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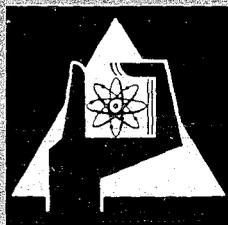
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The Calculation of Large Fast Reactors

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THE CALCULATION OF LARGE FAST REACTORS<sup>\*)</sup>

von

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<sup>\*)</sup> Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung mbH., Karlsruhe.



## THE CALCULATION OF LARGE FAST REACTORS <sup>\*)</sup>

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### Introduction

The development of large fast power reactors is one of the major topics of the atomic energy program of several countries. As far as the prediction of the physical behaviour of a fast breeder is concerned, the results presented at the Argonne Conference in 1965 showed no satisfactory agreement. In this paper the fundamental procedure used in Karlsruhe in calculating fast reactors is presented. The problems which arise with the preparation of multigroup constants, the determination of reactivity coefficients with the calculation of second criticality and the long-time behaviour of fast reactors are discussed in some detail. A comment to each topic is given on the necessity of further improvements. It should be possible to calculate fast reactors as accurate as thermal reactors in near future, say for instance to 1 % accuracy in  $k_{eff}$ . As an important consequence the competition of safety characteristics and economy may then be judged in a more reliable way.

### 1. Cross Sections and Group Constants

#### Microscopic Data

The basic nuclear data of the important materials for the calculation of fast and intermediate reactors are stored in the Karlsruhe nuclear data file KEDAK. The first KFK-120 [1] data have been revised and updated. A

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<sup>\*)</sup> Part of this paper was presented at the BNES Conference on Large Fast Reactors in London, May 1966.

second edition is being prepared<sup>\*)</sup>. The resonance data of the important fissile and fertile materials have been determined by analysing the known experiments and stored together with the important resonance data for structural and coolant materials on KEDAK.

There are continuous improvements, updating and complements of the nuclear data in Karlsruhe. Special interest should be given in general to the fission cross section of Pu 239, the absorption cross section of U 238, Fe and Ni and the inelastic data for Pu 239. Also data of the higher Pu-isotopes are not up-to-date. Recommended data of the higher Pu-isotopes are being prepared for the KEDAK-file by M. Segev, M. Caner and S. Yiftah (Israel) in contract with the Fast Breeder Association Euratom - Karlsruhe. In Karlsruhe just a reinvestigation of fission product cross sections has been carried out [2] for steam cooled reactors. The influence of the uncertainty of mentioned cross sections on integral fast reactor quantities is given in [3-4]. These data may influence the safety and breeding features significantly.

#### Preparation of Group Constants

Taking the first revision of KFK-120 data (status June 1965) as a basis, the first Karlsruhe group constants set KFK 26-10 was prepared with a weighting spectrum typical for a 1000 MWe sodium cooled fast reactor. Because we wanted to use the Russian-26-group constants [5] for the less important materials, we adopted the boundaries of the Russian set. The procedure of preparation and some characteristics of this set are given in [3] and [6]. The comparison of theoretical predictions for some integral data with experimental results of critical assemblies (ZPR/III, ZPR/VI) gives satisfactory agreement in critical mass and spectral indices [7].

The preparation of group constants is now being in the head program MIGROS including the part RESI in which the resonance self-shielding factors are determined.

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<sup>\*)</sup> Part I (Evaluation of Neutron Cross Sections for Fast Reactors) has been published during publication of this paper and contains the full information of data up to July 1966.

### Resonance Self-Shielding

From the Russian group set we adopted the definition of self-shielding factors  $f_i$ , following the concept of factorising the infinite dilute cross sections and  $f_i$ -factors dependent on the temperature and material composition. The shielding factors are tabulated for certain temperatures and values of the cross section  $\sigma_0$ , dependent on the material composition. The shielding factors are determined before every reactor calculation by a suitable interpolation formula [8]. Different definitions of the "composition" cross section  $\sigma_0$  have been investigated, but they practically result in negligible changes of the criticality factor. The influence on the lower energy part of the spectrum with the feedback and neutron lifetime and Doppler coefficient is just being studied.

The assumption of narrow resonance theory for the self-shielding effects of the heavy elements is quite satisfactory [9]. The resonance overlap effect of the same series gives sometimes remarkable contributions to the effective cross sections. For resolved resonances the appropriate method is to produce the temperature-dependent cross section from resonance data and using numerical integration to get the  $f_i$ -factor. In this energy region the J-function-method leads to difficulties. In the statistical region there is only a weak dependence of the  $f_i$  on the statistical parameters and this makes the use of  $f$ -factors very attractive because the infinite dilute values may be taken from the best known experimental results.

