KERNFORSCHUNGSZENTRUM

KARLSRUHE

Juli 1965

KFK 363

Institut für Neutronenphysik und Reaktortechnik

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GESELLSCHAFT FÜR KERNFORSCHUNG M.B.H.

KARLSRUHE

Sonderdruck aus
„Nukleonik", 7. Band, 6. Heft, 1965, S. 313—325
Springer-Verlag, Berlin · Heidelberg · New York

A Common Theory for Neutronic Noise Analysis Experiments in Nuclear Reactors*

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With 6 Figures in the Text

(Received January 23, 1965)

Summary. A common basic formula is developed for the description of neutronic noise analysis experiments in nuclear reactors. From this formula specific expressions are derived for techniques, such as frequency analysis of noise, Rossi-α-experiments, variance-determinations, and the analysis of counting statistics. Proposals for new techniques are included which seem advantageous from theoretical and experimental considerations. Although the basic theoretical formulas are of general validity and include space- and energy-dependence as well as delayed neutrons, the point-reactor model approximation has been used for the derivation of most specific expressions. The possibility of using noise analysis experiments for investigating the kinetics of coupled reactor systems is also discussed.

1. Introduction

The neutronic behaviour of a reactor is based on various nuclear reactions of stochastic nature. In a steady state reactor experiment the time averaged detector signals which are normally considered are related to the mean value of the neutron density. Apart from this mean value, additional information on reactor parameters can be gained from a detailed analysis of the statistical fluctuations in detector signals. Recently a number of papers have been published in which different methods of analysing the fluctuating signals are discussed. The range of experiments carried out extends from essentially unperturbed systems to reactors with strong external perturbations and from zero power assemblies to reactors at full power. It should be clearly recognized that the fluctuations in these experiments can have different origins.

In unperturbed zero power assemblies the fluctuations are due to the fact that the evolution of neutron chains and the detection and generation of neutrons are stochastic processes. Investigations done under these conditions are the Rossi-α-experiment [1, 2], the determination of the reduced variance (Feynman-experiment) [3, 19], the analysis of counting statistics (p_0-method) [4], and the frequency analysis of zero power reactor noise [5, 6, 7, 17, 18].

If on the other hand reactors at full power are considered, the noise is primarily due to statistical modulations of reactivity caused by vibration of fuel elements, turbulence of coolant flow and other similar sources, including internal and external feedback effects. The statistics of neutron chain processes add only a small contribution to the total noise in power reactors. In this field, the frequency analysis is normally applied to determine instabilities and to set up stability criteria for reactor performance [8].

Experiments of a third kind are performed by either modulating the reactivity of a reactor [9, 13] or by modulation of the neutron density with neutron generators in subcritical facilities [10]. In these cases the external excitation is applied in a quasistatistical manner and the known input signal is crosscorrelated with the detector output signal to obtain the response function of the system.

The following treatment is restricted to the description of experiments in unperturbed zero power reactors at steady state. Thus only the neutronic component of reactor noise is considered. The aim of this paper is to give a uniform theoretical description of neutron chain correlated fluctuations and their experimental evaluation, and to show the interconnections and specific features of the various techniques which are now being developed as part of the experimental reactor-physics programme at Karlsruhe. For this purpose a common basic equation has been developed from which expressions appropriate to the above mentioned experimental methods are derived.

* This work was performed within the association on fast reactor development between Gesellschaft für Kernforschung m.b.H., Karlsruhe and EURATOM.

2. A fundamental theorem on two crosscorrelated detector outputs

Consider a subcritical stationary reactor with parameters constant in time which is populated by particles of several types, e.g. neutrons, precursors of delayed neutrons, and gammas. The particles all participate in chains of reactions. For a complete specification of these particles let

y = vector specifying particle position, velocity, and type.

Particles can only be detected through the reactions they induce. In developing the fundamental theorem (2.12) we must carefully distinguish between single reactions, chains of reactions, and the particles taking part in them. A reaction should be specified completely by the position of its occurrence and the type of process, e.g. fission or scattering. As it is cumbersome and often unnecessary to write out such information in detail, we define

z = vector specifying position and type of a reaction.

In a subcritical stationary reactor we must also distinguish the *primary* reactions. Primary reactions are those which are not induced by some preceding reaction of a chain but act, themselves, as the spontaneous origins of these chains. Such primary reactions occur in Ra-Be-neutron sources, spontaneous fission and similar Poisson-distributed source reactions. We define

$c(z)$ = mean rate of z-reactions,

which comprises all reactions, and

$c_0(z)$ = mean rate of Poisson-distributed *primary* z-reactions which start chains.

From some z-reactions, e.g. pure capture processes, no particle will result. A scattering process will pro-

duce at least one scattered particle and a fission reaction a few fission neutrons, each of which can carry on the reaction chain.

Now let

$P(z, n, y_1, \ldots, y_n)$ = probability of producing n particles, $y_1, \ldots, y_n$, from a single z-reaction.

These probabilities contain all the relevant data pertaining to one reaction of type z. Regarding their normalization we will require that for all permutations of the n vectors $y_1, \ldots, y_n$ these probabilities be identical. Otherwise factors $n!$ would appear in the eqs. (2.1) and (2.2).

In the following derivations we need only the expected mean number of y-particles resulting from one z-reaction, $w_1(y, z)$, and also the mean number of *pairs*, (y_1, y_2), of particles from one z-reaction, $w_2(y_1, y_2, z)$. Such pairs of simultaneously produced particles occur mainly in fission as pairs of prompt

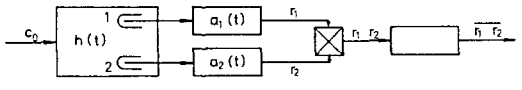

Fig. 1. Schematic diagram of a correlation experiment with 2 separate detectors (crosscorrelation)

fission neutrons and as neutron-precursor and neutron-gamma pairs. They are also produced in $(n, 2n)$, $(n, n'\gamma)$ and other branching processes.

$w_1(y_1, z)$ and $w_2(y_1, y_2, z)$ are obtained from the probability distribution $P(z, n, y_1, \ldots, y_n)$ by summation and integration over all variables not used as arguments of w_1 or w_2, giving

$$w_1(y, z) = \sum_{n=0}^{\infty} n \int dy_2 \ldots dy_n \, P(z, n, y, y_2, \ldots, y_n) \quad (2.1)$$

and

$$\left.\begin{aligned} &w_2(y_1, y_2, z) \\ &= \sum_{n=0}^{\infty} \frac{n(n-1)}{2} \int dy_3 \ldots dy_n \, P(z, n, y_1, \ldots, y_n). \end{aligned}\right\} \quad (2.2)$$

The factors n in eq. (2.1) account for the n possible positions of y as an argument of the probability $P(z, n, \ldots, y, \ldots)$. Correspondingly the factors $\frac{n(n-1)}{2}$ in eq. (2.2) are obtained. As the arguments y_1, y_2 of $P(z, n, y_1, y_2, \ldots)$ in (2.2) may be permutated, we have also

$$w_2(y_1, y_2, z) = w_2(y_2, y_1, z). \quad (2.3)$$

From these definitions we can now derive a few more quantities, viz. the primary source of y-particles,

$$s_0(y) = \int dz \, c_0(z) \, w_1(y, z); \quad (2.4)$$

the total rate of y-particles produced in all reactions of type z, including primary and induced reactions,

$$s_1(y, z) = c(z) \, w_1(y, z); \quad (2.5)$$

and the total rate of (y_1, y_2)-pairs produced in all reactions

$$s_2(y_1, y_2) = \int dz \, c(z) \, w_2(y_1, y_2, z). \quad (2.6)$$

Obviously

$$s_2(y_2, y_1) = s_2(y_1, y_2). \quad (2.7)$$

We may call $s_2(y_1, y_2)$ a *binary source* of (y_1, y_2)-pairs (of particles).

The theorem to be derived here concerns correlation experiments performed with two independent detectors (Fig. 1). As a logical sequence to this an extension to the single-detector-experiment (Fig. 2) is given, where both schemes represent limiting cases. In the first case (Fig. 1) we have a stationary reactor which is monitored by two independent instrument lines, each consisting of a detector followed by an electronic network. As far as neutron physics is concerned, the detectors must be treated as integral components of the reactor. The electronic circuits are assumed to be nearly linear elements. Their actual responses may show statistical variations, but no dead-time, saturation or similar interference effects are permitted. For an input at time 0, the response functions defined below always denote the average output expected after a delay time t. Let

$r_1 = r_1(t)$ = actual output on line 1 (Fig. 1) at time t,

$g_1(y, t)$ = GREEN's function, response of r_1 to the injection of one y-particle into the reactor.

The quantity $g_1(y, t)$ is the convolution of the response functions of reactor, detector, and electronic network, cf. (3.8). Contributions to $g_1(y, t)$ come from all detected z-reactions in the average reaction chain which started at time 0 by the release of one y-particle.

Fission chambers, proton recoil counters, and some other detectors emit secondary neutrons with each act of neutron detection. Other detectors, e.g. scintillation-counters, may register the gammas associated with the (α, n)-processes of a Ra-Be-neutron source. These special features add small correction terms to a correlation experiment. As an illustrative example the A-terms in the eqs. (5.9) and (5.10), for the Rossi-α-experiment, may be taken. To account for such effects, we have to complete our notation by introducing

$\varphi_1(z, t)$ = GREEN's function, response of r_1 to a single z-reaction.

$\varphi_1(z, t)$ involves only the response functions of detector 1 (to one z-reaction occurring in its sensitive volume) and the weighting function

$$a_1(t) = \int \frac{d\omega}{2\pi} A_1(\omega) \, e^{i\omega t} \quad (2.8)$$

of the associated linear electronic network. The weighting function $a_1(t)$ is the response function of the network to a δ-function input. Except for an arbitrary scaling factor, which may be chosen at convenience, $a_1(t)$ is a well-defined function.

Obviously the time-averaged signal on line 1 is

$$\bar{r}_1 = \int dz \, c(z) \int dt \, \varphi_1(z, t) \quad (2.9)$$

or

$$\left.\begin{aligned} \bar{r}_1 = \int dy \left[s_0(y) \int dt \, g_1(y, t) \right] + \\ + \int dz \left[c_0(z) \int dt \, \varphi_1(z, t) \right]. \end{aligned}\right\} \quad (2.10)$$

The second term of (2.10), i.e. a contribution from directly registered primary reactions, can normally be

neglected but may be important in special experiments [11].

