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ABSTRACT[+]

A method is presented for calculating the space, energy and detector dependent correlation function, which is the goal of noise analysis experiments in nuclear reactors. The diffusion and short time approximations to the dynamic Boltzmann equation are made for the sake of simplicity and clarity. A brief description is given of the salient features of the multigroup diffusion code used in these calculations, with the emphasis placed on those features required to calculate the correlation function. A simple reactor consisting of a U-235 spherical core surrounded by a graphite reflector was chosen to illustrate how higher modes of the correlation function may be observed in noise analysis experiments. In particular, it is seen that a Rossi-alpha experiment may measure two decay modes, corresponding to the fundamental mode and the most significant higher mode, or only the single fundamental mode, depending on the spectral sensitivity and the location of the two detectors used in the experiment.

[+] This paper was presented at the International Symposium on Neutron Noise, Waves and Pulse Propagation held at Gainesville, Florida, February 14 - 16, 1966

INTRODUCTION

In stationary zero-power reactors the correlation between the time-distributions of the pulses from two neutron-detectors can be measured in the well-known Rossi-alpha-experiment[1] and used to obtain the prompt time-decay constant

$$\alpha_1 = (1 - k_p)/l \tag{1}$$

of the reactor (k_p = prompt multiplication constant, l = prompt neutron generation time). In this paper only the short-time correlations are considered, delayed neutrons being neglected. If the reactor is sufficiently compact, only the fundamental dynamic mode $\phi_1(x, E)$ is significant for the statistical fluctuations of the neutron flux (x = position, E = energy). Then experiment and theory show that one obtains from two detectors in a suitable circuit a rate of delayed coincidences of pulses

$$\text{const} \cdot \Delta T + r_{12}(T) \cdot \Delta T$$

with T = delay time (> 0), ΔT = coincidence gate width, and a correlation function

$$r_{12}(T) = A_1 \cdot \exp(-\alpha_1 T). \tag{2}$$

However, Rossi-alpha-experiments on some ZPR III fast assemblies with moderating reflectors showed measured correlation functions[2] of the form

$$r_{12}(T) = A_1(x_1, x_2) \cdot \exp(-\alpha_1 T) + A_2(x_1, x_2) \cdot \exp(-\alpha_2 T) + \ldots \tag{3}$$

The coefficients $A_1(x_1, x_2)$ and $A_2(x_1, x_2)$ depend upon the position vectors x_1 of the first and x_2 of the second detector, as well as on their spectral sensitivities, cf. eq. (17 b). In this context first detector means that detector which delivers the first pulse of a delayed coincidence. The qualitative interpretation of such results is simple, α_1 and α_2 obviously being

time-decay constants of the fundamental and a transient mode of the neutron flux. In these moderator-reflected fast assemblies thermalized neutrons in the reflector are important for the slowly decaying fundamental mode, whereas the fast transient mode must be attributed to fast neutrons from the core. This argument[2] gives a satisfactory explanation of the observed space-dependence, which showed the preponderance of the slowly decaying first term of eq. (3) in experiments with detectors in the reflector.

Investigations have been started to reach a more quantitative understanding of these phenomena with the final aim of finding a reliable theoretical method for calculating these correlation functions in coupled fast-thermal reactors.

THEORY

The Rossi-alpha-experiment belongs to a class of correlation experiments which follow a scheme according to fig. 1. Two independent neutron-detectors monitor a stationary reactor with an average neutron flux $\phi_o(x, E)$ in space and energy. To guarantee stationary conditions the reactor is assumed slightly subcritical and equipped with a spontaneous neutron-source Q_o. The primary sequences of detector pulses are each passed through an electronic network with responses $a_1(t)$ and $a_2(t)$ to δ-function inputs. The resulting signals $r_1(t)$ and $r_2(t)$, with time averages $\overline{r_1}$ and $\overline{r_2}$, are multiplied, and the time-average of the product,

$$\overline{r_1 \cdot r_2} = \lim_{T \to \infty} \frac{1}{T} \int_o^T r_1(t) \cdot r_2(t)\, dt, \qquad (3)$$

is taken as the experimental information. According to a general theorem[3] one gets

$$\overline{r_1 \cdot r_2} - \overline{r_1} \cdot \overline{r_2} = \int Q_2(x') \int_o^\infty g_1(x',t)\, g_2(x',t)\, dt\, dx' \qquad (4)$$

This formula is valid for experiments with absorption detectors, such as BF_3-counters, otherwise small corrections must be made. The notation is

$g_i(x',t)$ = response of the signal $r_i(t)$, for i = 1, 2, to the injection at time t = 0 of one fission neutron into the reactor at x'; cf. eqs. (6 a) to (7).

$Q_2(x')$ = rate of pairs of neutrons produced simultaneously as prompt fission neutrons at x'.

