \sqrt{\mathcal{E}_o^2 + \mathcal{E}_i^2 + \mathcal{E}_{\varsigma}^2 + \mathcal{E}_{\Phi}^2} = \frac{A}{\Delta P} = \text{total random error} \qquad (8)$$

with:

$$A = \sqrt{\left[\left(\frac{\gamma}{2}+\gamma_r\right)\frac{nH}{G}\,\Delta\theta_o\right]^2 + \left[\left(\frac{\gamma}{2}-\gamma_r\right)\frac{nH}{G}\,\Delta\theta_i\right]^2 + \left[\sqrt{2}\,\frac{nH}{G}\,\delta k\right]^2 + \left[\sqrt{2}\;\delta P\right]^2} \qquad (9)$$

3.2 Choice of the power interval, ΔP

The power interval ΔP must be chosen in such a way that:

$$\left|\Gamma_{P_2}\right| < \left|G \mp \delta G\right| < \left|\Gamma_{P_1}\right| \tag{1}$$

where Γ_{P_1} and Γ_{P_2} are the differential global power coefficients respectively at power P_1 and P_2 ($P_2 > P_1$), G is the average global power coefficient in the interval between P_1 and P_2 and δG is the total absolute experimental random error. It is:

$$\left|\delta G\right| = E_r \left|G\right| \tag{2}$$

with E_r given by eq.(8) of para 3.1. From eq.(1) we can write:

$$\left|\delta G\right| < \left|G - \Gamma_P\right| \tag{3}$$

Introducing:

$$\sigma = \left|1 - \frac{\Gamma_P}{G}\right| \tag{4}$$

we obtain, taking into account (2):

$$E_r < \sigma \tag{5}$$

that is:

$$\frac{A}{\Delta P} < \sigma \tag{6}$$

In a fast reactor we can assume that the Doppler differential temperature coefficient Γ_f in first approximation is given by:

$$\Gamma_f = \frac{dk}{dT_{eff}} = \frac{C}{T_{eff}^{\,m}} \quad \text{(Appendix 1)} \tag{7}$$

with C and m constant and T_{eff} in °K.

By integrating eq.(7), we obtain:

$$k_f = B - \frac{C}{(m-1)T_{eff}^{m-1}} \tag{8}$$

Taking into account eq.(11) of para 2, eq.(8) becomes:

$$k_f = B - \frac{C}{(m-1)(\bar{\Theta} + \frac{A_fR_f}{nH} P)^{m-1}} = B - \frac{C}{m-1}\left(\frac{nH}{A_fR_f}\right)^{m-1} \frac{1}{(\frac{nH}{A_fR_f}\bar{\Theta} + P)^{m-1}} \tag{9}$$

and the differential Doppler power coefficient Γ_{Pf} is (Appendix 2 eq.49):

$$\Gamma_{Pf} = nH\left(\frac{\partial k_f}{\partial P}\right)_{\substack{\bar{\Theta} = const \\ \Theta_r = const}} = \frac{C}{(\frac{nH}{A_fR_f}\bar{\Theta} + P)^m} \cdot \left(\frac{nH}{A_fR_f}\right)^{m-1} \cdot nH \tag{10}$$

The average Doppler power coefficient, G_f, is given by:

$$G_f = - \frac{C(\frac{nH}{A_fR_f})^{m-1}}{m-1} \; \frac{\left[\frac{1}{\left[\frac{nH}{A_fR_f}\bar{\Theta} + P + \Delta P\right]^{m-1}} - \frac{1}{\left[\frac{nH}{A_fR_f}\bar{\Theta} + P\right]^{m-1}}\right]}{\Delta(P/nH)} \tag{11}$$

As we have already said in Sefor it is:

$$\Gamma_p \cong \Gamma_{Pf} \tag{12}$$

and

$$G \cong G_f \tag{13}$$

Taking into account (7) and (8), (12) and (13), eq.(4) becomes:

$$\sigma = \left| 1 - (m-1) \frac{\Delta P/(\frac{nH}{A_fR_f}\bar{\Theta} + P)}{1 - \frac{1}{\left[1 + \Delta P/(\frac{nH}{A_fR_f}\bar{\Theta} + P)\right]^{m-1}}} \right| \tag{14}$$

If

$$\frac{\Delta P}{\frac{nH}{A_f R_f}\bar{\Theta} + P} \ll 1 \qquad (15)$$

we have:

$$\sigma \cong \frac{m}{2} \, \frac{\Delta P}{\frac{nH}{A_f R_f}\bar{\Theta} + P} \qquad (16)$$

Condition (6) becomes:

$$\frac{A}{\Delta P} < \frac{m}{2} \, \frac{\Delta P}{\frac{nH}{A_f R_f}\bar{\Theta} + P} \qquad (17)$$

We can calculate the boundary value ΔP_b so defined that, if $\Delta P > \Delta P_b$, condition (17) is satisfied:

ΔP_b is given by the following equation:

$$\frac{m}{2\left(\frac{nH}{A_f R_f}\bar{\Theta} + P\right)} \, \Delta P_b^{\,2} - A = 0 \qquad (18)$$

from which:

$$\Delta P_b = \sqrt{\frac{2}{m} A\left(\frac{nH}{A_f R_f}\bar{\Theta} + P\right)} \qquad (19)$$

Putting (19) respectively in (16) and in eq.(8) of para 3.1, we get:

$$\sigma_b = E_{rb} = \sqrt{\frac{m}{2} \, \frac{A}{\frac{nH}{A_f R_f}\bar{\Theta} + P}} \qquad (20)$$

3.3 Numerical evaluations in the Sefor case

In the Sefor case we have:

$$nH = 500 \text{ m} \qquad (1)$$

$$A_f R_f \cong 1.585 \cdot 10^4 \ {}^oK \cdot m/MW \qquad (2)$$

$$\frac{A_c R_c}{A_f R_f} \cong 0.0167 \qquad (3)$$

$$\gamma_f = - 1.8 \cdot 10^{-3} \ \text{ø}/{}^oK \qquad (4)$$

$$G_f = - 28.6 \ \text{ø} \ m/MW \qquad (5)$$

$$\gamma_c = - 3 \cdot 10^{-3} \ \text{ø}/{}^oK \qquad (6)$$

$$G_c \cong - 0.8 \ \text{ø} \ m/MW \qquad (7)$$

$$G = G_f + G_c - 29.4 \ \text{ø} \ m/MW \qquad (8)$$

$$\gamma_{Na} = - 7.2 \cdot 10^{-4} \ \text{ø}/{}^oK \qquad (9)$$

$$\gamma_{st} = - 3.28 \cdot 10^{-3} \ \text{ø}/{}^oK \qquad (10)$$

$$\gamma = - 8.8 \cdot 10^{-3} \ \text{ø}/{}^oK \qquad (11)$$

$$\gamma_r \cong 0 \qquad (12)$$

$$NH \frac{\frac{\gamma}{2}+\gamma_r}{G} = 7.5 \cdot 10^{-2} \ MW/{}^oK \qquad (13)$$