Expressions equivalent to (2.8—10) are also valid for the output $r_2(t)$ of the second instrument line. We now turn to the product signal $r_1(t) \cdot r_2(t)$ (Fig. 1, right hand side), whose time-averaged value

$$\overline{r_1 \cdot r_2} = \lim_{T\to\infty} \frac{1}{T} \int_0^T dt\, r_1(t)\, r_2(t) \tag{2.11}$$

can be shown to be the relevant quantity obtained from most correlation experiments. The only known exception to this statement is the p-method [4], explained in section 7.

In Fig. 1 we have assumed that the instrument lines 1 and 2 are completely independent, i.e. no single z-reaction can produce a signal on both lines. This condition is met when the detectors are separated in space and when the associated networks are free from mutual electronic interference. Any particles emitted by one detector, e.g. a fission chamber, should be rigorously treated as contributing to reaction chains.

In the appendix (section 10) the following theorem on the time-averaged product signal $r_1(t) \cdot r_2(t)$ is proved.

$$\left.\begin{aligned} \overline{r_1 \cdot r_2} &= \overline{r_1} \cdot \overline{r_2} \\ &+ 2 \int dy_1\, dy_2\, s_2(y_1, y_2) \int dt\, g_1(y_1, t)\, g_2(y_2, t) \\ &+ \int dy\, dz\, s_1(y, z) \int dt\, g_1(y, t)\, \varphi_2(z, t) \\ &+ \int dy\, dz\, s_1(y, z) \int dt\, g_2(y, t)\, \varphi_1(z, t). \end{aligned}\right\} \tag{2.12}$$

The first term is the value expected for uncorrelated signals $r_1(t)$ and $r_2(t)$. The second term shows the correlation due to the binary source $s_2(y_1, y_2)$ of (y_1, y_2)-pairs generated in branching processes. The last two terms are small contributions due to the source of y-particles associated with registered z-reactions. These terms have to be considered in experiments using fission chambers, proton recoil counters etc., cf. section 5.

When an assembly has a high multiplication, we can restrict ourselves to the simplified expressions (2.15) and (2.16), which are rigorously valid for experiments with absorption detectors in which each detected particle is destroyed without successors. Such a detector will also be insensitive to any primary reaction, so that

$$c_0(z)\, \varphi_n(z, t) = 0, \qquad n = 1, 2, \tag{2.13}$$

$$\left.\begin{aligned} s_1(y, z)\, g_1(y, t)\, \varphi_2(z, t) &= 0, \\ s_1(y, z)\, g_2(y, t)\, \varphi_1(z, t) &= 0. \end{aligned}\right\} \tag{2.14}$$

Then (2.10) and (2.12) read simply

$$\overline{r_n} = \int dy\, s_0(y) \int dt\, g_n(y, t), \qquad n = 1, 2, \tag{2.15}$$

$$\left.\begin{aligned} \overline{r_1 \cdot r_2} = \overline{r_1} \cdot \overline{r_2} &+ 2 \int dy_1\, dy_2\, s_2(y_1, y_2) \times \\ &\times \int dt\, g_1(y_1, t)\, g_2(y_2, t). \end{aligned}\right\} \tag{2.16}$$

Eqs. (2.12) and (2.16) are generalizations of expressions obtained by MATTHES [12].

If we prefer to express the response functions by their Fourier-transforms,

$$g_n(y, t) = \int \frac{d\omega}{2\pi}\, G_n(y, \omega)\, e^{i\omega t}, \tag{2.17}$$

we get the equivalent expressions

$$\overline{r_n} = \int dy\, s_0(y)\, G_n(y, 0), \qquad n = 1, 2, \tag{2.15a}$$

$$\left.\begin{aligned} \overline{r_1 \cdot r_2} = \overline{r_1} \cdot \overline{r_2} &+ 2 \int dy_1\, dy_2\, s_2(y_1, y_2) \times \\ &\times \int \frac{d\omega}{2\pi}\, G_1(y_1, \omega)\, G_2(y_2, -\omega). \end{aligned}\right\} \tag{2.16a}$$

This general formalism can be extended to the single-detector-experiment shown in Fig. 2, which prevails in some applications. In this type of experiment we get an additional term to our formulas (2.12), (2.16), and (2.16a) from the fact that each registered reaction releases two synchronized responses contributing to the signals $r_1(t)$ and $r_2(t)$. Keeping in mind possible statistical variations in these responses we have to define

$\varphi_{12}(z, t)$ = mean product of $r_1(t)$, $r_2(t)$ as responses to a single z-reaction occurring at time 0, cf. (10.18).

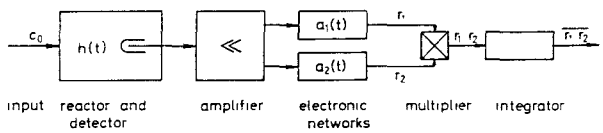

Fig. 2. Schematic diagram of a single-detector correlation experiment (autocorrelation)

Integrating over t and the total rate $c(z)$ of z-reactions we obtain the new term

$$\pi_{12} = \int dz\, c(z) \int dt\, \varphi_{12}(z, t). \tag{2.18}$$

This term must be inserted into an expression which corresponds to (2.16) but is valid for the single-detector-experiment (Fig. 2) with an absorption detector, giving

$$\left.\begin{aligned} \overline{r_1 \cdot r_2} = \overline{r_1} \cdot \overline{r_2} &+ \pi_{12} + 2 \int dy_1\, dy_2\, s_2(y_1, y_2) \times \\ &\times \int dt\, g_1(y_1, t)\, g_2(y_2, t). \end{aligned}\right\} \tag{2.19}$$

No generally applicable formula can be given for deriving π_{12}. It must, therefore, be evaluated for each specific experiment.

3. Specialized expressions for short-time correlations and for the point reactor model

Most correlation experiments are restricted to the study of the prompt kinetic behaviour of a reactor. Then the delayed neutrons can be approximately treated as an additional pseudo-random source. A further simplification of expressions (2.16, 16a) is obtained by neglecting $(n, 2n)$ $(n, n'\gamma)$, and similar branching reactions of minor importance.

The only binary source remaining consists of pairs of fission neutrons from spontaneous and induced fission. As a further approximation let us assume that (1) no angular and spectral correlations exist between pairs of fission neutrons and (2) that for all types of fission one energy spectrum $\chi(E)$ is applicable. With these assumptions, which are not very restrictive, we get a simple binary source (2.6)

$$\left.\begin{aligned} &s_2(x_1, E_1, \Omega_1, x_2, E_2, \Omega_2) \\ &\quad = S_2(x_1)\, \delta(x_2 - x_1)\, \frac{\chi(E_1)\, \chi(E_2)}{(4\pi)^2}, \end{aligned}\right\} \tag{3.1}$$

where x, E, Ω are position, energy, and direction of each fission neutron. These are, according to our assumptions, emitted isotropically with an energy spectrum $\chi(E)$. The δ-function assures that fission neutrons of one pair have the same origin x_1. The quantity

$$\left.\begin{aligned} S_2(x) = \tfrac{1}{2}\big[c_{sf}(x)\,\overline{\nu(\nu-1)}_{sf} + \\ + \int dE\,[\overline{\nu(\nu-1)}_p\,\Sigma_f\Phi]\,(x,E)\big] \end{aligned}\right\} \quad (3.2)$$

is the local rate of production of pairs of prompt fission neutrons, where $c_{sf}(x)$ is the local rate of spontaneous fission, and $\frac{1}{2}\overline{\nu(\nu-1)}$ the average number of prompt neutron pairs per fission.

For $n=1$ and 2 let

$$g_{fn}(x,t) = \int \frac{d\Omega}{4\pi}\, dE\,\chi(E)\,g_n(x,E,\Omega,t), \quad (3.3)$$

which is the response of signal r_n in Fig. 1 to the injection of one (average) fission neutron at x. The quantity $g_{fn}(x,t)$ is the convolution of the responses of reactor, detector, and linear electronic network. Insertion of (3.1—3) into (2.16) yields a more transparent expression,

$$\overline{r_1\cdot r_2} = \overline{r}_1\cdot\overline{r}_2 + 2\int dx\,S_2(x)\int dt\,g_{f1}(x,t)\,g_{f2}(x,t), \quad (3.4)$$

or, with the Fourier-transforms G_{fn} of g_{fn},

$$\left.\begin{aligned} \overline{r_1\cdot r_2} = \overline{r}_1\cdot\overline{r}_2 + \\ + 2\int dx\,S_2(x)\int \frac{d\omega}{2\pi}\,G_{f1}(x,\omega)\,G_{f2}(x,-\omega). \end{aligned}\right\} \quad (3.4\,a)$$

To obtain (3.4, 4a) we kept the space- and energy-dependence of the exact expression (2.16) and neglected delayed neutrons. On the other hand, neglecting the dependence of $g_n(y,t)$ on the space- and energy-components of the vector y amounts to the retention of the fundamental term of a modal expansion, cf. [14], and is legitimate in sufficiently compact, near-critical assemblies.

