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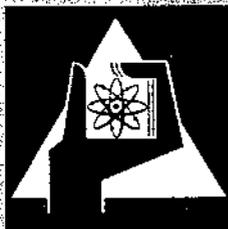
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Interpretation of Doppler Coefficient  
Measurements in Fast Critical Assemblies

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Karlsruhe

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Interpretation of Doppler Coefficient  
Measurements in Fast Critical Assemblies \*)

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## Introduction

This paper is concerned with the interpretation of Doppler experiments in which the reactivity change due to heating of a sample in a fast critical assembly is measured. Experiments of this kind were carried out at Argonne /1/, /2/ and Atomic International /3/, and are being planned at the critical assembly SNEAK at Karlsruhe /4/.

Probably the major difficulty in the theoretical treatment of such an experiment is that the flux dips due to the resonances are different in the sample and in the core because they are at different temperatures. At and near the boundary (very often the sample diameter is less than one mean free path, so that all of the sample volume is near the boundary) these fluxes overlap, and for the calculation of reaction rates in both regions one has to take account of this overlap. Clearly, this is a more complicated problem than the calculation of the standard Doppler coefficient, where the core is assumed to have uniform composition and temperature.

The first correct treatment of the problem just outlined is due to Storrer, Khairallah, and Ozeroff /5/. These authors assume that the sample has the same composition as the surrounding core, and they calculate the temperature dependent reaction rates in the sample and in the core for this special case.

However, in all the experiments mentioned above, the sample composition is entirely different from the one of the core. It is the main purpose of this paper to develop a formalism, valid in this more general case, which allows the calculation of the reactivity change due to heating of the sample. The formulas to be derived will be based on integral transport theory and perturbation theory. In order to illustrate the problem, a special case will be discussed first.

Preliminary Discussion of the Case of a Small Sample which has the same Composition as the Core

The case of a sample which is small compared to one mean free path, and which has the same composition as the core, is suitable to provide some physical insight, though it is of no practical importance. It is also a useful example to study the question whether a sample measurement is of any use to obtain the Doppler coefficient of a whole reactor. The following discussion is similar to the one given by Storrer et al. /5/, though a different approach will be taken.

The notation will be

- $V, s$  = volume, surface of the sample
- $\bar{l}$  =  $4 V/s$  mean chord length of the sample
- $p_{11}$  = probability that a neutron born in the sample makes its first collision in the sample ( $= \bar{l} \Sigma_{t1}$  for a small sample)
- $\Gamma_1$  = probability that a neutron impinging on the sample will make a collision in the sample ( $= \bar{l} \Sigma_{t1}$  for a small sample)
- $S$  = source density
- subscript 1 refers to the sample
- 2 refers to the core.

If  $(i,k)$  denotes the number of neutrons which are produced in region  $i$  and absorbed in region  $k$ , one can easily write down the absorption rates

$$(1,1) = SV p_{11} \frac{\Sigma_{a1}}{\Sigma_{t1}} = S \frac{s}{4} \bar{l}^2 \Sigma_{a1}$$

$$(1,2) = SV(1-p_{11}) \frac{\Sigma_{a2}}{\Sigma_{t2}} = S \frac{s}{4} \bar{l} \frac{\Sigma_{a2}}{\Sigma_{t2}} - S \frac{s}{4} \bar{l}^2 \Sigma_{t1} \frac{\Sigma_{a2}}{\Sigma_{t2}}$$

$$(2,1) = S \frac{s}{4 \Sigma_{t2}} \Gamma_1 \frac{\Sigma_{a1}}{\Sigma_{t1}} = SV \frac{\Sigma_{a1}}{\Sigma_{t2}}$$

$$(2,2) = S \frac{S}{4 \Sigma_{t2}} (1 - \Gamma_1) \frac{\Sigma_{a2}}{\Sigma_{t2}} = S \frac{S}{4} \frac{\Sigma_{a2}}{\Sigma_{t2}^2} - SV \frac{\Sigma_{t1} \Sigma_{a2}}{\Sigma_{t2}^2}$$

In the last two equations,  $Ss/4\Sigma_{t2}$  is the current of neutrons into the sample.

The terms (1,1) and (1,2) depend on the sample temperature only in the order  $\bar{l}^2$ , which can be neglected for small samples. Thus, the change in absorption rate when the sample is heated is entirely due to neutrons which were produced in the surrounding core, terms (2,1) and (2,2). Term (2,2) describes the "filter" effect of the sample: the number of neutrons which traverse the sample and are absorbed in the core depends on the sample temperature.