With σ_f = macroscopic fission cross section, $\overline{\nu(\nu-1)}_p$ = second moment of the numbers of prompt fission neutrons, $\phi_o(x',E)$ = stationary neutron flux

$$Q_2(x') = \int \overline{\nu(\nu-1)}_p(x',E)\ \sigma_f(x',E)\ \phi_o(x',E)\ dE \qquad (5)$$

The response functions $g_i(x',t)$ involve the responses of the neutron flux in the reactor, of the detector concerned and the associated network function, $a_i(t)$. In a reactor without sources and with an initial neutron population of one fission neutron at x', the subsequent neutron flux in x, E and time t is defined as F(x'; x, E, t). With v = v(E) = neutron velocity, χ = fission neutron spectrum, and $\underline{B}$ = approximate Boltzmann operator including diffusion, absorption, energy changes by scattering and fission neutron production, F(x'; x, E, t) obeys for $t \geq 0$

$$\underline{B}\ F(x';\ x,\ E,\ t) = \frac{\partial}{\partial t}\ \frac{1}{v}\ F(x';\ x,\ E,\ t), \qquad (6\ a)$$

$$F(x';\ x,\ E,\ 0) = \delta(x - x') \cdot \chi(E) \cdot v(E), \qquad (6\ b)$$

$$F(x';\ \bar{x},\ E,\ t) = 0 \quad \text{for } \bar{x} \text{ on a boundary} \qquad (6\ c)$$

The restriction of the operator $\underline{B}$ to the diffusion approximation is not essential.

When detector number i = 1,2 located at x_i has an effective cross section (in cm^2) $C_i(E)$ for neutrons of energy E, and with $a_i(t)$ for the response function of the associated network the concise definition of $g_i(x', t)$ is

$$g_i(x', t) = \int_0^\infty \int_0^t C_i(E)\, F(x'; x_i, E, t - t')\, a_i(t')\, dE\, dt'. \qquad (7)$$

In a Rossi-alpha-experiment the multiplication of pulse sequences is performed by a coincidence circuit. The first network in fig. 1 is a delay line introducing a positive delay T, the second network is missing, so that

$$a_1(t) = \delta(t - T), \qquad a_2(t) = \delta(t). \qquad (8)$$

Thus, for the Rossi-alpha-experiment eq. (4) becomes simply

$$r_{12}(T) = \int Q_2(x') \int_T^\infty \Big[\int C_1(E)\, F(x'; x_1, E, t - T)\, dE \times \int C_2(E')\, F(x'; x_2, E', t)\, dE' \Big] dt\, dx' \qquad (9)$$

If the time-dependent Boltzmann equation, here in diffusion approximation,

$$\underline{B}\, \phi(x, E, t) = \frac{\partial}{\partial t} \frac{1}{v} \phi(x, E, t) \qquad (10)$$

with the boundary condition (6 c) has solutions

$$\phi(x, E, t) = \phi_m(x, E) \cdot \exp(-\alpha_m t), \qquad (11\ a)$$

which are separable, then

$$\underline{B}\, \phi_m(x, E) + \frac{\alpha_m}{v} \phi_m(x, E) = 0. \qquad (11\ b)$$

We will now assume, that for m = 1, 2, ... these eigenfunctions $\phi_m(x, E)$ and the adjoint eigenfunctions $\phi_m^*(x, E)$ of the adjoint equation

$$\phi_m^*(x, E)\, \underline{B} + \frac{\alpha_m}{v} \phi_m^*(x, E) = 0, \qquad (11\ c)$$

also with the boundary condition (6 c), form a complete set with orthogonality relations (12 a) and normalization (12 b),

$$\int\int \frac{1}{v}\,\phi_m^{*}(x, E)\,\phi_n(x, E)\,dx\,dE = 0 \qquad m \neq n, \tag{12 a}$$

$$\int\int \frac{1}{v}\,\phi_m^{*}(x, E)\,\phi_m(x, E)\,dx\,dE = 1. \tag{12 b}$$

Note, that $\phi_o(x, E)$ has been defined as the stationary neutron flux in the reactor, cf. eq. (5), and is not included in this set of dynamic eigenfunctions.

The assumption of completeness is not generally proved, but is valid at least in one-group diffusion theory and also in the multigroup multipoint matrix schemes employed for numerical reactor codes. Then the solution of eq. (6 a) can be written for $t \geq 0$ as

$$F(x'; x, E, t) = \sum_{m=1}^{\infty} F_m^{*}(x')\,\phi_m(x, E)\,\exp(-\alpha_m t). \tag{13 a}$$

Inserting this into the initial condition (6 b) we get for $t = 0$

$$\sum_{m=1}^{\infty} F_m^{*}(x')\,\phi_m(x, E) = \chi(E)\cdot\delta(x - x')\cdot\nu(E). \tag{13 b}$$

Multiplication of this expression by $v^{-1}\cdot\phi_m^{*}(x, E)$ and integration over x and E leads in connection with the orthogonality (12 a) and normalization (12 b) to

$$F_m^{*}(x') = \int \chi(E)\,\phi_m^{*}(x', E)\,dE. \tag{14}$$

This specifies the coefficients $F_m^{*}(x')$ as importance functions averaged at x' over the fission neutron spectrum. Now we can insert the series (13 a) into eq. (9), perform the trivial integrations in t, and obtain

$$r_{12}(T) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\Big[\exp(-\alpha_n T)\int C_2(E')\,\phi_n(x_2, E')\,dE'$$

$$\times\frac{1}{\alpha_m + \alpha_n}\int Q_2(x')\,F_m^{*}(x')\,F_n^{*}(x')\,dx'\int C_1(E)\,\phi_m(x_1, E)\,dE\Big] \tag{15}$$