Branching reactions, which are of minor importance, are again neglected, but delayed neutrons are now treated rigorously. To make the derivation more transparent let us restrict ourselves to the realistic case where not more than one delayed neutron precursor is generated in any fission process. Let

$$\begin{aligned} h(t) &= \text{response of the neutron population to the injection of one (prompt) neutron,}\\ k &= \text{effective multiplication constant, including delayed neutrons,}\\ \beta_m &= \text{fraction of delayed neutrons of group } m,\\ \beta &= \sum_{m=1}^{6}\beta_m, \quad (3.5)\\ l &= \text{prompt neutron life-time,}\\ \lambda_m &= \text{decay constant of group } m \text{ precursors.} \end{aligned}$$

The response $h(t)$ is the solution of the point-reactor kinetics equation,

$$\left.\begin{aligned} \frac{d}{dt}\,h(t) &= \frac{k(1-\beta)-1}{l}\,h(t) + \Sigma\,\lambda_m C_m(t) + \delta(t),\\ \frac{d}{dt}\,C_m(t) &= -\lambda_m C_m(t) + \frac{k\beta}{l}\,h(t), \quad m=1,\ldots,6. \end{aligned}\right\} \quad (3.6)$$

To take account of delayed neutrons, we introduce the related response, $h_m(t)$, of the neutron population to the formation of one precursor belonging to group m. This response is the convolution of the precursor decay and $h(t)$, i.e.

$$h_m(t) = \int_0^\infty dt_1\, e^{-\lambda_m t_1}\,\lambda_m h(t-t_1). \quad (3.7)$$

It is advantageous to Fourier-transform eq. (3.7), giving

$$H_m(\omega) = \frac{\lambda_m}{\lambda_m + i\omega}\,H(\omega). \quad (3.7\,a)$$

The response function, $g_1(t)$, relating the response of r_1 to the injection of one source neutron into the reactor becomes

$$g_1(t) = \frac{W_1 k}{\nu l}\int dt_1\,h(t_1)\,a_1(t-t_1), \quad (3.8)$$

with the weighting function, $a_1(t)$, as in (2.8),

$$\begin{aligned} \nu &= \text{number of neutrons per fission, including delayed, and}\\ W &= \text{detector sensitivity (counts per fission).} \end{aligned}$$

Fourier-transforming (3.8) results in

$$G_1(\omega) = \frac{W_1 k}{\nu l}\,H(\omega)\,A_1(\omega). \quad (3.8\,a)$$

The appropriate response function to the formation of one precursor of group m is, cf. (3.7a),

$$G_{1m}(\omega) = \frac{\lambda_m}{\lambda_m + i\omega}\,G_1(\omega). \quad (3.9)$$

Similar formulas are valid for instrument line 2. Now, the total rate of (prompt) neutron pairs formed in fission becomes, accounting for permutations,

$$F\,\frac{\overline{\nu(\nu-1)_p}}{2}, \quad \text{where } F = \text{fission rate.}$$

The total rate of pairs (neutron, precursor m) and also of pairs (precursor m, neutron) is

$$F\,\frac{\beta_m \nu_m \nu}{2}, \quad \text{with}$$

ν_m = average number of prompt neutrons accompanying this group of precursors in fission.

When we insert these rates and (3.9) into (2.16a), we have

$$\left.\begin{aligned} \overline{r_1\cdot r_2} &= \overline{r}_1\cdot\overline{r}_2 +\\ &+ F\,\overline{\nu(\nu-1)}_p\int\frac{d\omega}{2\pi}\,G_1(\omega)\,G_2(-\omega) +\\ &+ F\sum_{m=1}^{6}(\beta_m\nu_m\nu)\int\frac{d\omega}{2\pi}\,G_1(\omega)\,G_2(-\omega)\,\frac{2\lambda_m^2}{\lambda_m^2+\omega^2}. \end{aligned}\right\} \quad (3.10)$$

With (3.8a) and inserting

$$H(\omega)H(-\omega) = |H(\omega)|^2 \text{ for } \omega \text{ real}, \quad (3.11)$$

eq. (3.10) yields the final expression

$$\left.\begin{aligned} \overline{r_1\cdot r_2} = \overline{r}_1\cdot\overline{r}_2 + F\,\frac{W_1 W_2 k^2}{l^2}\,\frac{\overline{\nu(\nu-1)_p}}{\nu^2}\times\\ \times\int\frac{d\omega}{2\pi}\,A_1(\omega)\,A_2(-\omega)\,|H(\omega)|^2\times\\ \times\left\{1+\sum_{m=1}^{6}\frac{2\beta_m\nu_m\nu}{\overline{\nu(\nu-1)}_p}\,\frac{\lambda_m^2}{\lambda_m^2+\omega^2}\right\}. \end{aligned}\right\} \quad (3.12)$$

In (3.12) delayed neutrons have been taken into account twice. First implicitely in the definition of $H(\omega)$, cf. (3.6), and secondly in the term within the brackets. However, the conditions

$$\left.\begin{aligned} 1 &< 1+\sum_{m=1}^{6}\frac{2\beta_m \nu_m \nu}{\nu(\nu-1)_p}\,\frac{\lambda_m^2}{\lambda_m^2+\omega^2} \\ &\leqq 1+\sum_{m=1}^{6}\frac{2\beta_m \nu_m \nu}{\nu(\nu-1)_p} \approx 1+2\beta \end{aligned}\right\} \tag{3.13}$$

suggest that in the following sections a simpler approximation of (3.12) may be used, viz.

$$\left.\begin{aligned} \overline{r_1 \cdot r_2} &= \bar{r}_1 \cdot \bar{r}_2 + \\ &+ F\,\frac{W_1 W_2 \chi_2 k^2}{l^2}\int \frac{d\omega}{2\pi} A_1(\omega)\, A_2(-\omega)\,|H(\omega)|^2. \end{aligned}\right\} \tag{3.14a}$$

Herein

$$\chi_2 = \frac{\overline{\nu(\nu-1)_p}}{\bar{\nu}^2} \tag{3.15}$$

is a nuclear parameter, F the fission rate, W a detector sensitivity (counts per fission), k the effective neutron multiplication constant, l the prompt lifetime, and $H(\omega)$ and $A(\omega)$ the Fourier-transforms of the reactor response $h(t)$, (3.6), and the network response $a(t)$, (2.8). When spontaneous fission does not contribute significantly to F,

$$\bar{r}_n = W_n F \int dt\, a_n(t) = W_n F A_n(0), \quad \text{for } n=1 \text{ and } 2. \tag{3.16}$$

In the three sections to follow, eq. (3.14a) and the equivalent

$$\left.\begin{aligned} \overline{r_1 \cdot r_2} &= \bar{r}_1 \cdot \bar{r}_2 + F\,\frac{W_1 W_2 \chi_2 k^2}{l^2} \times \\ &\times \int dt\, dt_1\, dt_2\, h(t_1)\, h(t_2)\, a_1(t-t_1)\, a_2(t-t_2) \end{aligned}\right\} \tag{3.14}$$

will be applied, with appropriate weighting functions, to some familiar correlation experiments.

4. Frequency analysis of ion-chamber currents

The frequency analysis of the noise spectrum obtained from boron-loaded ion-chambers or from fission chambers can be performed as a two-detector or as a single-detector-experiment. Single-detector-experiments have been treated extensively by many authors [5, 6, 7, 17]. In this paper, we want to give greater stress to the more flexible two-detector-experiment. Refer to Fig. 1 and assume that both detectors are boron-loaded ion-chambers, such that eq. (3.14a) applies. The block diagram of one instrument line is shown in Fig. 3. The output current of the ion-chamber is fed to a high-gain wide-band amplifier, followed by a filter of frequency characteristics $B(\omega)$. The mean output current of the amplifier is given by

$$\bar{i}_n = W_n F q_n, \quad \text{for } n=1 \text{ and } 2 \tag{4.1}$$

where n denotes the line number.

According to our previous definitions, $(W_n F)$ is the rate of neutron detections (pulses/sec) by the chamber n, and q_n is the average charge per detected neutron at the amplifier output. With this notation $A_n(\omega)$ becomes

$$A_n(\omega) = q_n B_n(\omega), \quad \text{for } n=1 \text{ and } 2 \tag{4.2}$$

so that eq. (3.14a) will be

$$\left.\begin{aligned} \overline{r_1 \cdot r_2} &= q_1 q_2 F^2 W_1 W_2 B_1(0)\, B_2(0) + \\ &+ q_1 q_2 F\,\frac{W_1 W_2 \chi_2 k^2}{l^2} \times \\ &\times \mathrm{Re}\int_0^\infty \frac{d\omega'}{\pi}\, B_1(\omega')\, B_2(-\omega')\,|H(\omega')|^2. \end{aligned}\right\} \tag{4.3}$$

The special feature of frequency analysis is the use of a tunable band pass filter in at least one instrument line, so that

$$B_1(\omega')\, B_2(-\omega') = 0 \tag{4.4}$$

outside a narrow frequency interval of width $\Delta\omega$ around a center frequency ω. If the resolution $\frac{\omega}{\Delta\omega}$ is sufficiently high, we can extract $|H(\omega)|^2$ from the integral and approximate

$$\left.\begin{aligned} &\mathrm{Re}\int_0^\infty \frac{d\omega'}{\pi}\, B_1(\omega')\, B_2(-\omega')\,|H(\omega')|^2 \\ &\qquad \approx \Gamma(\omega, \Delta\omega)\,|H(\omega)|^2 \end{aligned}\right\} \tag{4.5}$$

with

$$\Gamma(\omega, \Delta\omega) = \mathrm{Re}\int_0^\infty \frac{d\omega'}{\pi}\, B_1(\omega')\, B_2(-\omega'). \tag{4.6}$$

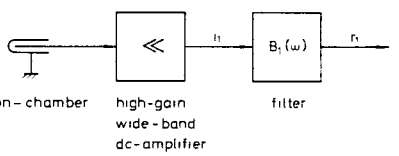

Fig. 3. Schematic diagram for one instrument line in a frequency analysis experiment

The function $\Gamma(\omega, \Delta\omega)$ is a parameter which characterizes the combination of filters used, and must be determined for each center frequency ω and band width $\Delta\omega$. Ratios $\frac{\omega}{\Delta\omega} > 2$ are normally used.

It follows from eq. (4.4) that $B_1(0)\, B_2(0) = 0$. Thus in this approximation eq. (4.3) becomes

$$\frac{\overline{r_1 \cdot r_2}(\omega, \Delta\omega)}{\Gamma(\omega, \Delta\omega)} = q_1 q_2 F\,\frac{W_1 W_2 \chi_2 k^2}{l^2}\,|H(\omega)|^2. \tag{4.7}$$

This expression is valid for the whole frequency range, including very low frequencies where the effect of delayed neutrons is important.

