KFK-296

KERNFORSCHUNGSZENTRUM

KARLSRUHE

Februar 1965

KFK 296

Institut für Experimentelle Kernphysik

On Isobaric Analogue States

Joachim Jänecke

GESELLSCHAFT FÜR KERNFORSCHUNG M.B.H.

KARLSRUHE

Kernforschungszentrum Karlsruhe

Februar 1965 KFK 296

Institut für

Experimentelle Kernphysik

On Isobaric Analogue States

Joachim Jänecke

Gesellschaft für Kernforschung mbH., Karlsruhe

Abstract:

The excitation energies of isobaric analogue states, i.e. the energetically lowest states with a given isobaric spin T, have been related to the symmetry and pairing energies of the respective nuclei. Symmetry parameters a(A) and pairing energies δ(A) were extracted from the experimental data. The parameter a(A) exhibits shell structure. An expression for the energetic difference between isobaric analogue states was established which reproduces the over 200 known values (known experimentally or calculated from the known masses of neighbouring isobars) up to A = 80 with a standard deviation of about 0.5 MeV. The same formula can be used to predict unknown excitation energies of isobaric analogue states and the masses of unknown neutron-rich and proton-rich nuclei. The above expression is in accordance with a theoretical expression based on an effective two-nucleon interaction. There exist small but systematic deviations from the supermultiplet model.

Submitted to Nuclear Physics

1. Introduction

Recently, isobaric analogue states have become of interest, both experimentally [1,2] and theoretically [3]. Such excited states, i.e. the energetically lowest states with a given isobaric spin T in a given nucleus, can be formed in direct nuclear reactions with a well defined change in isobaric spin, for instance certain (p,n), (p,d) or (p,t) reactions. It is of major interest to know what the excitation energies of these states are. Related is the problem of predicting and estimating the masses of unknown nuclei with a large excess of neutrons or protons. This is of particular interest for the very light nuclei like the tetra-neutron etc. because the Bethe-Weizsäcker[4] and related mass formulae [5-9] cannot be used.

Several procedures and methods [10-15] are given in the literature which permit an estimate of the masses of unknown light nuclei and of the excitation energies of isobaric analogue states. These methods, however, are applicable only over a restricted range of nuclei.

The method reported by Baz and Smorodinsky [11] is of particular interest with regard to the present paper. They evidenced the existence of regularities concerning the energy differences $\Delta_{TT'}$ between isobaric analogue states of isobaric spin T and T' and they showed empirically that Δ_{20}, Δ_{31} and also $\Delta_{3/2\ 1/2}$, $\Delta_{5/2\ 1/2}$ are relatively smooth functions of A while Δ_{10} and Δ_{21} split into two branches. From a reasonable interpolation or extrapolation of Δ_{20}, for instance, one can predict the excitation energies of the lowest T = 2 states in certain self-conjugate nuclei. By taking into account the different Coulomb energies one obtains approximate masses[11,14,16] for the nuclei with $T_z = \pm 2$ and the same A.

Franzini and Radicati [17] recently studied the excitation energies of isobaric analogue states up to A = 110 in terms of the supermultiplet model [18]. They came to the conclusion that this model appears to give a good interpretation of the ground state energy for a very large number of nuclei.

The purpose of the present paper (see also ref. 19) is to establish a semi-empirical equation which allows to predict within certain limits the excitation energy of any isobaric analogue state. The existence of regularities which interconnect all energetic differences $\Delta_{TT'}$ between analogue states will be shown. Remarks on the theoretical significance underlying these regularities will be made.

In section 2 the excitation energies of isobaric analogue states are calculated from the known masses of neighbouring nuclei. In sections 3 and 4 relations and regularities are deduced from the symmetry and pairing energies of the respective nuclei which extend far beyond the ones reported by Baz and Smorodinsky [11]. Section 5 gives empirical parameters which allow to predict unknown excitation energies and masses. The inversion of isobaric spin states and the validity of and the deviations from the relation $T = |T_z|$ for the ground states of nuclei is considered in section 6. The A-dependence of the above parameters is discussed in section 7. In section 8 the T-dependence of the above relations is discussed and attention is given to the question whether or not and to what extent the supermultiplet model or other theoretical expressions are compatible with the experimental data.

2. The Excitation Energies of Isobaric Analogue States

Fig. 1 on the left-hand side shows a plot of the masses (filled circles) of isobaric nuclei as a function of the z-component of the isobaric spin. Isobaric analogue states are shown as open circles. The energetic position of these states [15] for a given T depends quadratically on T_z. The figure corresponds to nuclei with an atomic weight which is a multiple of 4. Plots for nuclei with A = 4n+2 or with odd A look similar, except that for odd A because of $\delta = 0$ there is only one parabola-like curve. On the right-hand side of fig. 1 the states are shifted in such a way that corresponding analogue states have an equal energetic position. The situation is idealized because the Coulomb energies may depend not only on T_z and A but also on T, particularly when the configurations involve nucleons belonging to different shells. Effects due to a violation of charge independence of nuclear forces are also ignored. The curves show a cusp at $T_z = 0$. The energy differences $\Delta_{TT'}$ between the isobaric analogue states are indicated in fig. 1.

Most of the excitation energies Δ_{10} of the lowest T = 1 states in the self-conjugate nuclei are known experimentally [20, 21]. Additional excitation energies, $\Delta_{3/2\ 1/2}$ and Δ_{20} in particular, have also been measured recently [1, 2, 20, 22, 23]. These experimental values are plotted in fig. 2 for the odd-A nuclei and in fig. 3 for the even-A nuclei as a function of A as filled circles. In addition, about 200 values of $\Delta_{TT'}$ up to A = 80 and T = 4 were calculated from the known masses [24] of isobaric nuclei and an estimate [25] of the corresponding Coulomb energy difference. They are shown in figs. 2 and 3 as open circles. Errors are indicated when exceeding 0.4 MeV. The $\Delta_{TT'}$ appear as rather continuous functions of A with weak oscillations for odd A, and also for even A when T - T' is even. For even A but odd T - T', however, there are two such branches, one for the nuclei with A = 4n and the other for A = 4n + 2 (n integer). The significance of the curves shown in figs. 2 and 3 will be pointed out later.

3. Symmetry and Pairing Energies

The A- and T-dependence of the $\Delta_{TT'}$ as shown in figs. 2 and 3 can be described in terms of the corresponding symmetry and pairing energies. In good approximation one can express $\Delta_{TT'}$ as

$$\Delta_{TT'}(A) = (E_{sym} + E_{pair})_{T',A} - (E_{sym} + E_{pair})_{T,A}. \quad (1)$$

This representation implies (i) the validity of a Bethe-Weizsäcker type mass formula [4], i.e. the separability in an A-dependent term (for instance volume and surface energy), a Coulomb energy term and terms representing the symmetry and pairing energy and (ii) the T-independence of the Coulomb energy as mentioned before.

For E_{sym} and E_{pair} the following expressions will be used

$$E_{sym} = - \frac{a(A)}{A} \, (T^2 + b(A)\, T) \quad (2)$$

and

$$E_{pair} = \begin{cases} +\delta(A) & \text{for the ground states of even nuclei and its analogue states} \\ -\delta(A) & \text{for the ground states of odd nuclei and its analogue states} \\ 0 & \text{for odd-A nuclei.} \end{cases} \quad (3)$$

Eq. (3) with its secondary conditions can also be written as

$$E_{pair} = \frac{1}{2}\,(1 + (-1)^A)\,(-1)^{\frac{A}{2} - T}\,\delta(A). \quad (4)$$

The expression for the symmetry energy E_{sym} contains a term proportional to T^2 and a term proportional to T. The former term corresponds to the usual term in the Bethe-Weizsäcker mass formula which is proportional to $(N - Z)^2$

if one equates T with $|T_z|$ and extends the validity of the equation to the respective analogue states. The latter term corresponds to the term [18] which is proportional to $|(N - Z)|$. The quantity b(A) stands as an adjustable parameter which is expected to be either constant or only weakly A-dependent. Contradicting values of 1, about 2.5, and 4 were derived theoretically [26,8,18] for b(A).

Inserting eqs. (2) and (4) into eq. (1) immediately gives an explanation for the gross structure of the $\Delta_{TT'}$ as shown in figs. (2) and (3), namely the smooth A-dependence and the energetic difference between the two branches of $\Delta_{TT'}$ for even A and odd T - T' which is particularly obvious for Δ_{10}, i.e. for the excitation energies of the lowest T = 1 states in the self-conjugate nuclei. This energy difference becomes just $4\bar{\delta}(A)$. Thus, one has a method of deriving from experimental data the pairing energies $\delta(A)$ down to the lightest nuclei.

