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THE EMISSION OF PROTONS FROM LIGHT NEUTRON-DEFICIENT NUCLEI

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Abstract: The properties of the processes which lead to the emission of protons from light neutron-deficient nuclei are discussed. These processes are in particular proton decay, double proton decay and β-delayed emission of protons and di-protons. Masses of light nuclei with $Z \leq 14$ were calculated and a limit of stability with regard to the emission of protons is given. Candidates for double proton decay are the following light even-Z nuclei: O^{12}, Ne^{16}, Mg^{19} and possibly Si^{23}. The β-delayed emission of protons should occur subsequently to the β^{+}-decays of C^{9}, Ne^{17}, (Mg^{19}) and Mg^{20}. Emission of weak proton groups may also follow the β^{+}-decays of (O^{12}), O^{13}, Mg^{21}, Si^{24} and Si^{25}. Several of these proton groups have been observed. The findings of this paper supplement and extend previous predictions.

1. Introduction

The emission of heavy particles, i.e. α-particles, protons, neutrons, di-protons or di-neutrons from atomic nuclei takes place when the nuclei are either very heavy, very proton-rich or neutron-rich, or sufficiently highly excited. Of particular interest is decay (self-delayed emission) from the ground states and β-delayed emission. In this paper only the processes leading to the emission of protons from light nuclei are discussed.

Masses of neutron-deficient light nuclei were calculated and estimated by several authors [1-7]) and the existence and certain properties of many yet unknown isotopes were predicted. Limits of stability with regard to the emission of protons were given by Karnaukhov and Tarantin [3]), by Zeldovich [4]) and by Goldansky [5]). The predicted stability lines deviate slightly from each other.

The existence of the Coulomb barrier may lead to proton *decay* if the binding energy for the last proton is negative. Its characteristics have been discussed by Dželepov [1]) and Goldansky [6]). Close to the above-mentioned limit of stability, nuclei with an even number of protons which delay only by the simultaneously emission of two protons must exist. In contrast, the successive emission of two single protons is not possible energetically. There is a certain analogy to double β-decay. Several candidates for double proton decay (not all the same) were named by Goldansky [5,6]) and Zeldovich [4]). Properties of this decay mode were discussed by Goldansky [5,6]). (See also the review article by Baz, Goldansky and Zeldovich [8])). The effect has not yet been observed [8]).

Nuclei below the limit of stability undergo β^+-decay. If states of sufficiently high excitation in the daughter nucleus are populated, the subsequent emission of protons might become possible. The β-delayed emission of protons is expected from preferentially even-Z nuclei with a large proton excess. Protons from this decay mode have been observed [10-12]†.

This work was initiated to obtain more information on the properties of the various decay modes. Binding energies and masses of light neutron-deficient nuclei were calculated to permit more specific predictions about the nuclei under consideration.

2. The Masses of Light Neutron-Deficient Nuclei

The Bethe-Weizsäcker formula [13]) or other mass formulae [14-18]) are not suited to predict masses of nuclei below $A = 20$ or even $A = 40$ with sufficient accuracy. Therefore other methods are necessary to calculate approximate mass values. Several such methods are known [1,2,519,20]) which were also used by other authors [4-7]) and in this paper as well.

In method I, one uses the known mass of a neutron-rich nucleus (1) with an isobaric spin $T^{(1)} = T_z^{(1)}$. The mass of the proton-rich mirror nucleus (2) with the isobaric spin $T^{(2)} = T^{(1)}$ and $T_z^{(2)} = -T_z^{(1)}$ differs from the mass of nucleus (1) only by the Coulomb energy difference and the neutron-proton mass difference. Its mass can thus be calculated. In this paper a semi-empirical formula [7]) for the Coulomb energy differences was used. Shell effects and Coulomb pairing effects are included. The estimated uncertainty of the calculated masses is 0.3 to 0.4 MeV.

The masses of several light nuclei with $T = T_z = 2$ are not known and method I cannot be applied. Therefore method II was used to obtain the masses of the nuclei with $T = -T_z = 2$. The energy difference Δ_{20} between the energetically lowest states with $T = 2$ and $T = 0$ (in most cases the ground state) in the self-conjugate nuclei ($T_z = 0$) is a relatively smooth function [2, 20, 21]) of A. Specifically, Δ_{20} is rather independent of whether the respective nucleus is even or odd, i.e., $A = 4n$ or $A = 4n+2$. Thus, it is possible to interpolate or extrapolate the known energy differences Δ_{20} as a function of A. By adding Δ_{20} to the mass of the corresponding $T_z = 0$ nucleus and by adding or subtracting the proper Coulomb energy [6]) and n–p mass differences, one obtains approximate masses for nuclei with $T = 2$ and $T_z = \pm 2$. Due to the uncertainty of the extrapolated energies Δ_{20}, the accuracy of the calculated masses is appreciably lower than for method I. The energy difference Δ_{20} is to be interpreted [21]) as the energy needed to break up the "valence α-particle" of a nucleus in a characteristic way.

The mass of Li^4 is calculated similarly, except that it is based on the energy difference [22])† $\Delta_{10}^{A=4n} \approx 20$ MeV for He^4. The energies $\Delta_{10}^{A=4n}$ and $\Delta_{10}^{A=4n+2}$ are both [2,20,21,23]) relatively smooth functions of A, and a $T = 1$ state in this energy region was to be expected for $A = 4$.

† See note added in proof.

Method III, finally, was used in this paper to determine the masses of some nuclei with $T = -T_z = \frac{5}{2}$ where again the masses of the neutron-rich mirror nuclei with $T = T_z = \frac{5}{2}$ are not known. In principle method II is applicable since $\Delta_{\frac{3}{2}\frac{1}{2}}$ must be[21] a relatively smooth function of A. However, there exists practically no experimental information on $\Delta_{\frac{3}{2}\frac{1}{2}}$ for the light nuclei. Therefore the $(T_z)^2$-dependence for the binding energies from the Bethe-Weizsäcker mass formula [13] was used to extrapolate quadratically the known masses [24] for a given A and $|T_z| = \frac{1}{2}$ and $|T_z| = \frac{3}{2}$. Coulomb energies [7] were properly taken into account. The masses obtained this way are not very accurate and the errors are estimated to be of the order of $\pm$ 3 MeV. A more correct extrapolation must include a term with $|T_z|$.