Now let us take into consideration only the high frequency range, such that the effect of delayed neutrons can be neglected. In this region

$$H(\omega) = \frac{1}{\alpha + i\omega} \tag{4.8a}$$

and

$$|H(\omega)|^2 = \frac{1}{\alpha^2 + \omega^2} \tag{4.8b}$$

with

$$\alpha = \frac{1-k(1-\beta)}{l}. \tag{4.9}$$

Therefore in the high frequency approximation (4.7) becomes

$$\frac{\overline{r_1 \cdot r_2}(\omega, \Delta\omega)}{\Gamma(\omega, \Delta\omega)} = q_1 q_2 F\,\frac{W_1 W_2 \chi_2 k^2}{l^2(\alpha^2+\omega^2)}. \tag{4.10}$$

In the discussion of eq. (4.10) it is useful to consider two limiting cases. For sufficiently low fre-

quencies,

$$0 \ll \omega^2 \ll \alpha^2, \tag{4.11}$$

the term ω^2 in the denominator of eq. (4.10) can be neglected. Then the whole expression becomes practically constant at

$$\frac{\overline{r_1 \cdot r_2}(\omega, \Delta\omega)}{\Gamma(\omega, \Delta\omega)} \approx q_1 q_2 F \frac{W_1 W_2 \chi_2 k^2}{[1-k(1-\beta)]^2}. \tag{4.12}$$

On the other hand for high frequencies, where

$$\omega^2 \gg \alpha^2, \tag{4.13}$$

the function

$$\frac{\overline{r_1 \cdot r_2}(\omega, \Delta\omega)}{\Gamma(\omega, \Delta\omega)} \approx q_1 q_2 F \frac{W_1 W_2 \chi_2 k^2}{l^2 \omega^2} \tag{4.14}$$

is obtained, which falls off proportional to $\frac{1}{\omega^2}$. The two curves corresponding to the low frequency approximation (4.12) and the high frequency approximation (4.14) intersect at the so-called breaking frequency,

$$\omega_0 = \frac{1-k(1-\beta)}{l} = \alpha. \tag{4.15}$$

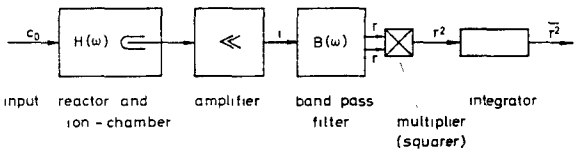

Fig. 4. Schematic diagram of a single-detector frequency analysis experiment

At this frequency, ω_0, the measured cross-power spectral density given by (4.10) has dropped to $\frac{1}{2}$ of its low frequency value (4.12). The prompt neutron decay constant α can be determined by both means. At delayed critical it is equal to $\alpha_c = \frac{\beta}{l}$.

In a critical thermal reactor with $\beta = 7.5 \cdot 10^{-3}$ and a prompt neutron life time of $l = 75$ μsec, the value $f_0 = \frac{\omega_0}{2\pi}$ becomes 16 cps. On the other hand, in a typical plutonium fuelled fast critical assembly with $\beta = 3 \cdot 10^{-3}$ and $l = 0.3$ μsec, $f_0 = 1.6$ kcps. Therefore, the instrumentation should cover frequencies up to $f_{max} = 1$ kcps for the investigation of thermal reactors and up to $f_{max} = 100$ kcps for fast assemblies. The determination of absolute reactor power is another area in which the frequency analysis of noise [17] is applied. It is only necessary to add the measurements of the mean currents $\bar{i}_1$ and $\bar{i}_2$ according to (4.1). These values may be inserted into (4.10) to eliminate the unknown products $q_1 W_1$ and $q_2 W_2$. The expression thus obtained for the fission rate F, is particularly simple if the experiment is performed in the low frequency range and at delayed critical. In this case

$$F = \frac{\chi_2}{\beta^2} \frac{\bar{i}_1 \cdot \bar{i}_2 \Gamma(\omega, \Delta\omega)}{\overline{r_1 \cdot r_2}(\omega, \Delta\omega)}. \tag{4.16}$$

A third application of the frequency analysis of noise is that of determination of reactivity. This possibility is clearly shown by the low frequency approximation (4.12). If we introduce the conventional definition of reactivity (in dollars)

$$\varrho = \frac{k-1}{k\beta}, \tag{4.17}$$

we may rearrange (4.12) to obtain

$$\frac{\overline{r_1 \cdot r_2}(\omega_1, \Delta\omega_1)}{\Gamma(\omega_1, \Delta\omega_1)} \approx \frac{q_1 q_2 F W_1 W_2 \chi_2}{\beta^2} \frac{1}{(1-\varrho)^2} \tag{4.12a}$$

which is valid for a low frequency ($0 \ll \omega_1 \ll \alpha$). For a sufficiently high frequency ($\omega_2 \gg \alpha$), eq. (4.14) is valid. The ratio of (4.12a) and (4.14) is

$$\frac{\overline{r_1 \cdot r_2}(\omega_1, \Delta\omega_1)\Gamma(\omega_2, \Delta\omega_2)}{\overline{r_1 \cdot r_2}(\omega_2, \Delta\omega_2)\Gamma(\omega_1, \Delta\omega_1)} = \frac{l^2 \omega_2^2}{k^2 \beta^2 (1-\varrho)^2} \tag{4.18}$$

and it depends strongly on ϱ. The practical application of this relation was shown by SCHULTZ, who measured reactivity values down to 10 dollars below delayed critical [18].

Finally, we want to treat the special case of a single-detector-experiment already discussed at the end of section 2. The block diagram in this case is very simple, as shown in Fig. 4. The additional contribution π_{12} of eq. (2.19) is approximately

$$\pi = W F \overline{q^2} \int_0^\infty \frac{d\omega'}{\pi} |B(\omega')|^2 = W F \overline{q^2} \Gamma(\omega, \Delta\omega). \tag{4.19}$$

This approximation is valid if the filter does not contribute noise components of its own. To account for the statistical fluctuations of the ionisation phenomena, BENNETT [7] has introduced a factor R, which is given by

$$R = \frac{\overline{q^2}}{\bar{q}^2} \geqq 1 \tag{4.20}$$

where q is the charge at the amplifier output which is released per detected neutron. With eqs. (4.7), (4.19) and (4.20) we obtain for a single-detector-experiment

$$\frac{\overline{r^2}(\omega, \Delta\omega)}{\Gamma(\omega, \Delta\omega)} = W F \bar{q}^2 \left[R + \frac{W \chi_2 k^2}{l^2} |H(\omega)|^2 \right]. \tag{4.21}$$

A comparison with eq. (4.7) for the two-detector-experiment shows a contribution due to uncorrelated events [the first term of (4.21)]. This term also contains the factor R, which is difficult to determine in practice. For these reasons, the two-detector-experiments may be generally more suitable, and particularly in all cases where the uncorrelated contribution predominates.

5. Rossi-α-experiment

The Rossi-α-experiment performed with two detectors (Fig. 2) consists of a direct measurement of the crosscorrelation function of detector counting rates

$$r_{12}(\tau) = \overline{r_1(t-\tau) \cdot r_2(t)} \tag{5.1}$$

where $r_1(t)$, $r_2(t)$ are the actual counting rates at time t, and $\tau \geqq 0$ is the delay time. The basic eqs. (3.14) and (3.14a) are equivalent, but for this problem (3.14) is more convenient than (3.14a), which is used for the frequency analysis of noise. The weighting functions $a_n(t)$ which are appropriate to this problem will be defined as

$$a_1(t) = \delta(t-\tau), \tag{5.2a}$$

$$a_2(t) = \delta(t). \tag{5.2b}$$

They have been scaled such that the average signals

$$\bar{r}_n = W_n F, \quad \text{for } n = 1 \text{ and } 2 \tag{5.3}$$

represent the mean counting rates of detector 1 and 2.

For the short-time approximation the point-reactor response function is given by

$$h(t) = e^{-\alpha t} \qquad t \geqq 0 \tag{5.4}$$

where α is the prompt decay constant defined in (4.9).

After insertion of expressions (5.2) and (5.4) into eq. (3.14), integrations can be performed easily in the order t_1, t_2, and t. We obtain thus for an experiment with 2 absorption detectors

$$\left.\begin{aligned} r_{12}(\tau)\,\delta\tau &= \overline{r_1(t-\tau)\cdot r_2(t)}\,\delta\tau \\ &= W_1 W_2 F\left[F + \frac{\chi_2 k^2}{2\alpha l^2}\cdot e^{-\alpha\tau}\right]\delta\tau. \end{aligned}\right\} \tag{5.5}$$

The practical measurement of the crosscorrelation function, $r_{12}(\tau)$, can best be demonstrated by referring to the equipment of ORNDOFF [1] shown in Fig. 5. Designed as a ten-channel-analyser, this apparatus measures the crosscorrelation function, $r_{12}(\tau_n)$, simultaneously for ten successive delay times, τ_n. Each single channel corresponds to one configuration according to Fig. 2. Time delays are introduced by delay cables and the logical analogue of a multiplication of pulse sequences is performed in coincidence circuits. If $\delta\tau$ is the small width of the delayed gate signals generated by detector 1, the counting rate of coincidence channel n is simply given by $r_{12}(\tau_n)\,\delta\tau$. When $r_{12}(\tau_n)$ is plotted on a semilogarithmic scale, the prompt decay constant α can be easily determined.

The absolute fission rate F, which gives us the reactor power, can also be obtained from this experiment. For this purpose, eq. (5.5), which is composed of two terms, can be used. The first term,

$$B = W_1 W_2 F^2\,\delta\tau, \tag{5.5a}$$

gives the rate of registered coincidence pulses due to uncorrelated events, which can be considered as background (B) in this experiment. The second term,

$$C = W_1 W_2 F\,\frac{\chi_2 k^2}{2\alpha l^2}\,e^{-\alpha\tau}\,\delta\tau, \tag{5.5b}$$

is the contribution of correlated events (C). The fission rate F is conveniently determined by measuring e.g. the decay constant at delayed critical, where $\alpha = \alpha_c = \frac{\beta}{l}$ and (5.5b) becomes

$$C = W_1 W_2 F\,\frac{\chi_2 \alpha_c}{2\beta^2}\,e^{-\alpha_c\tau}\,\delta\tau. \tag{5.5c}$$

From the ratio of (5.5c) to (5.5a) we thus get

$$F = \frac{B}{C}\,\frac{\chi_2}{\beta^2}\,\frac{\alpha_c}{2}\,e^{-\alpha_c\tau}. \tag{5.6}$$

This expression corresponds to (4.16) and contains, with the exceptions of χ_2 and β^2, only measured quantities on the right hand side.

So far Rossi-α-experiments performed only with absorption detectors have been considered. If other detectors (proton recoil counters or fission chambers) are used, neutrons will be released during the registration process. Let

Δ_n = Number of neutrons released in detector n for each registered neutron with
$\Delta = 0$ for absorbers, $\Delta = 1$ for recoil counters, and $\Delta = \nu$ for fission chambers.