4. Relations between the Energies $\Delta_{TT'}$

From eqs. (1), (2) and (4) one can easily verify relations like

$$\begin{aligned} \Delta_{20} &\approx 2\Delta_{3/2\;1/2} \\ \Delta_{31} &\approx 2\Delta_{5/2\;3/2} \end{aligned} \qquad (5)$$

or in general

$$\Delta_{T+2\;\;T} \approx 2\Delta_{T+3/2\;\;T+1/2} \qquad (6)$$

for integer $T \geq 0$ and neighbouring A. Eq. (6) is independent of the particular values of a(A) and b(A). From fig. 4 one can see that eq. (6) is indeed fulfilled, at least for T = 0, T = 1 and T = 2. The accuracy is of the order of 1 MeV. The lowest array of points represents the sum of the values of Δ_{10} for the two branches with A = 4n and A = 4n+2. This quantity which will be used below exhibits a similar A-dependence.

Based on eqs. (1), (2) and (4) one can show that the ratios $R_{TT'}$ and R which are defined below in eq. (7) depend on T, T' and b(A) only and <u>not</u> on a(A) and A (except for a possible A-dependence of b(A)).

$$R_{TT'} = \frac{\Delta_{T+2\ T}}{\Delta_{T'+2\ T'}} = \frac{\Delta_{T+3/2\ T+1/2}}{\Delta_{T'3/2\ T'+1/2}} = \frac{2T+2+b(A)}{2T'+2+b(A)} \tag{7}$$

$$R = \frac{\Delta_{20}}{\langle\Delta_{10}\rangle_{Av}} = \frac{\Delta_{20}}{1/2\left(\Delta_{10}^{A=4n} + \Delta_{10}^{A=4n+2}\right)} = \frac{4+2b(A)}{1+b(A)} .$$

These equations can be used to determine the parameter b(A). In fig. 5 the experimental values for the ratios R_{10}, R_{21}, R_{20}, and R are plotted as a function of A. The experimental ratios are indeed practically constant and independent of A. They are close to the constants b given on the right-hand side of the figure. Constants a, b, c, and d refer to b(A) = 0 (Bethe-Weizsäcker formula), b(A) = 1, b(A) = 2.5 and b(A) = 4 (supermultiplet model; see section 8). Averaging the experimental ratios one obtains

$R_{10} = 1.615 \pm 0.037$	$a_{10} = 2.00$
(A = 37 ... 65)	$b_{10} = 1.67$
	$c_{10} = 1.44$
	$d_{10} = 1.33$
$R_{21} = 1.385 \pm 0.027$	$a_{21} = 1.50$
(A = 47 ... 75)	$b_{21} = 1.40$
	$c_{21} = 1.31$
	$d_{21} = 1.25$
$R_{20} = 2.158 \pm 0.098$	$a_{20} = 3.00$
(A = 47 ... 65)	$b_{20} = 2.33$
	$c_{20} = 1.89$
	$d_{20} = 1.67$

$$R = 2.995 \pm 0.045 \quad (A = 8 \ldots 52) \qquad \begin{array}{l} a = 4.00 \\ b = 3.00 \\ c = 2.57 \\ d = 2.40 \end{array}$$

One clearly sees that the experimental ratios $R_{TT'}$ and R are best described by the constants b calculated from b(A) = 1. Consequently, the T-dependence of the symmetry energy E_{sym} of eq. (2) is of the form T(T + 1). It must be emphasized that the ratios R_{10}, R_{21}, R_{20} and R are more sensitive to the exact value of b(A) than the ratios used by Franzini and Radicati [17] in their analysis in terms of the supermultiplet model.

The experimental ratio R which comes close to the value of 3 proves that (i) the T(T + 1) dependence of the symmetry energy E_{sym} holds down to the very light nuclei (at least A = 8) and that (ii) the pairing energy E_{pair} can indeed be described by eq. (3) and (4).

Combining eqs. (1), (2) and (4) with b(A) = 1 gives

$$\Delta_{TT'} = \frac{a(A)}{A}\left(T(T+1)-T'(T'+1)\right)+\begin{cases} 0 & \text{for A odd and A even, T-T' even} \\ +2\,\delta(A) & \text{for A even, T-T' odd, } \frac{A}{2} - T' \text{ even} \\ -2\,\delta(A) & \text{for A even, T-T' odd, } \frac{A}{2} - T' \text{ odd} \end{cases} \tag{8}$$

or

$$\Delta_{TT'} = \frac{a(A)}{A}\left(T(T+1)-T'(T'+1)\right)+\frac{1}{2}(1+(-1)^{A})\left((-1)^{\frac{A}{2}-T'}-(-1)^{\frac{A}{2}-T}\right)\delta(A). \tag{9}$$

This is essentially a one-parametric equation for $\Delta_{TT'}$ because for odd A and also for even A when T - T' is even the second term vanishes. Only for even A and odd T - T' the pairing energy $\delta(A)$ is needed as a second parameter.

5. Empirical Parameters and the Prediction of Excitation Energies and Masses

The excitation energy of any isobaric analogue state[+)] and subsequently the mass of any neutron-rich or proton-rich light nucleus can be predicted from eq.(9) within certain limits if reasonable parameters a(A) and $\delta(A)$ can be derived theoretically or at least empirically. Empirical parameters will be extracted below.

Individual symmetry parameters a(A) and pairing energies $\delta(A)$ were calculated from all known $\Delta_{TT'}$ using eq.(9). Fig. 6 shows the result. Indeed, all a(A) and $\delta(A)$ are clustered and smooth curves can be drawn through the points. In principle, the filled circles and filled squares which are extracted from $\Delta_{10}^{A=4n}$ and $\Delta_{10}^{A=4n+2}$ alone are sufficient to obtain a(A) and $\delta(A)$ up to A=50. However, in order to obtain the best overall fit it appears most reasonable to use averaged parameters a(A) and $\delta(A)$ which are derived from <u>all</u> known $\Delta_{TT'}$. These parameters are shown in fig. 6 as full lines.

Energy differences $\Delta_{TT'}$ were calculated from eq.(9) using the averaged empirical symmetry parameter a(A) and

+) For the spin and parity of the states under consideration simple rules can be given: For the odd-A nuclei spin and parity J^{π} have a strong tendency towards j^{π} of the last unpaired nucleon (holds only if the state has lowest seniority). For the even-A nuclei J^{π} is equal to 0^{+} for $\frac{A}{2} - T$ = even and follows the revised Nordheim rules given by Brennan and Bernstein [35)] with positive parity for $\frac{A}{2} - T$ = odd.

pairing energy $\delta(A)$ from fig. 6. The results are shown in figs. 2 and 3 as full lines. There are apparently no obvious systematic deviations between the experimental and calculated values except, possibly, for the region slightly below A = 50. Deviations > 1 MeV appear for $\Delta_{3/2\ 1/2}$ at A = 13, 19, 29, and 65 for $\Delta_{5/2\ 1/2}$ at A =65, for $\Delta_{7/2\ 1/2}$ at A = 65, for $\Delta_{7/2\ 5/2}$ at A = 47, for Δ_{20} at A = 26 and 40, for Δ_{21} at A = 26, for Δ_{32} at A = 40, for Δ_{42} at A = 48 and 50, for Δ_{43} at A = 48. The values $\Delta_{3/2\ 1/2}$, $\Delta_{5/2\ 1/2}$ and $\Delta_{7/2\ 1/2}$ for A = 65 are too high which seems to indicate that the measured[27)] maximum ß-energy of the decay $Ge^{65} \rightarrow Ga^{65}$ of 3.7 ± 0.4 MeV is too low[+)] by about 1.35 MeV.

Fig. 7 shows the distribution function for the difference between the over 200 experimental values of $\Delta_{TT'}$ and the calculated values. The distribution has a Gaussian shape with a standard deviation of about 0.5 MeV. This means that the experimental $\Delta_{TT'}$ are reproduced by eq.(9) with the parameters a(A) and $\delta(A)$ from fig. 6 with a standard deviation of about 0.5 MeV. From this finding it follows that eq.(9) can as well be used to predict the excitation energies of unknown analogue states. These energies can be read directly from figs. 2 and 3. By adding or subtracting proper Coulomb energy differences the mass of any unknown nucleus can also be predicted with an estimated error of about $\pm$ 2 MeV up to A = 10 and $\pm$ 1 MeV up to A = 80. This procedure of estimating the masses of unknown nuclei is of particular interest when the mass of the higher-order mirror nucleus is not known, i.e. for all unknown neutron-rich and the unknown very proton-rich nuclei. Tables for the estimated masses and decay characteristics of unknown proton-rich and neutron-rich nuclei are in preparation[29)].

[+)] See also comment D for A = 65 of ref. 28.