$$NH \frac{\frac{\gamma}{2}-\gamma_r}{G} \cong 7.5 \cdot 10^{-2} \ MW/{}^oK \qquad (14)$$

$$\bar{\theta} = 404^oC = 677^oK \qquad (15)$$

We shall assume that inlet and outlet temperatures during the experiment are kept constant within ∓ 1°K. That is:

$$\Delta\theta_o = \Delta\theta_i = \mp 1^oK \qquad (16)$$

We shall also assume:

$$\delta k = \mp 5 \cdot 10^{-3} \ \text{ø} \qquad (17)$$

and

$$\delta P = 0.01\ P = 0.2\ \text{MW}\ (\text{when } P = 20\ \text{MW}) \qquad (18)$$

$$NH\ \frac{\sqrt{2}\delta k}{G} = -\ 12 \cdot 10^{-2}\ \text{MW} \qquad (19)$$

With the chosen values, we have:

$$A \cong 0.325\ \text{MW} \qquad (20)$$

$$\frac{nH}{A_f R_f}\ \theta = 21.4\ \text{MW} \qquad (21)$$

and since m=1 (Bibl.1)

$$\Delta P_b = \sqrt{2 \cdot 0.325(21.4+20)} \cong 5.2\ \text{MW} \qquad (22)$$

and

$$\sigma_b = E_{rb} = \frac{0.325}{5.2} \cong 6.3\ \% \qquad (23)$$

Since it is:

$$\varepsilon_K = 3.8\ \% \qquad \text{(Appendix 4)} \qquad (24)$$

We obtain:

$$E = 3.8\ \% + 6.3\ \% = 10.1\ \% \qquad (25)$$

It is interesting to see the magnitude of each error separately:

$$\varepsilon_o \cong 1.44\ \% \qquad (26)$$

$$\varepsilon_i \cong 1.44\ \% \qquad (27)$$

$$\varepsilon_\varrho \cong 2.3\ \% \qquad (28)$$

$$\varepsilon_\phi \cong 5.43\ \% \qquad (29)$$

$$\varepsilon_K \cong 3.8\ \% \qquad (30)$$

From this numerical evaluation it appears that the important errors are mainly the calibration error, ε_K, the random error, ε_ϕ, due to the measurement of the current produced by the neutron flux detector and ε_ξ. It must be taken in mind that the importance of ε_o; ε_i increases when the measurements are carried out at lower power levels.

4. The measurement of the global coolant temperature coefficient

4.1 Classification of the errors

As already stated in para 2, during this experiment, rise of coolant temperatures and power are held constant, as average temperature is changed. The global coolant temperature coefficient is than given by:

$$\gamma = \Delta k / \Delta\bar{\theta} \qquad (1)$$

From eq. (15) of para 2, we get the following errors:

$$\eta_p = \left| \frac{G}{\gamma} \frac{\Delta(P/nH)}{\Delta\bar{\theta}} \right| \qquad (2)$$

is the random error due to the not exact constancy of the power during the experiment.

$$\eta_r = \left| \frac{\gamma_r \Delta\bar{\theta}_r}{\gamma \Delta\bar{\theta}} \right| \qquad (3)$$

is the random error due to the not exact constancy of the reactor coolant temperature rise during the experiment

$$\eta_\xi = \left| \frac{\delta(\Delta k)}{\Delta k} \right| = \left| \frac{\sqrt{2}\delta k}{\Delta k} \right| = \left| \frac{\sqrt{2}\delta k}{\gamma \Delta\bar{\theta}} \right| \qquad (4)$$

is the random error due the measurement of the reactivity interval. "δk" is the error done in measuring each end of the interval.

$$\bar{\eta} = \left| \frac{\delta(\Delta\bar{\theta})}{\Delta\bar{\theta}} \right| = \left| \frac{\frac{\sqrt{2}}{2}\sqrt{(\delta\theta_o)^2+(\delta\theta_i)^2}}{\Delta\bar{\theta}} \right| \qquad (5)$$

is the random error due to the measurement of the average coolant temperature change, $\delta\theta$ is the error done in measuring the coolant temperature.

Taking into account the definition of the errors from (2) to (5) and their nature (systematic or random), we can write that the total experimental error, E_{Na}, is given by:

$$E_{Na} = \sqrt{\eta_P^2 + \eta_r^2 + \eta_\xi^2 + \bar{\eta}^2} \qquad (6)$$

4.2 Numerical evaluations in the SEFOR case

In the SEFOR case we have:

$$\eta_P = \left| \frac{G}{nH\gamma} \frac{\Delta P}{\Delta\bar{\theta}} \right| = 64.7 \left| \frac{\Delta P}{\Delta\bar{\theta}} \right| \qquad (1)$$

$$\eta_r \cong 0 \qquad (2)$$

$$\eta_\xi = \left| \frac{\sqrt{2}}{\gamma} \frac{k}{\Delta\bar{\theta}} \right| \doteq 1.6 \cdot 10^2 \left| \frac{\delta k}{\Delta\bar{\theta}} \right| \qquad (3)$$

$$\bar{\eta} = \left| \frac{0.705\sqrt{(\delta\theta_o)^2 + (\delta\theta_i)^2}}{\Delta\bar{\theta}} \right| \qquad (4)$$

By looking at eq. (1) it appears convenient to perform this experiment at low power level ($\nleq$4 MW). In this case we have:

$$\Delta P = 0.01 \cdot P = 0.01 \cdot 4 = 0.04 \text{ MW} \qquad (5)$$

Assuming:

$$\delta\theta_o = \Delta\theta_i = \mp 1^\circ K \qquad (6)$$

$$\delta k = \mp 5 \cdot 10^{-3} \% \qquad (7)$$

and putting:

$$\Delta\bar{\theta} = 20^\circ K \qquad (8)$$

we get:

$$\eta_P \cong 13 \% \qquad (9)$$

$$\eta_r \cong 0 \qquad (10)$$

$$\eta_\xi = 4 \% \qquad (11)$$

$$\bar{\eta} = 5\ \% \tag{12}$$

The total experimental error, E_{Na}, will be:

$$E_{Na} \cong 14.5\ \% \tag{13}$$

As already noticed this experiment cannot be carried out at full power because the result would be to inaccurate (η_P big). Nevertheless, from the point of view of the accuracy, this difficulty can be overridden if the power can be kept constant with better precision. (0.1 % instead of 1 % as stated in eq. 5).