With a definition of virtual neutron flux functions

$$\psi_n(x, E) = \sum_{m=1}^{\infty} \frac{1}{\alpha_m + \alpha_n} \phi_m(x, E) \int Q_2(x') F_m^{*}(x') F_n^{*}(x') dx' \qquad (16)$$

the expression (15) becomes finally

$$r_{12}(T) = \sum_{n=1}^{\infty} A_n(x_1, x_2) \exp(-\alpha_n T) \qquad (17\ a)$$

$$A_n(x_1, x_2) = \int C_1(E) \psi_n(x_1, E) dE \cdot \int C_2(E') \phi_n(x_2, E') dE'. \qquad (17\ b)$$

This solution verifies eq. (3). It becomes clear, how the time-decay constants α_n, the spectral sensitivities $C_1(E)$ and $C_2(E')$ of the detectors and space-dependence come into play. But for practical applications the summation in eq. (16) is not feasible and must be avoided. In a previous paper[4] it was demonstrated that $\psi_n(x, E)$ obeys a modified Boltzmann equation with a source-term,

$$(\underline{B} - \frac{\alpha_n}{v}) \psi_n(x, E) + \chi(E) Q_2(x) F_n^{*}(x) = 0 \qquad (18)$$

with the boundary condition (6 c). The Boltzmann-operator $\underline{B}$ is modified by introducing an additional, fictitious (1/v)-absorption term in all regions. To prove eq. (18) we distinguish its formal solution $\psi_n'(x, E)$ by a prime and express it as a series

$$\psi_n'(x, E) = \sum_{m=1}^{\infty} \psi_{nm} \phi_m(x, E). \qquad (19)$$

When this is inserted into eq. (18) and eq. (11 b) is applied, we get

$$-\sum_{m=1}^{\infty} (\frac{\alpha_m + \alpha_n}{v}) \psi_{nm} \phi_m(x, E) + \chi(E) Q_2(x) F_n^{*}(x) = 0. \qquad (20)$$

The scalar product of this expression with $\phi_m^{*}(x, E)$ yields in view of the relations (12 a), (12 b) and (14)

$$-(\alpha_m + \alpha_n) \psi_{nm} + \int Q_2(x) F_m^{*}(x) F_n^{*}(x) dx = 0 \qquad (21)$$

The unprimed expression (16) and the primed solution of eq. (18) have the same expansion coefficients and are therefore identical.

According to the eqs. (17 a) and (17 b) in the Rossi-alpha-experiment both detectors participate in the selection of modes of time-decay. If the second detector is put into a node of the eigenfunction $\phi_n(x, E')$, then this eigenfunction does not contribute to the correlation function $r_{12}(T)$. This is true also, when the second detector has a high sensitivity $C_2(E')$ only for energies E' which are not significant for $\phi_n(x, E')$.

The same applies for the first detector and the functions $\psi_n(x, E)$. Most of these functions have low values, e.g. when the source terms $Q_2(x) \cdot F_n^*(x)$ in eq. (18) have one or more nodes and/or caused by the high value of the additional absorption α_n/v. When α_n is high, the values of $\psi_n(x, E)$ will be mainly low for thermal and near-thermal energies E, which are most affected by this additional absorption term. This applies to all eigenvalues α_n of fast transients, so that these transients cannot contribute to the measured correlation functions, when position and sensitivity of the first detector accentuate thermalized neutrons.

This is, once more, a qualitative argument confirming the findings from experiments on moderator-reflected fast assemblies.

NUMERICAL PROCEDURE, CODE

For numerical evaluation of the foregoing equations the flexible one-dimensional multigroup diffusion code of the Karlsruhe Nuclear Program System NUSYS is used. It has been written in FORTRAN II for a 10 K version of the IBM 7074. Basic input data for the code are geometry parameters, dimensions and material compositionsof all reactor zones. Computations follow a three-step routine.

Radius iteration, binary source. In the first step the dimension of one zone, which is indicated in the input data is adjusted until $k = k_{eff}$ has, within a given tolerance, a desired value. Then the reactor shows the correct prompt kinetic behaviour. Mostly as input for the desired k the value $(1 - \beta)$ is chosen (β = effective delayed neutron fraction).

Output of this stage are the exact dimensions of the reactor, stationary neutron flux $\phi_o(x, E)$ and the binary source $Q_2(x)$ according to eq. (5). So far, as a good approximation $Q_2(x)$ is assumed proportional to the ordinary fission neutron

source $Q_1(x)$,

$$Q_1(x) = \int \nu(x, E)\, \sigma_f(x, E)\, \phi_o(x, E)\, dE \qquad (22)$$

Search for eigenvalues α_n. In the second step the eigenvalue problems (11 b) and (11 c) are solved for different modes, resulting in α_n, $\phi_n(x, E)$, $\phi_n^*(x, E)$. An iterative algorithm allows the computation of each real mode, including higher modes with negative flux components. Input are an estimate α_n' and a tolerance for the eigenvalue α_n. With this estimate an additional absorption cross section α_n'/v is introduced into all zones and a $k_{eff}(\alpha_n')$-calculation performed with a number of source-iterations limited by input. If $k = k_{eff}(\alpha_n') \neq 1$, the difference $(k - 1)$ and the associated flux and importance functions are inserted into a perturbation functional to improve the estimate of α_n. This continues until $k(\alpha_n) = 1$.