### Elastic Moderation

In addition to the investigations concerning the calculation of elastic removal cross sections described in [3] two different weighting spectra for the elastic removal cross sections have been used for core and blanket. The results for a spherical reactor with core composition typical for a large sodium cooled breeder (case No. 8 of [6], see appendix) without fission products are summarized in Table 1. The values for the internal and external breeding ratios and the reactivity change for 50 % loss of sodium out of the core remain practically unchanged if an extra weighting spectra is used for the blanket. But the power distribution in

the blanket has increased by about 4 %. Compared with the procedure given in [5] a better description of the anisotropic scattering in the centre of mass system is now being investigated. Most of the angular distributions are taken from [10] together with more recent data.

In conclusion one can say that the effect of different weighting spectra (i.e. different group sets) in the zones of a multilayer fast reactor is not negligible for a proper description of the power distribution and of the blanket management and blanket optimization features. Also the radiation exposure to the steel vessel is influenced by using different sets. This effect is confirmed by other methods, which will be discussed in section 2. The results require special attention to the space dependency of weighting spectra. Especially the use of fundamental mode calculations with many energy groups as done in ELMOE-type codes have to be reinvestigated according to space-dependent problems. For routine calculations we feel that the description of elastic slowing down as given in [3] is sufficient. For the reactor chosen to be built more refined methods describing elastic moderation have to be used. In contract with the Fast Breeder Association Euratom - Karlsruhe M. Segev in Israel is investigating a semi-analytical representation of the collision density in the Doppler energy range for pure elastic down-scattering. The more general space-dependent problem including absorption is being studied in Karlsruhe now.

### Transport Cross Section

The problems related to the appropriate weighting and shielding of the transport cross section are well known [11]. Here another effect is mentioned which may be remarkable in some cores. If one considers the space-independent slowing-down mechanism in a consistent  $P_1$ -approximation, the evaluation of the linear anisotropic scattering-down integral  $\int_E^{E/\alpha} K_1(E',E)j(E')dE'$ , results for  $\alpha < 1$  (say  $A \geq 10$ ) in different transport cross sections for different assumptions on the energy dependence of  $j(E)$ . It is  $\alpha = (A-1)^2/(A+1)^2$ ,  $j(E')$  = net current,  $K_1(E',E)$  = linear anisotropic moment of the scattering kernel for neutrons scattered by atoms at rest. Assuming for simplicity isotropic scattering in the centre of mass system and a smooth variation of the scattering cross section  $\Sigma_s(E)$  we have for one isotope A

$$\Sigma_{tr}^0 = \Sigma_a + \Sigma_s \left(1 + \frac{2}{3A\alpha}\right) \quad \text{for } \frac{j(E)}{E} \quad \text{as slowly varying function}$$

$$\Sigma_{tr}^1 = \Sigma_t \quad \text{for } j(E) \quad \text{as slowly varying function}$$

$$\Sigma_{tr}^2 = \Sigma_a + \Sigma_s \left(1 - \frac{2}{3A}\right) \equiv \Sigma_a + \Sigma_s (1 - \bar{\mu}) \quad \text{for } j(E) \cdot E \quad \text{as slowly varying function}$$

In a fast sodium cooled reactor the current is roughly proportional to  $E$  up to 1 keV, then roughly constant up to some 100 keV. Because the assumptions made above are true in the range below 50 keV for C, O, Na, Fe, Ni, we see that the leakage due to the light materials is reduced in the lower energy groups, where for instance the Doppler effect is important. The mentioned light materials contribute more than the 50 % to the total cross section in this energy range.

We note that the transport cross section is very sensitive to the assumed energy dependence of the current. Especially for a steam cooled oxyde breeder the effect is important. In this case the transport cross section increases roughly proportional to  $1/\sqrt{E}$  when approaching lower energies. Again as in the preceding section and even more severely the space dependency of weighting spectra plays a role.