5. The measurement of the secondary radial expansion temperature coefficient

5.1 Classification of the errors involved in the measurements

As already stated in para 2, during this experiment, average coolant temperature and power are held constant, as inlet and outlet coolant temperatures are changed of the same amount but in opposite directions ($\Delta\theta_o = - \Delta\theta_i$)

$$\gamma_r = \frac{\Delta k}{\Delta\theta_r} \tag{1}$$

From eq. (15) of para 2, we get the errors associated to eq. 1

$$\xi_P = \left| \frac{G}{nH\gamma_r} \frac{\Delta P}{\Delta\theta_r} \right| \tag{2}$$

is the random error due to the not exact constancy of the power during the experiment.

$$\bar{\xi} = \left| \frac{\gamma}{\gamma_r} \frac{\Delta\theta}{\Delta\theta_r} \right| \tag{3}$$

is the random error due to the not exact constancy of the average coolant temperature during the experiment. This means that the condition $\Delta\theta_o = - \Delta\theta_i$ is not precisely satisfied.

$$\xi_\rho = \left| \frac{\delta(\Delta k)}{\Delta k} \right| = \left| \frac{\sqrt{2}}{\gamma_r} \frac{\delta k}{\Delta\theta_r} \right| \tag{4}$$

is the random error due to the measurement of the reactivity interval. δk is the error done in measuring each end of the interval.

$$\xi_r = \left|\frac{\delta(\Delta\theta_r)}{\Delta\theta_r}\right| = \left|\frac{\sqrt{2}\sqrt{(\delta\theta_o)^2+(\delta\theta_i)^2}}{\Delta\theta_r}\right| \qquad (5)$$

is the random error due to the evaluation of the change in the reactor coolant temperature rise. $\delta\theta$ is the error done in measuring the coolant temperature.

Taking into account the definition of the errors from (2) to (5) and their nature (systematic or random), we can write that the total experimental error, E_i, is given by:

$$E_r = \sqrt{\xi_p^{\,2} + \bar{\xi}^2 + \xi_\rho^2 + \xi_r^{\,2}} \qquad (6)$$

5.2 Numerical evaluations in the SEFOR case

Since in SEFOR it is expected to have $\gamma_r = 0$, it has no meaning to evaluate relative errors. It makes instead sense to evaluate absolute errors.

We have:

$$|\gamma_r|\,\xi_p = 5.9 \cdot 10^{-2} \left|\frac{\Delta P}{\Delta\theta_r}\right| ¢/^{\circ}K \qquad (1)$$

$$|\gamma_r|\,\bar{\xi} = 8.8 \cdot 10^{-3} \left|\frac{\Delta\bar{\theta}}{\Delta\theta_r}\right| ¢/^{\circ}K \qquad (2)$$

$$|\gamma_r|\,\xi_\rho = 1.41 \left|\frac{\delta k}{\Delta\theta_r}\right| ¢/^{\circ}K \qquad (3)$$

$$|\gamma_r|\,\xi_r = \left|\frac{\sqrt{2}\sqrt{(\delta\theta_o)^2(\delta\theta_i)^2}}{\Delta\theta_r}\right| \quad \gamma_r \cong 0 \qquad (4)$$

In this case too, by looking at eq. (1), it appears convenient to perform the experiment at low power level ($\leqslant 4$ MW).

We have:

$$\Delta P = 0.01 \cdot P = 0.01 \cdot 4 = 0.04 \text{MW} \tag{5}$$

Assuming:

$$\Delta \bar{\theta} = \mp 1^{\circ}\text{K} \tag{6}$$

$$\delta k = 5 \cdot 10^{-3} \text{ ¢} \tag{7}$$

and putting:

$$\Delta \theta_i = - 20^{\circ}\text{K}; \quad \Delta \theta_o = + 20^{\circ}\text{K} \tag{8}$$

and therefore:

$$\Delta \theta_r = 40^{\circ}\text{K} \tag{9}$$

we have:

$$\left| \gamma_r \right| \xi_P = 5.9 \cdot 10^{-5} \text{ ¢}/^{\circ}\text{K} \tag{10}$$

$$\left| \gamma_r \right| \bar{\xi} = 2.2 \cdot 10^{-4} \text{ ¢}/^{\circ}\text{K} \tag{11}$$

$$\left| \gamma_r \right| \xi_{\varrho} = 1.76 \cdot 10^{-4} \text{ ¢}/^{\circ}\text{K} \tag{12}$$

$$\left| \gamma_r \right| \xi_r = 0 \tag{13}$$

The total experimental error, $\delta \gamma_r$, will be:

$$\left| \delta \gamma_r \right| = \left| \gamma_r \right| E_i \cong 2.88 \cdot 10^{-4} \text{ ¢}/^{\circ}\text{K} \tag{14}$$

6. Considerations on the regulation and control of the reactor during the experiments

In order to perform the experiments in the best way, it is reasonable to ask for the most precise regulation of the important parameters.

The parameters which must be regulated are:

the reactor power, P
the primary coolant reactor outlet temperature, θ_o and
the primary coolant reactor inlet temperature, θ_i.

The parameters which control the system are:

reactivity, Δk
primary coolant flow, μ and
secondary coolant flow ν.

It seems reasonable to regulate θ_i by means of ν.
Two possibilities are given to regulate θ_o and P.

Case 1: To regulate θ_o with Δk and P with μ
Case 2: To regulate θ_o with μ and P with Δk

We shall define as better the regulation which gives the finest control, that is the minimum change of the controlled parameter due to the minimum possible change of the controlling parameter.

Taking into account eq. (15) of para 2, we have in case 1:

$$\left|\delta\theta_o\right|_{1\,\min} = \left|\frac{\partial\theta_o}{\partial k}\right|_{\substack{P=\text{const}\\ \theta_i=\text{const}}} \cdot \left|\delta k\right|_{\min} = \left|\frac{1}{\frac{\gamma}{2}+\gamma_r}\right|\left|\delta k\right|_{\min} \qquad (1)$$

It is:

$$P = c\mu\,(\theta_o - \theta_i) \qquad (2)$$

"c" being the specific heat capacity of the coolant.

We have therefore:

$$\frac{1}{P}\left|\delta P\right|_{1\,\min} = \frac{\mu}{P}\left|\frac{\partial P}{\partial\mu}\right|_{\substack{\theta_o=\text{const}\\ \theta_i=\text{const}}} \cdot \left|\frac{\delta\mu}{\mu}\right|_{\min} = \left|\frac{\delta\mu}{\mu}\right|_{\min} \qquad (3)$$

From eq. (2), we have in case 2:

$$\left|\delta\theta_o\right|_{2\,\min} = \mu\left|\frac{\partial\theta_o}{\partial\mu}\right|_{\substack{P=\text{const}\\ \theta_i=\text{const}}} \cdot \left|\frac{\delta\mu}{\mu}\right|_{\min} = \frac{P}{c\mu}\cdot\left|\frac{\delta\mu}{\mu}\right|_{\min} = (\theta_o-\theta_i)\cdot\left|\frac{\delta\mu}{\mu}\right|_{\min} \qquad (4)$$

From eq. (15) of para 2 we get in case 2:

$$\frac{1}{P}\left|\delta P\right|_{2\,\min} = \frac{1}{P}\left|\frac{\partial P}{\partial k}\right|_{\substack{\theta_o=\text{const}\\ \theta_i=\text{const}}} \cdot \left|\partial k\right|_{\min} = \frac{1}{P}\left|\frac{nH}{G}\right|\cdot\left|\delta k\right|_{\min} \qquad (5)$$