Up to A = 8 one obtains the following values with an estimated accuracy of about ± 2 MeV:

Δ_{10}	$\approx$ 22.5 MeV	for A = 4
$\Delta_{3/2\ 1/2}$	$\approx$ 20.0 MeV	for A = 3
$\Delta_{3/2\ 1/2}$	$\approx$ 17.8 MeV	for A = 5
$\Delta_{3/2\ 1/2}$	$\approx$ 15.9 MeV	for A = 7
Δ_{20}	$\approx$ 38.2 MeV	for A = 4
Δ_{20}	$\approx$ 33.9 MeV	for A = 6
Δ_{20}	$\approx$ 30.0 MeV	For A = 8.

From these energies it follows that the tri-neutron and tetra-neutron are expected to be highly unstable with regard to disintegration into single neutrons by about 12,3 MeV and 10,7 MeV, respectively. No conclusive statements can be made concerning H^4, H^5, and He^8. Of these the nucleus He^8 experimentally appears to be the only one which is stable with regard to the emission of neutrons and undergoes a ß-decay[30] instead. The nuclei H^6 and He^7 (see ref. 31) are expected to be unstable with regard to the spontaneous emission of a neutron by about 9.6 MeV and 5.2 MeV, respectively.

6. The Inversion of Isobaric Spin States

Eq.(9)describes within the accuracy shown in fig. 7 the inversion of the lowest T = 0 and T = 1 states in the odd self-conjugate nuclei. The ground states of Cl^{34}, Sc^{42}, V^{46}, and possibly Mn^{50} and Co^{54} are known experimentally [21,32-34] to have isobaric spin T = 1. The calculated energy difference $\Delta_{10}^{A=4n+2}$, on the other hand, is indeed close to zero or slightly negative for A = 34 ... 58. For larger A the quantity $\Delta_{10}^{A=4n+2}$ becomes more negative and consequently the ground states of the T_z = 0 nuclei (Ga^{62}), As^{66}, Br^{70}, Rb^{74} etc. are most likely to have T = 1. They then have spin and parity 0^+ and undergo a super-allowed pure Fermi ß-decay with ft= 3100 sec. Isomerism is likely in these nuclei [34,35].

Inversion occurs whenever Δ_{T+1T} is negative. From eq.(9) one can easily derive the conditions for such an inversion in terms of the parameters a(A) and δ(A). One obtains

$$\Delta_{10}^{A=4n+2} < 0 \qquad \text{for} \qquad A_1 > \frac{a(A_1)}{\delta(A_1)}$$

$$\Delta_{21}^{A=4n} < 0 \qquad \text{for} \qquad A_2 > 2\,\frac{a(A_2)}{\delta(A_2)} \tag{10}$$

$$\Delta_{32}^{A=4n+2} < 0 \qquad \text{for} \qquad A_3 > 3\,\frac{a(A_3)}{\delta(A_3)}$$

The first inequality has been discussed before. Using the parameters a(A) and δ(A) from refs. 8 and 36 one estimates $108 \lessapprox A_2 \lessapprox 124$, $A_2 \gtrapprox 192$ and $A_3 \gtrapprox 290$.

The obove considerations show that the relation $T = |T_z|$ holds for the ground states of most nuclei. There are only a few exceptions when $T = |T_z| + 1$.

The relation $T = |T_z|$ holds

- (i) for all odd-A nuclei
- (ii) for all even nuclei
- (iii) for most odd nuclei except for the nuclei with

$T_z = 0$; $A = 4n+2$; A = 34, 42, 46, 50 (?), 54 (?) and $\gtrapprox 62$

$T_z = \pm 1$; $A = 4n$; $108 \lessapprox A \lessapprox 124$ (uncertain) and $A \gtrapprox 192$

$T_z = \pm 2$; $A = 4n + 2$; $A \gtrapprox 290$

The exceptions from the rule $T = |T_z|$ are of practical interest only for $T_z = 0$.

7. Discussion of the Empirical Parameters

The parameters a(A) and δ(A) are shown in fig. 6. The symmetry parameter a(A) exhibits shell structure with maxima at A = 16, 28, 40 and 56. This is reasonable. The configurations of the states under consideration are very simple because they are analogue to the ground states of the neighbouring isobars with $T_z = \pm T$. In a simplified picture the energies $\Delta_{3/2\ 1/2}$ or Δ_{20}, for instance, can be interpreted as the energies needed to raise one or two nucleons into higher orbits without changing the number of antisymmetrically coupled pairs. This energy is indeed expected to be higher near closed shells. In the same picture one can also, at least qualitatively, understand relations like $\Delta_{20} \approx 2\Delta_{3/2\ 1/2}$ or other similar relations between the various $\Delta_{TT'}$.

For small A the parameter a(A) becomes small and $a(A) \rightarrow 0$ for $A \rightarrow 0$. As a consequence, $\frac{a(A)}{A}$ remains finite for small A and so do the excitation energies of the analogue states. For larger A up to A = 80 the parameter a(A) can be compared with the parameters given for the symmetry term proportional to $(N - Z)^2 = 4\ T_z^2$ of known semi-empirical mass formulae [4-9]. There is qualitative agreement in the range of overlapping A. The parameter a(A) of this paper exhibits shell structure though, while the known parameters do not.

The pairing energies δ(A) shown in fig. 6 were extracted from the energy differences Δ_{10}, Δ_{21} etc., i.e. from $\Delta_{TT'}$ for even A and odd T - T'. There are two branches, one for nuclei with A = 4n and the other for nuclei with A = 4n + 2. The separation energy between these branches is equal to $4\,\delta$(A). From eq.(9)it becomes clear that there exist additional relations which allow the extraction of values for δ(A). The simplest relations are

$$\begin{aligned} 6\ \delta(A) &= 3\,\Delta_{10}(A) - \Delta_{20}(A) && \text{for } A = 4n \\ 6\ \delta(A) &= \Delta_{20}(A) - 3\,\Delta_{10}(A) && \text{for } A = 4n+2. \end{aligned} \tag{11}$$

These relations are of special interest because recently experimental values for the excitation energies of the lowest T = 2 states in T_z = 0 nuclei have become available[2].

Nemirovsky and Adamchuk [36] calculated the pairing energies δ_n and δ_p of two neutrons and two protons for nuclei from A = 10 to A = 252. They used the second differences of the known binding energies $E_{Z,N}$ of adjacent nuclei (isotopes and isotones, not isobars) and applied corrections due to the curvature of the mass surface. For the corrections they considered the surface energy, the Coulomb energy and the symmetry energy. Below A = 40 the pairing energy $\delta(A)$ of our paper is appreciably smaller than given in the detailed analysis by Nemirovsky and Adamchuk [36]. This discrepancy is at least in part due to the symmetry energy, which was taken by the authors to be proportional to T^2 and thus results in a not quite adequate description of the actual curvature of the mass surface.

8. Discussion of the T-dependence

Eq.(9) and its T-dependence in particular can be compared with corresponding expressions which were obtained theoretically [18,8,26]. In this paper the symmetry energy is given with a T-dependence of the form T(T+1). The empirical Bethe-Weizsäcker mass formula [4] uses only a T^2-dependence. Contradicting dependences of the form T(T+4), T(T+ appr. 2.5) and T(T+1) were derived theoretically.

Franzini and Radicati have shown that the supermultiplet model[18] gives a good interpretation of the ground state energy for a large number of nuclei. The supermultiplet model leads to a proportionality between $\delta(A)$ and a(A). This fact has the advantage that the $\Delta_{TT'}$ then can be given in a completely one-parametric form [17]. From the analysis given in the present paper, however, it follows that there exist small but systematic deviations between the experimental data and the description in terms of the supermultiplet model.

(i) The T-dependence of the symmetry energy does not seem to be of the form $T(T+4)$ but of the form $T(T+1)$.

(ii) The pairing energy $\delta(A)$ does not seem to be proportional to the quantity $a(A)/A$; Instead $\delta(A) < a(A)/A$ for $A < 34$, $\delta(A) \approx a(A)/A$ for $34 \leqq A \leqq 58$, and $\delta(A) > a(A)/A$ for $A > 58$.

(iii) The relation $\Delta_{10}^{A=4n+2} = 0$ does not hold, i.e. the lowest $T = 1$ and $T = 0$ states in the odd self-conjugate nuclei are not degenerate.

In the supermultiplet model Wigner and Majorana forces only are used. The above deviations show that Bartlett and Heisenberg forces cannot be neglected, i.e. the supermultiplet model represents an approximation only.