Binding energies and mass excesses were calculated using the described methods.

TABLE 1

Calculated binding energies BE and mass excesses $\Delta M = M - Am_0$ [with $\Delta M(C^{12}) = 0$; $m_0 = 931.4$ MeV]

(1) Nucleus	(2) BE (MeV)	(3) ΔM (MeV)	(4) Method	(5) ΔM Baz [2]) (MeV)	(6) ΔM Goldansky [5]) (MeV)
Li^4	7.2	22.7	II		
Be^5	6.1	31.1	I		
B^7	20.0	32.6	[a])		29.4±0.5
C^8	21.8±3.5	38.1±3.5	I		<38 ±4
C^9	38.7±1.0	29.3±1.0	I		
N^{10}	31.1±3.0	44.1±0.3	II		
N^{11}	57.8±0.3	25.5±0.3	I		25.3
O^{11}	28.8	53.7	III		
O^{12}	60.2±3.0	30.4±3.0	II		<34.8
O^{13}	75.3±0.3	23.4±0.3	I		23.5
F^{14}	74.5±3.0	31.5±3.0	II		
F^{15}	97.0±0.3	17.0±0.3	I		
Ne^{15}	71.0	42.2	III		
Ne^{16}	97.2±0.4	24.1±0.4	I		23–24.3
Ne^{17}	113.1±0.3	16.3±0.3	I	16.6	16.4
Na^{18}	109.6±2.0	27.1±2.0	II		
Na^{19}	131.9±0.3	12.9±0.3	I	12.8	12.7
Mg^{19}	111.9	32.1	III		
Mg^{20}	134.6±0.4	17.4±0.4	I		17.2
Mg^{21}	148.5±0.3	10.8±0.3	I	7.1	10.8
Al^{22}	147.9±2.0	19.5±2.0	II		
Al^{23}	168.8±0.3	6.7±0.3	I	6.7	
Si^{23}	147.3	27.4	III		
Si^{24}	171.9±0.4	10.9±0.4	I		10.8
Si^{25}	187.1±0.5	3.7±0.5	I	3.2	3.6
Ca^{38}	312.4±0.3	−21.3±0.3	I	−23.0	
Sc^{39}	311.7±0.3	−13.3±0.3	I	−14.8	−14.4

[a]) If He^7 is just stable with regard to the emission of a neutron [31]) one has to assume that the level at 9.6 MeV in Li^7 has $T = \frac{3}{2}$. Based on this energy one obtains using method I for B^7 a mass excess of 26.9±1.0 MeV. However, there are regularities [21]) with respect to the masses of light nuclei which seem to indicate that the mass excess of B^7 and He^7 is about 5.7 MeV larger.

They are shown in table 1, columns 2 and 3. The mass excesses ΔM are given in C^{12} units, i.e. $\Delta M(C^{12}) = 0$, as in the mass tables of König *et al.* [24]). Column 4 indicates the method used for calculation. The masses of heavier nuclei can be obtained in the same way. As an example, the masses of Ca^{38} and Sc^{39} are given. The masses calculated by Baz [2]) (column 5) and Goldansky [5]) (column 6) are in good agreement with our values.

3. Proton Decay

The emission of protons from the ground state or an excited state of a nucleus B according to $B^{(*)} \rightarrow C+p$ is possible if the binding energy for the last proton is negative, i.e.

$$\mathrm{BE(B)}-\mathrm{BE(C)} < 0, \tag{1}$$

or

$$\mathrm{BE(B^*)}-\mathrm{BE(C)} = \mathrm{BE(B)}-E_x-\mathrm{BE(C)} < 0. \tag{2}$$

The emission may be self-delayed due to the Coulomb and centrifugal barrier. Transitions with half-lives $T_{\frac{1}{2}} \gg 10^{-21}$ sec are to be called proton decay †. Hence the process which leads to the formation of $B^{(*)}$ and the decay process are independent.

If $B^{(*)}$ is formed via strong interaction, i.e. through a nuclear reaction, one has to consider the process $A+a \rightarrow B^{(*)}+b$ (b stands for any number of particles). This notation includes compound nuclear reactions (b = 0) and a preceding decay (a = 0), for instance a preceding α-particle decay. As stated before, formation and decay can indeed be treated separately for $T_{\frac{1}{2}} \gg 10^{-21}$ sec.

The proton emitting nucleus $B^{(*)}$ may also be produced via weak interaction, i.e. through a preceding β-decay. Because of its particular characteristics this decay mode is called β-delayed emission of protons and will be treated separately in sect. 5.

Proton decay [1,3,5]) of the ground states or of excited isomeric states of light and medium heavy nuclei corresponds to some extent to α-particle decay of heavy nuclei. Mono-energetic particles penetrate the Coulomb and centrifugal barrier and are emitted with a characteristic half-life. There are, however, also significant differences related to the Coulomb and centrifugal barriers and to the structure of the emitted particle, of the initial and of the final nucleus.

(i) The Coulomb barrier of light nuclei for an emitted particle of charge 1 is much smaller than for heavy nuclei and an emitted particle of charge 2. Consequently the half-lives for proton decay are smaller than those for α-particles decay by many orders of magnitude.

(ii) For the same angular momentum of the emitted particle the hindrance factor is much more significant in proton decay of light nuclei than is the case in α-particle decay. Because of the smaller mass of the emitted particle and the smaller radius of the decaying nucleus, the centrifugal barrier is larger in proton decay on an absolute scale and even more compared to the smaller Coulomb barrier. A non-zero angular momentum of the emitted proton increases the half-life considerably (see end of this section.)

† See note added in proof.

