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A Gamma-Ray Cascade Model for the Calculation of Average  $\gamma$ -Ray Multiplicities and Isomeric Cross Section Ratios

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### A Gamma-Ray Cascade Model for the Calculation of Average γ-Ray Multiplicities and Isomeric Cross Section Ratios

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The calculations of the  $\gamma$ -ray cascades following the decay of a compound state were carried out using all levels with known values of energy, spin and parity at low energy and statistical assumptions for the distribution of the nuclear levels in the energy range between the initially occupied compound state and the low-lying levels. Dipole and quadrupole transitions were taken into account. The model works without the introduction of an artificial average transition energy. The model was applied to the calculation of average  $\gamma$ -ray multiplicities and of isomeric cross section ratios. The applications to the determination of level density parameters are discussed. It was shown that it is not allowed to neglect the contribution of quadrupole radiation in the cascade as was done in other models in the past. Only in the case of  $\ln^{115}(n, \gamma) \ln^{116}$  enough data are available to determine statistical parameters. We get for the level density parameter  $a=18.6 \text{ MeV}^{-1}$ , for the spin cut-off factor  $\sigma=4.8$  and a contribution of about 5% quadrupole radiation in the cascade.

#### 1. Introduction

HUIZENGA and VANDENBOSCH<sup>1</sup> have published in 1960 a paper concerning the calculation of isomeric cross section ratios. They have used a model based on the assumptions

a) that only E1-transitions occur in a  $\gamma$ -ray cascade following the decay of a compound state,

b) that the levels down to the ground state can be described by a level density formula, e.g. that given by  $BLOCK^2$ ,

c) that the transition probabilites between the levels are proportional only to the spin dependent part of the level density of the occupied levels,

d) that all  $\gamma$ -ray cascades consist of  $\overline{n}_{\gamma}$  transitions where  $\overline{n}_{\gamma}$  is the average  $\gamma$ -ray cascade multiplicity and

e) that the last transition feeds the isomeric or the ground state depending on which transition has the smaller spin change.

This model gives a qualitative description of the final population of the isomeric and the ground state and was applied in a large number of

<sup>&</sup>lt;sup>1</sup> HUIZENGA, I.R., and R. VANDENBOSCH: Phys. Rev. 120, 1305 (1960).

<sup>&</sup>lt;sup>2</sup> BLOCK, C.: Phys. Rev. 93, 1094 (1954).

works to compare measured isomeric cross section ratios with theoretically predicted ones<sup>3</sup>. In a few cases it was used also to determine the spin cut-off factor  $\sigma$  occuring in the level density formula<sup>4,5</sup> or to give predictions of the spin values of the initial compound state<sup>6</sup>. Estimations of the spin value for the isomeric state of the nucleus with this model are also possible<sup>7</sup>. In a newer work VONACH, VANDENBOSCH and HUI-ZENGA<sup>8</sup> take into account the energy dependence of the nuclear level density and the transition probability. This means a variation only of the assumption c).

In 1961 TROUBETZKOY<sup>9</sup> published a paper concerning the calculations of the spectra of prompt  $\gamma$ -rays following neutron capture or neutron inelastic scattering. His model was also based on the assumptions that only dipole transitions occur, but in contrast to the assumptions b) and c) TROUBETZKOY used the following ones:

b) The levels down to an energy limit  $E_{\rm th}$  can be described by a level density formula. Below this limiting value, levels are used with parameters  $E_i$ ,  $J_i$ ,  $\pi_i$  which are known from experimental research.

c) The transition probability in the level continuum is proportional only to the energy dependent part of the nuclear level density formula.

The calculations give good agreement between the measured and the calculated  $\gamma$ -ray spectra.

It is the purpose of this paper to give a  $\gamma$ -ray cascade model for the calculation of the spectra of a) multiplicities and of b) the low level population. It is easy to caculate from these results the average  $\gamma$ -ray multiplicity of the cascade and the isomeric cross section ratio. We use a combination and extention of the models given by TROUBETZKOY<sup>9</sup> and by HUIZENGA and VANDENBOSCH<sup>1</sup>. The advantage of the model suggested here is the direct and straightforeward manner in which the calculation of the  $\gamma$ -ray cascade can be carried out without the introduction of artificial average transition energies and multiplicities, and the consideration of the low-lying levels as far as they are known.