This method raises two problems, (1) the large number of possible eigenvalues α_n (in the maximum group-number times meshpoint-number) and (2) finding good estimates for α_n. These problems are somewhat less difficult on account of the fact, observed in practical applications, that inexact eigenvalue estimates often lead to convergence to some physically significant eigenvalue with omission of all less significant modes. Still, in some cases with many energy-groups (26) the search for higher eigenvalues was unsuccessful. Output data of this step are for each mode α_n, $\phi_n(x, E)$ and $F_n^*(x)$ according to eq. (14).

Inhomogeneous equation. Input for the third step are α_n, $F_n^*(x)$ and $Q_2(x)$. In this step for the desired mode the inhomogeneous eq. (18) is solved by iteration, yielding $\psi_n(x, E)$. For this iterative process the initial estimate is provided by computing the projection of the unknown function $\psi_n(x, E)$ into the solution of the associated homogeneous k_{eff}-problem. In near-critical reactors this yields the main part of the solution, which otherwise would make convergence of the inhomogeneous problem very slow. With α_n and $\phi_n(x, E)$ from the second and $\psi_n(x, E)$ from the third step the contribution of the n-th mode in eqs. (17 a) and (17 b) is known.

RESULTS FOR A GRAPHITE-REFLECTED U-235 SPHERE

As a first simple case computations were done for a sphere of U-235 (4.0 10^{22} atoms/cm^3) surrounded by a 15 cm shell of graphite (8.4 10^{22} atoms/cm^3). An 8-group-set of cross-sections condensed from the 26-group-set of Bondarenko et al.[5] was used; energy limits for the groups are given in table 1.

The computer code yields a radius of 8.2 cm for the uranium-sphere, i.e. the fast core, for a desired k = 0.993. The asymptotic eigenfunction $\phi_1(x, E)$ and one significant fast transient mode $\phi_2(x, E)$ were found, with time-decay constants

$$\alpha_1 = 4630 \text{ sec}^{-1} \qquad \text{and} \qquad \alpha_2 = 68750 \text{ sec}^{-1}.$$

One more real eigenfunction $\phi_3(x, E)$ was found similar to $\phi_2(x, E)$, with

$$\alpha_3 = 57320 \text{ sec}^{-1},$$

but considerably less significant. The significance of these modes is estimated from their contributions to the stationary flux function

$$\phi_o(x, E) = \sum_{n=1}^{\infty} P_n \, \phi_n(x, E) \tag{23}$$

In this case the computed $\phi_1(x, E)$ and $\phi_2(x, E)$ contribute almost equally to the fast groups of $\phi_o(x, E)$, but make up only about 45 per cent of the fast flux. Therefore, other significant eigenfunctions, possibly with complex time-decay constants α_n are still missing.

The eigenfunctions ϕ_1 and ϕ_2 corresponding to the lowest eigenvalue and the most significant higher real eigenvalue are virtually identical in the core, where the spectrum is quite hard and the pseudo-absorption term (- α/v) is relatively unimportant. The difference between the eigenfunctions becomes noticeable in the reflector, where the spectrum is softer and the (α/v)-term assumes more importance. Fig. 2 shows the distinction between the two eigenfunctions at x = 13.7 cm, i.e. 5.5 cm into the reflector.

Fig. 3 illustrates the behaviour of the functions $\psi_1(x, E)$ and $\psi_2(x, E)$, which according to eqs. (17 a) and (17 b) determine how the first detector's position $x_1 = x$ and its spectral sensitivity $C_1(E)$ select the contributions of ϕ_1 and ϕ_2 to the correlation function. In fig. 3 two curves $\psi_1(x)$ and $\psi_2(x)$ are presented, which are integrals over the second energy group and refer to a fast neutron detector. Two other curves $\psi_1(x)$ and $\psi_2(x)$, which have been drawn to another scale, are the corresponding integrals over the thermal group.

The effect of detector location and spectral sensitivity is clearly shown in two sample calculations. These computed correlation functions include all three real eigenfunctions found. Other contributions are assumed as either excluded by the experimental arrangement or subtracted from the raw data. Normalization is arbitrary.

Fig. 4 shows the correlation function $r_{12}(T)$ for the case where the first detector is at the center of the core and the second detector is in the reflector. Both detectors are sensitive to fast (group 2) neutrons. One recognizes two distinct exponentials corresponding to the fundamental and to the important higher mode.

On the other hand, fig. 5 shows the correlation function $r_{12}(T)$ for two detectors in the reflector at $x = 13.7$ cm. Both detectors have been chosen as 1/v absorbers. In this case the contribution of the higher mode is very small, and it is unlikely that a Rossi-alpha-experiment would be able to distinguish any modes other than the fundamental mode with such a detector arrangement.

Finally in fig. 6 the computed ratio $\psi_2(x, E)/ \psi_1(x, E)$ is shown as a function of the spectral sensitivity $C_1(E)$ of the first detector and for two positions $x = 0$ in the core and $x = 13.7$ cm in the reflector. This ratio is measured with monoenergetic detectors.

CONCLUSIONS

The results presented show that the modal symthesis of Rossi-alpha data can be applied to interprete important features of experiments performed on moderator-reflected fast assemblies. Further numerical and theoretical investigations are

in progress for reactor models whose physical build-up insures that the consideration of only one transient mode besides the fundamental mode is justified.