The contribution of such neutrons to the signal $\overline{r_1 \cdot r_2}$ has already been considered in the fundamental eq. (2.12). For positive delays ($\tau > 0$) only the initiating detector with its response $a_1(t) = \delta(t-\tau)$ contributes, and this can be seen to be the last term of eq. (2.12). In the space and energy independent model considered here, this term can be obtained from

$$\left.\begin{aligned} &\int dy\,dz\,s_1(y,z)\int dt\,g_2(y,t)\,\varphi_1(z,t) \\ &\quad = \int dt\,g_2(t)\int dz\,S_1(z)\,\varphi_1(z,t) \\ &\quad = \int dt\,g_2(t)\,W_1 F \Delta_1\,\delta(t-\tau). \end{aligned}\right\} \tag{5.7}$$

By inserting the appropriate response function, $g_2(t)$, we get

$$\int dt\,g_2(t)\int dz\,S_1(z)\,\varphi_1(z,t) = \frac{W_2 k}{\nu l}\,e^{-\alpha\tau}\,W_1 F \Delta_1. \tag{5.8}$$

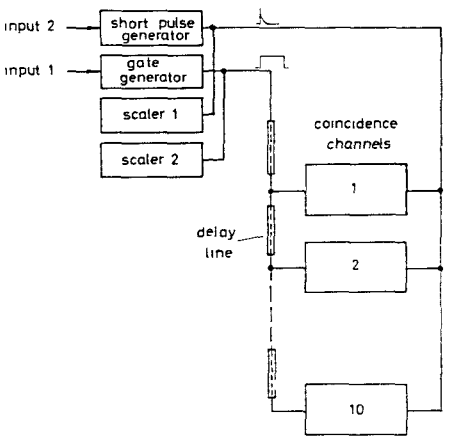

Fig. 5. ORNDOFF's setup for Rossi-α-experiments [1]

The total crosscorrelation function $r_{12}(\tau)$ of two detectors, including this contribution in (5.8), becomes thus

$$r_{12}(\tau) = W_1 F\left[W_2 F + \frac{W_2}{l}\left\{\frac{\chi_2 k^2}{2\alpha l} + \frac{\Delta_1 k}{\nu}\right\}e^{-\alpha\tau}\right]. \tag{5.9}$$

In a single-detector-experiment where the autocorrelation function of the detector's counting rate is measured, a trivial coincidence is produced for each count at zero delay ($\tau = 0$). The autocorrelation function becomes, according to eq. (5.9),

$$\left.\begin{aligned} r_{11}(\tau) &= \overline{r(t-\tau)\cdot r(t)} \\ &= WF\left[\delta(\tau) + WF + \frac{W}{l}\left\{\frac{\chi_2 k^2}{2\alpha l} + \frac{\Delta k}{\nu}\right\}e^{-\alpha\tau}\right]. \end{aligned}\right\} \tag{5.10}$$

Finally a comment has to be made about the different experimental techniques used for measuring the crosscorrelation function. The setup of ORNDOFF corresponds completely to the theory developed here. From a practical point of view, however, severe problems arise if this design principle has to be applied to the large number of channels and wide range of channel widths necessary for investigations of flexible critical assemblies. In such systems the α-values can vary over a wide range due to changes in core composition. For these reasons, other experimental designs allowing for a greater versatility have been applied. The apparatus of BRUNSON [2], for example, uses a stop-watch technique, measuring the time intervals between a selected initiating count from detector 1 and the successive counts (one or more) from detector 2. This

mode of operation might introduce some intrinsic preselection of the initiating counts, whereas ORNDOFF's apparatus gives equal weight to each reference count from detector 1.

Although this feature should not be overestimated, we believe that expressions (5.9) and (5.10) derived above do not strictly apply to the stop-watch technique. There will be a need for corrections when high detector sensitivities are encountered in near critical systems. In such experiments detector 1 will sometimes register two or more correlated counts from a single neutron chain. In the stop-watch technique only one count from detector 1 is admitted as a time reference for counts from detector 2. Therefore, the ratio of correlated (C) to uncorrelated counts (B) will be lower than obtained from (5.5a, b).

6. Variance (Feynman-experiments)

In this section the variance of counting rates is determined from which reactor-physics parameters can be evaluated. As in the preceeding paragraphs the point-reactor model is used, delayed neutrons are not taken into consideration and contributions due to spontaneous fission are neglected. For simplicity the formulas are derived for absorption detectors only. Consider first an experiment with two independent detectors. Let the response functions of the two systems be

$$a_1(t)=a_2(t)=\begin{cases}\dfrac{1}{T}, & \text{for } 0\leqq t\leqq T\\ 0, & \text{otherwise.}\end{cases} \tag{6.1}$$

When T is the length of a time interval and $n_1(t-T,t)$ the number of counts by detector 1 in the interval $(t-T,t)$, $r_1(t)$ is given by

$$r_1(t)=\frac{n_1(t-T,t)}{T} \tag{6.2}$$

and correspondingly

$$r_2(t)=\frac{n_2(t-T,t)}{T}. \tag{6.3}$$

The product of (6.2) and (6.3),

$$r_1(t)\cdot r_2(t)=\frac{n_1(t-T,t)\cdot n_2(t-T,t)}{T^2}, \tag{6.4}$$

gives the number of pairs of counts from two detectors in the time interval $(t-T,t)$, divided by T^2. The mean counting rates from detectors 1 and 2 are given by

$$\overline{r_1}=W_1F=\frac{\overline{n_1}(T)}{T} \tag{6.5}$$

$$\overline{r_2}=W_2F=\frac{\overline{n_2}(T)}{T} \tag{6.6}$$

and the mean product becomes

$$\overline{r_1\cdot r_2}(T)=\frac{\overline{n_1\cdot n_2}(T)}{T^2}. \tag{6.7}$$

Fourier-transformation of (6.1) results in

$$A_1(\omega)=A_2(\omega)=\frac{1-e^{-i\omega T}}{i\omega T}. \tag{6.8}$$

The Fourier-transformed response function of the neutron population to the injection of one prompt neutron, $h(t)$, is given by

$$H(\omega)=\frac{1}{\alpha+i\omega} \tag{6.9}$$

in the high-frequency approximation [cf. (4.8a)], where α has been defined in (4.9). As in the case of the frequency analysis of ion-chamber currents and the Rossi-α-experiment the common eq. (3.14), or (3.14a), is used for the calculation of the signal $\overline{r_1\cdot r_2}$. Insertion of the expressions (6.8) and (6.9) into the integral of (3.14a) and integration yields

$$\int\frac{d\omega}{2\pi}\,\frac{(1-e^{-i\omega T})(1-e^{i\omega T})}{\omega^2T^2(\alpha+i\omega)(\alpha-i\omega)}=\frac{e^{-\alpha T}-1+\alpha T}{\alpha^3T^2}. \tag{6.10}$$

With (6.5), (6.6), (6.7), and (6.10) we obtain from (3.14a) the expression

$$\left.\begin{aligned}\overline{r_1\cdot r_2}(T)&=\frac{\overline{n_1\cdot n_2}(T)}{T^2}\\ &=W_1F\left[W_2F+\frac{W_2\chi_2k^2}{al^2}\cdot\frac{e^{-\alpha T}-1+\alpha T}{(\alpha T)^2}\right].\end{aligned}\right\} \tag{6.11}$$

Insertion of (6.5) and (6.6) leads to

$$\left.\begin{aligned}&\frac{\overline{n_1\cdot n_2}(T)-\overline{n_1}(T)\cdot\overline{n_2}(T)}{T^2}\\ &\quad=\frac{\overline{n_1}(T)}{T}\cdot\frac{W_2\chi_2k^2}{al^2}\cdot\frac{e^{-\alpha T}-1+\alpha T}{(\alpha T)^2}.\end{aligned}\right\} \tag{6.12}$$

For small (αT)-values a useful approximation is

$$\left.\begin{aligned}&\frac{\overline{n_1\cdot n_2}(T)-\overline{n_1}(T)\cdot\overline{n_2}(T)}{T^2}\\ &\quad=\frac{\overline{n_1}(T)}{T}\cdot\frac{W_2\chi_2k^2}{2\alpha l^2}\left[1-\frac{\alpha T}{3}+\frac{\alpha^2T^2}{12}\right].\end{aligned}\right\} \tag{6.12a}$$

Another derivation of (6.12) can easily be verified by recognizing that $\frac{\overline{n_1\cdot n_2}(T)}{T^2}$ is an average of the cross-correlation function (5.5). Thus

$$\frac{\overline{n_1\cdot n_2}(T)}{T^2}=\frac{1}{T^2}\int_0^T dt_1\int_0^T dt_2\,r_{12}(t_2-t_1). \tag{6.13}$$

This expression can also be used to derive the appropriate formula in the case of the single-detector-experiment. In this experiment an absorption detector with $A=0$ is again chosen for simplicity. If the autocorrelation function (5.10) is inserted into (6.13) it follows

$$\frac{\overline{n_2}(T)}{T^2}=\frac{1}{T^2}\int_0^T dt_1\int_0^T dt_2\,r_{11}(t_2-t_1). \tag{6.14}$$

After integration, in which the symmetry of $r_{11}(\tau)$ is important,

$$\frac{\overline{n^2}(T)}{T^2}=WF\left[\frac{1}{T}+WF+\frac{W\chi_2k^2}{al^2}\,\frac{e^{-\alpha T}-1+\alpha T}{(\alpha T)^2}\right]. \tag{6.15}$$

Using (6.5) and rearranging (6.15) we find an expression containing the variance $\overline{n^2}(T)-\overline{n}^2(T)$ in the following form

$$\left.\begin{aligned}&\frac{\overline{n^2}(T)-\overline{n}^2(T)-\overline{n}(T)}{T^2}\\ &\quad=\frac{\overline{n}(T)}{T}\,\frac{W\chi_2k^2}{al^2}\,\frac{e^{-\alpha T}-1+\alpha T}{(\alpha T)^2}.\end{aligned}\right\} \tag{6.16}$$

The left hand side of (6.16) contains the difference between the measured variance $(\overline{n^2}-\overline{n}^2)$ and the variance of a Poisson distribution $\overline{n}$ in the numerator. A slight rearrangement of these terms leads to an equivalent formulation

$$\left.\begin{aligned}&\frac{\overline{n(n-1)}(T)-\overline{n}^2(T)}{T^2}\\ &\quad=\frac{\overline{n}(T)}{T}\,\frac{W\chi_2k^2}{al^2}\,\frac{e^{-\alpha T}-1+\alpha T}{(\alpha T)^2},\end{aligned}\right\} \tag{6.17}$$

which is particularly useful for evaluation purposes. To determine, for instance, the decay constant α_c at delayed critical, in which case (6.17) reduces to

$$\left.\begin{aligned}\overline{n(n-1)}(T)-\bar{n}^2(T) \\ T^2 \\ =\frac{\bar{n}(T)}{T}\,\frac{W\chi_2\alpha_c}{\beta^2}\,\frac{e^{-\alpha_c T}-1+\alpha_c T}{(\alpha_c T)^2}\end{aligned}\right\}, \quad (6.18)$$

the experimental task is to measure the quantities $\overline{n(n-1)}$ and $\bar{n}$ for various lengths T of the time interval. If this has been done, the decay constant can be found from a fitted curve. The absolute fission rate, F, can also be determined from this experiment by elimination of the detector sensitivity, W, and rearrangement of (6.17) in analogy to the frequency analysis and the Rossi-α-experiment.