Ayres et al.[8] theoretically derived for the quantity b(A) in $E_{sym} = \frac{a(A)}{A}\,(T^2 + b(A)\;T)$ the expression

$$b(A) = 2\;\frac{3+\alpha}{3-\alpha}\,(\,1 + \text{small terms}). \qquad (12)$$

($b(A) = a_i/2\;a_a$ in their notation). Here $\alpha = {}^{13}V/{}^{31}V$ is the ratio of the singulet-even to triplet-even forces. The small terms are slightly A-dependent and, in addition, contain the quantities $\beta = {}^{33}V/{}^{31}V$ and $\gamma = {}^{11}V/{}^{31}V$ which are the ratios of the triplet-odd and singulet-odd forces to the triplet-even forces. With an $\alpha = 0.754$ as given by Ayres et al.[8] the quantity b(A) varied for $A = 20$ to $A = 267$ from 2.4 to 2.9. It is only for an unreasonable ratio α of about -1 that the quantity b(A) becomes + 1.

De Shalit and Talmi[26] have given a theoretical expression for the binding energies of n nucleons in a given j-shell based on effective two-nucleon interactions. The T-dependence derived empirically in the present paper is in agreement with the above expression. Therefore it becomes possible to equate the quantities $a(A)/A$ and $\delta(A)$ with the respective coefficients. These coefficients are functions of j and the effective two-nucleon interactions which according to fig. 6 appear not to be completely constant within given shells. It has been shown before from the experimental data that $\delta(A) \approx a(A)/A$ over a range of atomic weights A. This fact leads to an additional relation between the effective interactions+).

Brief mention shall be made of the fact that the level structure of the isobaric analogue states (analogue to the ground states of the nuclei with $T_z = \pm T$) as shown in fig. 8 and described by eq.(9) bears close similarity to rotational bands. T stands for J and $a(A)/A$ stands for the rotational energy. For even A the pairing energy $\delta(A)$ must be added or subtracted in a way which is similar to the decoupling term in K = 1/2 rotational bands. Thus, one may at least call the level structure of the isobaric analogue states "isobaric spin rotational bands" which seem to exist in all atomic nuclei.

After the completion of this work Zeldes, Gronau and Lev[37] published a shell - model semi - empirical nuclear mass formula which reproduces the experimental masses of not too light nuclei very well.

+) In the notation of De Shalit and Talmi [26] this relation can be written as

$$(\bar{V}_2 - \bar{V}_1) \begin{matrix} > \\ \approx \\ < \end{matrix} 2\,\frac{2j+3}{2j+1}\,(V_0 - \bar{V}_2) \quad \text{for} \begin{cases} A < 34 \\ 34 \leqq A \leqq 58 \\ A > 58 \end{cases}$$

References:

1) J.D. Anderson and C. Wong, Phys. Rev. Lett. 7 (1961) 250;

J.D. Anderson, C. Wong and J.W. MacClure, Phys. Rev. 129 (1963) 2718;

R. Sherr, M.E. Rickey and C.G. Hoot, Bull. Amer. Phys. Soc. 9 (1964) 458

2) G.T. Garvey, J. Cerny and R.H. Pehl, Phys. Lett. 12 (1964) 234; Phys. Rev. Lett. 12 (1964) 726

3) A.M. Lane and J. Soper, Phys. Lett. 1 (1962) 28; Nuclear Physics 37 (1962) 506; Nuclear Physics 37 (1962) 663

4) C.F. von Weizsäcker, Z.f.Phys. 96 (1935) 431; H.A. Bethe, Phys. Rev. 50 (1936) 332; H.A. Bethe and R.F. Bacher, Revs. Mod. Phys. 8 (1936) 82

5) A.G.W. Cameron, Chalk River Report AECL 433 (1957)

6) H.B. Levy, Phys. Rev. 106 (1957) 1265

7) P.A. Seeger, Nuclear Physics 25 (1961) 1

8) R. Ayres, W.F. Hornyak, L.Chan and H. Fann, Nuclear Physics 29 (1962) 212

9) B.S. Dzhelepov and G.V. Dranitsyna, Systematics of ß-decay energies (Academy of Sciences of the USSR, Leningrad 1960; Pergamon Press, Oxford 1963)

10) B.S. Dzhelepov, Izv. Akad. Nauk USSR (ser. fyz.) 15 (1951) 496

11) A.I. Baz and J.A. Smorodinsky, Usp. Fiz. Nauk 55 (1955) 215; Der Isospin von Atomkernen, ed. by J. Schintlmeister (Akademie Verlag, Berlin 1960)

A.I. Baz, Atomn. Energ. 6 (1959) 571; J. Atom. Energy 6 (1959) 422

12) V.I. Goldansky, JETP 38 (1960) 1637; JETP (Soviet Physics) 11 (1960) 1179

13) V.I. Goldansky, JETP 39 (1960) 497; JETP (Soviet Physics) 12 (1961) 348; Nuclear Physics 19 (1960) 482

14) J. Jänecke, Nuclear Physics 61 (1965) 326; Kernforschungszentrum Karlsruhe report KFK-185 (1963)

15) D.H. Wilkinson, Phys. Lett. 11 (1964) 243; Phys. Lett. 12 (1964) 348

16) Ya.B. Zeldovich, JETP 38 (1960) 1123; JETP (Soviet Physics) 11 (1960) 812;

17) P. Franzini and L.A. Radicati, Phys. Lett. 6 (1963)322

18) E.P. Wigner, Phys. Rev. 51 (1937) 106;
J.M. Blatt and V.I. Weisskopf, Theoretical Nuclear Physics (John Willey and Sons, New York, 1952) p. 222

19) J. Jänecke, International Conference on Nuclear Physics, Paris (1964) Vol. II, p. 359

20) F. Ajzenberg-Selove and T. Lauritsen, Nuclear Physics 11 (1959) 1

21) P.M. Endt and C. van der Leun, Nuclear Physics 34 (1962) 1

22) J.C. Hardy and R.I. Verrall, Phys. Lett. 13 (1964) 148;
J.C. Hardy and R.I. Verrall, Phys. Rev. Lett. 13 (1964) 764;
P.L. Reeder, A.M. Poskanzer and R.A. Esterlund, Phys. Rev. Lett. 13 (1964) 767;
R. McPherson and J.C. Hardy, Can. J. Phys. 43 (1965) 1

23) R.D. MacFarlaine and A. Siivola, Nuclear Physics 59 (1964) 168

24) L.A. König, J.H.E. Mattauch and A.H. Wapstra, Nuclear Physics 31 (1962) 18

25) J. Jänecke, Z.f.Phys. 160 (1961) 171

26) I. Talmi, Revs. Mod. Phys. 34 (1962) 704;
A. de-Shalit and I. Talmi, Nuclear Shell Theory (Academic Press, New York and London, 1963) p. 457

27) N.T. Porile, Phys. Rev. 120 (1960) 927

28) Nuclear Data Sheet of the National Academy of Sciences

29) H. Behrens and J. Jänecke, to be published

30) B.M.K. Nefkens, Phys. Rev. Lett. 10 (1963) 243

31) V.V. Balashov, Atomn. Energ. 9 (1960) 48; J. Atom. Energy 9 (1961) 544

32) P.C. Rogers and G.E. Gordon, Phys. Rev. 129 (1963) 2653
J.W. Nelson, J.D. Oberholtzer, and H.S. Plendl, Nuclear Physics 62 (1965) 434

33) D.C. Sutton, H.A. Hill and R. Sherr, Bull. Amer. Phys. Soc. 4 (1959) 278;
D.C. Sutton, Thesis, Princeton University Technical Report PUC-1961-50 (unpublished)

34) J. Jänecke, Nuclear Physics 30 (1962) 328

35) M.H. Brennan and A.M. Bernstein, Phys. Rev. 120 (1960) 927

36) P.E. Nemirovsky and Yu.V. Adamchuk, Nuclear Physics 39 (1962) 551

37) N. Zeldes, M. Gronau and A. Lev, Nuclear Physics **63** (1965) 1

Fig. 1 Relationship between the masses (filled circles) of nuclei with A = 4n(n integer) and $T_z = \frac{1}{2}$ (N - Z). The left-hand side is without any corrections, the right-hand side is corrected with regard to the n-p mass difference and the different Coulomb energies. The quantity δ is the pairing energy. Isobaric analogue states are shown as open circles. The energy differences $\Delta_{TT'}$ between these states are indicated.

Figs. 2 and 3 Energy difference $\Delta_{TT'}$ between the energetically lowest states of isobaric spin T and T' as a function of A. Fig. 2 is for odd-A nuclei, fig. 3 for even-A nuclei. Experimental values are shown as filled circles. Values calculated from the known masses of isobars are shown as open circles. Errors are indicated when exceeding 0.4 MeV. The curves are calculated from eq. (9) with the help of the semi-empirical symmetry parameter a(A) and pairing energy δ(A) of fig. 6.