(iii) In α-particle decay, the structure of the emitted α-particle is rather complex, and one needs to know the probability for the preformation of an α-particle at the nuclear surface. On the other hand, α-transitions take place between nuclei of the same character, i.e. even, odd, odd neutron or odd proton nuclei decay to even, odd, odd neutron or odd prcton nuclei, respectively, and in many cases the emitted α-particle carries no angular momentum. In proton decay the situation is quite

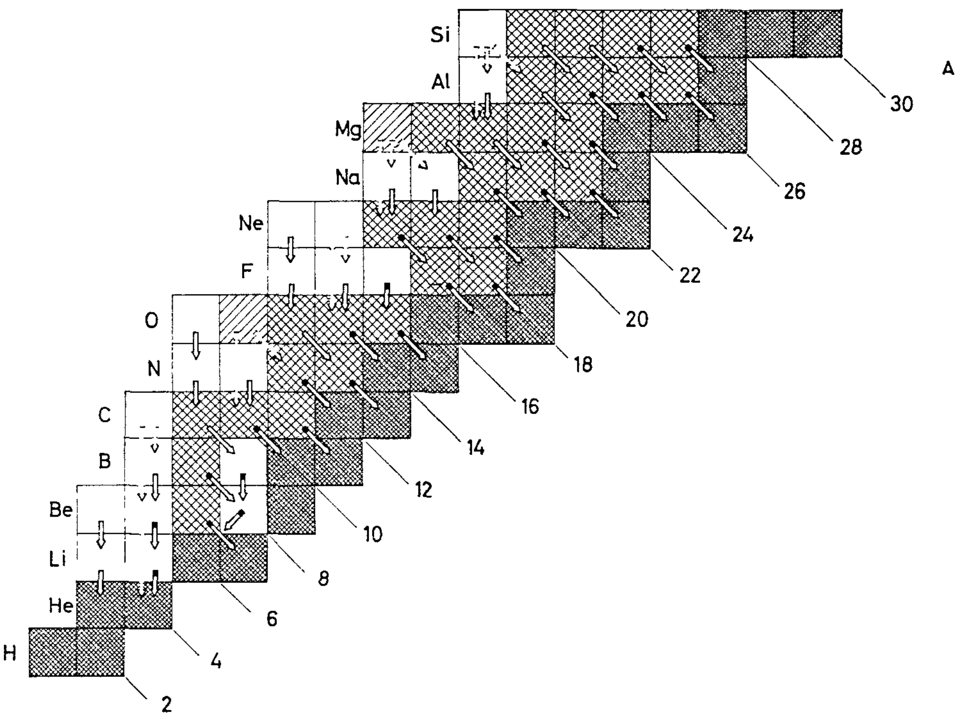

Fig. 1. Chart of the light neutron-deficient nuclei. The β-decay, proton decay (emission), and double proton decay (emission) are indicated by arrows. Known decays are shown by a black area on the shaft of the arrow. Uncertain decays are dotted. The stable nuclei, the β-unstable nuclei and the proton-unstable nuclei are shown by different shadings (the decay character of O^{12} and Mg^{19} is uncertain, which is indicated by a shading intermediate between the latter two). The boundary line between the β-unstable and proton-unstable nuclei represents the limit of stability with regard to the emission of protons, as discussed in the text.

opposite. The emitted particle is just a single nucleon and the spectroscopic factor is much more directly related to the structure of the initial and final states. On the other hand, even for single proton states (spectroscopic factor equals 1) the angular momentum is of importance, since $s_{\frac{1}{2}}$-, $p_{\frac{3}{2}\text{ or }\frac{1}{2}}$-, $d_{\frac{5}{2}\text{ or }\frac{3}{2}}$-, or $f_{\frac{7}{2}\text{ or }\frac{5}{2}}$- protons have to penetrate different centrifugal barriers with $l = 0$, 1, 2 or 3.

An approximate limit of stability with regard to the emission of protons was calculated by Karnaukhov and Tarantin [3]). More accurate limits of stability were given by Zeldovich [4]) and by Goldansky [5]). With the help of the masses given in table 1, the decay characteristics of the light neutron-deficient nuclei have been studied [25]). The resulting limit of stability with respect to the emission of protons is shown in fig. 1. In the case of O^{12} and Mg^{19} the calculated mass values are not accurate enough to decide definitely whether there is double proton decay or β^+-decay. The stability limit of fig. 1 deviates slightly from those given by Zeldovich [4]) and by Goldansky [5]). Fig. 1 shows that proton instability starts with a somewhat smaller proton excess for even-Z nuclei than for odd-Z nuclei.

Assuming single proton states (spectroscopic factor equals 1), the half-lives $T_{\frac{1}{2}}$ and level widths Γ of nuclei emitting protons can be estimated by the semi-classical expression

$$\frac{\Gamma}{\hbar} = \frac{1}{\tau} = \frac{v}{2R} P_l(E). \tag{3}$$

Here, v is the velocity of the emitted particle within the nucleus, R is the nuclear radius, and $P_l(E)$ is the penetration factor for particles with the energy E and the angular momentum l through Coulomb and centrifugal barrier. For $P_2(E)$ one may use the expression [25, 26]) which is obtained in WKB approximation with an infinite square well of radius $R = r_0(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$ and a Coulomb *and* centrifugal barrier. The proper shell model angular momentum l_{sm} has to be used. Table 2 shows a comparison

TABLE 2

Calculated energy widths Γ and half-lives $T_{\frac{1}{2}}$ for some light proton-instable nuclei and comparison with known values