In the SEFOR case at full power and flow, we have:

$$\left|\delta\theta_o\right|_{1\ min} = 227\left|\delta k\right|_{min} \tag{6}$$

$$\frac{1}{P}\left|\delta P\right|_{1\ min} = \left|\frac{\delta\mu}{\mu}\right|_{min} \tag{7}$$

$$\left|\delta\theta_o\right|_{2\ min} = 67\left|\frac{\delta\mu}{\mu}\right|_{min} \tag{8}$$

$$\frac{1}{P}\left|\delta P\right|_{2\ min} = 0.85\left|\delta k\right|_{min} \tag{9}$$

Assuming for $\left|\delta k\right|_{min}$ and $\left|\frac{\delta\mu}{\mu}\right|_{min}$ the following values:

$$\left|\delta k\right|_{min} = 5\cdot 10^{-3}\ \$ \tag{10}$$

$$\left|\frac{\delta\mu}{\mu}\right|_{min} = 1\ \% \tag{11}$$

we get:

$$\left|\delta\theta_o\right|_{1\ min} \cong 1.14^\circ C \tag{12}$$

$$\left|\frac{\delta P}{P}\right|_{1\ min} = 1\ \% \tag{13}$$

$$\left|\delta\theta_o\right|_{2\ min} = 0.67^\circ C \tag{14}$$

$$\left|\frac{\delta P}{P}\right|_{2\ min} = 0.425\ \% \tag{15}$$

From (12); (13); (14) and (15) it appears that case "2" gives a finer control of both outlet coolant temperature "θ_o" and power "P". Case two is also more convenient from the point of view of the speed of the regulation. In fact μ acts on θ_o quicker than on P, and Δk acts on P quicker than on θ_o.

We can therefore conclude that it is convenient to compensate:

changes of θ_o by means of μ

and

changes of P by means of Δk

7. Conclusions

The global power coefficient, G, is expected to be measured in SEFOR with 10 % accuracy, when normal instrumentation is used. The power coefficient, so obtained, is averaged over power intervals, ΔP, which increase as power increases (eq. 19 of para 3.2). At full power (20 MW) ΔP is about 5 MW.

From the analysis developed in para 3.3 it appears that the main source of error is expected to be that due to the measurement of the current produced by the neutron detector ($\mathcal{E}_{\phi}$). The accuracy can therefore be improved (or the power interval reduced) by using a more refined neutron flux detection system.

The global coolant temperature coefficient, γ, can be also measured by means of static experiments. The expected precision is about 15 % over intervals of average coolant temperature of 20°C. It will also possible to check that the secondary radial expansion temperature coefficient, γ_r, is about zero as it is expected according to theoretical calculations.

The measurement of γ and γ_r must be carried out at low power level (≤ 4 MW). At full power the measurements would be too inaccurate. However, if the neutron flux can be controlled within 0.1 % of the power level (instead of 1 %), it is possible to perform the experiments also at full power with about the same accuracy.

The analysis developed in para 6 gives a guide to the control phylosophy of the reactor system during the experiments. It is convenient to regulate:

(i) the outlet primary coolant temperature, Θ_o, by means of the primary coolant flow, μ

(ii) the inlet primary coolant temperature, Θ_i, by means of the secondary coolant flow, ν

(iii) the reactor power, P, by means of the reactivity Δk.

With this type of control phylosophy a fine change of 0.5 % of power is achievable when changes of $5 \cdot 10^{-3}$ ¢ in Δk are possible.

Appendix 1

Dependence of the Doppler reactivity effect upon the effective fuel temperature, T_{eff}

The infinitesimal reactivity change, dk_f, due to infinitesimal changes, dT, of fuel temperatures, is given by:

$$dk_f = \frac{\int_{\text{all rods}} \Phi \, \Phi^* \, \Gamma_f' \, dT dV}{\int_{\text{reactor}} \Phi \, \Phi^* \, dV} \qquad (1)$$

where:

Φ = flux

Φ^* = adjoint flux

Γ_f' = local fuel reactivity coefficient

V = volume

We define the effective fuel temperature, T_{eff}, as follows:

$$T_{eff} = \frac{\int_{\text{all rods}} \Phi \, \Phi^* \, T dV}{\int_{\text{all rods}} \Phi \, \Phi^* \, dV} = \frac{\int_{\text{all rods}} \Phi \, \Phi^* \, T dV}{\int_{\text{reactor}} \Phi \, \Phi^* \, dV} \; \frac{V_{reactor}}{V_f} \qquad (2)$$

where V_f = volume of fuel in reactor.

In the simple case in which Γ_f' is constant, introducing (2) in (1), we obtain:

$$dk_f = \Gamma_f' \, dT_{eff} \; \frac{V_f}{V_{reactor}} \qquad (3)$$

We can therefore define a differential reactivity temperature coefficient, Γ_f, as follows:

$$\Gamma_f = \frac{dk_f}{dT_{eff}} = \Gamma_f' \frac{V_f}{V_{reactor}} \qquad (4)$$

In the most general cases Γ_f' is a function of the fuel temperature. We can develop $\Gamma_f'(T)$ in a Taylor series:

$$\Gamma_f' = \alpha_o + \sum_{l=1}^{l=\infty} \left[\frac{\alpha_l}{l!} (T-T_{eff})^l \right] \qquad (5)$$

where:

$$\alpha_o = \left[\Gamma_f' \right]_{T=T_{eff}} \qquad (6)$$

$$\alpha_l = \left[\frac{d^l \Gamma_f'}{dT^l} \right]_{T=T_{eff}} \qquad (7)$$

Introducing (5) in (1) and taking into account (2), we have:

$$dk_f = \frac{V_f}{V_{reactor}} \alpha_o \, dT_{eff} + \sum_{l=1}^{l=\infty} \left[\frac{\alpha_l}{l!} \frac{\int_{all\ rods} \Phi \Phi (T-T_{eff})^l \, dTdV}{\int_{reactor} \Phi \Phi^+ \, dV} \right] \qquad (8)$$

In the practical cases it is in eq. (8):

$$\sum_{l=1}^{l=\infty} \left[\frac{\alpha_l}{l!} \frac{\int_{all\ rods} \Phi \Phi (T-T_{eff})^l \, dTdV}{\int_{reactor} \Phi \Phi^+ \, dV} \right] \ll \frac{V_f}{V_{reactor}} \alpha_o \, dT_{eff} \qquad (9)$$

Taking into account expression (9), eq. 8 becomes:

$$dk_f \cong \frac{V_f}{V_{reactor}} \alpha_o \, dT_{eff} \qquad (10)$$

We can therefore define the differential reactivity temperature coefficient, Γ_f, as follows:

$$\Gamma_f = \frac{dk_f}{dT_{eff}} = \frac{V_f}{V_{reactor}} \alpha_o \qquad (11)$$

If α_o is given by

$$\alpha_o = \left[\Gamma_f' \right]_{T=T_{eff}} = \frac{\Gamma_o}{T_{eff}^{\ m}} \qquad (12)$$

then

$$\Gamma_f = \frac{C}{T_{eff}^{\ m}} \tag{13}$$

where:

$$C = \frac{V_f}{V_{reactor}} \Gamma_o \tag{14}$$

Appendix 2

Calculation of the effective temperature of the fuel

We define the effective fuel temperature, T_{eff}, as follows:

$$T_{eff} = \frac{\int_{\text{all rods}} \bar{\Phi}\,\bar{\Phi}^{*}\ T dV}{\int_{\text{all rods}} \bar{\Phi}\,\bar{\Phi}^{*}\ dV} \tag{1}$$

where:

