A Reevaluation of the Coupling Constants in Beta Decay

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A precise determination of the $\beta$ decay coupling constants is of great importance with respect to the universal Fermi interaction and the CVC theory. The vector coupling constant $g_V$ is obtained from the $ft$ value of $0^{14}$ and other $0^+\rightarrow 0^+$ transitions and the ratio $g_A/g_V$ from the $ft$ value of the neutron. For this purpose the $f$ function must be known precisely. In the past calculations for a point charge [1] were improved by applying corrections for the nuclear size and the screening by orbital electrons. Because of the approximations used some of the results, especially for the screening correction, differed considerably [2].

One of us [3] has developed an exact method to calculate the solution of the Dirac equation (for an extended nuclear charge) taking electron screening into account. With the electron wave functions obtained in this way the function

$$ f = \int_{W_0} W_0^{W_0} F_o L_o p W(W - W)^2 dW $$

was calculated (for the definition of $F_o L_o$ see [11] and [3]). It is thought that the $f$ values derived here are to be preferred to those evaluated in the past on the basis of various approximations.

The results for the neutron and $0^{14}$ are shown in the table together with values published by other authors.
It might be interesting to mention that the effect of screening is very small ($< 2 \times 10^{-4}$) for the $f_t$ value of $^{14}$O. The influence of a change of the nuclear radius $R = (0.428 \pm 0.005) \, \text{cm}$ has also been investigated since this is the only parameter relevant to the calculations. A variation of $R$ by 10% in the case of $^{14}$O or by 20% in the case of the neutron changes the $f_t$ value by less than $10^{-4}$.

Several additional effects have to be taken into account. The most important are the radiative corrections which were calculated by Kinoshita, Berman and Sirlin [4, 5], using perturbation theory. Durand et al. [6] evaluated the radiative corrections with dispersion relations but in this way only results for partially renormalized coupling constants are obtained. Therefore the numbers deduced by Kinoshita et al. were used (see table). Corrections originating from the nucleon structure and from the competition of $K$ capture were also applied. Corrections due to relativistic effects and contributions from second forbidden matrix elements are small and can be neglected [2, 6].

The $f_t$ values computed from the exact electron wave functions and with the corrections mentioned above are shown in the table. From these and the experimental half lives the $f_t$ values can be deduced. The results are also given in the table.

The constant $g_V$ can be calculated from the relation

$$g_V^2 \left| M_F \right|^2 f_t 14 = 2\pi^3 \hbar^7 \ln \frac{2}{m_o^5} c^4.$$
The matrix element $M_F$ can be computed for $0^+ - 0^+$ transitions if isospin is a good quantum number. Although the influence of Coulomb effects on the matrix element is not yet completely clarified (see [2], [6]) we shall assume $|M_F|^2 = 2$ for $0^{14}$. We shall not consider other $0^+ - 0^+$ transitions since their matrix elements are known with even less reliability.

From the fit value given in the table one obtains

$$\bar{E}_V = (1.4061 \pm 0.0034) \times 10^{-49} \text{ erg cm}^3.$$  

For the axial vector coupling constant one has

$$\lambda^2 = \left| \frac{C_A}{C_V} \right|^2 = \frac{1}{3} \left( \frac{2 \pi}{f_{t_n}^{14}} \right) = 1.38 \pm 0.04$$  

and

$$\lambda = \left| \frac{C_A}{C_V} \right| = 1.175 \pm 0.02.$$  

This value is somewhat lower than the values accepted thus far, $\lambda = 1.19$ or 1.25 [see 6] and higher than a recent value derived by Bhalla, $\lambda \approx 1.15$ (reported by C.S. Wu at the Paris Conference 1964).

We do not yet understand the difference between the value given by Bhalla and ours, since there is no screening in the case of the neutron and therefore the method of Bhalla [11] is equivalent to ours.

If $\bar{E}_V$ is compared to $\bar{E}_V = (1.4350 \pm 0.0011) \times 10^{-49}$ erg cm$^3$ [2] one obtains $(\bar{E}_V - E_\mu)/E_\mu = (2.02 \pm 0.25)\%$. This difference is in reasonable agreement with the prediction of Cabbibo's theory [9] of 3.3\%. It should be noted that the error of $\bar{E}_V$ does not contain the uncertainty of the radiative correction which might be 0.5\%.
Literature

[1] Tables for the Analysis of Beta Spectra, NBS Ser. 13 (1952)


[8] Freeman, White, Montagne, Murray and Burcham, Phys. Lett. 8, 115 (1964)


Table

<table>
<thead>
<tr>
<th>n</th>
<th>0.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>f uncorrected point charge</td>
<td>$1.688 \pm 0.006$ [1]</td>
</tr>
<tr>
<td>f nuclear size, screening (Rose)</td>
<td>-</td>
</tr>
<tr>
<td>f nuclear size, screening (Reitz)</td>
<td>-</td>
</tr>
<tr>
<td>f exact wave function</td>
<td>$1.695 \pm 0.005$</td>
</tr>
</tbody>
</table>

\[ \frac{\Delta f}{f} \]

| radiative corrections | + 1.8 % [4] | + 1.7 % [5] |
| finite nucleon size | - 0.002 % [6] | 0.110 % [6] |
| competition from K capture | - | 0.090 % [6] |
| total $\Delta f/f$ | + 1.8 % | + 1.90 % |

\[ f (\text{exact, corrected}) \]

| | $1.725 \pm 0.005$ | $43.6 \pm 1.5$ |
| t (sec) | $702 \pm 18$ [7] | $71.36 \pm 0.09$ [7] |
| ft (sec) | $1211 \pm 37$ | $3111 \pm 15$ |