<sup>5</sup> Albold, A., and P. v. Blankenhagen: EANDC-E-55 (1964).

<sup>9</sup> TROUBETZKOY, E.S.: Phys. Rev. 122, 212 (1961).

<sup>&</sup>lt;sup>3</sup> SEHGAL, M.L.: Phys. Rev. **128**, 761 (1962). — KEISCH, B.: Phys. Rev. **129**, 769 (1963). ALEXANDER, K.F., u. H.F. BRINKMANN: Ann. Physik **12**, 225 (1963). (See also other references cited in these three works).

<sup>&</sup>lt;sup>4</sup> VANDENBOSCH, R., and I.R. HUIZENGA: Phys. Rev. **120**, 1313 (1960). — SCHULT, O.W.B., B.P.K. MAIER, U. GRUBER U. R. KOCH: Z. Physik **185**, 295 (1965).

<sup>&</sup>lt;sup>6</sup> BISOP, C.T.: ANL-6405 (in press).

<sup>&</sup>lt;sup>7</sup> FETTWEIS, P.: Phys. Letters **3**, 40 (1962). — ALEXANDER, K.F., H.F. BRINK-MANN, F. DÖNAU, and H.R. KISSENER: Phys. Letters **4**, 302 (1963). — PÖNITZ, W.P.: Nuclear Phys. **66**, 297 (1965).

<sup>&</sup>lt;sup>8</sup> VONACH, H.K., R. VANDENBOSCH, and I.R. HUIZENGA: Nuclear Phys. **60**, 70 (1964). — VONACH, H.K., and I.R. HUIZENGA: Phys. Rev. **138**, B 1372 (1965).

#### 2. The y-Ray Cascade Model

For the calculations we use the following general assumptions and notations: The level scheme consists of the low-lying levels which are characterized by the terms energy  $E_i$ , spin  $J_i$  and parity  $\pi_i$  and of an upper range of a level continuum described by the density  $\rho(E, I_k, \pi_e)$ . This continuum range is bounded by the energy value  $E_{\rm th}$  and the excitation energy  $E_c$ . For a particular parity value in the level density formula, the situation is shown in Fig. 1 for a special example of  $(E_i, J_i, \pi_i)$ -values.



Fig. 1. Level scheme of the model

At the beginning of our calculations only one compound state is excited. Thus the occupation probability is given by

$$W_{n=0}(E,J,\pi) = \delta(E-E_c) \cdot \delta_{JJ_c} \cdot \delta_{\pi\pi_c} \tag{1}$$

where  $\delta(x)$  is the Dirac delta function and  $\delta_{xx'}$  is the Kronecker symbol. The index *n* designates the number of  $\gamma$ -rays which have occured in the past in the cascade. The occupation probability after *n*  $\gamma$ -transitions in the continuum range is determined by the occupation probability before the last transition (multiplicity n-1) and by the transition probability *S*. We must integrate over the energy range above *E* and sum over all possible spin values and the two parities:

$$W_{n}(E, J_{k}, \pi_{m}) = \sum_{e} \sum_{j} \int_{E}^{E_{e}} W_{n-1}(E', J_{e}, \pi_{j}) \cdot S(E', J_{e}, \pi_{j} \to E, J_{k}, \pi_{m}) dE'.$$
(2)

For the discrete levels we get:

$$W_{n}(E_{i}, J_{i}, \pi_{i}) = \sum_{e} \sum_{j} \int_{E_{th}}^{E_{e}} W_{n-1}(E', J_{e}, \pi_{j}) \cdot S(E', J_{e}, \pi_{j} \to E_{i}, J_{i}, \pi_{i}) dE' + \sum_{k > i} W_{n-1}(E_{k}, J_{k}, \pi_{k}) \cdot S_{ki}(E_{k}, J_{k}, \pi_{k} \to E_{i}, J_{i}, \pi_{i}).$$
(3)