The net change in absorption rate upon heating of the sample is

$$\delta A' = SV \int dE \left( \frac{\Sigma_{a1}}{\Sigma_{t2}} - \frac{\Sigma_{t1} \Sigma_{a2}}{\Sigma_{t2}^2} \right).$$

The quantity of interest, however, is the change in absorption rate that occurs if the whole reactor is heated, namely

$$\delta A'' = SV \int dE \left( \frac{\Sigma_{a1}}{\Sigma_{t1}} - \frac{\Sigma_{a2}}{\Sigma_{t2}} \right).$$

These two quantities shall now be compared, assuming that the temperature change is small; one can then put  $\Sigma_1 = \Sigma_2 + \delta \Sigma$  and retain only terms of first order in  $\delta \Sigma$ . One obtains

$$\begin{aligned} \delta A' &= SV \int dE \left( \frac{\Sigma_{a2} + \delta \Sigma_a}{\Sigma_{t2}} - \frac{(\Sigma_{t2} + \delta \Sigma_t) \Sigma_{a2}}{\Sigma_{t2}^2} \right) \\ &= SV \int dE \left( \frac{\delta \Sigma_a}{\Sigma_{t2}} - \frac{\Sigma_{a2} \delta \Sigma_t}{\Sigma_{t2}^2} \right) \end{aligned}$$

and

$$\delta A'' = SV \int dE \left( \frac{\Sigma_{a2} + \delta \Sigma_a}{\Sigma_{t2} + \delta \Sigma_t} - \frac{\Sigma_{a2}}{\Sigma_{t2}} \right) = SV \int dE \left( \frac{\delta \Sigma_a}{\Sigma_{t2}} - \frac{\Sigma_{a2} \delta \Sigma_t}{\Sigma_{t2}^2} \right)$$

Thus,  $\delta A' = \delta A''$ , and we have the result: In an experiment where only a small sample is heated, the changes in absorption rate are due to neutrons which have the resonance structure of the environment ("cold" neutrons). If the whole reactor is heated, the changes are due to neutrons which have the resonance structure of the sample ("hot" neutrons). It has been shown that the two changes are equal for small temperature changes. Therefore, a sample measurement is, in principle, suitable to measure the Doppler coefficient of a fast reactor, if the temperature changes are kept small.

Derivation of a Perturbation Expression  
for the Reactivity Change

In this section, an expression for the reactivity change will be derived, which is rigorous except for the use of perturbation theory. More specifically, the following equations will be based on integral transport theory, which means that isotropic scattering will be assumed. The narrow resonance approximation, which is standard in most Doppler calculations in fast spectra, will also be made.

The integral transport equation reads

$$\Sigma_t(E, r) \phi(E, r) = \int_V S(E, r') p(r' \rightarrow r, E) dV' \quad (1)$$

In this and the following equations, the notation is

$\Sigma(E,r)$	macroscopic cross section, $\text{cm}^{-1}$
t,s,f,p,c	refer to the cross section for total collisions, scattering, fission, potential scattering, formation of the compound nucleus
$S(E,r)$	source density of neutrons due to scattering and fission, per unit volume, energy and time, $\text{n/cm}^3\text{-ev-sec}$
$S_{xi}$	rate of reactions of type x in the energy group i
$g^{j \rightarrow i}$	rate of (elastic or inelastic) scattering events in group j, with moderation of the neutrons into group i
$F_j$	fission rate in the energy group j
V	total reactor volume
$\langle \rangle$	average over an energy group
$\bar{\Sigma}_{p1} = \Sigma_{p1} + \frac{1}{V}$	equivalent potential cross section of the sample

The function

$$p(r' \rightarrow r, E) = \frac{\Sigma_t(E,r)}{4\pi} \frac{e^{-L(r',r,E)}}{|r'-r|^2} \quad (2)$$

in eq. (1) is the probability that a neutron born at  $r'$  will make its next collision in the unit volume at  $r$ ;  $L$  is the "optical path" between  $r$  and  $r'$ .

We now multiply equation (1) by the cross section ratio  $\Sigma_x/\Sigma_t$ , and integrate over the energy group  $i$ .