Fig. 4 Plot of the quantities Δ_{20}, Δ_{31}, and Δ_{42} (filled circles) as well as $2\Delta_{3/2\ 1/2}$, $2\Delta_{5/2,\ 3/2}$, and $2\Delta_{7/2\ 5/2}$ (open circles) as functions of A. The points are arranged in three groups which proves eq. (6) for T = 0, T = 1 and T = 2. The A-dependence of $(\Delta_{10}^{A=4n} + \Delta_{10}^{A=4n+2})$ (open triangles) is of a similar structure as compared with the other quantities.

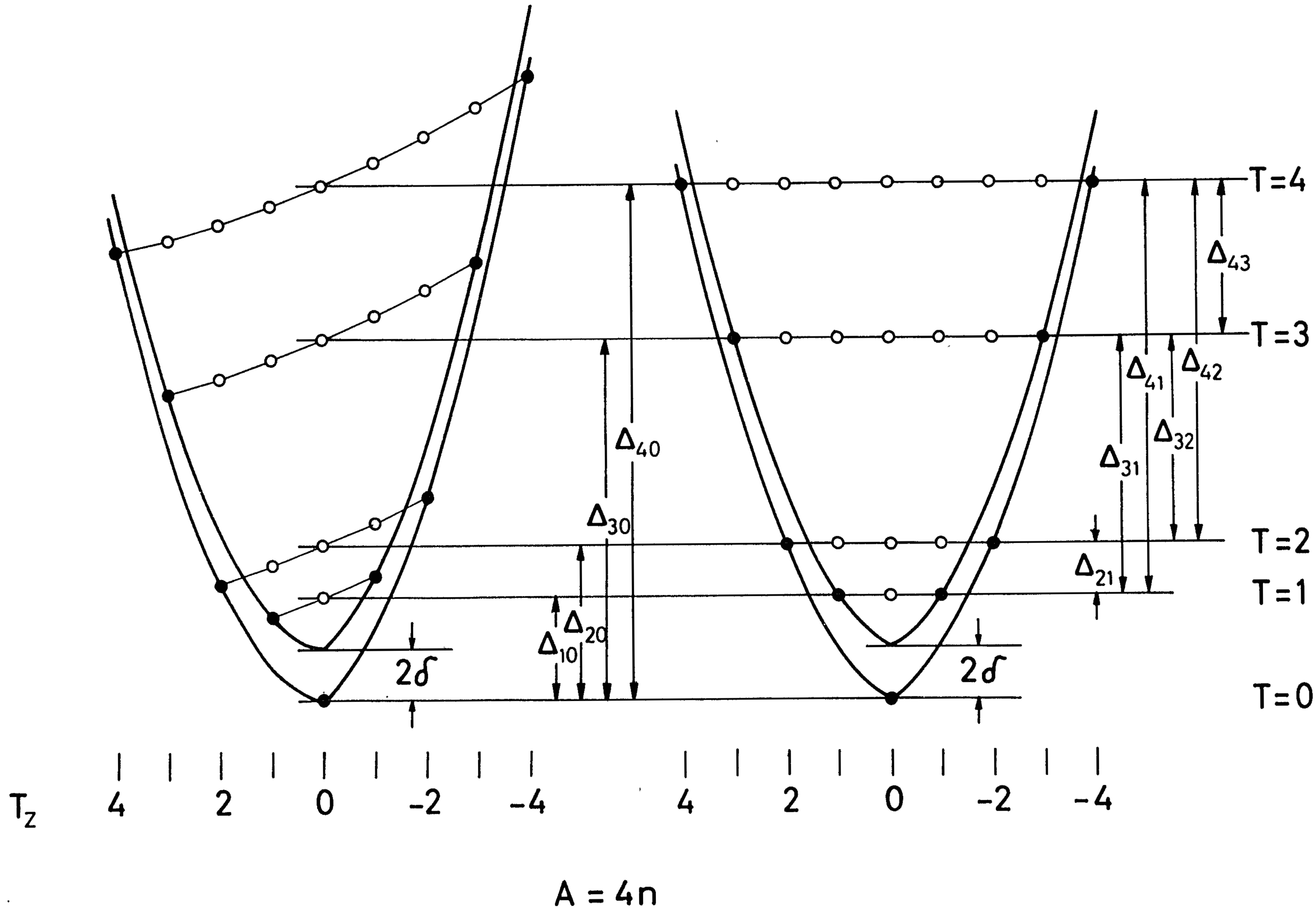
Fig. 5 Plot of the experimental ratios R_{10}, R_{21}, R_{20}, and R (for definition see eq. (7)) as functions of A. The constants a, b, c, and d given on the right-hand side refer to the constants calculated with b(A) = 0 (Bethe-

Weizsäcker formula), b(A) = 1, b(A) = 2.5 and b(A) = 4 (supermultiplet model). The experimental points come closest to the respective constants b calculated with b(A) = 1.

Fig. 6 Symmetry parameters a(A) and pairing energies δ (A) calculated from the experimental energy differences $\Delta_{TT'}$. Values for a(A) derived from even-A and odd-A nuclei are shown as circles and triangles, respectively, Values for δ(A) are shown as squares. Values for a(A) and δ(A) derived from $\Delta_{10}^{A=4n}$ and $\Delta_{10}^{A=4n+2}$, in particular, are shown as filled circles and squares. Averaged curves are shown as full lines.

Fig. 7 Distribution function for the difference between experimental and calculated $\Delta_{TT'}$. The distribution is compatible with a Gaussian function with a standard deviation of about 0.5 MeV.

Fig. 8 Energetic position of the isobaric analogue states for odd-A and even-A nuclei. The energy scale is in units of a(A)/A. The pairing energy δ(A) in the figure was arbitrarily chosen equal to 0.9 a(A)/A. The level structure resembles rotational bands.



T=4
T=3
T=2
T=1
T=0
Δ_{43}
Δ_{42}
Δ_{41}
Δ_{32}
Δ_{31}
Δ_{21}
Δ_{40}
Δ_{30}
Δ_{20}
Δ_{10}
2δ
2δ
T_z
4 2 0 -2 -4
4 2 0 -2 -4
A = 4n