The limit  $n \to \infty$  means that all promptly decaying levels are deexcited and we get:

$$W_{n=\infty}(E, J, \pi) = \sum_{m} T_{m} \cdot \delta(E - E_{m}) \cdot \delta_{JJ_{m}} \cdot \delta_{\pi \pi_{m}} + T_{g} \cdot \delta(E - E_{g}) \cdot \delta_{JJ_{g}} \cdot \delta_{\pi \pi_{g}}$$
(4)

where the index m designates isomeric states as far as they exist and g signifies the ground state. Because of the normalization, the following holds:

$$\sum_{m} T_m + T_g = 1.$$
 (5)

The  $T_m$  and  $T_g$ 's are the occupation probabilities of the isomeric states or the ground state. We define the ratio

$$R = \frac{T_m}{T_g} \tag{6}$$

and call it the isomeric cross section ratio. For a given nucleus (this means given values of  $E_i$ ,  $J_i$  and  $\pi_i$ ; i=m, g) R depends only on the quantum values  $E_c$ ,  $J_c$ ,  $\pi_c$  of the initially occupied state.

The difference between the occupation probabilities of the isomeric and the ground state after n and  $n-1 \gamma$ -rays in a cascade gives the frequency h(n) of the multiplicity n in the cascade:

$$h(n) = \sum_{m} W_{n}(E_{m}, J_{m}, \pi_{m}) + W_{n}(E_{g}, J_{g}, \pi_{g}) - \sum_{m} W_{n-1}(E_{m}, J_{m}, \pi_{m}) - W_{n-1}(E_{g}, J_{g}, \pi_{g}).$$
(7)

h(n) is the spectrum of the  $\gamma$ -ray cascade multiplicities, by which one can calculate the average multiplicity  $\overline{n}_{\gamma}$ .

The population probability of a low-lying level is given by

$$T_i = \sum_n W_n(E_i, J_i, \pi_i).$$
(8)

However, to calculate values of R, h(n) and  $T_i$  we still need assumptions about the transition probabilities S and  $S_{ki}$ . We use, as did TROUBETZKOY<sup>9</sup>, experimental values for the transition probability  $S_{ki}(E_k, J_k, \pi_k \rightarrow E_i, J_i, \pi_i)$  between the discrete levels. Furthermore we assume for the

energy region (see Eq. (13)). Both effects leads to the same direction. This breakdown of R is a "model-effect" and vanishes if we use constant  $\sigma$ -values and no quadrupole radiation.

The contributions of M1- and M2-transition probabilities in the cascades are smaller than the E1- and E2-contributions, by about two



Fig. 2. The isomeric cross section ratio of  $Rh^{103}(n, \gamma) Rh^{103}(n, \gamma) Rh^{104}$  as a function of the continuum threshold energy  $E_{th}$ 

orders of magnitudes<sup>15</sup>, and were regarded only due to the possible different parities of the discrete low-lying levels. Because the influence of the parity is small (the max difference of R which we have observed by our calculations is about 5%) the ratios  $C_{M1}/C_{E1}$  and  $C_{M2}/C_{E1}$ are unimportant for the following considerations: The choice of  $C_{E2}/C_{E1}$ (this means the choice of the contribution of quadrupole radiation in the cascade) is difficult. The single particle model predicts<sup>15</sup>  $C_{E2}/C_{E1} \approx 2 \cdot 10^{-5}$ but an analysis of absolute transition probabilities of transitions between low energetic levels gives <sup>15</sup>  $C_{E2}/C_{E1} > 1$ .

In Fig. 3 the isomeric cross section ratio of Rhodium is shown as a function of  $C_{E2}/C_{E1}$ . There are two asymptotic constant values of R which are due to only dipole or only quadrupole transitions in the cascades. It is of interest that small contributions of quadrupole radiation (10%) can change the isomeric cross section ratio by a factor of about 2.