This gives

$$S_{xi}(r) = \int \Sigma_x(E,r) \phi(E,r) dE = \int dV' S_i(r') \left\langle \frac{\Sigma_x}{\Sigma_t}(E,r) p(r' \rightarrow r, E) \right\rangle_j \quad (3)$$

In deriving equation (3) we have made the narrow resonance approximation. As a consequence of it, the source density  $S(E,r)$  is a slowly varying function of energy, and can be taken out of the averaging process over energy.

By definition, the group integrated source density which occurs in eq. (3) is the sum of the moderation and fission source

$$S_i(r) = \sum_{j \leq i} S_j^{j \rightarrow i}(r) + \chi_i \sum_j \nu F_j(r). \quad (4)$$

Obviously, we can express the reaction rates on the right side of eq. (4) through eq. (3), and thus we obtain

$$S_i(r) = \sum_{j \leq i} \int dV' S_j(r') \left\langle \frac{\sum_{s \rightarrow i}^{j \rightarrow i}}{\sum_t} (E, r) p(r' \rightarrow r, E) \right\rangle_j + \chi_i \sum_j \int dV' S_j(r') \left\langle \frac{\nu \sum_f}{\sum_t} (E, r) p(r' \rightarrow r, E) \right\rangle_j \quad (5)$$

These are the multigroup transport equations in a form in which the source density  $S_i$  is the variable; this form has the advantage that it is obvious which quantities have to be averaged over the resonances in space dependent problems. The meaning of the variable  $S_i$  is made obvious by eq. (4).

The notation can now be simplified somewhat by introducing the following group averaged probability distributions

$$P_{xi}(r', r) = \left\langle \frac{\sum_x}{\sum_t} (E, r) p(r' \rightarrow r, E) \right\rangle_i \quad (6)$$

Then we can write the equations (5)

$$S_i(r) = \sum_{j \leq i} \int_V dV' S_j(r') P_S^{j \rightarrow i}(r', r) + \chi_i \sum_j \int_V dV' S_j(r') \nu P_{fj}(r', r) \quad (7)$$

We can now proceed to find the perturbation equations based on (7) and on the following equations which are adjoint to (7)

$$S_i^+(r) = \sum_{j \neq i} \int_V dV' S_j^+(r') P_s^{i \rightarrow j}(r, r') + \sum_j \chi_j \int_V dV' S_j^+(r') \nu P_{fj}(r, r') \quad (8)$$

We assume that there are small perturbations  $\delta P_s$  in the  $P_x$

$$P_{xi}(r'r) = P_{xi}^{(0)}(r', r) + \delta P_{xi}(r', r)$$

and that the effect of these  $\delta P_x$  is compensated in first order by a change  $\delta \nu$  in  $\nu$ . Then we obtain, by standard manipulations, the following expression for the first order reactivity effect of the perturbation

$$= \frac{-\frac{\delta \nu}{\nu} \sum_{i,j \neq i} \int dV' \left( \nu S_i^+(r) \delta P_s^{j \rightarrow i}(r', r) S_j(r') + \sum_{i,j} \delta \nu \right) \nu S_i^+(r) \nu P_{fj}(r', r) S_j(r')}{\sum_{i,j} \int dV' \left( \nu \chi_i S_i^+(r) \nu P_{fj}(r', r) S_j(r') \right)} \quad (9)$$

Within the frame of first order perturbation theory, this equation gives the reactivity effect of any small enough perturbation in a reactor which can be described by the multigroup transport equations. We shall apply it to the interpretation of Doppler effect measurements of the type mentioned above.

Application to the Case where the Sample  
has the same Composition as the Core

We consider first the case where a small central portion of the core is heated, or in other words the case treated by Storrer et al. /5/, where the sample has the same composition as the core. The unperturbed reactor is homogeneous and at uniform temperature.

Let us assume a reactor large enough so that both the  $S$  and  $S^+$  can be considered as constant over the region of perturbation, which is obviously the sample volume and the region around it, within two or three mean free paths. Then the double integrals in (9), which have to be extended over the region of perturbation, can be split up into four integrals in the following way

$$\begin{aligned} & \int_V dV' \int_V dV P_x(r', r) = \\ & = \left\langle \frac{\sum x_1}{\sum t_1}(E) \int_{V_1} dV' \int_{V_1} dV p(r' \rightarrow r, E) \right\rangle + \left\langle \frac{\sum x_1}{\sum t_1}(E) \int_{V_2} dV' \int_{V_1} dV p(r' \rightarrow r, E) \right\rangle \\ & + \left\langle \frac{\sum x_2}{\sum t_2}(E) \int_{V_1} dV' \int_{V_2} dV p(r' \rightarrow r, E) \right\rangle + \left\langle \frac{\sum x_2}{\sum t_2}(E) \int_{V_2} dV' \int_{V_2} dV p(r' \rightarrow r, E) \right\rangle. \quad (10) \end{aligned}$$

Obviously, the integral  $\int_{V_a} dV' \int_{V_b} dV$  represents the contribution of neutrons, which are moderated to energy  $E$  in region  $a$  and make the next collision in region  $b$ , to the rate of reactions of type  $x$ . Thus, we have the four terms which were already discussed in the first section.