FIG. 1

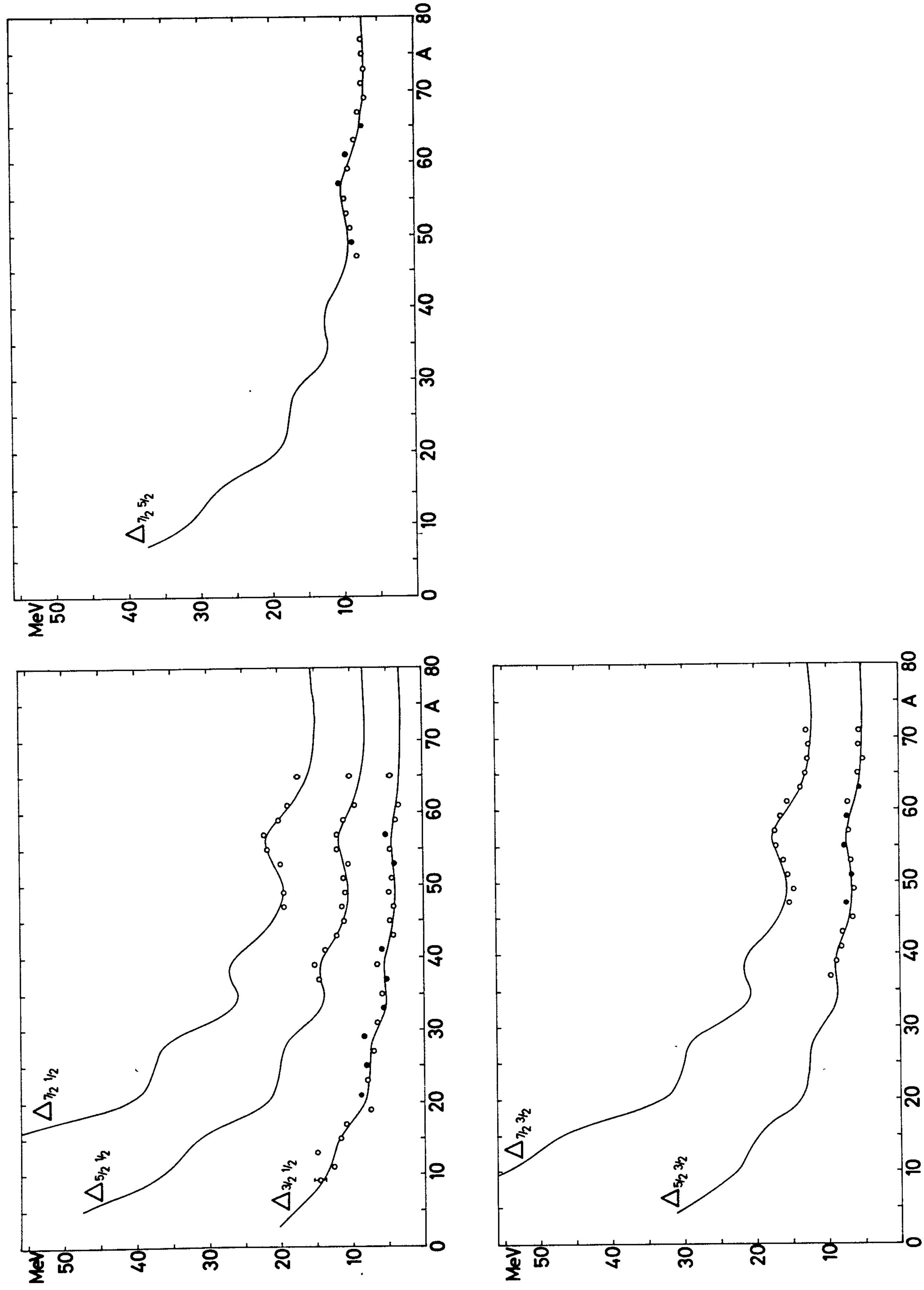
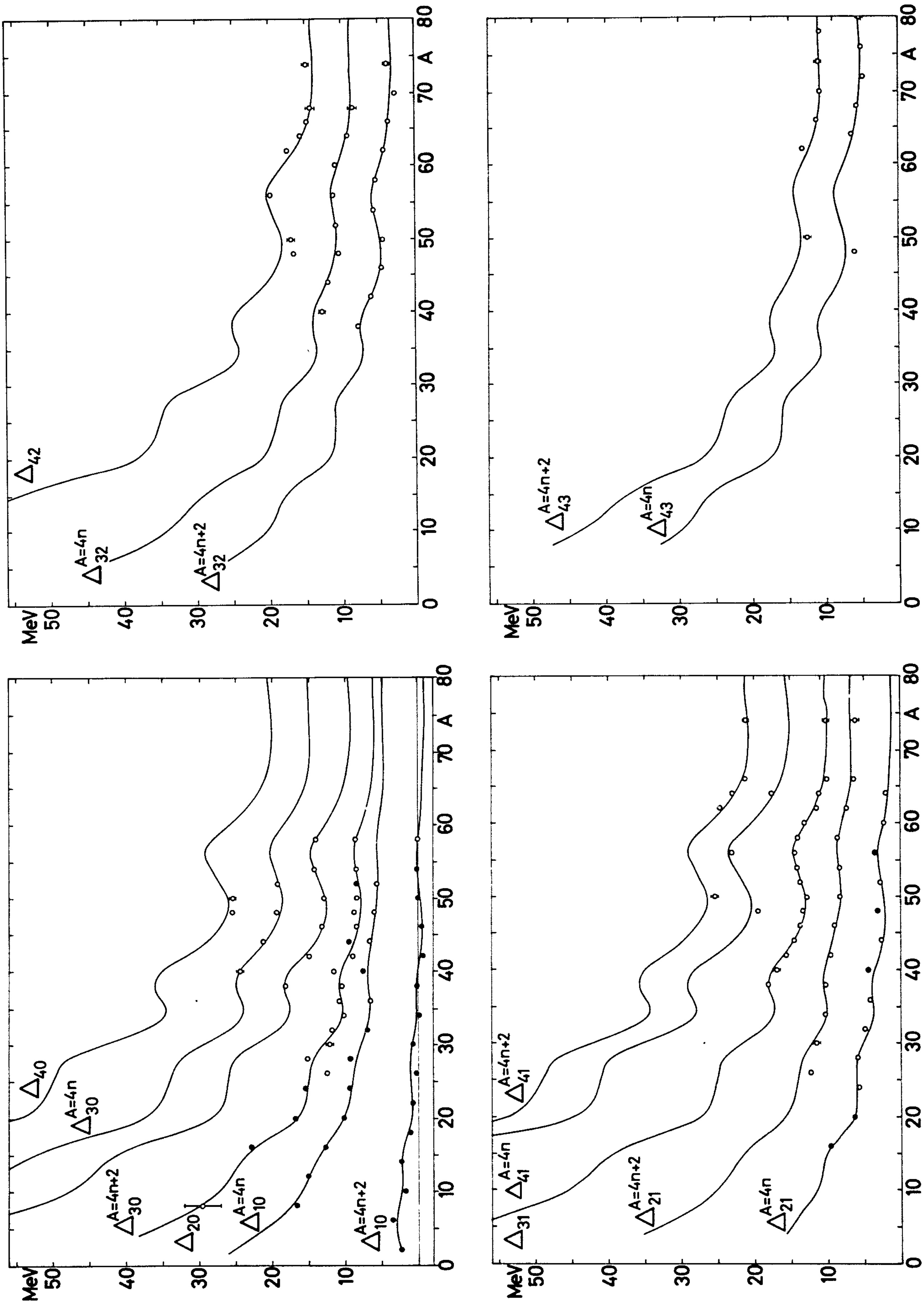

MeV
A
$\Delta_{7/2\ 5/2}$
$\Delta_{7/2\ 1/2}$
$\Delta_{5/2\ 1/2}$
$\Delta_{3/2\ 1/2}$
$\Delta_{7/2\ 3/2}$
$\Delta_{5/2\ 3/2}$

FIG. 2



MeV
A
Δ40
Δ30 A=4n
Δ30 A=4n+2
Δ20
Δ10 A=4n
Δ10 A=4n+2
Δ42
Δ32 A=4n
Δ32 A=4n+2
Δ31
Δ41 A=4n+2
Δ41 A=4n
Δ21 A=4n+2
Δ21 A=4n
Δ43 A=4n+2
Δ43 A=4n