The level density formula contains the parameters  $a_0$  and  $\sigma_0$  (see Eqs. (11), (12) and (13)) which are up to now undetermined. ABDELMALEK and STAVINSKY<sup>11</sup> have fitted the experimental values of a and get a constant  $(a_0)_A = (0.095 \pm 0.007) \text{ MeV}^{-1}$ . Fig. 4 shows the dependence of the isomeric cross section ration R on the values of a for Rhodium. The behaviour can be understood easily: For higher a-values the level density in the continuum range is greater and therefore transitions to

<sup>&</sup>lt;sup>15</sup> WILKINSON, D.H.: Analysis of gamma decay data. In: Nuclear spectroscopy, part B. New York and London: Academic Press 1960.

the continuum are more probable. This means that the cascade "lives" longer and will be broader when it reaches the low-lying levels.

Fig. 5 shows the same ratio as a function of  $\sigma_0$ . We have related  $\sigma_0$  to  $(\sigma_0)_M = 0.845$  given by MALYSHEV<sup>16</sup> (this value includes  $(a_0)_A$ 



Fig. 3. The isomeric cross section ratio of  $Rh^{103}(n, \gamma) Rh^{104}$  as a function of the quadrupole radiation contribution in the cascade



Fig. 4. The isomeric cross section ratio of Rh<sup>103</sup>  $(n, \gamma)$  Rh<sup>104</sup> as a function of the level density parameter a

as cited above). The ratio increases if  $\sigma$  increases because for larger  $\sigma$ -values levels with high spin become more probable. Because the isomeric cross section ratio is often used to determine the spin cut-off factor, it should be noted that increasing values of a or  $\sigma$  lead to the same effect in R. We will discuss this in the last section.

Due to the application of the model to the strength function determintation<sup>17</sup> the dependence of R on the excitation energy  $E_c = E_B + E_n$ 

<sup>&</sup>lt;sup>16</sup> MALYSHEV, A.V.: Soviet Phys. JETP 18, 221 (1964).

<sup>&</sup>lt;sup>17</sup> PÖNITZ, W.P.: Diss. Karlsruhe (1966).

is of interest.  $E_n$  is the energy of the incoming neutron in the  $(n, \gamma)$ -reaction and  $E_B$  the binding energy. In Fig. 6 this influence is shown. The increase in the energy range 0 to 100 keV is only about 1% and can be ignored.



Fig. 5. The isomeric cross section ratio of Rh<sup>103</sup> $(n, \gamma)$  Rh<sup>104</sup> as a function of the ratio  $\sigma_0/(\sigma_0)_M$ .  $\sigma_0$  is the constant of the spin cut-off factor  $\sigma$  in Eq. (13) and  $(\sigma_0)_M = 0.845$ 



Fig. 6. The isomeric cross section ratio of  $Rh^{103}(n, \gamma) Rh^{104}$  as a function of the excitation energy  $E(E_n=E-E_B)$ 

The dependence of the isomeric cross section ratio on the inital compound state spin value  $J_c$  is very strong and is shown in Fig. 7 also for Rhodium.

The  $\gamma$ -ray cascade statistics given in this work depend, just as those of HUIZENGA and VANDENBOSCH<sup>1,8</sup>, on the assumption of the statistical behaviour of the level density. However, the largest violation of this assumption is expected for the low energy region. Therefore we consider in the case of Dy<sup>165</sup> the influence of the low lying levels on the isomeric

cross section ratio. Using only two states (the isomeric and the ground state) and the parameters  $(a_0)_A$  and  $(\sigma_0)_M$  given by ABDELMALEK and STAVINSKY<sup>11</sup> and MALYSHEV<sup>15</sup> we get R=1.92. BISHOP, VONACH and HUIZENGA<sup>18</sup> get R=1.77. The experimental value<sup>19</sup> is  $R=1.94\pm0.20$ .