In order to evaluate the integrals, we introduce the probability

$$P_1 = \frac{1}{V_1} \int_{V_1} dV' \int_{V_1} dV p(r' \rightarrow r) \quad (11)$$

that the first collision of a neutron born in the sample will be in the sample.

Then one can easily evaluate the two integrals

$$\int_{V_1} dV' \int_{V_1} dV p(r' \rightarrow r) = V_1 P_1$$

$$\int_{V_1} dV' \int_{V_2} dV p(r' \rightarrow r) = V_1 (1 - P_1)$$

The other two integrals in (10) can be evaluated if the reciprocity theorem

$$V_1 \sum_{t1} P_{1 \rightarrow 2} = V_2 \sum_{t2} P_{2 \rightarrow 1} \quad (12)$$

is used.

One obtains by simple manipulations

$$\int_{V_2} dV' \int_{V_1} dV p(r' \rightarrow r) = V_1 \frac{\sum_{t1}}{\sum_{t2}} (1 - P_1)$$

and further

$$\int_{V_2} dV' \int_{V_2} dV p(r' \rightarrow r) = V_2 - V_1 \frac{\sum_{t1}}{\sum_{t2}} (1 - P_1) - \int_{V_2} dV' \int_{V_\infty - V_1 - V_2} dV p(r' \rightarrow r)$$

The last term on the right side represents the leakage out of the region of perturbation, which is in good approximation independent of the sample temperature, and so is the first term,  $V_2$ . As we are interested only in the temperature dependence of  $P_x$ , we omit these two terms, and collect all the other terms. We call the resulting temperature dependent integral  $\tilde{P}_x$ , and obtain

$$\tilde{P}_x = V_1 \left\langle \frac{\sum_{x1}}{\sum_{t1}} + (1 - P_1) \left( \frac{\sum_{x2}}{\sum_{t2}} - \frac{\sum_{x1}}{\sum_{t1}} \right) \left( 1 - \frac{\sum_{t1}}{\sum_{t2}} \right) \right\rangle \quad (13)$$

Thus, we have found an explicit expression for the integrals in eq. (9) in our special problem. All that remains is to insert the quantities  $\delta \tilde{P}_x$ , which are changes in the  $\tilde{P}_x$  defined by eq. (13), into the general eq. (9). Thus we obtain

$$\frac{\delta v}{v} = - \frac{1}{D} \left[ \sum_{i,j} \chi_i S_i^+ \delta \tilde{P}_{fj} S_j - \sum_j S_j^+ \delta \tilde{P}_{aj} S_j + \sum_j (S_{j+1}^+ - S_j^+) \delta \tilde{P}_s^{j \rightarrow j+1} S_j \right] \quad (14)$$

where use has been made of the fact that

$$\sum_{tj} = \sum_{aj} + \sum_s^{j \rightarrow j} + \sum_s^{j \rightarrow j+1}$$

in energy groups where the Doppler effect is important. D is the denominator of equation (9).

Expression (14) is the reactivity change associated with the heating of the sample.

The three terms on the right side of eq. (14) are the effects of changes in the fission rates, absorption rates, and moderation rates, respectively, in both the sample and the surrounding core. It can be shown that the third term is small and can be neglected in all cases of interest.

#### Discussion of these Equations

1. The perturbation expression (9), which is "exact" in the sense that it was derived with only a few minor approximations mentioned in the introduction, can be further evaluated in the case where the unperturbed reactor is homogeneous, and large enough, so that in the center, where the perturbation occurs, the asymptotic solution of the transport equation is valid. This is exactly the case considered in the paper by Storrer et al. /5/. Our equation (13) contains essentially the same results as those obtained in /5/, and for a detailed discussion and numerical evaluation in special cases we refer to /5/. Equation (14) represents the link between the reaction rates and the reactivity change, which is different from the standard perturbation expression in that it contains the source densities and not the fluxes.