FIG. 3

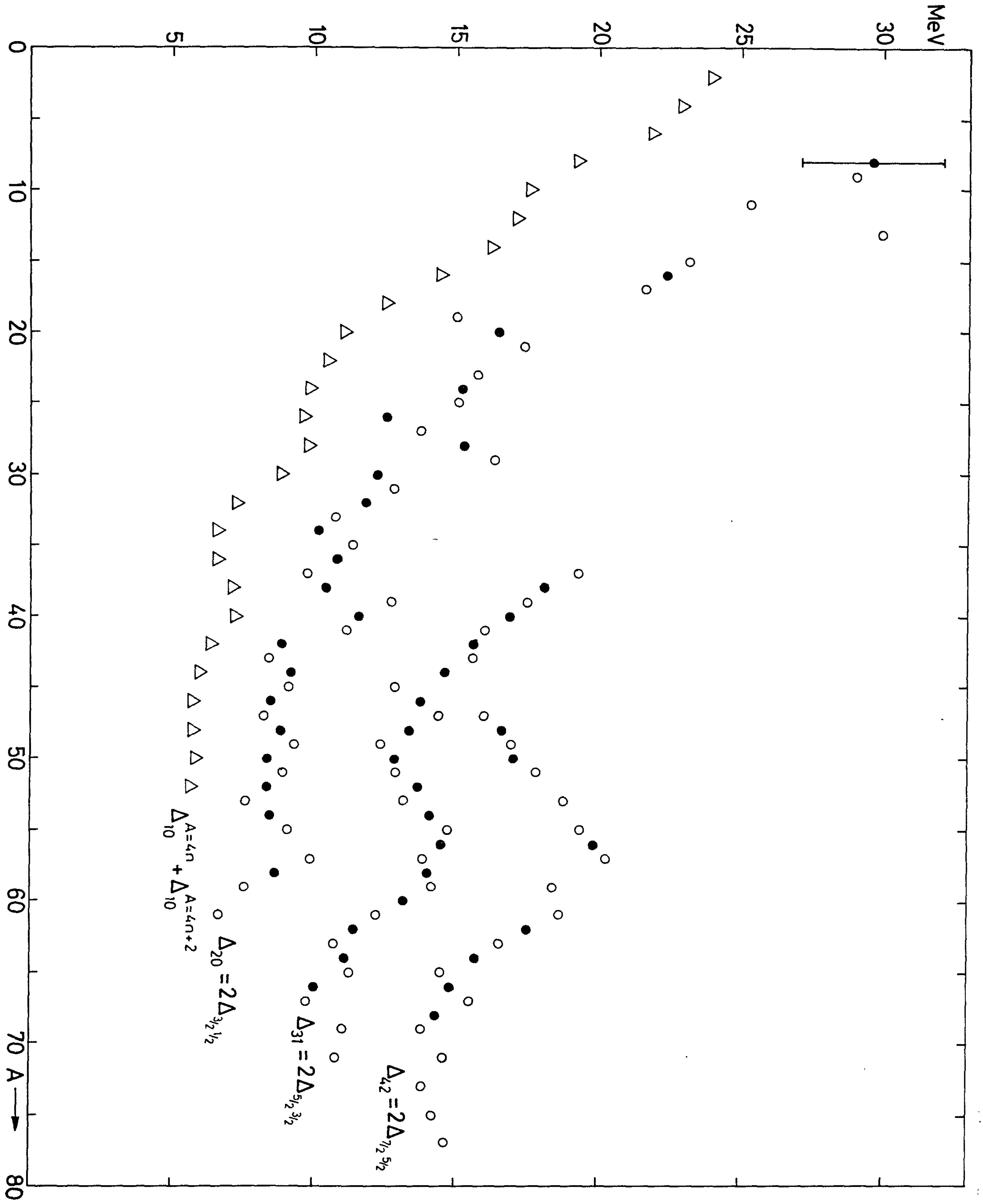
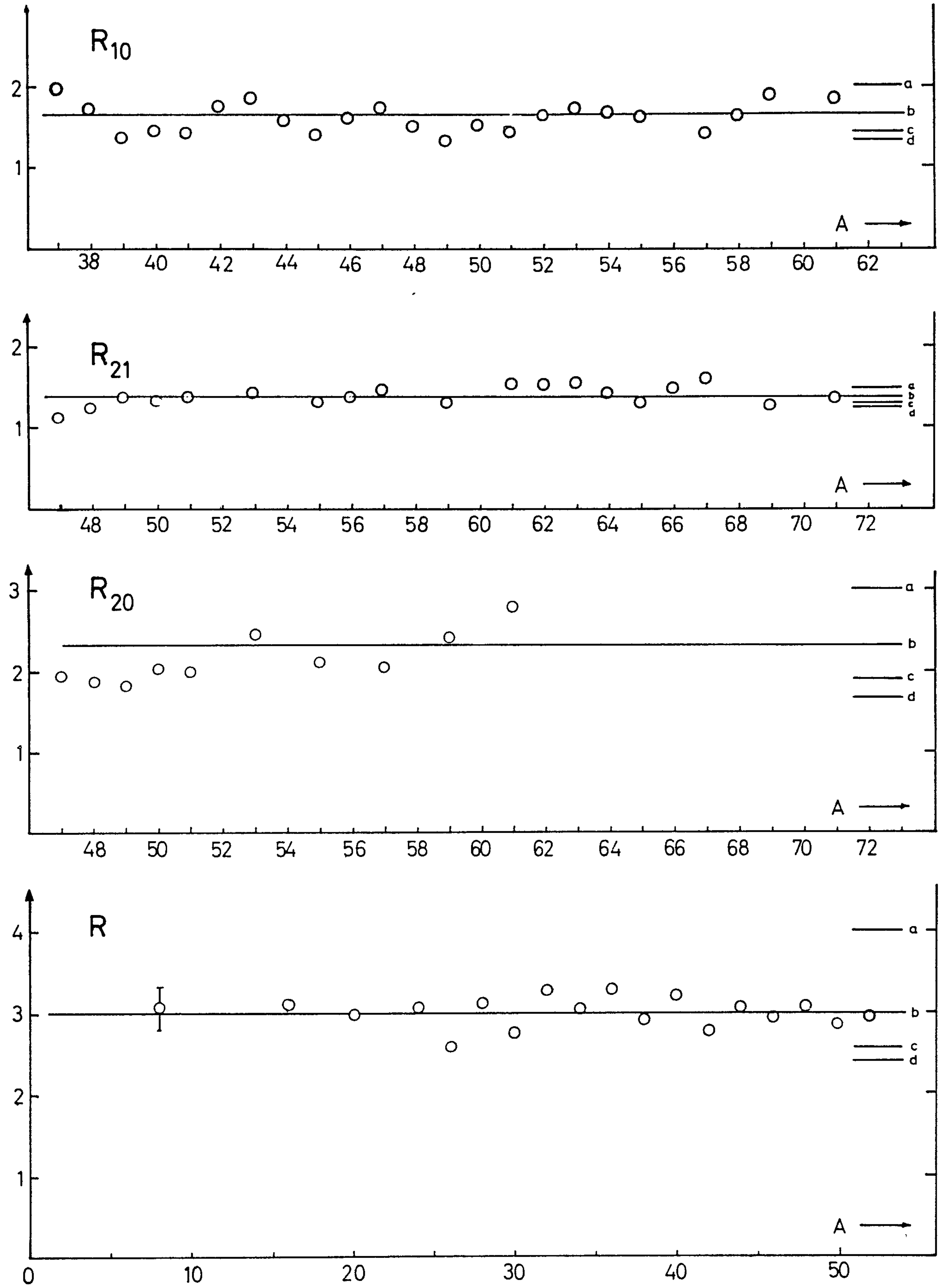

MeV
A
$\Delta_{10}^{A=4n} + \Delta_{10}^{A=4n+2}$
$\Delta_{20} = 2\Delta_{3/2\,1/2}$
$\Delta_{31} = 2\Delta_{5/2\,3/2}$
$\Delta_{42} = 2\Delta_{7/2\,5/2}$

FIG.4



R_{10}
2
1
a
b
c
d
A
38
40
42
44
46
48
50
52
54
56
58
60
62
R_{21}
2
1
a
b
c
d
A
48
50
52
54
56
58
60
62
64
66
68
70
72
R_{20}
3
2
1
a
b
c
d
A
48
50
52
54
56
58
60
62
64
66
68
70
72
R
4
3
2
1
a
b
c
d
A
0
10
20
30
40
50