Fig. 7. The isomeric cross section ratio of Rh<sup>103</sup>  $(n, \gamma)$  Rh<sup>104</sup> as a function of the initial compound spin value  $J_c$ 



If we use 20 states at low energy with known  $E_i$ ,  $I_i$ ,  $\pi_i$  and the relative transition probabilities<sup>20</sup> we get R=1.13. If we use  $(a_0)_A$  and  $(\sigma_0)_M$  as cited above and 8% quadrupole radiation in the  $\gamma$ -ray cascades we get R=1.21, whereas if we use 1% quadrupole radiation but  $\sigma_0=0.75 \cdot (\sigma_0)_M$  we get R=1.20 and for  $\sigma_0=0.5 \cdot (\sigma_0)_M$ , R=1.47.

We will consider the meaning of this disagreement in connection with the application of the cascade statistics in the last section.

In Fig. 8 the  $\gamma$ -ray multiplicity spectra are shown for the two compound spin values I=4 and 5 which can be achieved by *s*-wave capture in the reaction  $\ln^{115}(n, \gamma) \ln^{116}$ . The spectrum of the multiplicities depends only slightly on the compound spin value. The average multiplicities change to  $\overline{n_{\gamma}}=3.90$  and  $\overline{n_{\gamma}}=3.94$  for only 1% quadrupole radiation and to  $\overline{n_{\gamma}}=3.82$  and  $\overline{n_{\gamma}}=3.83$  for  $\sigma_0=0.75$  ( $\sigma_0$ )<sub>M</sub>. However, the dependen-

<sup>&</sup>lt;sup>18</sup> BISHOP, C.T., H.K. VONACH, and I.R. HUIZENGA: Nuclear Phys. 60, 241 (1964).

<sup>&</sup>lt;sup>19</sup> WEBER, G.: Z. Naturforsch **9** A, 115 (1954). – TORNAU, R.: Z. Physik **159**, 101 (1960).

 <sup>&</sup>lt;sup>20</sup> SCHULT, O. W.B., P.B.K. MAIER u. U. GRUBER: Z. Physik 182, 171 (1964).
 19a Z. Physik, Bd. 197

ce of  $\overline{n}_{\gamma}$  on *a* is stronger. This can be explained easily by the larger population of the levels in the continuum range due to the higher level density. For  $a_0 = 1.4 \cdot (a_0)_A$  we get  $\overline{n}_{\gamma} = 4.91$  and  $\overline{n}_{\gamma} = 4.90$ .

#### 4. Applications of the *γ*-Ray Cascade Model

Experimental values of R are available with sufficient accuracy for several reactions and target nuclei. For  $(n, \gamma)$ -reactions R-values have been measured almost exclusively for thermal neutrons and only in a few cases for different spin values of resonances in the eV-energy region. Also available are data with adequate accuracy for  $\overline{n}_{\gamma}$  for the  $(n, \gamma)$ reaction for some nuclei in the thermal and resonant neutron energy range.

As far as we know,  $\gamma$ -ray cascade multiplicity spectra have not been measured up to now. However, a few values of h(n=1) are available, but only with large experimental error.

Low level population probabilities have been measured only for a few nuclei with large A; the results which are usable for our considerations are further restricted because the spin values and parities of the levels are not known in the most cases.

#### 4.1. Calculations of the Average $\gamma$ -Ray Cascade Multiplicities

Table 1 contains the experimental values of  $\overline{n}_{\gamma}$  for the  $(n, \gamma)$ -reaction for nuclei with A > 100. DRAPER and SPRINGER<sup>21</sup> have measured their values in different resonances of the  $(n, \gamma)$ -reaction in the eV-energy region using the time-of-flight method. The values measured by MUEHL-HAUSE and by GROSHEV at thermal neutron energy are cited in ref. 21.

Target nuclei	Compound spin	$\overline{n}_{\gamma}(\exp)$	$\tilde{n}_{\gamma}$ (theor)			
		Draper	MUELHOUSE	GROSHEV	a)	b)
Ag <sup>109</sup>	1	$5.0 \pm 0.3$	2.9	4.0	3.93	
Cd <sup>113</sup>	1	$4.1 \pm 0.3$	4.1	4.0	3.49	
In <sup>115</sup>	5	$4.4 \pm 0.2$	3.3	4.4	3.94	5.05
	4	$5.6 \pm 0.4$			3.90	4.91
	5	$4.2 \pm 0.4$			3.94	5.05
Sm <sup>149</sup>	4	$6.2 \pm 0.3$	5.6	4.7	4.40	
	4	$5.0 \pm 0.4$			4.40	
Sm <sup>152</sup>	12	$4.1 \pm 0.5$			4.01	
Dy <sup>164</sup>	1 2	$3.1 \pm 0.2$	3.7	3.9	3.71	
Au <sup>197</sup>	2	3.8 + 0.3	3.5	3.9	4.44	