Decay	E_p (MeV)	l_{sm}	Γ (keV) known	Γ (keV) calc.	$T_{\frac{1}{2}}$ (sec) known	$T_{\frac{1}{2}}$ (sec) calc.
$Li^5 \rightarrow He^4+p$	1.97 ± 0.04	1	≈ 1500 [a])	2600	$\approx 3 \cdot 10^{-22}$	$1.8 \cdot 10^{-22}$
$B^7 \rightarrow Be^6+p$	≈ 6.9	1		≈ 7800		$6.8 \cdot 10^{-23}$
$B^9 \rightarrow Be^8+p$	0.19 ± 0.01	1	≈ 0.5 [b])	2.2	$\approx 9 \cdot 10^{-19}$	$2.1 \cdot 10^{-19}$
$N^{11} \rightarrow C^{10}+p$	2.55 ± 0.3	1		2700		$1.7 \cdot 10^{-22}$
$F^{15} \rightarrow O^{14}+p$	1.7 ± 0.3	2		54		$8.5 \cdot 10^{-21}$
$F^{16} \rightarrow O^{15}+p$	0.75 ± 0.02	2	≈ 7	1.3	$\approx 10^{-19}$ [c])	$3.5 \cdot 10^{-19}$
$Na^{18} \rightarrow Ne^{17}+p$	3.5 ± 2.0	2		560		$8.1 \cdot 10^{-22}$
$Al^{22} \rightarrow Mg^{21}+p$	1.4 ± 2.0	2		6.9		$6.6 \cdot 10^{-20}$

[a]) See ref. [29]). [b]) See ref. [32]) [c]) Also calculated, see ref. [29]).

between calculated and a few known values. The agreement is much better than is normally the case in α-particle decay. Except for B^9 and F^{16} the level widths Γ of the nuclei chosen are of importance with regard to the existence or non-existence of a double proton decay of the neighbouring nuclei with one more proton (see sect. 4). The calculated half-lives of the sample are of the order of 10^{-19} sec to 10^{-22} sec.

This result shows that one can hardly expect proton *decay* in the light nuclei, at least none with a half-life accessible to a direct measurement.

The lightest nucleus which undergoes proton decay and has a half-life which might be measurable directly is Sc^{39}. Karnaukhov and Tarantin [3]) and Goldansky [5]) pointed out that Sc^{39} is likely to be proton unstable. The masses given in table 1 for Sc^{39} and Ca^{38} lead to the same conclusion with an energy for the emitted proton of $E_p = 0.7 \pm 0.5$ MeV. In the decay $Sc^{39} \rightarrow Ca^{38} + p$, a $1f_{\frac{7}{2}}$ proton must penetrate the Coulomb barrier *and* a centrifugal barrier with $l = 3$. In fig. 2 the calculated half-

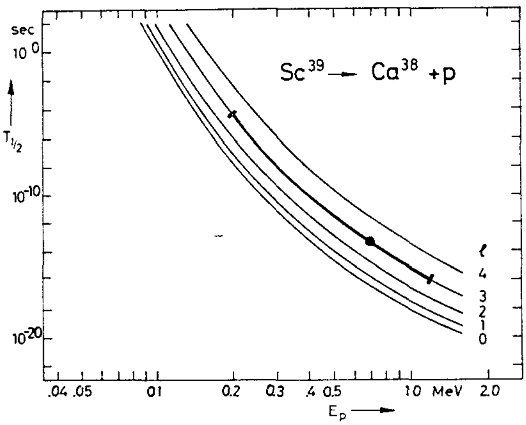

Fig. 2. Half-lives for the proton decay $Sc^{39} \rightarrow Ca^{38}+p$ as a function of the decay energy E_p and for assumed angular momenta $l = 0, 1, 2, 3$ and 4 of the emitted proton. The most probable energy range and range of half-lives is marked.

lives are shown for a range of decays energies E_p and with assumed angular momenta l as parameters. As stated before, the angular momentum strongly affects the half-life, and centrifugal barriers with $l = 1, 2, 3$ and 4 lengthen the half-life by factors of about 4, 40, 1200 and 59 000. Similar factors were given by Karnaukhov and Tarantin [3]). The half-life is strongly energy dependent. With $E_p = 0.7 \pm 0.5$ MeV and $l = 3$, one obtains a half-life $T_{\frac{1}{2}} \approx 10^{-14^{-2}_{+9}}$ sec. For $E_p < 0.4$ MeV, the half-life should be accessible to a direct measurement. The nucleus Sc^{39} is relatively easy to produce through the reactions $Ca^{40}(p, 2n)Sc^{39}$ or $Ca^{40}(d, 3n)Sc^{39}$.

4. Double Proton Decay

Double proton decay (self-delayed di-proton emission) was predicted by Goldansky [5,6]). Due to the pairing energy of even-Z nuclei close to the limit of stability mentioned before, the energetic conditions may be such that the subsequent emission of protons $A \xrightarrow{p} B \xrightarrow{p} C$ is not possible ($A \xrightarrow{p} B$ is not possible) while the simultaneous emission of two protons $A \xrightarrow{(2p)} C$ is possible. The following inequalities

$$\mathrm{BE(B)} + \tfrac{1}{2}\Gamma_A + \tfrac{1}{2}\Gamma_B < \mathrm{BE(A)} < \mathrm{BE(C)} \tag{4}$$

represent the conditions for double proton emission or decay in terms of the binding energies BE of the nuclei ${}_{Z=2m}A_{N}^{A}$, ${}_{Z=2m-1}B_{N}^{A-1}$ and ${}_{Z=2m-2}C_{N}^{A-2}$. The restricting level widths Γ_A and Γ_B have been included in the formula.