2. The source densities  $S$ , which have to be calculated for the unperturbed reactor, are related to standard quantities by eq. (4). From a consideration of neutron balance, it follows that the source density is the sum of the total collision density and the leakage rate. In the unperturbed reactor, this can be loosely designated by  $\Sigma_t \phi + DB^2 \phi$ . In all homogeneous cores of interest the term  $DB^2$  is much smaller than  $\Sigma_t$  and may be neglected. In the same approximation,  $S^+$  is equal to the ordinary adjoint flux, and we may approximate the terms in eq. (14) in the following way

$$S^+ \mathcal{J} \tilde{P} S \approx \bar{\phi}^+ \mathcal{J} \tilde{P} \left\langle \frac{\phi}{\Sigma_t} \right\rangle \quad (14a)$$

where  $\phi$  means the group flux. However, though this is a good approximation for homogeneous cores, it might not be good in cases where the sample is different from the core, and strongly absorbing, so that some group fluxes show a strong curvature in the vicinity of the sample.

3. Storrer et al. /5/ broke up the right side of eq. (13) into two terms, namely

$$\left\langle \frac{\Sigma_{x1}}{\Sigma_{t1}} \right\rangle \quad (15a)$$

and

$$\left\langle \left(1 - P_1\right) \left( \frac{\Sigma_{x2}}{\Sigma_{t2}} - \frac{\Sigma_{x1}}{\Sigma_{t1}} \right) \left(1 - \frac{\Sigma_{t1}}{\Sigma_{t2}}\right) \right\rangle \quad (15b)$$

and showed that in representative cases, heating of the sample causes a change of second order in the temperature difference in (15b), whereas the change in (15a) is of first order. Therefore, one can consider the term (15b) as a small correction, and from insertion into (14) one can see that the sample measurement gives just the Doppler coefficient of the core, except for the correction

(15b) and a statistical weight factor. In this sense, one can say that the Doppler coefficient of the core can be determined directly by a sample measurement, provided that the sample has the same composition as the core. If the sample is different in composition from the core, no such interpretation is possible, because one measures the Doppler coefficient appropriate to the equivalent potential cross section of the sample.

Evaluation of the Perturbation Expression  
in the Two-Region Approximation

In the more general case where the sample is different in composition from the core, the evaluation of the reactivity change is much more complicated. In fact, the calculation of the "unperturbed" source densities, with the sample at room temperature, is then already a difficult problem of transport theory. Thus, in this case of practical importance, the theoretical treatment becomes rather involved.

The simplest approach is to assume that the source densities  $S$  and  $S^+$  are flat in the sample, and in the core region around the sample. Then a two-region approximation can be used to evaluate eq. (9). Let us further assume that the sample contains only one fuel isotope. Then, the same analysis which leads to eq. (13) gives

$$\int dV' \int dV S^+ \int P_x S = V \rho \left\langle \frac{\Sigma_{x1}}{\Sigma_{t1}} S_1^+ S_1 + (1-P_1) \left( \frac{\Sigma_{x2}}{\Sigma_{t2}} S_2^+ - \frac{\Sigma_{x1}}{\Sigma_{t1}} S_1^+ \right) \left( S_1 - \frac{\Sigma_{t1}}{\Sigma_{t2}} S_2 \right) \right\rangle \quad (16)$$

In this more general case, the second term in the brackets is no longer a correction of second order in the temperature difference, and, therefore, it appears somewhat artificial to isolate the term  $\Sigma_{x1}/\Sigma_{t1}$ . Therefore, the equation shall be re-written in a different form.

If, for simplicity, Wigner's rational approximation

$$P_1 = \frac{\bar{I} \Sigma_{t1}}{1 + \bar{I} \Sigma_{t1}}$$

is introduced at this point, one obtains

$$\int dV' \int dV S^+ \mathcal{P}_x S = v_1 \int \left\langle \frac{\Sigma_{x1}}{1 + \bar{I} \Sigma_{t1}} s_1^+ s_1 + \right. \\ \left. + \frac{1}{\bar{I}(1 + \bar{I} \Sigma_{t1})} \left( \frac{\Sigma_{x2}}{\Sigma_{t2}} s_2^+ s_1 + \frac{\Sigma_{x1}}{\Sigma_{t2}} s_1^+ s_2 - \frac{\Sigma_{x2} \Sigma_{t1}}{\Sigma_{t2}^2} s_2^+ s_2 \right) \right\rangle \quad (16a)$$

One can easily recognize the four terms (i,k) in the form (16a) of the equation.