## 2. Methodical Procedure for Neutronic Calculation

### Codes in Operation in Karlsruhe

Most of the codes used in Karlsruhe are combined in the "nuclear head system" NUSYS [12]. The subcodes are compatible in their formats so that they can always be arranged for the special purpose wanted. NUSYS contains the necessary codes requested for the calculation of fast reactors. 1x1 synthesis, 2-dimensional diffusion, 1-dimensional  $S_N$ , perturbation, Doppler effect routine, reaction rates, breeding ratios, heterogeneity corrections for the calculation of critical assemblies, 0- and 1-dimensional burn-up codes are in operation.

Transport codes are needed in the calculation of large fast reactors for the determination of blanket properties (about 7 % change in the power distribution for the middle of the blanket is observed going from diffusion to  $S_4$  approximation. For the calculation of the relative large control rods diffusion approximation gives only a small error. In the context of final prediction of design values besides  $S_N$  also the evaluation of Monte-Carlo codes [13] is interesting.

### Some Numerical Questions

In view of the discrepant results presented by the intercomparison in Argonne [6] it was felt to prove the insensitiveness of typical integral fast reactor quantities on the number of meshpoints and accuracy of source distribution used in a reactor calculation. For this investigation we used two different one-dimensional diffusion codes [12, 14]. In the MGP-code the diffusion equation is solved by quadrature, in Diff (NUSYS)-code difference techniques are used. For a sodium cooled fast reactor (No. 9 of [6]) a reduction from 95 core points and 45 blanket points to 30 and 15, respectively gives errors in breeding ratio and reactivity change for total of Na loss out of the core which are less than 0,2 %. (In all cases  $k_{\text{eff}}$  was accurate to  $10^{-4}$ ).

In the cases of small numbers of meshpoints the fluxes in the soft part of the spectrum (group 25 and 26) may become negative because of the large removal cross sections. This is a normal property of the used codes.

Improving the accuracy of the source distribution from  $10^{-2}$  to  $10^{-5}$  for two consecutive iterations practically no change in local source and fission density and breeding ratio was observed. For routine calculations of fast sodium cooled reactors only a small number of meshpoints is required.

### Comparison of One- and Two-Dimensional Methods in Calculating a Cylindrical Reactor

Reactor No. 9 of [6] was investigated. For all calculations we condensed the Karlsruhe group set KFK 26-10 to four groups always weighting with the same spectrum.  $k_{\text{eff}}$  and  $\Delta k_{\text{Na-loss}}$  (total sodium loss of core) have been calculated with the two-dimensional diffusion code DIXY (NUSYS) [12] (about 1500 meshpoints have been used).

Using the one-dimensional diffusion code Diff (NUSYS), we determined successively the reflector savings and generated group-dependent bucklings. By interpolation of the corresponding  $k_{\text{eff}}$ -values we got the approximated  $k_{\text{eff}}$ -values.

	1-D. Diff.	2-D. Diff.
$k_{\text{eff}}$	$1.0073 \pm 1 \cdot 10^{-4}$	$1.0065 \pm 1 \cdot 10^{-4}$
$k_{\text{Na void}}$	$0.0430 \pm 2 \cdot 10^{-4}$	$0.0428 \pm 2 \cdot 10^{-4}$

There is a good agreement between the results of the one- and two-dimensional methods. Also the breeding ratios show small differences only: about 1 % in  $BR_{\text{int}}$  and 3 % in  $BR_{\text{ext}}$ . The deviations of "flux shape factors" resulting from one- and two-dimensional methods do not exceed 5 % [15].

A more convenient way in simulating cylindrical reactors is the use of flux synthesis methods, either using the Ritz- or the Kantorowitsch procedure. A method automatized by using a code for the simplest case (separation of variables) is just completed [16]. For the 3-dimensional diffusion calculation for a reactor with inserted control rods a Kantorowitsch variational technique is being developed.

#### Investigation of Different Condensation Spectra and Different Numbers of Condensed Groups

In calculating two-dimensional problems it is necessary in almost all cases to collapse drastically the original group set because of the limited capacity of the computer. Many problems arise in finding out the most suitable condensation spectra. In order to get an estimation of the error involved in this procedure, we used two different fundamental-mode spectra (spectrum 1 :  $B^2 = 0$ ; spectrum 2 :  $B^2 = 7,38 \cdot 10^{-4}$ ) collapsing the 26-group-set to 13, 10 and 5 groups. The resulting group cross sections were used to calculate a spherical reactor, No. 9 of [6], without changing the geometry.