On the other hand, direct application of this method as a routine tool seems unfeasible when many transient modes have to be included.

TABLE 1

Structure of condensed 8-group-set

Group	energy region	groups in ref. 5
1	1.4 - 10.5 MeV	1 - 4
2	.1 - 1.4 MeV	5 - 8
3	4.65 - 100 keV	9 - 12
4	.215 - 4.65 keV	13 - 16
5	10 - 215 eV	17 - 20
6	.465 - 10 eV	21 - 24
7	.215 - .465 eV	25
8	thermal	26

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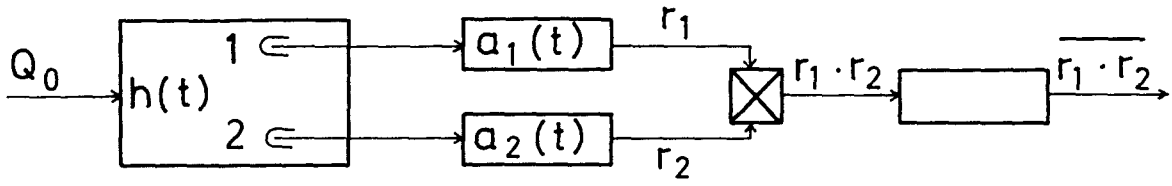

Fig. 1 Scheme of a correlation experiment

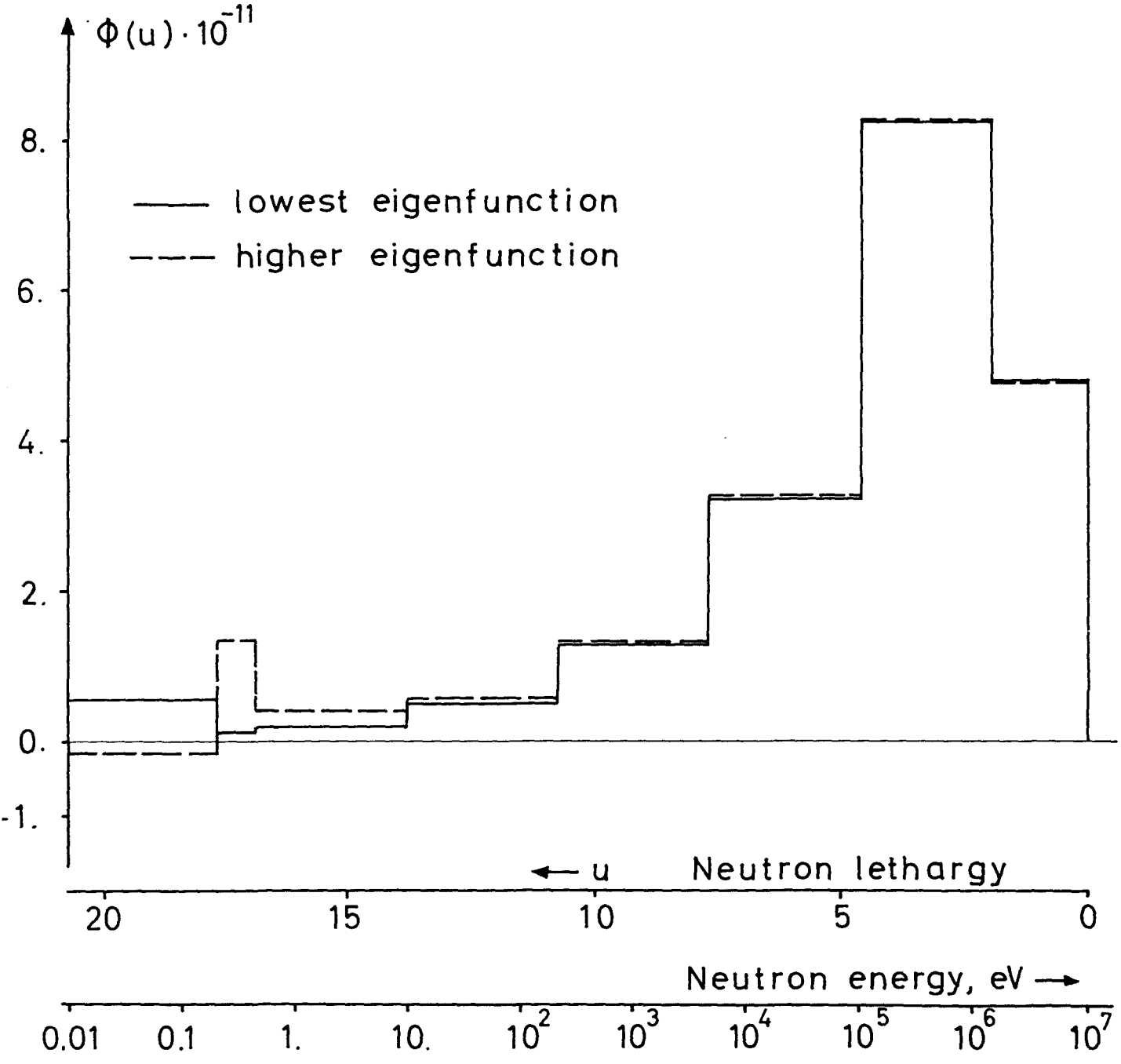

Fig.2 Neutron spectra in graphite reflector

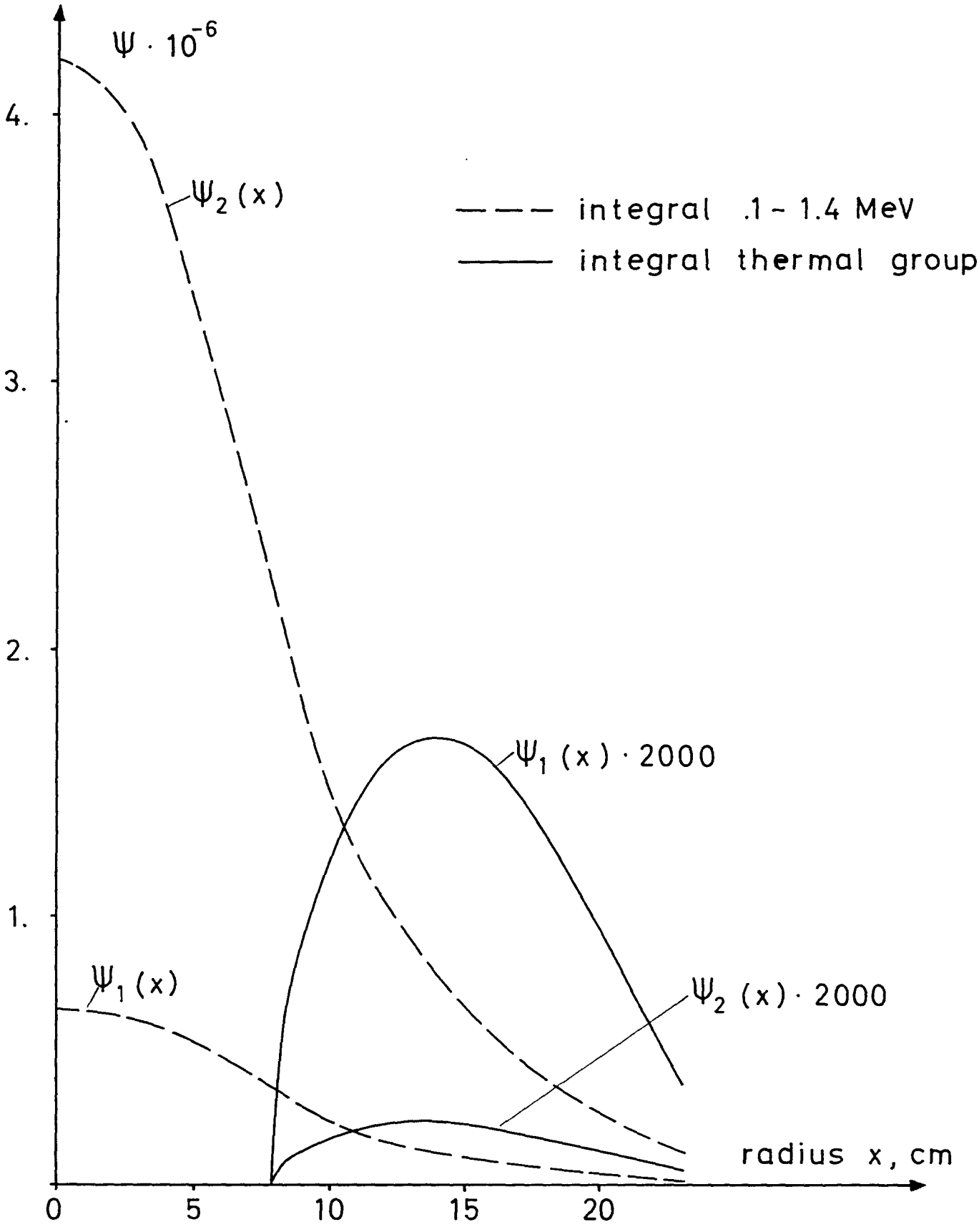

Fig. 3 Selection of modes by position and sensitivity of the first detector

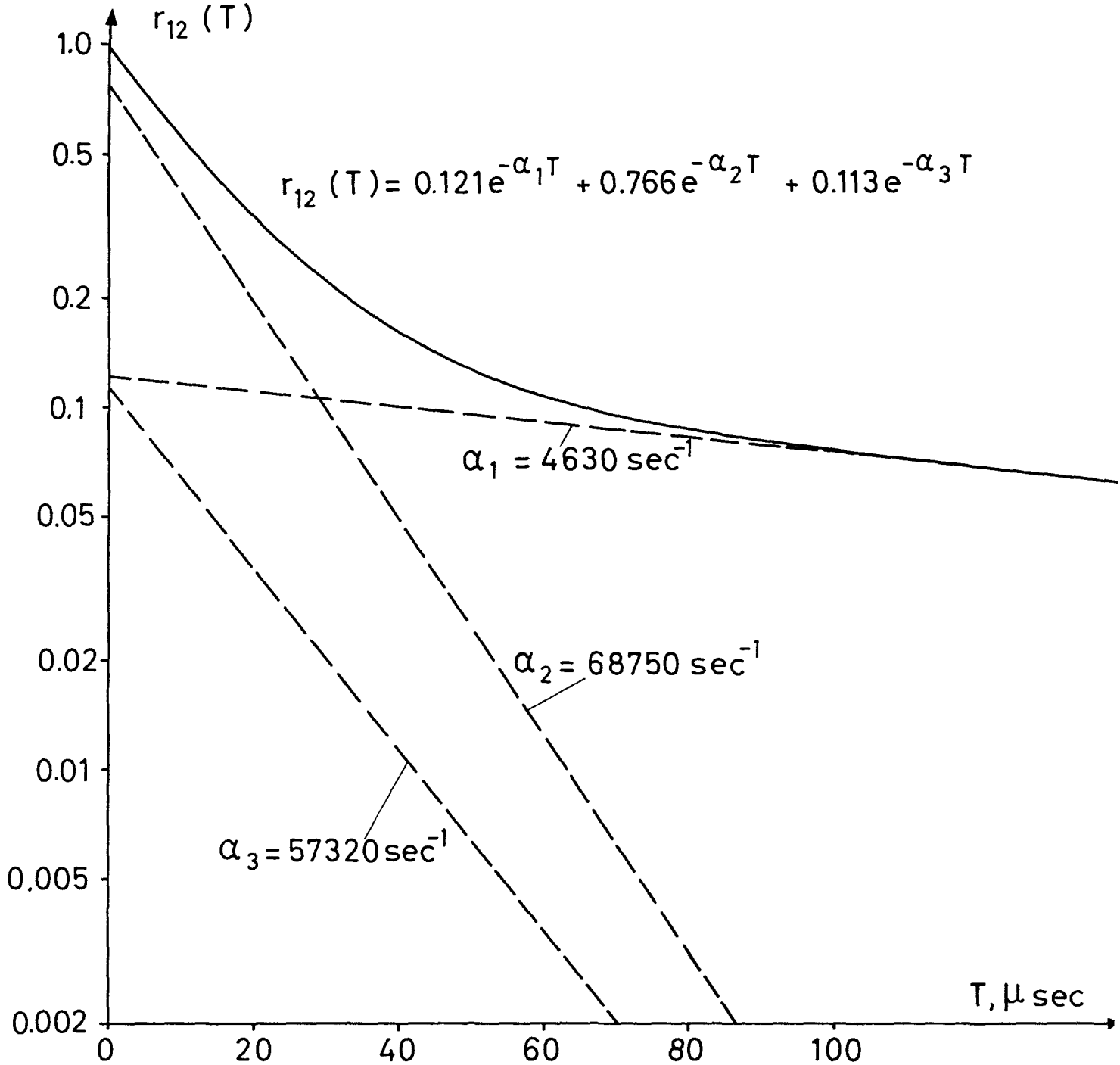

Fig.4 Correlation function for two fast neutron detectors, first detector in core, second in reflector