FIG. 5

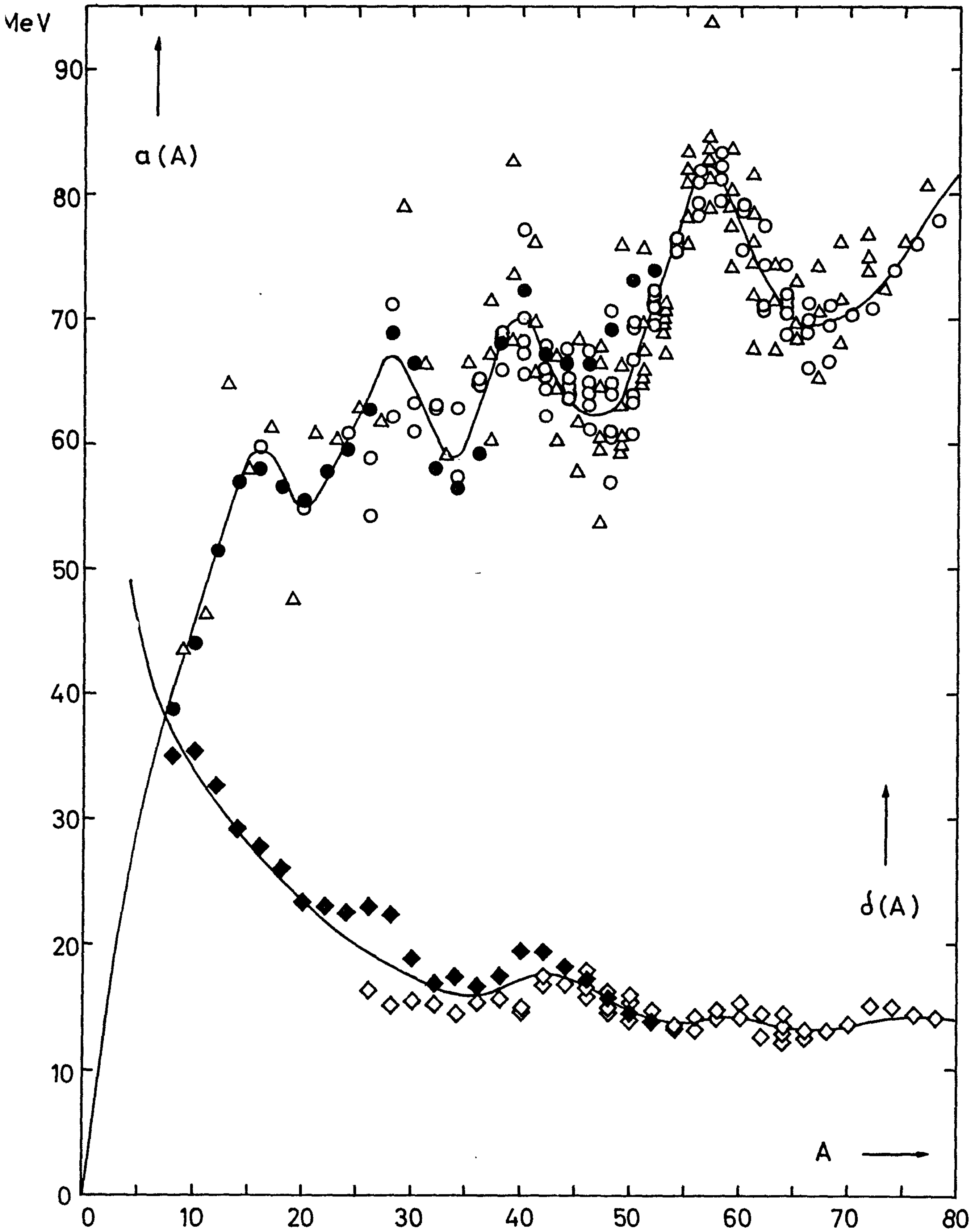
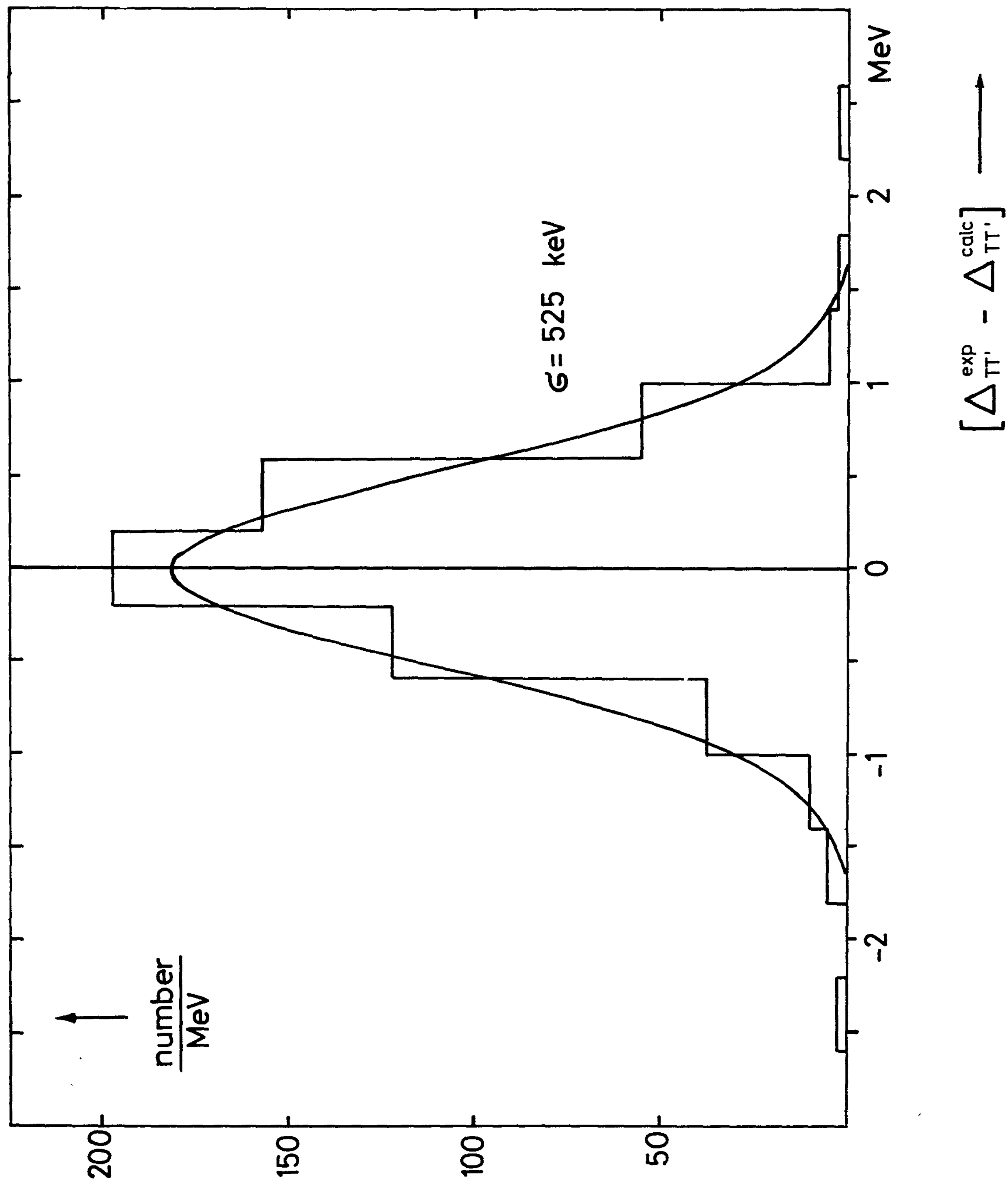
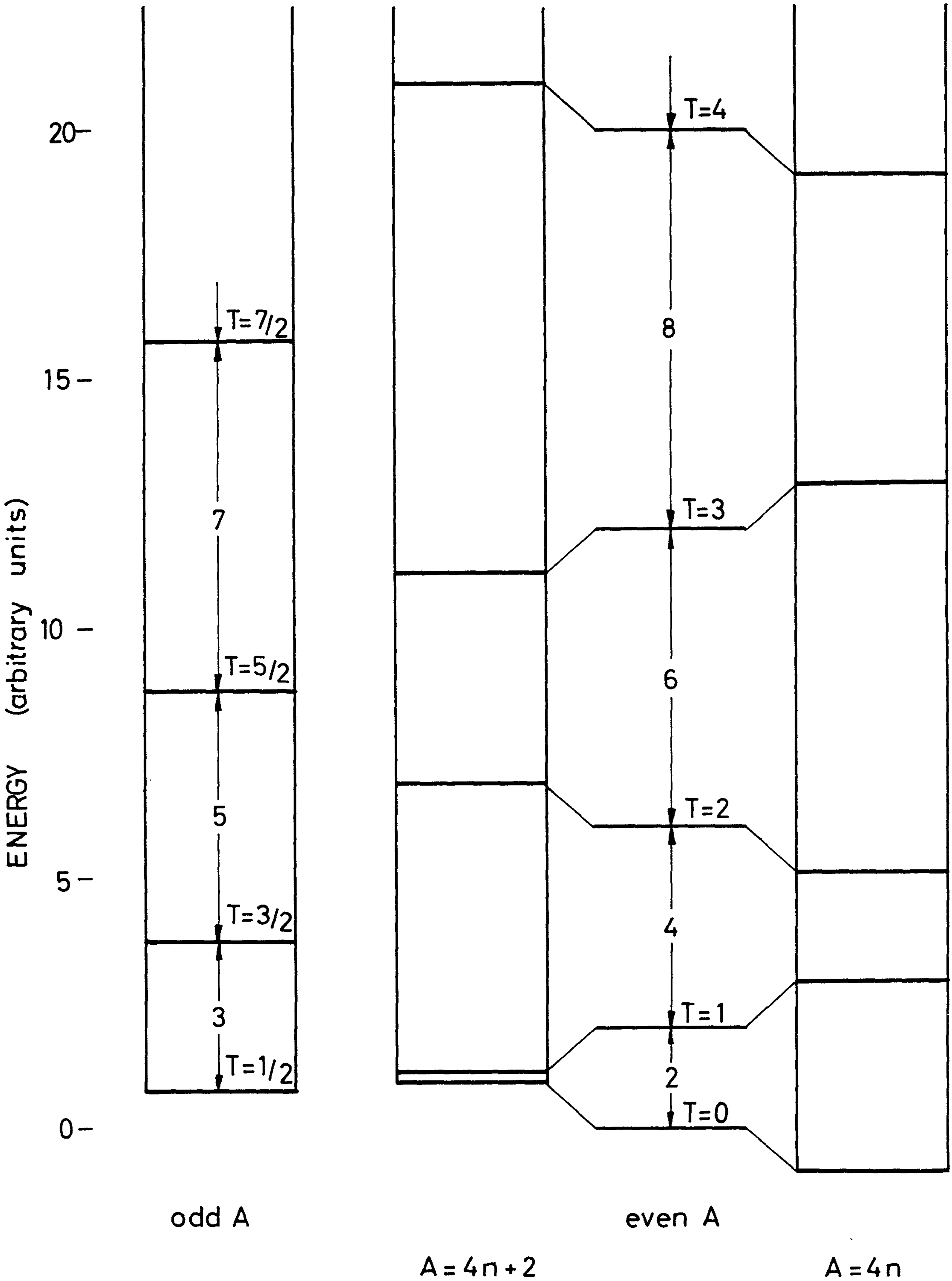

MeV
90
80
70
60
50
40
30
20
10
0
a (A)
MeV
5
4
3
2
1
0
δ (A)
A
0
10
20
30
40
50
60
70
80

FIG. 6



number/MeV
σ = 525 keV
200
150
100
50
-2
-1
0
1
2
MeV
$[\Delta^{exp}_{TT'} - \Delta^{calc}_{TT'}]$

FIG.7



20
15
10
5
0
ENERGY (arbitrary units)
T=7/2
T=5/2
T=3/2
T=1/2
7
5
3
T=4
T=3
T=2
T=1
T=0
8
6
4
2
odd A
even A
A=4n+2
A=4n

FIG 8