An experimental technique for recording the actual number of counts registered in a large number of individual time intervals of length T has been described by ALBRECHT [19]. The counts obtained from a gating scaler were punched on cards and for each time interval of variable length T the variance was calculated by a computer. For this type of experimental setup only moderate counting rates can be allowed, which limits the reactor power. These limitations can be widely reduced by using a more sophisticated system. Such a system will be described in the next section (cf. Fig. 6).

Apart from fast pulse techniques there may also be a possibility of using analogue techniques with an exponential response function, for instance. Again a two-detector-experiment with $A=0$ (absorber) is considered. The response functions shall be of exponential nature described by

$$a_1(t)=a_2(t)=\left\{\begin{array}{ll}\frac{1}{T}\exp(-t/T), & t\geqq 0\\ 0, & \text{otherwise}\end{array}\right\} \quad (6.19)$$

$$A_1(\omega)=A_2(\omega)=\frac{1}{1+i\omega T}. \quad (6.19a)$$

With the response function $H(\omega)$ defined in (6.9) and (6.19a) we find for the integral part of (3.14a)

$$\left.\begin{aligned}\int\frac{d\omega}{2\pi}\cdot\frac{1}{(\alpha+i\omega)(\alpha-i\omega)(1+i\omega T)(1-i\omega T)} \\ =\frac{1}{2\alpha}\,\frac{1}{1+\alpha T}\end{aligned}\right\}. \quad (6.20)$$

Due to the normalization chosen in (6.19), the mean values of r_1 and r_2 are the same as in (6.5) and (6.6). Thus

$$\overline{r_1\cdot r_2}(T)=W_1F\left[W_2F+\frac{W_2\chi_2 k^2}{2\alpha l^2}\,\frac{1}{1+\alpha T}\right]. \quad (6.21)$$

In this case, one finds a very simple formula for the evaluation of reactor parameters. We should like to conclude with this demonstration of the possibility for investigating the features of new techniques.

7. Analysis of counting statistics (p-method)

Correlated detector counts in chain-reacting systems are responsible for the dependence of the variance of counts, registered in specified time intervals T, on reactor-physics parameters, i.e. the constants k and α, as well as on the detector sensitivity, W. More generally, we observe a deviation of the counting statistics from POISSON's law.

When the distribution of probabilities $p_n(T)$ for registering n counts in specified time intervals of variable length T can be directly measured, the average $\bar{n}(T)$, the variance $\left(\overline{n^2}(T)-\bar{n}^2(T)\right)$, the quantity $\left(\overline{n(n-1)}(T)-\bar{n}^2(T)\right)$, and higher moments are easily calculated. For instance,

$$\bar{n}(T)=\sum_{n=1}^{\infty}p_n(T)\,n, \quad (7.1)$$

$$\overline{n^2}(T)=\sum_{n=1}^{\infty}p_n(T)\,n^2, \quad (7.2)$$

$$\left.\begin{aligned}\overline{n(n-1)}(T)-\bar{n}^2(T) \\ =\sum_{n=2}^{\infty}p_n(T)\,n(n-1)-\left[\sum_{n=1}^{\infty}p_n(T)\,n\right]^2.\end{aligned}\right\} \quad (7.3)$$

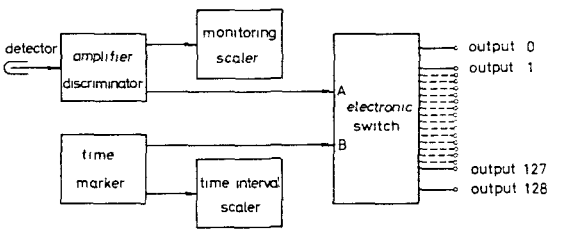

Fig. 6. Schematic diagram of a probability analyser [20]

An experimental setup for the direct measurement of the probabilities $p_n(T)$ (n ranging from 0 to 128) is schematically shown in Fig. 6 [20]. Pulses from a detector are fed into the input A of a fast acting electronic step switch, which proceeds by one step for each incoming pulse. Pulses from a time marker terminating the length T of the time interval are fed into input B. According to the position of the electronic switch, a gate is opened for the time pulse which gives a signal to the scaler connected with this output. If, for instance, 3 counts came in during the interval, output 3 of the switch opens gate 3 so that the pulse from the time marker delivers a signal into scaler 3. Thus 3 counts have been registered during this time interval. The step switch is reset and the probability analyser is opened for a new cycle.

After a sufficiently long time the run for one T-value is finished and the probabilities $p_n(T)$ may be obtained directly by dividing the number of counts in scaler n by the number of counts in the time interval scaler.

The exploitation of this setup for the Feynman-experiment is, of course, straightforward. We need only apply eqs. (7.1—3) and insert these quantities into eqs. (6.16), (6.17) or (6.18).

One simple check of the performance of the apparatus is furnished by the monitor scaler, which registers each count so that the average, $\bar{n}(T)$, can be obtained independently from (7.1). Additional checking of the apparatus and improved evaluation methods are possible because the general behaviour of $p_n(T)$ is known from theoretical expressions derived by ZOLOTUKHIN and MOGILNER [4] and by PÁL [21]. They introduced a probability generating function (abbr. p.g.f.).

$$R(T,u)=\sum_{n=0}^{\infty}p_n(T)\,u^n \quad (7.4)$$

(T = time interval, u = auxiliary variable). It is easy to prove that the p.g.f. $R_p(T,u)$ for the numbers

of Poisson-distributed counts would be defined by

$$\ln R_p(T, u) = \bar{n}(T)(u-1). \tag{7.5}$$

These authors [4, 21] obtain the approximate expression

$$\ln R(T,u) = \bar{n}(T)(u-1)\left\{\frac{1-\varphi}{z(u-1)} - \frac{2}{\alpha T z(u-1)} \ln\left[1 + \frac{(\varphi-1)^2}{4\varphi}(1-e^{-\alpha T\varphi})\right]\right\} \tag{7.6}$$

for $R(T, u)$, where

$$\varphi = \sqrt{1-2z(u-1)}, \tag{7.7a}$$

and

$$z = \frac{W\chi_2 k^2}{(\alpha l)^2}. \tag{7.7b}$$

All symbols have the same meaning as in the previous sections 3—6. This expression (7.6) is obtained within the usual limits of the point reactor model, prompt kinetics, the use of absorption detectors and the neglection of spontaneous fission. Furthermore, the underlying branching process is slightly simplified, leading to a dependence of the probabilities $p_n(T)$ on only 3 parameters, i.e. the average $\bar{n}(T)$, αT, and the parameter z of (7.7b). For a more detailed discussion of these mathematical simplifications the reader is referred to the original papers [4, 21].

As each p.g.f. (7.1) is normalized to unity at $u=1$, we may perform an expansion of the form

$$\ln R(T, u) = \sum_{m=1}^{\infty} a_m(T)(u-1)^m. \tag{7.8}$$

In practice this series can always be truncated after the M-th term, giving

$$\ln R(T, u) \approx \sum_{m=1}^{M} a_m(T)(u-1)^m. \tag{7.8a}$$

One can prove [22] that when M is taken as the practical limit to the number of (correlated) detector counts obtainable from a single neutron chain, $a_M(T)$ is the last significant term. Vice versa, the number M of terms needed in (7.8a) to get a good approximation of the expression (7.6) indicates how many detector counts may be expected from a single chain.

From (7.6) we obtain

$$a_1(T) = \bar{n}(T) \tag{7.9a}$$

$$a_2(T) = \frac{\bar{n}(T)z}{2}\,\frac{e^{-\alpha T} - 1 + \alpha T}{\alpha T} \tag{7.9b}$$

$$a_3(T) = \frac{\bar{n}(T)z^2}{2}\,\frac{(2+\alpha T)e^{-\alpha T} - 2 + \alpha T}{\alpha T}. \tag{7.9c}$$

In conclusion we will make two statements, the second one being a corollary to the first:

1. $a_1(T)$ and $a_2(T)$ are not affected by the above-mentioned simplification of the branching process leading to the expression (7.6), but are rigorously valid in the point-reactor model.

2. The approximate p.g.f. $R(T, u)$ according to (7.6) yields the correct average $\bar{n}(T)$ and the mean square $\overline{n^2}(T)$. This involves the correct variance $\left(\overline{n^2}(T) - \bar{n}^2(T)\right)$ and $\overline{n(n-1)}(T)$, of course.

For a proof, we apply the familiar technique for the derivation of moments from the p.g.f.

$$R(T, u) = \exp \sum_{m=1}^{\infty} a_m(T)(u-1)^m. \tag{7.8a}$$

Obviously, from (7.1) and (7.8a) we get

$$\frac{dR}{du}(T, 1) \equiv \bar{n}(T) = a_1(T), \tag{7.10}$$

which is identical with (7.9a). To confirm (7.9b) we differentiate (7.4) and (7.8a) twice at $u=1$, giving

$$\frac{d^2R}{du^2}(T, 1) \equiv \overline{n(n-1)}(T) = a_1^2(T) + 2a_2(T). \tag{7.11a}$$

With $a_1(T)$ from (7.10) this yields

$$a_2(T) = \tfrac{1}{2}\left[\overline{n(n-1)}(T) - \bar{n}^2(T)\right], \tag{7.11b}$$

and we need only insert eq. (6.17) and the parameter z from (7.7b) in order to confirm (7.9b). We have thus verified our first statement, and the corollary (2) follows directly from the form in which $\bar{n}(T)$ and $\overline{n^2}(T)$ enter the expressions (7.10) and (7.11b).