The light candidates for double proton decay given by Goldansky [5,6]) are Be^6, Ne^{16}, Mg^{17} or Mg^{18}, and Si^{21} or Si^{22}. The candidates given by Zeldovich [4]) are O^{12}, Ne^{16} and Mg^{19}. From the calculated masses given in table 1 the following light nuclei can be named as candidates for double proton decay: Be^6, C^8, O^{12}, Ne^{16}, Mg^{19} and Si^{23}. This list partially overlaps with the lists of candidates given by Goldansky [5,6]) and Zeldovich [4]). The decay schemes obtained for the considered nuclei are shown in fig. 3. Because of the uncertainty of the calculated masses there is still

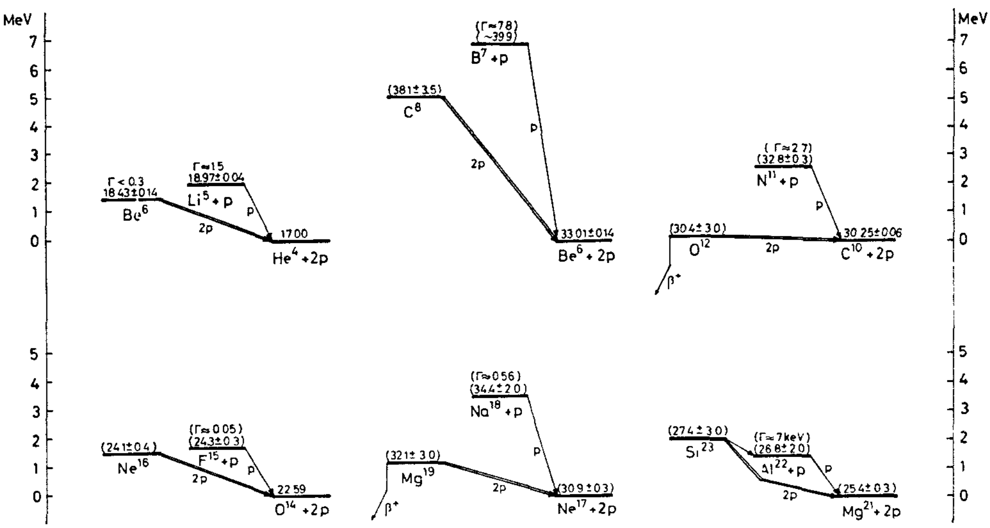

Fig. 3. Light candidates for double proton decay with proposed decay schemes. Mass excesses ΔM and approximate energy widths Γ are given in MeV. Calculated values are shown in parentheses. In O^{12} and Mg^{19}, also possibly in Si^{23}, there is competition between double proton decay (emission) and β^+-decay. In Be^6, C^8, Ne^{16} and Si^{23} there is competition between double proton decay (emission) and (single) proton emission. The most favourable candidates for double proton decay are O^{12}, Ne^{16} and Mg^{19}.

competition between several decay modes. In O^{12} and Mg^{19} double proton decay (emission) or β^+-decay must take place. In Be^6, C^8 and Ne^{16} double proton decay (emission) or proton decay (emission) are possible. In Si^{23}, finally, all three decay modes are possible. As one can see from fig. 3, the nuclei O^{12}, Ne^{16} and Mg^{19}, also possibly Si^{23}, are most favourable for a double proton decay. The existence for non-existence of a double proton decay (emission) of O^{12} and Mg^{19} can be inferred from the non-existence or existence of a β^+-decay of these nuclei.

Goldansky [5,6]) discussed certain properties of double proton decay. He showed two ways to calculate the probability for the simultaneous emission of two protons.

Either one calculates the penetration factor for a particle of charge two, i.e. a di-proton with energy E_0 through the corresponding Coulomb barrier or one calculates the product of the penetration factors for two single protons of energy E and E_0-E. The latter product has a maximum value for $E = \frac{1}{2}E_0$, and Goldansky [5,6]) pointed out that both methods then lead to the same result. Thus, the di-proton may decompose into its constituents either within or outside the Coulomb barrier. This conclusion, however, holds only for $l_{sm} = 0$, i.e. for di-protons with the shell model configuration $[(\pi s_{\frac{1}{2}})^2]_{0^+}$, for instance from the decay of a sulphur isotope. Otherwise, for instance in the case of Ne^{16}, when a di-proton with the configuration $[(\pi d_{\frac{5}{2}})^2]_{0^+}$ is emitted, the centrifugal barrier for the protons comes into effect and suppresses the uncorrelated emission of the two protons. On the other

TABLE 3

Calculated penetration factors and penetration factor ratios for di-protons of energy E_0 and $E_0-\varepsilon_0$ and angular momenta $L = 0$, and for protons of energy $\frac{1}{2}E_0$ and angular momenta $l = 0$ and $l = l_{sm}$

	(1) Decay	(2) $\bar{E}_0$ (MeV)	(3) Configuration	(4) l_{sm}
1	$Be^6 \rightarrow He^4 + (2p)$	1.42	$[(\pi 1p_{\frac{3}{2}})^2]_{0^+}$	1
2	$C^8 \rightarrow Be^6 + (2p)$	5.10	$[(\pi 1p_{\frac{3}{2}})^2]_{0^+}$	1
3	$O^{12} \rightarrow C^{10} + (2p)$	0.15	$[(\pi 1p_{\frac{1}{2}})^2]_{0^+}$	1
4	$Ne^{16} \rightarrow O^{14} + (2p)$	1.51	$[(\pi 1d_{\frac{5}{2}})^2]_{0^+}$	2
5	$Mg^{19} \rightarrow Ne^{17} + (2p)$	1.20	$[(\pi 1d_{\frac{5}{2}})^2]_{0^+}$	2
6	$Si^{23} \rightarrow Mg^{21} + (2p)$	2.00	$[(\pi 1d_{\frac{5}{2}})^2]_{0^+}$	2

	(5) $P^{(2p)}_{L=0}(E_0)$	(6) $P^{(2p)}_{L=0}(E_0-\varepsilon_0)$	(7) $P^{(p)}_{l=0}(\frac{1}{2}E_0)$	(8) $P^{(p)}_{l=l_{sm}}(\frac{1}{2}E_0)$
1	$9.4 \cdot 10^{-1}$	$9.1 \cdot 10^{-1}$	$9.2 \cdot 10^{-1}$	$5.8 \cdot 10^{-2}$
2	$1.0 \cdot 10^{0}$	$1.0 \cdot 10^{0}$	$1.0 \cdot 10^{0}$	$4.3 \cdot 10^{-1}$
3	$1.0 \cdot 10^{-13}$	$1.4 \cdot 10^{-17}$	$1.4 \cdot 10^{-7}$	$7.4 \cdot 10^{-9}$
4	$3.2 \cdot 10^{-3}$	$2.5 \cdot 10^{-3}$	$4.3 \cdot 10^{-2}$	$2.1 \cdot 10^{-4}$
5	$2.0 \cdot 10^{-5}$	$1.2 \cdot 10^{-5}$	$3.1 \cdot 10^{-3}$	$1.7 \cdot 10^{-5}$
6	$3.9 \cdot 10^{-4}$	$3.0 \cdot 10^{-4}$	$1.4 \cdot 10^{-2}$	$1.7 \cdot 10^{-4}$