Table 1. Comparison of experimental and theoretical values of the average  $\gamma$ -ray cascade multiplicity  $\overline{n}_{\gamma}$ . Values are calculated with: a) 1% quadrupole radiation  $(a_0)_A$  and  $(\sigma_0)_M$  and b) 1% quadrupole radiation,  $a_0=1.4 \cdot (a_0)_A$  and  $(\sigma_0)_M$ 

<sup>21</sup> DRAPER, I.E., and T.E. SPRINGER: Nuclear Phys. 16, 27 (1960).

We have omitted all nuclei with unknown compound spin values. As far as we know, values of  $\overline{n}_{\gamma}$  are not calculated by any other authors up to now.

The theoretical values are in general in agreement with the experimental ones. However, it is of interest that DRAPER and SPRINGER<sup>21</sup> have measured two different values of  $\overline{n}_{\gamma}$ , in the Sm<sup>149</sup>-resonances with the same spin value, whereas the statistical theory gives exactly the same value. They get also a difference of about 20% for  $\overline{n}_{\gamma}$  in the resonances of In<sup>116</sup> with the spin values 4 and 5. The statistical theory predicts a very small difference of  $\overline{n}_{\gamma}$  for different spin values. This is also true if we change the various parameters.

#### 4.2. The Isomeric Cross Section Ratio

In Table 2 we compare experimental and theoretical values of R for  $(n, \gamma)$ -reactions for nuclei with A > 100. We have omitted nuclei with

Table 2. Comparison of experimental and theoretical values of the isomeric cross section. Brackets mean that the spin value is unknown. Ref. a KEISCH<sup>3</sup>, Ref. b BISHOP<sup>18</sup> or values referred in this work. Theoretical values: a)  $(a_0)_A$  and  $(\sigma_0)_M$  as cited in Sec. 3 and  $C_{E2}/C_{E1}=0.01$ , b)  $\sigma_0=1.2 (\sigma_0)_M$ , c)  $C_{E2}/C_{E1}=0.06$  and d)  $a_0=1.4 (a_0)_A$ 

Target nuclei	Com- pound spin	R (exp)	Ref.	R Bishop <sup>18</sup>	R (this work)			
					a)	b)	c)	d)
Rh <sup>103</sup>	0			0	0.024	0.031	0.046	
	1	$(0.080 \pm 0.001)$	а	0.078	0.058	0.073	0.083	
Pd <sup>108</sup>	$\frac{1}{2}$	$0.020 \pm 0.006$	b	0	0.001	0.002	0.006	
Ag <sup>109</sup>	0	—		0	0.019	0.020	0.025	
	1	$0.032 \pm 0.009$	b	0.039	0.035	0.053	0.052	
In <sup>115</sup>	4	$1.95 \pm 0.10$	5	2.70	2.02	2.81	1.80	2.16
	5	$3.75 \pm 0.10$	5	8.09	4.83	7.29	3.72	4.75
$Sn^{124}$	<u>1</u> . 2	$38 \pm 19$	b	77	24.1	18.7	16.8	
Te <sup>126</sup>	$\frac{1}{2}$	$0.15 \pm 0.04$	Ъ	0.029	0.043	0.057	0.067	
Te <sup>128</sup>	$\frac{1}{2}$	$0.095 \pm 0.030$	b	0.031	0.045	0.059	0.069	
Te <sup>130</sup>	1/2	0.053	b	0.027	0.038	0.050	0.060	
Cs <sup>133</sup>	3			0	0.005	0.010	0.010	
	4	$(0.081 \pm 0.010)$	b	0.033	0.023	0.039	0.032	
Eu <sup>151</sup>	2	(0.56 + 0.01)	b	0.219	0.205	0.183	0.197	
				0.075	0.115	0.099	0.122	
$Dy^{164}$	1 2	1.94 + 0.20	b	1.77	1.92	1.44	1.26	
Pt <sup>196</sup>	1 2	$0.069 \pm 0.014$	b	0	0.015	0.022	0.023	

known experimental R-values if the experimental error was very large or other contradictory values existed. If several experimental values of R are available we have calculated the average.