Furthermore, it is obvious from eq. (16a) that the resonance neutrons do not really "see" the potential cross section  $\Sigma_{p1}$  of the sample, if  $\Sigma_{p1}$  is different from  $\Sigma_{p2}$ . Rather, they see the equivalent cross section  $\bar{\Sigma}_{p1}$  of the sample and, of course, the cross section  $\Sigma_{p2}$ , which appear in the denominators of eq. (16a). Therefore, the term  $\Sigma_{x1}/\Sigma_{t1}$  of eq. (16) has no physical meaning.

We now split the term (2,1) (absorption in the sample of neutrons produced in the core) into two terms, say (2,1)a and (2,1)b, namely

$$(2,1) = \frac{s_1^+ s_2}{\bar{I} \Sigma_{p2}} \left[ \int \left\langle \frac{\Sigma_{x1}}{1 + \bar{I} \Sigma_{t1}} \right\rangle - \left\langle \frac{\Sigma_{c2}}{\Sigma_{t2}} \int \left\langle \frac{\Sigma_{x1}}{1 + \bar{I} \Sigma_{t1}} \right\rangle \right\rangle \right] \quad (17)$$

In eq. (16a), the two terms (1,1) and (2,1)a have the form of resonance integrals, and can be calculated by the usual methods of Doppler coefficient calculations.

The remaining 3 terms, however, are more complicated. They disappear if there is no resonance absorption in the core; let us call them core absorption terms.

They can only be evaluated analytically at high neutron energies, where the so called "High Energy Approximation" used in Doppler coefficient calculations is valid. Indeed, one obtains in this approximation, for example, for the term (1,2)

$$\left\langle \frac{\Sigma_{x2}}{1/\bar{\Sigma}_{t2}} \int \left( \frac{1}{1/\bar{\Sigma}_{t1}} \right) \right\rangle = - \frac{1}{1/\bar{\Sigma}_{p1}} \sum_s \frac{Q_{xs} (E_x^{(s)} - e^{(s)})}{\sqrt{2\bar{\pi}} \langle \bar{\sigma}_{p1} + \bar{\sigma}_{c1} \rangle \langle \bar{\sigma}_{t2} \rangle \Delta'} \left( \frac{1}{\Delta'} - \frac{\sqrt{2}}{\sqrt{\Delta'^2 + \Delta''^2}} \right) \quad (18)$$

Similar expressions are obtained for the other terms. The notation in (18) is the usual one in Doppler coefficient calculations. The Doppler widths  $\Delta'$  and  $\Delta''$  refer to the core temperature and sample temperature, respectively. It is interesting to compare eq. (18) with the "High Energy Approximation" of the term (1,1) namely

$$\int \left\langle \frac{\Sigma_{x1}}{1/\bar{\Sigma}_{t1}} \right\rangle = \sum_s \frac{Q_{xs} (E_x^{(s)} - e^{(s)})}{\sqrt{2\bar{\pi}} \langle \bar{\sigma}_{p1} + \bar{\sigma}_{c1} \rangle^2} \left( \frac{1}{\Delta'} - \frac{1}{\Delta''} \right) \quad (18a)$$

In eq. (18), the cross sections of both regions appear in the denominator. If they are equal, (18) is, for small temperature changes, just one half of (18a), because only one of the two regions is heated. It should be mentioned that all the 3 terms (1,2), (2,1)b, and (2,2) give positive contributions to the change in reaction rates in this approximation, and they are of the same size as the other two terms.

In the spectra typical for fast breeders, most of the Doppler effect is at low energies where the so called "isolated resonance approximation" is valid. In this energy region, the core absorption terms can only be evaluated numerically. Calculations show that they can be of either sign, but are small compared to the terms (1,1) and (2,1)a.

It has been shown in this section that the analysis of a sample experiment, in addition to requiring a lot of numerical work, involves additional uncertainties, which do not appear in the usual Doppler coefficient calculation. The major uncertainty is in the calculation of the group fluxes in the cold sample. It is, therefore, desirable to design experiments such that the perturbation of the group fluxes by the cold sample is small.