	(9) $\frac{[P^{(p)}_{l=0}(\frac{1}{2}E_0)]^2}{P^{(2p)}_{L=0}(E_0)}$	(10) $\frac{[P^{(p)}_{l=l_{sm}}(\frac{1}{2}E_0)]^2}{P^{(2p)}_{L=0}(E_0)}$	(11) $\frac{[P^{(p)}_{l=0}(\frac{1}{2}E_0)]^2}{P^{(2p)}_{L=0}(E_0-\varepsilon_0)}$	(12) $\frac{[P^{(p)}_{l=l_{sm}}(\frac{1}{2}E_0)]^2}{P^{(2p)}_{L=0}(E_0-\varepsilon_0)}$
1	0.91	$3.6 \cdot 10^{-3}$	0.94	$3.7 \cdot 10^{-3}$
2	1.00	$1.9 \cdot 10^{-1}$	1.00	$1.9 \cdot 10^{-1}$
3	0.18	$5.4 \cdot 10^{-4}$	$1.28 \cdot 10^{3}$	$3.8 \cdot 10^{0}$
4	0.57	$1.4 \cdot 10^{-5}$	0.75	$1.8 \cdot 10^{-5}$
5	0.48	$1.5 \cdot 10^{-5}$	0.79	$2.4 \cdot 10^{-5}$
6	0.53	$7.6 \cdot 10^{-5}$	0.70	$1.0 \cdot 10^{-4}$

The energy ε_0 of the virtual level of the singlet proton-proton interaction [27]) was assumed to be 50 keV.

hand, when a di-proton, i.e. a pair of correlated protons, is emitted, not the full energy E_0 but only the energy $E_0 - \varepsilon_0$ is available as decay energy, since an energy ε_0 is needed for the breakup of the di-proton outside the barrier. Here, ε_0 is the energy of the virtual level of the singulet proton-proton interaction †. Thus, for small E_0 the emission of a di-proton is also strongly depressed.

Penetration factors and penetration factor ratios were calculated [25,26]) for the candidates for double proton decay listed above. Table 3 shows the results for the most likely decay energies. In column 3 the shell model configuration of the emitted di-proton is given. Column 4 gives the angular momenta l_{sm} of the protons according to the shell model if the two protons are emitted uncorrelated. Columns 5–8 give the penetration factors for di-protons of energy E_0 and with angular momenta $L = 0$, for di-protons of energy $E_0 - \varepsilon_0$ and with angular momenta $L = 0$, for protons of energy $\frac{1}{2}E_0$ and with angular momenta $l = 0$, and for protons of energy $\frac{1}{2}E_0$ and with angular momenta $l = l_{sm}$. Columns 9–12 show the corresponding penetration factor ratios, i.e. between uncorrelated protons and a di-proton (correlated protons). In column 10 the angular momentum l_{sm} is taken into account, in column 11 a finite value of 50 keV for ε_0 is taken into account, and in column 12, both are taken into account. The ratios of column 9 are close to the value 1 derived by Goldansky [5,6]). The deviation from 1 comes from the radius $R = r_0(A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}})$, which is different for the emission of protons ($A_2 = 1$) and di-protons ($A_2 = 2$). For $A_2 = 0$ one obtains precisely the value 1. Columns 10–12, however, show that both a non-zero angular momentum l_{sm} and a non-vanishing energy ε_0 strongly influence the decay mode and with it the decay probability. Generally, in double proton decay a di-proton with angular momentum $L = 0$ (two protons in a singlet state) penetrates the Coulomb barrier and the two protons separate outside the barrier. Only for $E_0 \lesssim 2\,\varepsilon_0$ and in case the di-proton inside the decaying nucleus has the configuration $[(\pi s_{\frac{1}{2}})^2]_{0^+}$, is there a preference for the uncorrelated emission of the protons.

Assuming an undisturbed double proton decay (no admixture from single proton decay) and assuming the probability for the preformation of the di-proton in the nucleus equals 1 (spectroscopic factor equals 1), the half-lives for the various candidates can be estimated as functions of the decay E_0 on the basis of eq. (3). The decay probabilities for the correlated and the uncorrelated emission of protons have to be added. This addition, however, is of significance only near the transition region at about $E_0/\varepsilon_0 \approx 2$ to 3 (see C^8 and O^{12}). Fig. 4 shows the result of the calculations. The half-lives are strongly energy-dependent and it is only in the case of Be^6 and Ne^{16} that a more precise estimate of the half-life can be made. The half-lives are very small and in only a few, if any, cases is the condition†† $T_{\frac{1}{2}} \gg 10^{-21}$ sec for double proton *decay* (self-delayed emission) fulfilled. It seems that O^{12} is the only nucleus for which one can hope to measure the half-life directly. The estimated disintegration energy of $E_0 \approx 150$ keV leads to a half-life $T_{\frac{1}{2}} \approx 10^{-6}$ sec. For E_0 between about

† See ref. [27]) and ref. [26]), p. 418.
†† See note added in proof.

100 keV and 200 keV the half-life should be measurable. One has to note, however, that perticularly in the case of O^{12} the penetration factor is very sensitive to the exact value of ε_0.