$\bar{\Phi}$ = flux

$\bar{\Phi}^{*}$ = adjoint flux

T = fuel temperature

V = volume

We consider the reactor as devided in "n" vertical channels each including a fuel element and its associated coolant. The fuel temperature in the channel "q" is given by:

$$T(q;\tfrac{r}{R};\tfrac{z}{H}) = \Theta_i + \frac{P}{c\mu}\ \frac{N(q)}{Q(q)} \cdot \int_{-1/2}^{\frac{z}{H}} M(\tfrac{z}{H})\ d(\tfrac{z}{H}) + R_s(T;\tfrac{r}{R})\ M(\tfrac{z}{H})\ N(q)\ \frac{P}{nH} \tag{2}$$

where:

z = vertical coordinate ($z=o$ at the middle plane of the channel)

H = height of the fuel rod

r = radial coordinate of the fuel rod

R = radius of the fuel rod

Θ_i = inlet coolant temperature

P = reactor power

c = specific heat capacity of the primary coolant

μ = primary coolant flow

$N(q)$ = normalized flux(power) distribution in the radial direction

$$\left[\frac{1}{n}\sum_{q=1}^{q=n} N(q) = 1\right]$$

$Q(q)$ = normalized flow distribution in the radial direction

$$\left[\frac{1}{n}\sum_{q=1}^{q=n} Q(q) = 1\right]$$

$R_s(T\,\frac{r}{R})$ = equivalent thermal resistance between the temperature of the points at distance "r" from the axis of the fuel rod and the coolant temperature

$M(\frac{z}{H})$ = normalized flux (power) distribution in the axial direction

$$\left[\int_{-1/2}^{+1/2} M(\tfrac{z}{H})\; d(\tfrac{z}{H}) = 1\right] \tag{3}$$

We can put:

$$R_s(T;\tfrac{r}{R}) = R_c + R_{sf}(T;\tfrac{r}{H}) \tag{4}$$

where R_c is the thermal resistance between the fuel surface temperature and the coolant temperature and R_{sf} depends on the fuel thermal conductivity λ. R_c can be supposed to be constant. R_{sf} instead is in general function of the temperature because λ is function of the temperature.

Taking into account (2) and (4), eq.(1) becomes:

$$T_{eff} = \Theta_i + \frac{P}{c\mu} B + A_f R_f \frac{P}{nH} \tag{5}$$

where:

$$B = \pi R^2 \; \frac{\displaystyle\sum_{q=1}^{q=n} \left\{ \frac{N(q)}{Q(q)} \left[\int_{-1/2}^{+1/2} \Phi\, \Phi^* \left(\int_{-1/2}^{\frac{z}{H}} M(\tfrac{z}{H})\; d(\tfrac{z}{H}) \right) d(\tfrac{z}{H}) \right] \right\}}{\displaystyle\int_{all\ rods} \Phi\, \Phi^*\; dV} \tag{6}$$

$$R_f = R_c + \frac{\displaystyle\sum_{q=1}^{q=n} \left\{ N(q) \left[\int_{-1/2}^{+1/2} R_{av}\; M(\tfrac{z}{H})\; d(\tfrac{z}{H}) \right] \right\}}{n} \tag{7}$$

$$A_f = \frac{V_f}{\int_{\text{all rods}} \Phi\,\Phi^* \,dV} \; \frac{R_c \sum\limits_{q=1}^{q=n} \left\{ N(q) \left[\int_{-1/2}^{+1/2} \Phi\,\Phi^* M(\tfrac{z}{H})\, d(\tfrac{z}{H}) \right] \right\} + \sum\limits_{q=1}^{q=n} \left\{ N(q) \left[\int_{-1/2}^{+1/2} R_{av} M(\tfrac{z}{H})\; \Phi\,\Phi^* \, d(\tfrac{z}{H}) \right] \right\}}{nR_c + \sum\limits_{q=1}^{q=n} \left\{ N(q) \left[\int_{-1/2}^{+1/2} R_{av} M(\tfrac{z}{H})\; d(\tfrac{z}{H}) \right] \right\}} \tag{8}$$

and

$$V_f = \text{volume of fuel in reactor} = nH\pi R^2 \tag{9}$$

$$R_{av} = \frac{1}{\pi} \int_0^1 R_{sf}\; 2\pi \frac{r}{R}\, d(\tfrac{r}{R}) = 2 \int_0^1 \frac{T - T_s}{N(q) M(\frac{z}{H}) P/nH}\; \frac{r}{R}\, d(\tfrac{r}{R}) = \frac{T_{av} - T_s}{N(q) M(\frac{z}{H}) P/nH} \tag{10}$$

T_{av} being the average fuel temperature in a section of a fuel rod given by:

$$T_{av} = \frac{1}{\pi} \int_0^1 T\; 2\pi \frac{r}{R}\, d(\tfrac{r}{R}) \tag{11}$$

The average fuel temperature, $\bar{T}$, is obtained by the same procedure with the exception of the term $\Phi\,\Phi^*$ which is equal to 1.
It is:

$$\bar{T} = \Theta_i + \frac{P}{c\mu} B' + R_f \frac{P}{nH} \tag{12}$$

where:

$$B' = \frac{\sum\limits_{q=1}^{q=n} \left\{ \frac{N(q)}{Q(q)} \int_{-1/2}^{+1/2} \left[\int_{-1/2}^{z/H} M(\tfrac{z}{H})\; d(\tfrac{z}{H}) \right] d(\tfrac{z}{H}) \right\}}{V_f}\; \pi R^2 \tag{13}$$

In the Sefor (and in most of the experimental fast reactors), flow and flux distributions in the radial direction are similar because of special design.
It is therefore:

$$N(q) = Q(q) \tag{14}$$

The flux distribution along the axis of a fuel rod is in general symmetrical in respect to the middle plane of the core. That is:

$$M(\tfrac{z}{H}) = M(-\tfrac{z}{H}) \tag{15}$$

Taking into account (15) and the definition of $M(\frac{z}{H})$ (eq.3) we have:

$$\int_{-1/2}^{z/H} M(\tfrac{z}{H})\, d(\tfrac{z}{H}) = \frac{1}{2} + \frac{1}{2}\int_{-z/H}^{+z/H} M(\tfrac{z}{H})\, d(\tfrac{z}{H}) \qquad \text{if } z > 0 \tag{16}$$

and

$$\int_{-1/2}^{z/H} M(\tfrac{z}{H})\, d(\tfrac{z}{H}) = \frac{1}{2} - \frac{1}{2}\int_{-z/H}^{+z/H} M(\tfrac{z}{H})\, d(\tfrac{z}{H}) \qquad \text{if } z < 0 \tag{17}$$

Introducing (14); (16) and (17) in eq.(13), we have:

$$B' = \frac{1}{2} \tag{17'}$$

$\Phi\, \Phi^*$ has also in general an axial distribution which is symmetrical in respect to the middle plane of the core. In this case, taking into account (15) and (17), if $N(q) = Q(q)$, eq.(6) gives:

$$B = B' = \frac{1}{2} \tag{18}$$

Taking into account (14), the average coolant temperature, $\bar{\Theta}$, is given by:

$$\bar{\Theta} = \Theta_i + \frac{P}{2c\mu} \tag{19}$$

and therefore from (5); (18) and (19) it follows:

$$T_{eff} = \bar{\Theta} + A_f R_f \frac{P}{nH} \tag{20}$$

and

$$\bar{T} = \bar{\Theta} + R_f \frac{P}{nH} \tag{21}$$

Let us consider now R_f and A_f, given respectively by eqs.(7) and (8), in the two particular cases:

I) the thermal conductivity, λ, of the fuel element is constant

II) the thermal conductivity, λ, follows the Scott's law (Bibl.8) that is:

$$\lambda = \frac{\beta}{T + \alpha} \tag{22}$$

with β and α constant.