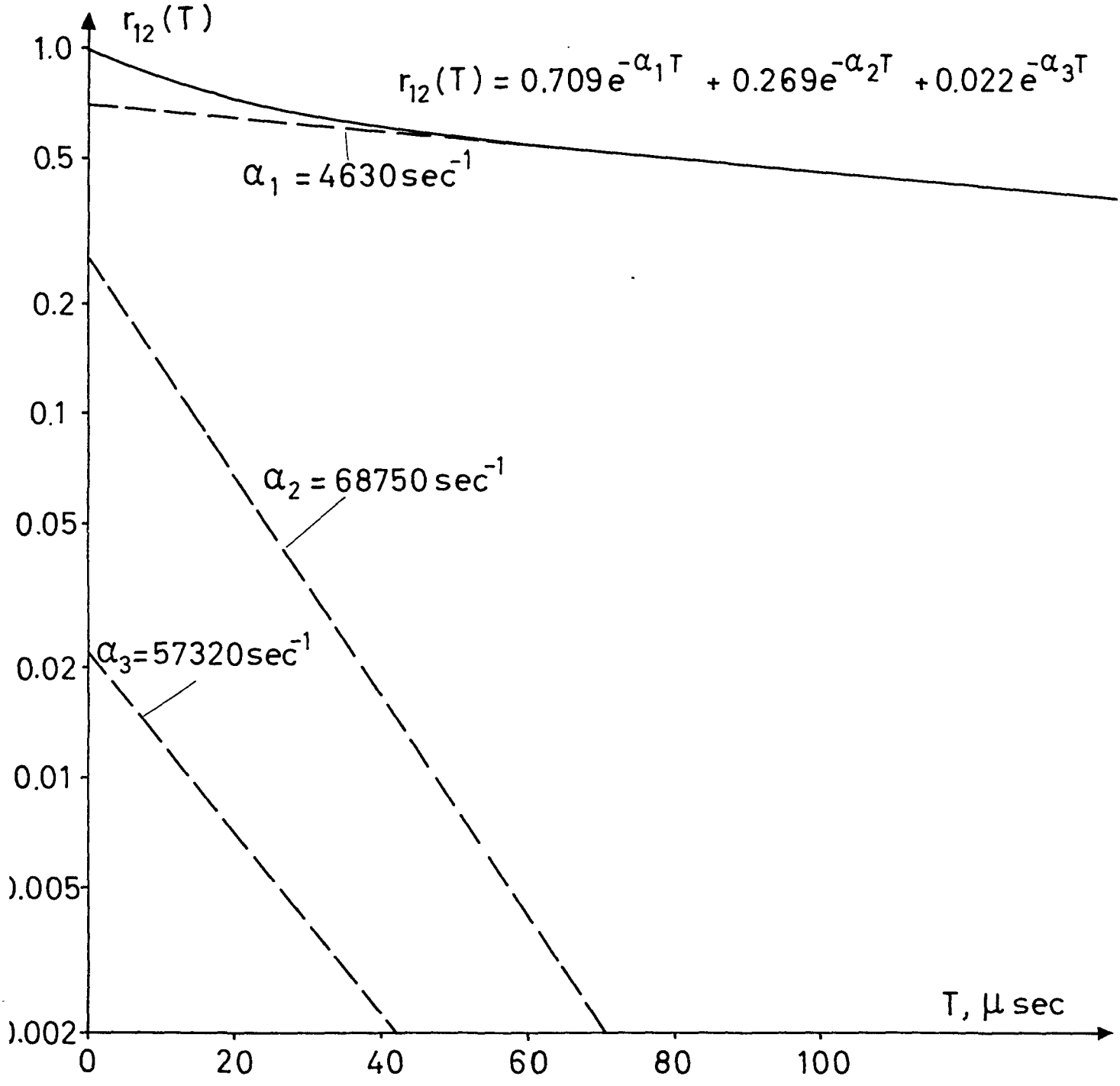

Fig.5 Correlation function for two 1/v-detectors