The agreement between the experimental and the theoretical values is good for a few nuclei. However, it becomes better for most nuclei if we 19\* use larger  $\sigma$ -values or larger contributions of quadrupole radiation in the cascade. As we can see in the case of  $In^{115}$  we get the same effect if we use larger *a*-values.

#### 5. Discussion

In the following we will discuss the results of the  $\gamma$ -ray cascade model and its application to the determination of quantum numbers and of parameters in the nuclear level density formula.

In our model the behaviour of a  $\gamma$ -ray cascade was calculated with the assumption of a level continuum between the low lying levels and the initial occupied compound state and with the assumption of transitions due only to dipole and quadrupole radiation. The transitions to levels in the continuum range are proportional to the level density at the end point of the transition. For the level density we have taken into account the Fermi gas model. However, it is possible to use any other model, e. g. the superconductor model as was done by VONACH, VANDENBOSCH and HUIZENGA<sup>8</sup>. The last model would reduce the influence of the somewhat artificial energy limit  $E_{\rm th}$  on the isomeric cross section ratio because in the superconductor model the spin cut-off factor does not break down at low energy.

A very important point is the dependence of the  $\gamma$ -ray cascade statistics on the level density parameters a and  $\sigma$  and on the quadrupole contributions to the  $\gamma$ -ray transitions. It was shown in sections 3 and 4 that it is not appropriate to neglect the quadrupole radiation in the cascade if we use the model to determine the parameter  $\sigma$ . Only 5% quadrupole radiation corresponds to a change of about 30% in  $\sigma$ . We have used for our calculations the *a*-values given by ABDELMALEK and STAVINSKY<sup>15</sup> because this parameter can be determined well by inelastic scattering experiments. However, it was shown that *a* influences the results of *R* in the same direction as a change in  $\sigma$ . As was shown in the case of Dy<sup>164</sup>, it is necessary also to take into account the low-lying states of the compound nucleus.

This means that we cannot determine from the isomeric cross section ratio the values  $\sigma$ , a and  $C_{E2}/C_{E1}$  individually. To determine these three values we need more experimental data than are presently available:

a) Average  $\gamma$ -ray multiplicities of the cascades, which are very sensitive to the level density parameter *a*.

b) The isomeric cross section ratio for different initially occupied compound states. This seems to be sensitive to the contribution of quadrupole radiation in the cascade as was shown in the case of  $In^{115}$ .

c) The low-lying level occupation probabilities or isomeric cross section ratios to determine  $\sigma$ .

The most data are available for  $\ln^{115}(n, \gamma) \ln^{116}$ . We utilize this data to fit the parameters  $a, \sigma$  and  $C_{E2}/C_{E1}$ . We get

 $a = 18.6 \text{ MeV}^{-1}$ ,  $\sigma = 4.8$ ,  $C_{E2}/C_{E1} = 0.05$ .

The  $\sigma$ -value is valid for the energy range in the neighbourhood of the neutron binding energy.

Another point is the determination of quantum numbers using  $\gamma$ -ray cascade models. As was shown in the case of  $\operatorname{In}^{115}(n, \gamma) \operatorname{In}^{116}$  the isomeric cross section ratio for the initial compound spin value 5 is larger than that for the initial compound spin value 4. This is also valid for the theoretically calculated values, in whatever way we alter the parameters of the model. Furthermore the effect of different initial compound state spin values on R is large. Therefore one can determine the spin value of resonances in the  $(n, \gamma)$ -cross section if one measures different isomeric cross section values or low level population probabilities in several resonances. Experiments of this kind are in preparation.

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