The angular and energy distributions and correlations of the two protons emitted in double proton decay † reflect the pair interaction of the two particles when inside and outside the nucleus. There exists a maximum opening angle ϑ_0 when the protons are emitted correlated. Openingangles ϑ between 0 and ϑ_0 with $\mathrm{tg}\,\frac{1}{2}\vartheta_0 = \sqrt{\varepsilon_0/(E_0-\varepsilon_0)}$ occur and the distribution for ϑ shows a pronounced preference for the largest possible angles. The energy distribution for the protons is zero outside the limits $E = \frac{1}{2}E_0 \pm \sqrt{\varepsilon_0(E_0-\varepsilon_0)}$ and the most probable energy is $E = \frac{1}{2}E_0$. When the protons are emitted

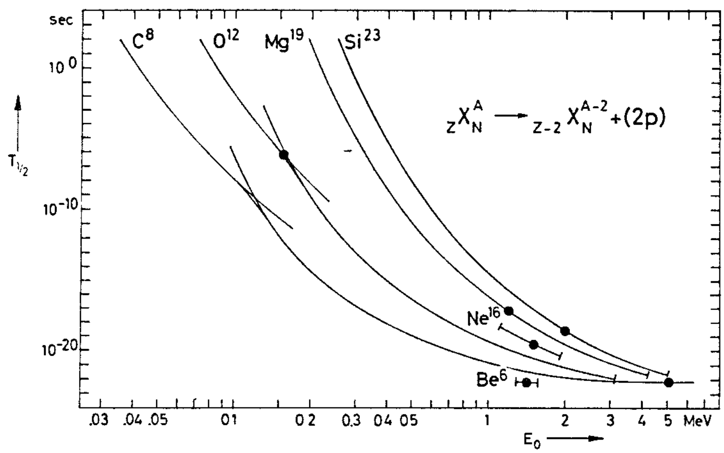

Fig. 4. Half-lives for the candidates for double proton decay as functions of the decay energies. The most probable decay energy is marked. The curves for C^8 and O^{12} consist of two branches and a relatively small transition region. The two branches correspond to the uncorrelated and correlated emission of the two protons.

uncorrelated the distribution for the opening angles ϑ should be more isotropic. The energy distribution for the protons depends on the Coulomb barrier, as pointed out by Goldansky [4, 5]), and on the centrifugal barrier. The most probable energy is $E = \frac{1}{2}E_0$ and the width of the distribution is a function of the parameters of the barrier.

The above remarks on the angular and energy distributions and correlations should be considered only as qualitative. A more detailed analysis must include at least the Coulomb repulsion between the protons and the recoil energy of the final nucleus. It must be stressed that the experimental inquiry into the angular and energy distributions and correlations of the protons emitted in double proton decay should yield detailed information on the proton-proton interaction.

† In certain proton induced pickup reactions [28]) a singlet di-proton or singlet deuteron might be formed in an intermediate step, thus leading to a (p, 2p) or (p, pn) reaction with a strong angle and energy correlation between the outgoing particles. The correlations in these processes and in double proton decay should correspond to some extent.

5. β-Delayed Emission of Protons

The β-delayed emission of protons has been discussed [5,6]) and has been observed [10–12]) in Ne^{17}, O^{13}, Mg^{21} and Si^{25}. The study of β-delayed protons including the measurement of half-lives, energy spectra, excitation functions, branching ratios, prompt and delayed β^+p-coincidences supplements and extends the information one obtains from $\beta\gamma$-spectroscopy. The β-delayed emission of proton corresponds to β-delayed emission of neutrons and α-particles. When β-transitions can populate sufficient highly excited states in the daughter nucleus, i.e.

$$E_x > \mathrm{BE(B)} - \mathrm{BE(C)}, \tag{5}$$

the β-delayed emission of protons becomes possible according to $A \xrightarrow{\beta^+} B^{(*)} \rightarrow C + p$. Even if possible energetically, the percentage of β-delayed protons, however, depends strongly on the branching ratios of the preceding β-decay. In general, there are strong allowed β-branches to the low excited states of the daughter nucleus with energies of the order of 10 MeV. For nuclei with $Z > N$ there is always a super-allowed component.

Nuclei lying below the limit of stability shown in fig. 1 undergo β-decay. Using the calculated masses of table 1 and the known masses and excitation energies of the neighbouring nuclei [24,29,30]), the properties of the nuclei which may emit β-delayed protons have been reviewed [25]). As an example the decay characteristics of C^9 are shown in fig. 5. The nucleus C^9 seems to be the lightest nucleus emitting β^+-delayed protons. For comparison, the known β^--decay of the mirror nucleus Li^9 with the subsequent emission of neutrons is also shown in the figure. The β^+-decay of C^9 has not yet been observed. A strong β^+-branch to the ground state of B^9 is expected. Since even the ground state of B^9 is proton unstable, this β^+-transition must be followed by the emission of protons of about 0.17 MeV. The β^+-transitions to higher excited states including the super-allowed transition to the $T = \frac{3}{2}$ state at about 15 MeV can lead to the delayed emission of higher energy protons. Primary emission of delayed α-particles is followed by the emission of a broad proton group of about 1.6 MeV. Thus it is evident that the β^+-decay of B^9 is always followed by a breakup into three particles, one proton and two α-particles. The breakup involves either a cascade process as described before or a direct transition.

A high percentage of delayed protons can be expected subsequently to the β^+-decays of C^9, Ne^{17}, (Mg^{19}) and Mg^{20}. Delayed protons can also be expected from the decays of (O^{12}), O^{13}, Mg^{21}, Si^{24} and Si^{25}. The intensities, however, are smaller and depend strongly on the branching ratios of the preceding β^+-decay. In Na^{20}, Al^{23} and Al^{24}, β-delayed emission of protons is unlikely, though possible energetically for β-transitions to highly excited states. The nuclei experimentally identified so far as emitters of β-delayed protons [10–12]) are indeed contained among the listed nuclei. Pairing energy considerations lead to the conclusion that β-delayed emission of protons should occur in preference subsequently to β-decays of even-Z nuclei with a high proton excess. The classification given above confirms this expectation.