8. Approximate treatment of space-dependence in coupled reactors

So far only the point-reactor eqs. (3.14, 14a) have been applied. For a rigorous treatment of genuine space- and energy-dependent problems a modal expansion is recommended. Relying on this technique, a formalism applicable to the Rossi-α-experiment was developed previously [14]. It shall not be reproduced here, although a much simpler derivation starting from (3.2, 4) is now possible.

Interesting examples of space-dependent reactor kinetics are those of coupled reactors, which were first studied by AVERY [15]. In an idealized case these assemblies are divided into N loosely coupled core regions, e.g. a 2-slab-core-configuration of the Argonout type reactor [16, 23]. Then the kinetical characteristics allow for a nodal treatment instead of the rigorous and laborious eigenfunction expansion mentioned above.

Now make the explicit assumption that the interaction between two core regions is a slow process, e.g. neutrons diffuse through zones of pure moderator in the space between cores. The term "slow" means slow in comparison with the relaxation phenomena in each single core region. Often in practice all assumptions are not well fulfilled, e.g. in fast-thermal coupled systems. Nevertheless, this mode of investigation leads to useful results even in such cases.

For each core region the point-reactor concept will be considered to be a good approximation, and the kinetics of the complete assembly of N cores can be described by general multiplication functions $k_{mn}(t)$. Within the frame of prompt kinetics we define these first generation coupling functions as

$k_{mn}(t)$ = rate of prompt fission neutrons produced after a delay t in region m as direct successors to one fission neutron from region n.

A set of prompt fission-source response functions $q_{mn}(t)$, $1 \leqq m,\ n \leqq N$, is defined through integral eqs.,

$$q_{mn}(t) = k_{mn}(t) + \sum_{j=1}^{N} \int dt_1\, q_{mj}(t-t_1)\, k_{jn}(t_1). \tag{8.1}$$

The quantity $q_{mn}(t)$ denotes the total rate of (prompt) fission neutrons produced after a delay t in region m which are related to one fission neutron injected into

region n at time 0. Whereas $k_{mn}(t)$ refers only to the first daughter generation, $q_{mn}(t)$ takes account of all generations.

Denoting the Fourier-transforms of $k_{mn}(t)$, $q_{mn}(t)$ by $K_{mn}(\omega)$, $Q_{mn}(\omega)$ we obtain from (8.1)

$$Q_{mn}(\omega) = K_{mn}(\omega) + \sum_{j=1}^{N} Q_{mj}(\omega)\, K_{jn}(\omega) \qquad (8.1\,a)$$

or in matrix form

$$\boldsymbol{Q}(\omega)[\boldsymbol{I} - \boldsymbol{K}(\omega)] = \boldsymbol{K}(\omega) \qquad (8.1\,b)$$

with $\boldsymbol{I}$ for the unit matrix.

The solution to (8.1 b) is

$$\boldsymbol{Q}(\omega) = \boldsymbol{K}(\omega)[\boldsymbol{I} - \boldsymbol{K}(\omega)]^{-1} = \sum_{n=1}^{\infty} \boldsymbol{K}^n(\omega). \qquad (8.2)$$

According to our assumption, the interaction between core regions is a slow process. Thus all off-diagonal elements of $\boldsymbol{K}(\omega)$ can be neglected at high frequencies. This sort of decoupling leads, for high ω, to

$$Q_{mn}(\omega) \approx 0 \qquad m \neq n, \qquad (8.3)$$

$$Q_{nn}(\omega) \approx \frac{K_{nn}(\omega)}{1 - K_{nn}(\omega)}\,. \qquad (8.4)$$

Of course, we cannot without reservations insert one-group expressions for $K_{nn}(\omega)$ into (8.4), as the decoupling of core n from other core-regions at high frequencies does not automatically imply decoupling from the surrounding moderator. Such a reflector may cause considerable deviations from one-group kinetics. We will, nevertheless, introduce one-group kinetics as a model into (8.4). In this approximation

$$k_{nn}(t) = \frac{K_{nn}(0)}{l_{nn}} \exp(-t/l_{nn}), \qquad t > 0 \qquad (8.5\,a)$$

$$K_{nn}(\omega) = \frac{K_{nn}(0)}{1 + i\omega l_{nn}} \qquad (8.5\,b)$$

where $K_{nn}(0)$ has been defined as a prompt multiplication constant, and l_{nn} is in this model the effective neutron life-time in region n. Then

$$Q_{nn}(\omega) \approx \frac{K_{nn}(0)}{l_{nn}} \frac{1}{\alpha_{nn} + i\omega}, \qquad (8.4\,b)$$

with

$$\alpha_{nn} = \frac{1 - K_{nn}(0)}{l_{nn}}\,. \qquad (8.6)$$

When detector 1 (cf. Fig. 1) is put into region m, its sensitivity is W_{1m} (counts per fission in region m). The response of signal r_1 to the injection of one fission neutron into region n is written $g_1(m,n,t)$. Let $G_1(m, n, \omega)$ be its Fourier-transform and ν_m the number of prompt neutrons per fission in region m. Then

$$G_1(m, n, \omega) = \frac{W_{1m}}{\nu_m} A_1(\omega)\, Q_{mn}(\omega). \qquad (8.7)$$

If F_n is the fission rate in region n, the binary source of prompt fission neutron pairs in that region is

$$S_{2n} = F_n \frac{\overline{\nu(\nu-1)_n}}{2}\,. \qquad (8.8)$$

We insert (8.7, 8) and the corresponding term for instrument line 2 (Fig. 1) into (3.4a) to obtain expressions for the crosscorrelated output signals $r_1(t)$ and $r_2(t)$ with detector 1 in region m and detector 2 in region n:

$$\left.\begin{aligned} \overline{r_1 \cdot r_2}(m,n) &= \overline{r_1}(m) \cdot \overline{r_2}(n) + \\ &+ \int \frac{d\omega}{2\pi} A_1(\omega)\, A_2(-\omega)\, B_{mn}(\omega), \end{aligned}\right\} \qquad (8.9)$$

$$\left.\begin{aligned} \overline{r_1}(m) &= W_{1m} F_m A_1(0), \\ \overline{r_2}(n) &= W_{2n} F_n A_2(0), \end{aligned}\right\} \qquad (8.10)$$

$$\left.\begin{aligned} B_{mn}(\omega) &\\ &= \frac{W_{1m} W_{2n}}{\nu_m \nu_n} \sum_{j=1}^{N} F_j \overline{\nu(\nu-1)_j}\, Q_{mj}(\omega)\, Q_{nj}(-\omega). \end{aligned}\right\} \qquad (8.11)$$

In general we have to rely on the numerical solution of (8.2) to obtain the $B_{mn}(\omega)$. But with the assumptions leading to (8.3) we get for sufficiently high frequencies

$$B_{mn}(\omega) \approx 0 \qquad m \neq n, \qquad (8.12)$$

and, with the additional approximation (8.5a, b),

$$B_{nn}(\omega) = F_n \frac{W_{1n} W_{2n} K_{nn}^2(0)}{l_{nn}^2} \frac{\overline{\nu(\nu-1)_n}}{\nu_n^2} \frac{1}{\alpha_{nn}^2 + \omega^2}\,. \qquad (8.13)$$

Eqs. (8.9—13) are to be compared with (3.14a) for the point-reactor. This example should serve as an illustration of how correlation experiments can be applied to the study of coupled reactors. Of special interest will be the frequencies, at which decoupling (8.12) occurs. They yield, for a specified system, a certain insight into the coupling mechanism and the associated travelling times of neutrons between the core regions.

9. Conclusions

In the preceding sections it has been clearly demonstrated that neutronic noise analysis experiments in zero power reactors can be theoretically treated by one common basic formula. In its general form this expression is still sufficiently transparent to include delayed neutrons as well as space- and energy-dependence. Nevertheless, for the sake of simplicity and clearness, we restricted ourselves mostly to the point-reactor model and the short time approximation when comparing different experimental methods. A common treatment of three types of correlation experiments has been used by inserting appropriate pairs of weighting-functions into the basic integral. This offers the possibility of investigating new techniques which may use either a continuous signal or discrete pulses.

In all expressions thus derived two terms can be distinguished. One term is mainly governed by the decay constant α; the other contains the detector sensitivity, the fission rate, the multiplication constant, and nuclear parameters. In frequency analysis experiments, particularly, advantage is often taken of the complete information, contained in both terms. But from the expressions derived it can be concluded that each type of correlation experiment should be applied also to the determination of reactivity and absolute fission rate.

Acknowledgements

The authors express their gratitude to Mr. D. SANITZ for his continuous interest and his adive on mathematical statistics; to Mr. M. EDELMANN and Mr. W. VÄTH for helpful discussions about experimental problems. We are also very grateful to Mr .B. A. HUTCHNIS who carefully corrected the English version of this paper.

10. Appendix. Proof of eqs. (2.12) and (2.19)

Refer to the definitions and notation of section 2. Define the joint probability for the output signals, $r_1(t)$ and $r_2(t)$, of the two instrument lines (Fig. 1 or Fig. 2) to lie simultaneously in intervals (r_1, r_1+dr_1) and (r_2, r_2+dr_2) as $p(r_1, r_2)\,dr_1\,dr_2$. We want to apply the familiar technique of generating functions (g.f.) and define

$$f(u_1, u_2) = \int dr_1\,dr_2\,p(r_1, r_2)\exp(u_1 r_1 + u_2 r_2). \quad (10.1)$$

The g.f. $f(u_1, u_2)$ obeys

$$f(0, 0) = 1, \quad (10.2)$$

and

$$\frac{\partial f}{\partial u_1}(0, 0) = \int dr_1\,dr_2\,p(r_1, r_2)\,r_1 = \overline{r_1}. \quad (10.3)$$

Similarly $\overline{r_2}$ may be obtained and

$$\frac{\partial^2 f}{\partial u_1\,\partial u_2}(0, 0) = \overline{r_1 \cdot r_2}. \quad (10.4)$$

Eqs. corresponding to (10.1—4) are used to obtain averages and second moments from the group of g.f.s defined below.