In the first case (λ = const) R_{av} is also constant and we have from eq.(7):

$$R_f = R_c + R_{av} = \text{const.} \tag{23}$$

Eq.(8) gives:

$$A_f = \frac{\sum_{q=1}^{q=n} \left\{ N(q) \int_{-1/2}^{+1/2} \Phi \, \Phi^* \, M(\tfrac{z}{H}) \, d(\tfrac{z}{H}) \right\}}{\int_{\text{all rods}} \Phi \, \Phi^* \, dV} \; \frac{V_f}{n} \tag{24}$$

We can conclude that, if the thermal conductivity, λ, of the fuel element is constant, R_f is constant and A_f depends only on the distribution of the flux and the adjoint flux in the reactor.

In the second case (λ follows the Scott's law), it is (from eq.10):

$$R_{av} = \frac{T_{av} - T_s}{N(q) \cdot M(\frac{z}{H}) \cdot P/nH} \tag{25}$$

where:

T_{av} = average fuel temperature in a section of a fuel rod

T_s = surface fuel temperature

If we use for λ the expression (22), $T_{av} - T_s$ is given by:

$$T_{av} - T_s = (T_s + \alpha) \left[\frac{4\beta}{N(q)M(\frac{z}{H})P/\pi nH} \left(e^{\frac{N(q)M(\frac{z}{H})P/\pi nH}{4\beta}} -1 \right) -1 \right] \tag{26}$$

In addition, it is:

$$T_s = \Theta + R_c \, M(\tfrac{z}{H}) \, N(q) \, P/nH \tag{27}$$

Taking into account (26) and (27), eq.(25) becomes:

$$R_{av} = \left(R_c + \frac{\Theta + \alpha}{N(q)M(\frac{z}{H})P/nH} \right) \cdot \left[\frac{4\,\beta\,\pi}{N(q)M(\frac{z}{H})P/nH} \left(e^{\frac{N(q)M(\frac{z}{H})P/nH}{4\,\beta\,\pi}} -1 \right) -1 \right] \tag{28}$$

Eq.(28) can be developed in series:

$$R_{av} = R_c \sum_{m=1}^{m=\infty} \frac{1}{(m+1)!} \left(\frac{N(q)M(\frac{z}{H})P/nH}{4\beta\pi} \right)^m + \frac{\Theta + \alpha}{8\pi\beta} \left[1 + 2 \sum_{m=1}^{m=\infty} \frac{1}{(m+2)!} \left(\frac{N(q)M(\frac{z}{H})P/nH}{4\beta\pi} \right)^m \right] \tag{29}$$

We also remember that:

$$\Theta = \Theta_i + \frac{P}{c\mu} \int_{-1/2}^{z/H} M(\tfrac{z}{H}) \, d(\tfrac{z}{H}) \tag{30}$$

$$\bar{\Theta} = \Theta_i + \frac{P}{2c\mu} \tag{31}$$

$$\sum_{q=1}^{q=n} \left[N(q) \int_{-1/2}^{+1/2} M(\tfrac{z}{H}) \, d(\tfrac{z}{H}) \right] = n \tag{32}$$

$$\sum_{q=1}^{q=n} \left[N(q) \cdot \int_{-1/2}^{+1/2} M(\tfrac{z}{H}) \left(\int_{-1/2}^{z/H} M(\tfrac{z}{H}) \, d(\tfrac{z}{H}) \right) d(\tfrac{z}{H}) \right] = \frac{n}{2} \tag{33}$$

$$\sum_{q=1}^{q=n}\left[\left[N(q)\right]^{m}\int_{-1/2}^{+1/2}\left[M(\tfrac{z}{H})\right]^{m}\left(\int_{-1/2}^{z/H}M(\tfrac{z}{H})\,d(\tfrac{z}{H})\right)d(\tfrac{z}{H})\right]=\frac{1}{2}\sum_{q=1}^{q=n}\left[\left[N(q)\right]^{m}\int_{-1/2}^{+1/2}\left[M(\tfrac{z}{H})\right]^{m}d(\tfrac{z}{H})\right] \quad (34)$$

Putting (29) and (30) in eq.(7), and taking into account (31), (32), (33) and (34), we get:

$$R_f = R_c \sum_{m=o}^{m=\infty} a_m \left(\frac{P}{nH}\right)^m + \frac{\bar{\Theta}+\alpha}{8\pi\beta}\sum_{m=o}^{m=\infty} b_m \left(\frac{P}{nH}\right)^m \qquad (35)$$

where:

$$a_m = \frac{1}{(m+1)!}\left(\frac{1}{4\pi\beta}\right)^m \frac{1}{n}\sum_{q=1}^{q=n}\left\{\left[N(q)\right]^{m+1}\int_{-1/2}^{+1/2}\left[M(\tfrac{z}{H})\right]^{m+1}d(\tfrac{z}{H})\right\} \qquad (36)$$

and

$$b_m = \frac{2}{m+2}\,a_m \qquad (37)$$

It is:

$$\sum_{q=1}^{q=n}\left\{\left[N(q)\right]^{m}\cdot\int_{-1/2}^{+1/2}\left[M(\tfrac{z}{H})\right]^{m}\Phi\Phi^{*}\left(\int_{-1/2}^{z/H}M(\tfrac{z}{H})\,d(\tfrac{z}{H})\right)d(\tfrac{z}{H})\right\}=\frac{1}{2}\sum_{q=1}^{q=n}\left\{\left[N(q)\right]^{m}\left(\int_{-1/2}^{+1/2}\left[M(\tfrac{z}{H})\right]^{m}\Phi\Phi^{*}d(\tfrac{z}{H})\right)\right\} \quad (38)$$