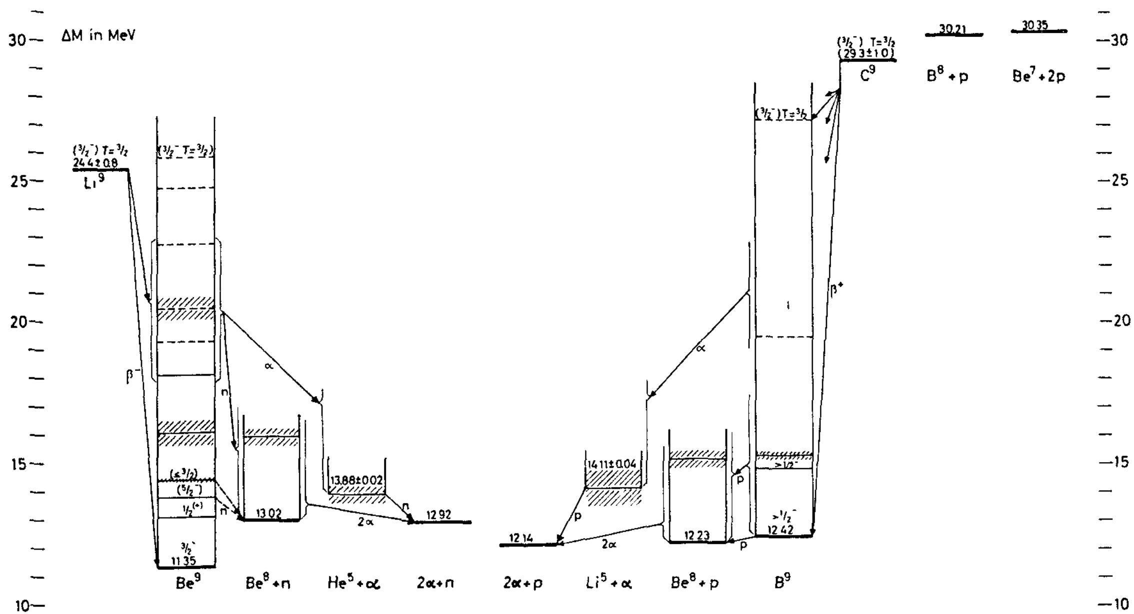

Fig. 5. Proposed and known decay characteristics of the mirror nuclei C^9 and Li^9. A β-delayed emission of protons, neutrons and α-particles takes place. Mass excesses ΔM are given in MeV. Calculated values are shown in parentheses.

In the list given above, O^{12} and Mg^{19} are shown in parentheses. As mentioned before, these two nuclei are among the candidates for double proton decay. The calculated masses, however, are not accurate enough to exclude one or the other decay mode. In the case of O^{12} and Mg^{19} one can establish the non-existence (existence) of a double proton decay (respectively the emission of a di-proton) by demonstrating the existence (non-existence) of β-delayed protons. The statement in parentheses should be correct for Mg^{19} and is very likely correct for O^{12}.

Brief mention must be made of another decay mode, namely β-delayed emission of di-protons [9]. As noted before, β-delayed emission of protons occurs mainly in proton-rich even-Z nuclei and is less likely in proton-rich odd-Z nuclei. In proton-rich odd-Z nuclei, however, β-delayed emission of di-protons might become possible. This decay mode occurs when instability with regard to the emission of di-protons starts at a lower excitation energy of the even-Z daughter nucleus than instability with regard to the emission of protons. Thus, though very infrequently, there may exist proton rich odd-Z nuclei with β-transitions to certain excited states in the daughter nucleus which subsequently emit di-protons.

Among the proton-rich nuclei with $Z \leqq 14$ there are no candidates for β-delayed di-proton emission.

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Note added in proof: The proposed isobaric spin assignment $T = 1$ for the state at 20 MeV excitation energy in He^4 seems to be inconsistent [33]. Consequently another state in the region of excitation energies between 20 and 25 MeV is the lowest state with $T = 1$. The mass of Li^4 as given in Table 1 must therefore be increased by a corresponding amount.

The definition of proton and double proton decay (i.e. radioactivity) involves a condition pertaining to energy and a condition related to the life times or the widths of the decaying states. The latter condition is not well defined. The most restrictive criterion one can think of is that $T_{\frac{1}{2}}$ shall be accessible to a direct measurement. The other extreme $T_{\frac{1}{2}} \gg 10^{-21}$ sec was used in this paper. Goldansky [34,35] pointed out that a definition in between the two extremes which excludes the decay of compound nuclei is most reasonable. It is difficult, however, to come to a distinct differentiation because of the gradual transition between proton decay and compound nucleus decay.

Several papers appeared recently [35–37] on the β-delayed emission of protons in light and medium heavy nuclei. The conclusions and experimental results are in accord with the findings of this paper.

Fleroc *et al.* [36] bombarded Pd^{102} with ions of Si^{28} and produced nuclei (see also Karnaukhov and Ter-Akopyan [37]) which emit β-delayed protons. From the energy of one particular proton group and from an estimate of the Coulomb barrier they calculated a half-life of 10^{-12} sec for the emission of the protons. This proton emitter could then be classified as a nucleus undergoing proton *decay*.

An experimental attempt was undertaken [38]) to detect a double proton decay of Ne^{16}. For the half-life an upper limit of 10^{-8} sec was established.

A theory for double proton decay was developed by Galitzky and Cheltsov [39]) for the case of protons carrying no angular momentum. Goldansky [34]) pointed out that a theory for double proton decay which includes the centrifugal barrier should give life times which are longer than the ones calculated in this paper. This is because (in a classical picture) the pairing energy of the two protons inside the decaying nucleus is released *gradually* during the passage through the barrier and not *at once* at the inner boundary of the barrier. Only the latter simplified case was considered in this paper.

Recently [40]) the reaction $p+d \rightarrow p+p+n$ has been investigated and peaks in the cross section were observed at an energy near the break up threshold in the intermediate singlet deuteron system and at a higher energy in the intermediate singlet di-proton system (see also Bilanuk and Slobodrian [41])).

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