$f_0(z, t, u_1, u_2)$ = g.f. for the distribution of signals r_1, r_2 at time t, due to all chains of reactions initiated by one z-reaction at time 0. Similarly,

$f_1(y, t, u_1, u_2)$ = g.f. for the signals due to chains of reactions starting with the release of one y-particle at time 0,

$\psi(z, t, u_1, u_2)$ = g.f. for the signals due to a single z-reation at time 0.

For negative as well as for very large delay times t these g.f.s are equal to 1. We define time-weighted g.f.s which are zero at $u_1 = u_2 = 0$,

$$F_0(z, u_1, u_2) = \int dt\,[f_0(z, t, u_1, u_2) - 1], \quad (10.5)$$

$$F_1(z, u_1, u_2) = \int dt\,[f_1(z, t, u_1, u_2) - 1], \quad (10.6)$$

$$\Psi(z, u_1, u_2) = \int dt\,[\psi(z, t, u_1, u_2) - 1]. \quad (10.7)$$

Extensive use will be made of the following two fundamental properties of these g.f.s which are immediate consequences of the defining eq. (10.1):

1. When $f_1(u_1, u_2)$ and $f_2(u_1, u_2)$ are the g.f.s for two independent random vectors, (r_1, r_2) and (s_1, s_2), then

$$f(u_1, u_2) = f_1(u_1, u_2)\cdot f_2(u_1, u_2) \quad (10.8)$$

is the g.f. for the random sum vector $(r_1+s_1,\ r_2+s_2)$.

2. Let $f_1(z, u_1, u_2)$ be the g.f. for the random vectors $(r_1, r_2)_z$, restricted by a certain condition z; and $f_2(u_1, u_2)$ the unconditional, general g.f. for (r_1, r_2). If $p(z)$ is the probability for z, then

$$f_2(u_1, u_2) = \int dz\,p(z)\,f_1(z, u_1, u_2). \quad (10.9)$$

A direct application is furnished by the signals r_1 and r_2 observed at time 0 and due to chains of reactions initiated by primary reactions occurring in a time interval $(-T, 0)$. If these primary z-reactions are Poisson-distributed with mean rates $c_0(z)$, the appropriate g.f. will be

$$\left.\begin{aligned} f_2(T, u_1, u_2) &= \exp\Big[-T\int dz\,c_0(z)\Big]\times \\ &\times \sum_{n=0}^{\infty}\frac{1}{n!}\left[\int dz\,c_0(z)\int_0^T dt\,f_0(z, t, u_1, u_2)\right]^n, \end{aligned}\right\} \quad (10.10)$$

$$\left.\begin{aligned} &f_2(T, u_1, u_2) \\ &= \exp\left[\int dz\,c_0(z)\int_0^T dt\,[f_0(z, t, u_1, u_2) - 1]\right]. \end{aligned}\right\} \quad (10.11)$$

In the limit of $T\to\infty$ eq. (10.11) yields, cf. (10.5),

$$f(u_1, u_2) = \exp\int dz\,c_0(z)\,F_0(z, u_1, u_2). \quad (10.12)$$

From (10.12) we obtain by differentiation at the origin, cf. (10.3, 4),

$$\overline{r_1} = \int dz\,c_0(z)\frac{\partial F_0}{\partial u_1}(z, 0, 0), \quad (10.13)$$

$$\overline{r_1 \cdot r_2} = \overline{r_1}\cdot\overline{r_2} + \int dz\,c_0(z)\frac{\partial^2 F_0}{\partial u_1\,\partial u_2}(z, 0, 0). \quad (10.14)$$

We will now introduce the expressions previously defined in connection with (2.9, 10, 18), viz.

$$\frac{\partial f_1}{\partial u_1}(y, t, 0, 0) = g_1(y, t), \quad (10.15)$$

$$\frac{\partial F_1}{\partial u_1}(y, 0, 0) = \int dt\,g_1(y, t), \quad (10.16)$$

$$\frac{\partial \psi}{\partial u_1}(z, t, 0, 0) = \varphi_1(z, t), \quad (10.17)$$

$$\frac{\partial^2 \psi}{\partial u_1\,\partial u_2}(z, t, 0, 0) = \varphi_{12}(z, t), \quad (10.18)$$

$$\frac{\partial \Psi}{\partial u_1}(z, 0, 0) = \int dt\,\varphi_1(z, t), \quad (10.19)$$

$$\frac{\partial^2 \Psi}{\partial u_1\,\partial u_2}(z, 0, 0) = \int dt\,\varphi_{12}(z, t). \quad (10.20)$$

For a neutron of specified position and velocity we have a certain probability that its next collision will occur in a certain position and will be a reaction of a specified type, e.g. fission or scattering. Generally, any y-particle has a probability $K(y, z)$ that the next observed reaction of this particle will be a process of type z. Therefore, cf. (10.9), we can express F_1 in terms of F_0,

$$F_1(y, u_1, u_2) = \int dz\,K(y, z)\,F_0(z, u_1, u_2). \quad (10.21)$$

On the other hand, we can use the relations (10.8, 9) to express $f_0(z, t, u_1, u_2)$ in terms of $\psi(z, t, u_1, u_2)$, which is related only to the initiating z-reaction, and of $f_1(y, t, u_1, u_2)$, which is related to the y-particles which are released in this primary z-reaction and able to carry on the chain of reactions. Using the definitions (2.1—3),

$$\left.\begin{aligned} f_0(z, t) = \psi(z, t)\cdot\sum_{n=0}^{\infty}\Big[&\int dy_1 \ldots dy_n\times \\ &\times P(z, n, y_1, \ldots, y_n)\prod_{k=1}^{n} f_1(y_k, t)\Big]. \end{aligned}\right\} \quad (10.22)$$

$$\left.\begin{aligned} f_0(z, t) = \psi(z, t)\Big[1 &+ \int dy\,w_1(y, z)[f_1(y, t) - 1] + \\ &+ \int dy_1\,dy_2\,w_2(y_1, y_2, z)\times \\ &\times [f_1(y_1, t) - 1][f_1(y_2, t) - 1] + \cdots\Big]. \end{aligned}\right\} \quad (10.23)$$

In these expressions the arguments u_1, u_2 have been omitted. Also in (10.23) those terms are omitted

which will not contribute to moments of second or lower order. In all subsequent derivatives of g.f.s the arguments $u_1 = u_2 = 0$ are implied and will not be written. From (10.5—7), (10.15—20) and (10.23)

$$\frac{\partial F_0(z)}{\partial u_1} = \int dy\, w_1(y,z) \frac{\partial F_1(y)}{\partial u_1} + \int dt\, \varphi_1(z,t), \quad (10.24)$$

$$\left.\begin{aligned} \frac{\partial^2 F_0(z)}{\partial u_1 \partial u_2} &= \int dy\, w_1(y,z) \frac{\partial^2 F_1(y)}{\partial u_1 \partial u_2} + \\ &+ \int dt\, \varphi_{12}(z,t) + 2 \int dy_1\, dy_2\, w_2(y_1, y_2, z) \times \\ &\times \int dt\, g_1(y_1,t)\, g_2(y_2,t) + \int dy\, w_1(y,z) \times \\ &\times \int dt [g_1(y,t)\, \varphi_2(z,t) + g_2(y,t)\, \varphi_1(z,t)]. \end{aligned}\right\} \quad (10.25)$$

With (10.13) and (2.4)

$$\left.\begin{aligned} \overline{r_1} = \int dz\, c_0(z) \frac{\partial F_0(z)}{\partial u_1} &= \int dy\, s_0(y) \frac{\partial F_1(y)}{\partial u_1} + \\ &+ \int dz\, c_0(z) \int dt\, \varphi_1(z,t). \end{aligned}\right\} \quad (10.26)$$

When we insert (10.16) into this expression, we get a verification of (2.10). On the other hand, we may insert (10.21) into (10.24) and get

$$\frac{\partial F_0(z)}{\partial u_1} = \int dz' \frac{\partial F_0(z')}{\partial u_1} K_1(z', z) + \int dt\, \varphi_1(z,t) \quad (10.27)$$

with the kernel

$$K_1(z', z) = \int dy\, K(y, z')\, w_1(y, z). \quad (10.28)$$

This kernel can be recognized to give the distribution of z'-reactions which are directly related to a z-reaction, i.e. as first successors. Therefore, it may also be applied in the computation of the total reaction rate $c(z)$ from the rate $c_0(z)$ of primary reactions,

$$c(z) = \int dz'\, K_1(z, z')\, c(z') + c_0(z). \quad (10.29)$$

A comparison of (10.27) with (10.29) shows that the integral operators are adjoints of each other. Therefore, if

$$c(z) = \int dz'\, K_2(z, z')\, c_0(z') \quad (10.30)$$

is a formal solution of (10.29) with a certain kernel $K_2(z, z')$, then the adjoint kernel $K_2(z', z)$ solves (10.27). Thus

$$\frac{\partial F_0(z')}{\partial u_1} = \int dz \left[\int dt\, \varphi_1(z,t)\right] K_2(z, z'). \quad (10.31)$$

Combining (10.30) and (10.31) we obtain finally

$$\overline{r_1} = \int dz'\, c_0(z') \frac{\partial F_0(z')}{\partial u_1} = \int dz\, c(z) \int dt\, \varphi_1(z,t), \quad (10.32)$$

which is the verification of (2.9).

In the same way we can deal with the mixed derivative (10.25).

Using (10.14) we can write

$$\left.\begin{aligned} \overline{r_1 \cdot r_2} - \overline{r_1} \cdot \overline{r_2} &= \int dz\, c(z) \Big[\int dt\, \varphi_{12}(z,t) + \\ &+ 2 \int dy_1\, dy_2\, w_2(y_1, y_2, z) \times \\ &\times \int dt\, g_1(y_1,t)\, g_2(y_2,t) + \int dy\, w_1(y,z) \times \\ &\times \int dt [g_1(y,t)\, \varphi_2(z,t) + g_2(y,t)\, \varphi_1(z,t)]\Big]. \end{aligned}\right\} \quad (10.33)$$

When the definitions (2.5, 6, 18) are now inserted, we obtain directly (2.12) for an experiment with two independent detectors (Fig. 1, $\pi_{12} = 0$) or (2.19) for a single-detector experiment (Fig. 2).

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Druck der Universitätsdruckerei H. Stürtz AG., Würzburg