$k_{\text{eff}}$  and the breeding ratios (BR) are not very sensitive to group collapsing. The deviations from the case of 26 groups are small (spectrum 1, maximal deviations:  $\Delta k = 0,5 \%$ ,  $\Delta BR_{\text{int}} = 1 \%$  and  $\Delta BR_{\text{ext}} = 3 \%$ ). The deviations of values corresponding to spectrum 2 are generally smaller (maximal deviations:  $\Delta k = 0,3 \%$ ,  $\Delta BR_{\text{int}} = 0,2 \%$ ,  $\Delta BR_{\text{ext}} = 2,5 \%$ ).

A greater sensitivity is given for the zero of the local sodium void coefficient (variations up to 3 cm) and the power distributions mainly in the blanket. The deviations in the power distribution are positive and negative depending not only on the used spectrum for condensation but also on the manner of combining the groups.

That means that for a rough estimation of integral values it is possible to use condensed cross sections, however, for spatial-dependent values (danger coefficients) one should be more carefully. These effects are more pronounced in multizoned reactors, especially in the investigation of blanket properties (i.e. external breeding ratios, breeding- and power distribution). To get reasonable accuracy in the blanket power distribution it is necessary to use two blanket condensation spectra [17].

### 3. Reactivity Coefficients

#### Coolant Loss

The reactivity change due to coolant loss is very sensitive to the used basic data and methods. In Karlsruhe we may use for routine calculations the same group-constant set for the two necessary criticality calculations (but with changed shielding factors according to the different material compositions in the normal case and in the loss case) or two different group sets (different in the used weighting spectrum for the two cases), as described in [3].

For very specific determination of the coolant loss coefficients one has to describe the slowing-down process properly by using ELMOE-type or even improved ELMOE-calculations. But the intercomparison [6] shows that even results obtained with these refined methods may be discrepant by about 300 % due to the different basic data involved. Furthermore, attention has to be paid to the fuel temperature of the voided region. This is especially significant for steam cooled reactors. In testing the calculation procedure with experimental results of critical assemblies one has to be very careful; especially the heterogeneity effects have to be considered properly. The sign of the reactivity coefficient may change in different arrangements of the fuel and coolant plates [18]. A systematic study on the calculation of Na-void is under way.

Doppler Coefficient

In Karlsruhe a code for the calculation of the Doppler effect (DCP) has been established on the basis of perturbation theory [19]. The Doppler coefficient (DC) is given by

$$DC = -\sum_i \left( \phi_i^+ \frac{d}{dT} \left[ \frac{\Sigma_{vi}^+ \Sigma_{fi}^+}{\Sigma_{vi}^+ \Sigma_{fi}^+} \phi_i - \phi_i^+ \chi_i \cdot \sum_k v_k \Sigma_{fk}^+ \phi_k \right] \right) dV$$

$\Sigma_{xi}^+$  is the effective group cross section of type x in group i. The method of calculating  $d\Sigma^+/dT$  is derived in such a way that it calculates in a short time the correct  $d\Sigma^+/dT$  for sodium cooled fast breeder reactors now under discussion.

The method of calculating

$$\frac{d}{dT} \Sigma_x^+ = \frac{d}{dT} \left\{ \left( \int \frac{\Sigma_x^+}{\Sigma_t^+} dE \right) : \left( \int \frac{1}{\Sigma_t^+} dE \right) \right\}$$

is based in principle on R.B. Nicholson's paper [20], where only statistical parameters are used. For the determination of the integrals  $\int \Sigma_x^+ / \Sigma_t^+ dE$  and  $\int 1 / \Sigma_t^+ dE$ , we have splitted the denominator  $\Sigma_t^+$  into two parts. The first part contains the main contributions to  $\Sigma_t^+$ . These are the large resonances of the first U-238-series ( $\Sigma_{res}^{U238}$ ) and an energy-independent part, representing a kind of averaged cross section of all other materials ( $C_x$ ). The second part of  $\Sigma_t^+$  describes the deviation  $\Sigma^{rest} = \Sigma_t^+ - \Sigma_{res}^{U238} - C_x$ .