both in reflector

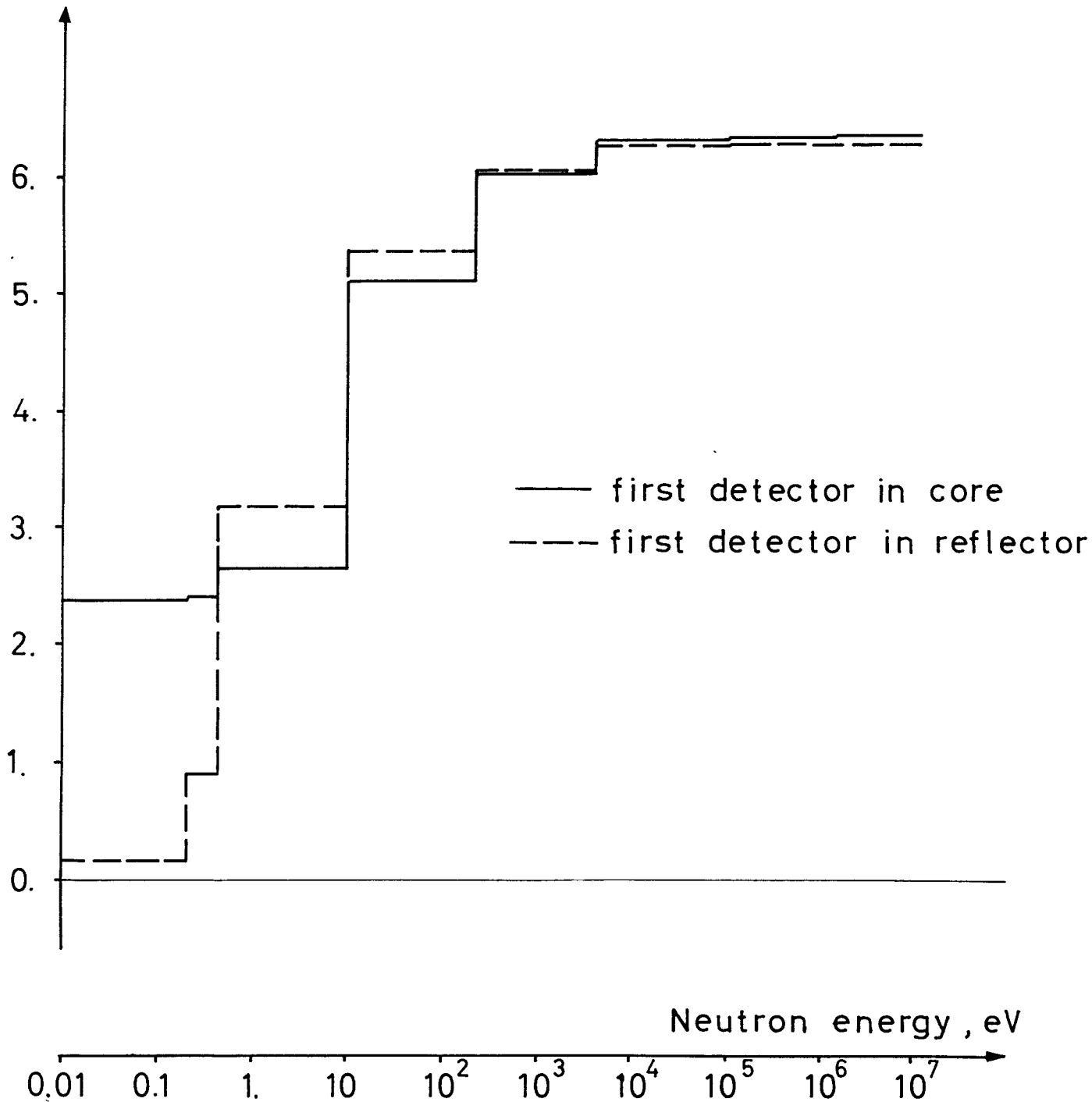

Fig. 6 Ratio Ψ_2 / Ψ_1