The method appropriate to the analysis of sample experiments is based on ratios of reaction rates, rather than on effective group cross sections, as can be seen from (16a). It is, therefore, an extension of the "reaction rate method" used by Froelich and Ott /6/, and by Hummel and Hwang /7/, to space dependent problems. The overlap correction between resonances of different isotopes, which is small in the method of effective cross sections, may be quite large in the method of reaction rates. Therefore, the analysis is simplified if the sample contains only one fuel isotope.

### Numerical Results

The numerical work at Karlsruhe to evaluate the above expressions is still in progress. The results to be presented here are for samples of  $U(238)O_2$ , of the type used in the experiments on ZPR-3 /1/ and ZPR-6 /2/. The multigroup diffusion calculations were done with a 26 group cross section set by Abagyan et al. /8/, which includes selfshielding. The statistical method was used for the Doppler calculations.

A code was written to evaluate the core absorption terms for isolated resonances, using the asymptotic expression for the  $\Psi$ -function.

The temperature dependence of both terms (1,2) and (2,1)b is described by the function

$$\left\langle \frac{\Sigma_{a2}}{\Sigma_{t2}} \int \left( \frac{\Sigma_{c1}}{\Sigma_{p1} + \Sigma_{c1}} \right) \right\rangle = \frac{\sqrt{s}}{\langle s \rangle} \left\langle \int_0^{\infty} dx \frac{\gamma_2}{\beta_2 + \gamma_2} \int \left( \frac{\gamma_1}{\beta_1 + \gamma_1} \right) \right\rangle \quad (19)$$

which is plotted in Fig. 1 for different values of  $\bar{\sigma}_{p2}$ , the potential cross section per atom in the surrounding core. The sample is  $U(238)O_2$ , 1 inch in diameter, the energy  $E=1.47$  kev (group 14 of the cross section set).

For the usual values of  $\bar{\sigma}_{p2}$  the curves start with a negative tangent, but are strongly curved, and then change sign. The term (2,2) leads to a different function, but shows the same qualitative behaviour. Fig. 2 shows all the five terms for  $\bar{\sigma}_{p2}=30.8$  b, which corresponds to the core composition of ZPR-6. The core absorption terms are small in the temperature range of interest.

In order to understand the behaviour of the function (19), let us look, for example, at the term (1,2). Then the second factor,  $\Sigma_{c1}/(1+\Sigma_{t1})$  in equation (19), is the temperature dependent spectrum of neutrons which leave the sample without a collision. This factor increases with temperature in the wings, and decreases in the center of a resonance, leaving a net increase of the integral. The neutrons with this spectrum are subjected to the absorption probability  $\Sigma_{a2}/\Sigma_{t2}$  in the core, which is largest in the center of the resonance and may, therefore, change the above balance into a net decrease. Thus, the sign of the term is very sensitive to the parameters, and the net balance is small. Only at high temperatures and/or large potential cross sections, the net effect in (19) is always negative, giving a positive contribution to the term (1,2).

Table 1 shows the calculated reactivity change for an experiment on ZPR-6, Assembly 4Z /2/. The sample is natural uranium oxide, 1 inch in diameter, and expansion constrained (measurement No. 7 of /2/). The calculated values are for a sample heated to 500°K and 800°K. The unperturbed group fluxes

were used, no correction was made for the Debye temperature. The calculated values are 10% and 13% smaller than the measured values. As fast reactor spectra are not well known, this may be considered a fair agreement. It is important to note that without the core absorption terms the agreement would be much worse. These terms contribute about 20% in the relatively hard spectrum of Assembly 4Z, where almost 50% of the Doppler coefficient comes from energies above 10 kev.

There is, however, a strong disagreement with a measurement where U-238 was replaced by molybdenum in the environment of the sample. The result should be smaller, because the core absorption terms are missing. However, the experiment gave a larger result. Possible explanations are that the group fluxes change, or that the calculated spectrum is too hard. More work is required to understand these effects.

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Table 1:            Results for the ZPR-6 Experiment,  
1-inch-sample

	-10 <sup>6</sup> δk/kg U-238			
	293°K-500°K		293°K-800°K	
	above 10 kev	below 10 kev	above 10 kev	below 10 kev
(1,1)	0.23	0.44	0.38	0.84
(1,2)	0.08	0.01	0.16	0.05
(2,1)a	0.32	0.59	0.53	1.14
(2,1)b	0.20	0.02	0.39	0.13
(2,2)	0.10	0.00	0.18	0.03
Total calculated	1.99		3.83	
Experimental	2.22 ±0.09		4.38 ±0.09	