Putting (29) and (30) in eq.(8) and taking into account (31) to (38), we get:

$$A_f = \frac{R_c\cdot\sum_{m=o}^{m=\infty} c_m\left(\frac{P}{nH}\right)^m + \frac{\bar{\Theta}+\alpha}{8\pi\beta}\sum_{m=o}^{m=\infty} d_m\left(\frac{P}{nH}\right)^m}{R_c\sum_{m=o}^{m=\infty} a_m\left(\frac{P}{nH}\right)^m + \frac{\bar{\Theta}+\alpha}{8\pi\beta}\sum_{m=o}^{m=\infty} b_m\left(\frac{P}{nH}\right)} \quad \frac{V_f}{\int_{\text{all rods}}\Phi\,\Phi^{*}\,dV} \qquad (39)$$

where:

$$c_m = \frac{1}{(m+1)!}\left(\frac{1}{4\pi\beta}\right)^m \frac{1}{n}\sum_{q=1}^{q=n}\left\{\left[N(q)\right]^{m+1}\int_{-1/2}^{+1/2}\left[M(\tfrac{z}{H})\right]^{m+1}\Phi\Phi^{*}\,d(\tfrac{z}{H})\right\} \qquad (40)$$

and

$$d_m = \frac{2}{m+2} \; c_m \tag{41}$$

It is:

$$A_f R_f = \frac{V_f}{\int_{\text{all rods}} \Phi \, \Phi^* \, dV} \left[R_c \sum_{m=o}^{m=\infty} c_m \left(\frac{P}{nH}\right)^m + \frac{\bar{\Theta} + \alpha}{8\pi\beta} \sum_{m=o}^{m=\infty} d_m \left(\frac{P}{nH}\right)^m \right] \tag{42}$$

In the case $\Phi\Phi^* = 1$, it is:

$$c_m = a_m \tag{43}$$

and

$$d_m = b_m \tag{44}$$

From eqs.(35) to (42) we conclude that, if the fuel thermal conductivity "λ" follows the Scott's law, A_f and R_f depend on reactor power, P, average coolant temperature $\bar{\Theta}$ and on flux and adjoint flux distribution in the reactor.

In Appendix 1 we have seen that the fuel temperature effect on reactivity can be referred to an effective fuel temperature, T_{eff}, which is defined by eq.(1).

In this Appendix we have also found that T_{eff} depends directly upon P, $\bar{\Theta}$ and flux and adjoint flux distribution. This means that changes in the distribution of fuel temperatures, which do not effect P, $\bar{\Theta}$ and flux and adjoint flux distribution, do not effect also T_{eff}. This has been demonstrated only in two cases: λ is constant or follows the Scott's law(eq.22).

However, this two cases are important if these conclusions are applied to ceramic fuel elements.

Ceramic fuel elements have a thermal conductivity which decreases with the Scott's law until 600°C, from 600°C to 2000°C is more or less constant, and from 2000°C tends to increase. The region in which we are interested is from room temperature to 2000°C. We can extrapolate the conclusions of our theoretical analysis and we can therefore say that in ceramic fuel reactors the effective temperature of the fuel, T_{eff}, for a given flux distribution, depends mainly on $\bar{\theta}$ and P. The dependence on $\bar{\theta}$ is linear while that on P becomes linear as we move from the low temperature to the high temperature region (not beyond 2000°C). Any other effect on T_{eff} depending on the change of the temperature distribution (with flux distribution unchanged) is a second order effect which can be neglected. It is therefore:

$$T_{eff} = T_{eff}\,(\bar{\theta}\ ;\ P) \tag{45}$$

The first practical observation is that, also if we are unable to measure T_{eff}, we can control its constancy by controlling the constancy of $\bar{\theta}$ and P.

In the most general cases, we can write:

$$dk_f = \Gamma_{fr}\, d\theta_r + \Gamma_{fc}\, d\bar{\theta} + \Gamma_{Pf}\, d\left(\frac{P}{nH}\right) \tag{46}$$

where:

$\Gamma_{fr} = \left(\frac{\partial k_f}{\partial \theta_r}\right)_{\substack{P=\text{const.}\\ \bar{\theta}=\text{const.}}}$ = fuel coefficient referred to reactor coolant temperature rise, $\theta_r = \theta_o - \theta_i$ (47)

$\Gamma_{fc} = \left(\frac{\partial k_f}{\partial \bar{\theta}}\right)_{\substack{P=\text{const.}\\ \bar{\theta}_r=\text{const.}}}$ = fuel coefficient referred to average coolant temperature (48)

$\Gamma_{Pf} = nH\left(\frac{\partial k_f}{\partial P}\right)_{\substack{\bar{\theta}=\text{const.}\\ \theta_r=\text{const.}}}$ = fuel power coefficient (49)

It is also:

$$dk_f = \Gamma_f dT_{eff} = \Gamma_f \, d\left(\bar{\Theta} + A_f R_f \frac{P}{nH}\right) = \tag{50}$$

$$= \Gamma_f \cdot \left[1 + \frac{P}{nH}\left(\frac{\partial A_f R_f}{\partial \bar{\Theta}}\right)_{P=const}\right] d\bar{\Theta} + \Gamma_f \left[A_f R_f + \frac{P}{nH}\left(\frac{\partial A_f R_f}{\partial (P/nH)}\right)_{\bar{\Theta}=const}\right] d\left(\frac{P}{nH}\right)$$

By comparing (46) and (50) we get:

$$\Gamma_{fr} = 0 \tag{51}$$

$$\Gamma_{f_c} = \Gamma_f \left[1 + \frac{P}{nH}\left(\frac{\partial A_f R_f}{\partial \bar{\Theta}}\right)_{P=const}\right] \tag{52}$$

and

$$\Gamma_{Pf} = \Gamma_f \left[A_f R_f + \frac{P}{nH}\left(\frac{\partial A_f R_f}{\partial (P/nH)}\right)_{\bar{\Theta} = const.}\right] \tag{53}$$

In the two particular cases:

(a) no power production in the reactor $(P = 0)$

(b) $A_f R_f$ constant

We get from eqs. (52) and (53)

$$\Gamma_{fc} = \Gamma_f \tag{54}$$

and

$$\Gamma_{Pf} = \Gamma_f A_f R_f \tag{55}$$

For a finite interval, we have:

$$\Delta k_f = \gamma_f \, \Delta T_{eff} = \gamma_{fc} \, \Delta \bar{\Theta} + G_f \, \Delta \left(\frac{P}{nH}\right) \tag{56}$$

where:

$$\gamma_f = \frac{1}{\Delta T_{eff}} \int_1^2 \Gamma_f \, dT_{eff} \qquad (57)$$

$$\gamma_{fc} = \frac{1}{\Delta \bar{\Theta}} \int_1^2 \Gamma_{fc} \, d\bar{\Theta} \qquad (58)$$

and

$$G_f = \frac{1}{\Delta(P/nH)} \int_1^2 \Gamma_{Pf} \, d(P/nH) \qquad (59)$$

Appendix 3

Determination of the calibration error

The neutron flux detectors will be calibrated by means of the well known calorimetric method. The power P is given by:

$$P = c\mu\ (\Theta_o - \Theta_i) \tag{1}$$

where:

c = specific heat capacity of the coolant

μ = primary coolant flow

Θ_o = outlet coolant temperature

Θ_i = inlet coolant temperature

The neutron flux detector produces a current, I, proportional to P. It is:

$$P = K\ I \tag{2}$$

where K is the calibration constant which must be determined.