Fig. 1 The Function  $\frac{\Gamma_p}{\langle S \rangle} \left\langle \int dx \frac{\gamma_2}{\beta_2 + \gamma_2} \delta\left(\frac{\gamma_1}{\beta_1 + \gamma_1}\right) \right\rangle$   
 for different  $\sigma_{p2}$   
 Sample: U(238)O<sub>2</sub>, 1 inch, E = 1.47 kev

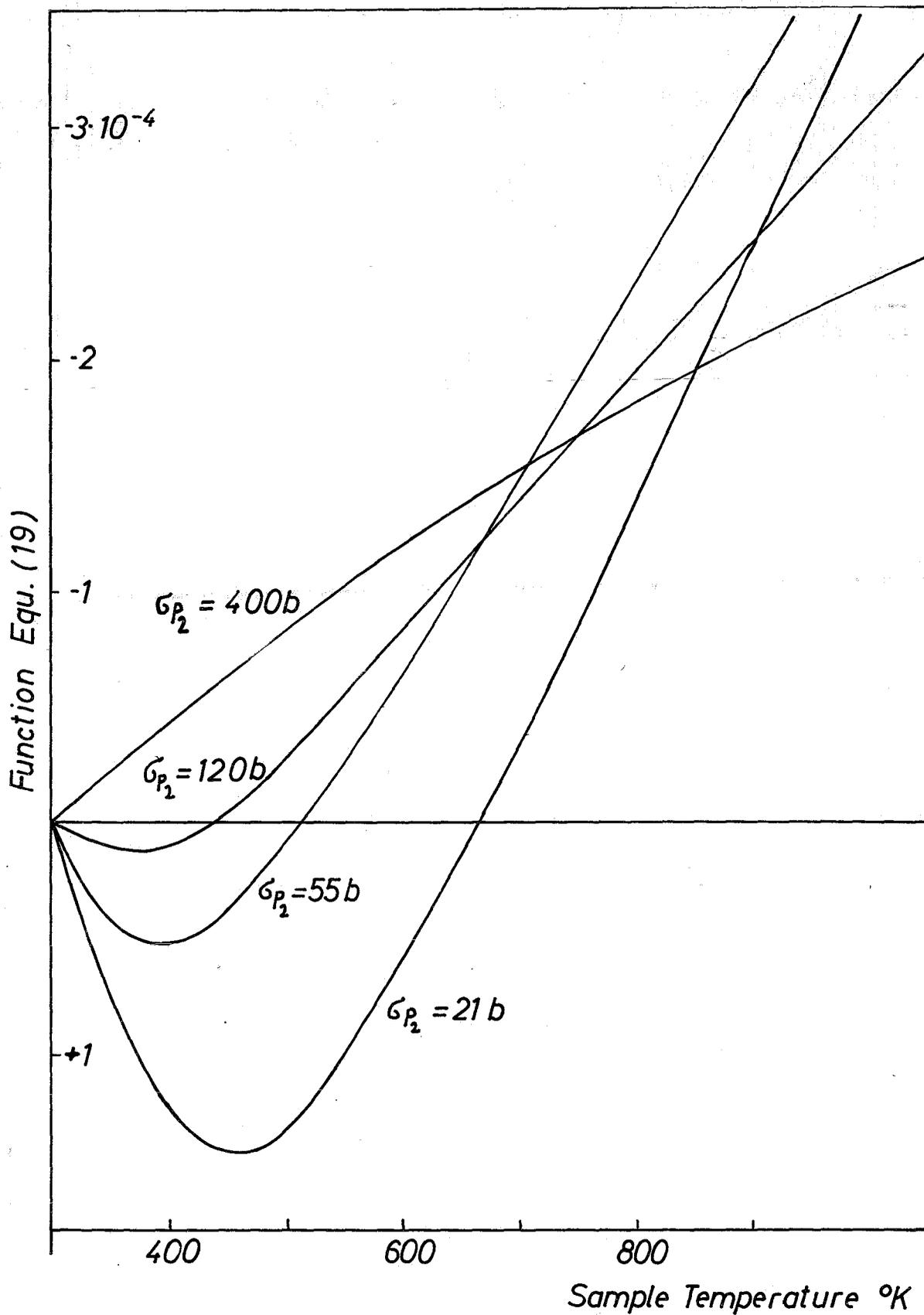


Fig. 2 Contribution of the Different Terms to  $\delta k$   
Sample:  $U(238)O_2$ , 1 inch,  $E = 1.47$  kev