From eqs.(1) and (2), we get:

$$K = \frac{c\mu\ (\Theta_o - \Theta_i)}{I} \tag{3}$$

The probable calibration error, $\left|\frac{\delta K}{K}\right|$, will be:

$$\left|\frac{\delta K}{K}\right| = \left|\frac{\delta c}{c}\right| + \sqrt{\left|\frac{\delta\mu}{\mu}\right|^2 + \left|\frac{\delta\Theta_o}{\Theta_o-\Theta_i}\right|^2 + \left|\frac{\delta\Theta_i}{\Theta_o-\Theta_i}\right|^2 + \left|\frac{\delta I}{I}\right|^2} \tag{4}$$

where $\left|\frac{\delta c}{c}\right|$ is a systematic error.

From the point of view of the accuracy it is convenient to carry out the calibration with the maximum value of $\Theta_o - \Theta_i$, that is at full power.

I is:

$$(\theta_o - \theta_i)_{max} = 67^\circ C \qquad (5)$$

Assuming:

$$\left|\frac{\delta c}{c}\right| = 0.5\ \% \qquad (6)$$

$$\left|\frac{\delta \mu}{\mu}\right| = 2.5\ \% \qquad (7)$$

$$\left|\delta \theta_o\right| = \left|\delta \theta_i\right| = 1^\circ C \qquad (8)$$

$$\left|\frac{\delta I}{I}\right| = 1\ \% \qquad (9)$$

we get:

$$\left|\frac{\delta K}{K}\right| = 10^{-2}\left[0.5 + \sqrt{(2.5)^2 + (1.4)^2 + (1.4)^2 + 1^2}\right] \cong 3.8\ \% \qquad (10)$$

Bibliography

(1) K.Cohen, P.Greebler, K.M.Horst, B.Wolfe
The Southwest Experimental Fast Oxide Reactor
GE-APED-4281 (June 1963)

(2) M.J.McNelly, K.P.Cohen, B.Wolfe
Experimental Fast Oxide Reactor
Proceedings of Symposium on Power Reactor Experiments, Vienna (October 1961 -SM-21/18)

(3) P.Liewers
Reaktivitätsmessungen - Probleme, Methoden und Anwendungsmöglichkeiten
Kernenergie 4 (1961), page 593-518

(4) R.D.Brooks, A.L.Rosenblatt
Design and Performance of Liquid-Metal Heat Exchangers and Steam Generators
Mechanical Engineering (May 1953) page 365

(5) Southwest Experimental Fast Oxide Reactor,
Proposal by GE-APED (Jan.1963)

(6) W.Häfele, K.Ott, L.Caldarola, W.Schikarski, K.P.Cohen, B.Wolfe, P.Greebler, A.B.Reynolds
Static and Dynamic Measurements on the Doppler Effect in an Experimental Fast Reactor
Geneva Conference 1964

(7) F.Storrer
Measurement of the Doppler Coefficient in Large Fast Power Reactors using a Fast Critical Assembly and an Experimental Fast Reactor
Part II: (by J.Ladet, I.Puig, and L.Stevens)
Use of RAPSODIE for the measurement of the Doppler Coefficient of Fast Power Reactors
Conference on Breeding, Economics and Safety in Large Fast Power Reactors, Argonne National Laboratory, Oct.1963

(8) R.Scott
Thermal Conductivity of UO_2
AERE-M/R-2526

(9) P.Greebler, B.Hutchins

The Doppler Effect in a Large Fast Oxide Reactor. Its Calculation and Significance for Reactor Safety

Proceedings of the Physics of Fast and Intermediate Reactors sponsored by IAEA 3rd - IIth August 1961 in Vienna, - Vol.3 pages 121-138

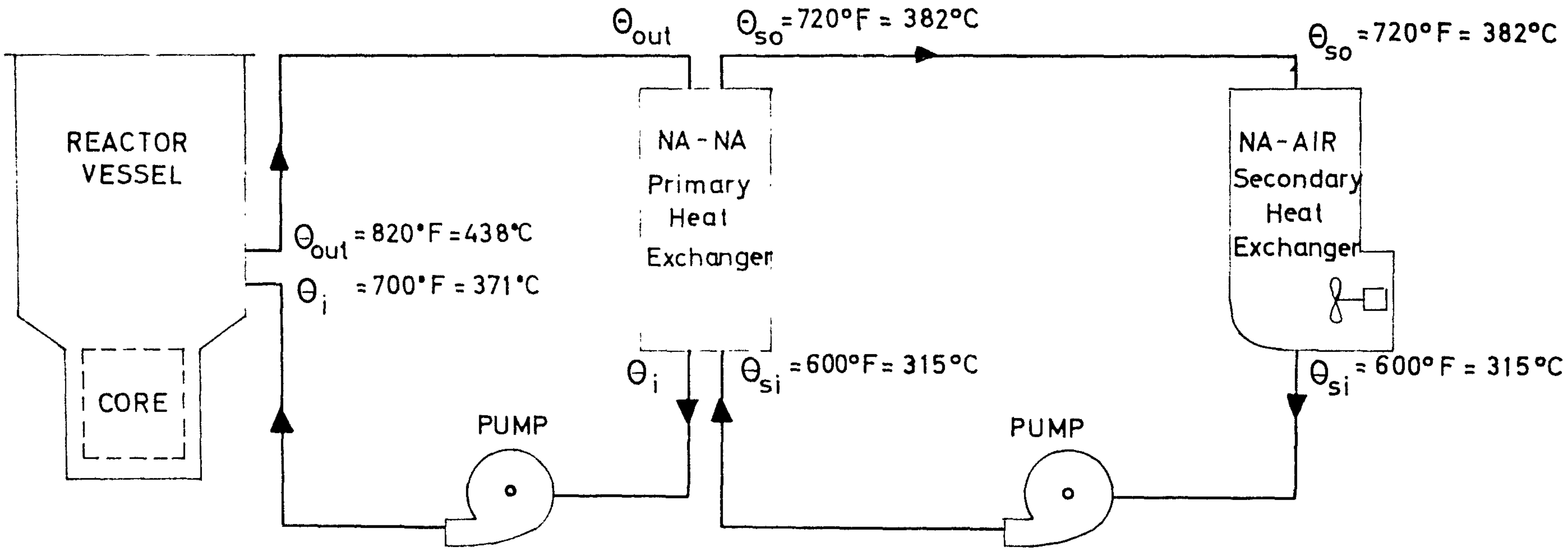
(10) SEFOR Development Program First Quarterly Report GEAP-4594

(11) SEFOR Development Program Second Quarterly Report GEAP-4742

(12) SEFOR Development Program Third Quarterly Report GEAP-4799

(13) SEFOR Development Program Fourth Quarterly Report GEAP-4864

(14) SEFOR Development Program Fifth Quarterly Report GEAP-4934



Θ_i = reactor inlet temperature

Θ_{out}= reactor outlet temperature

Θ_{si}= secondary coolant inlet temperature

Θ_{so}= secondary coolant outlet temperature

Fig 1 SCHEMATIC REACTOR FLOW DIAGRAM
(SEFOR)