In the important energy regions of the discussed fast reactors the second part of  $\Sigma_t^+$  is nearly everywhere small compared with the first part so that the following expansion is allowed:

$$\frac{1}{\Sigma_t^+} = \frac{1}{\Sigma_{res}^{U238} + C_x + \Sigma^{rest}} \approx \frac{1}{\Sigma_{res}^{U238} + C_x} \left( 1 - \frac{\Sigma^{rest}}{\Sigma_{res}^{U238} + C_x} + \dots \right)$$

This expansion is inserted into the integrals

$$I_1 = \int \frac{\Sigma_x^{U238}}{\Sigma_t^+} dE, \quad I_2 = \int \frac{\Sigma_x^{REST}}{\Sigma_t^+} dE \quad \text{and} \quad I_3 = \int \frac{1}{\Sigma_t^+} dE$$

( $\Sigma_x^{U238}$  indicates the contribution of the first U238 series to  $\Sigma_x^+$  and  $\Sigma_x^{REST}$  indicates all other contributions to  $\Sigma_x^+$ ). We get equations for the determination of  $C_x$  by the condition that the integrals of the first expansion term of  $I_k$  exactly disappears.

For  $I_1$  and  $I_3$  the  $C_x$  are equal and have the form

$$C_x^{I_{1,3}} = \frac{1}{\Delta E} \int (\Sigma_t - \Sigma_{res}^{U238}) dE$$

while for  $I_2$

$$C_x^{I_2} = C_x^{I_{1,3}} + \frac{\int_{\Sigma_x}^{\Sigma_x^{REST}} \Sigma_{res}^{REST} dE}{\int_{\Sigma_x}^{\Sigma_x^{REST}} dE} - \frac{1}{\Delta E} \int_{\Sigma_{res}}^{\Sigma_{res}^{REST}} dE$$

Regarding the repulsion effect of resonances within the same series, we can carry out a second evaluation so that we have one resonance in the denominator of the expressions  $1/(\Sigma_{res}^{U238} + C_x)$  only. The overlap effect of resonances of the same series is so taken into account in the first-order term of this expansion. With these two evaluations we can express the upper integrals and their temperature derivations by J-functions averaged over all possible neutron widths and terms of the form  $\int_{\Sigma_x} \Sigma_{res} dE$  and their corresponding temperature derivations. The J-function  $J = \int \psi / (\beta + \psi) dx$  is available in form of a table. This procedure results in a relative short computing time for the calculation of Doppler coefficients of fast reactors.

As far as the validity of the assumptions is concerned, enough resonances must be in a group, because statistical parameters are used only. This is e.g. not the case in the Russian group constants [5] for energy groups lower than 100 eV. But this region is not very important for the Doppler coefficient of fast reactors, at least for the sodium cooled type. Besides this another limitation exists, caused by the first type of evaluation. This limitation expresses that no Doppler-material besides U 238 must have such a nuclear concentration that the resonances of this material are higher than the average total cross section. These two conditions are fulfilled in the presently discussed fast breeder reactors. Comparisons of the results with other teams who calculated Doppler coefficients showed no remarkable differences. These comparisons are described for  $d\bar{\Sigma}/dT$  and for the total Doppler coefficients in [21].

The overlap effect of resonances of different series, which is included in our calculation method, shows no remarkable influence on the Doppler-coefficient [22].

Besides the DCP-program there exists a first version of a program for the interpretation of reaction-rate measurements.

The influence of solid-state properties on the DC [23-24] can be taken into account by a correction factor [24]. If more accurate measurements of the crystal-frequency spectrum of Dolling et al. [25] are used, the result is that a fictitious Debye temperature of about 600°K can describe the solid-state influence on DC in the interesting temperature range. Curve 1 of Fig. 1 shows the Debye temperature which gives the temperature-dependent correction factor between DC with solid-state properties and DC without solid-state properties. The curve 2 shows the Debye temperature which describes the specific heat in a correct way. One recognises a strong dependency of the Debye temperature on the temperature and the way how it is calculated. Fig. 2 shows the magnitude of this correction factor ( $DC^{\text{with}}/DC^{\text{without}}$ ).

In conclusion we state that the calculation of the Doppler coefficient gives satisfactory results. Differences mainly result from the use of different basic group sets. Assuming  $1/T$  dependency of the DC, there is also good agreement between successive  $k$  calculations and the method described above. Therefore, it seems that a special DC code for the resolved resonance region is not necessary, as we have codes (RESI) for the temperature-dependent group constants in this range.

#### 4. Investigation of Core Disassembly and Second Criticality

##### General Remarks

The safety analysis of large reactors usually is divided into two parts:

- 1) As a model for the accident situation a time-dependent reactivity insertion e.g. reactivity ramp is evaluated.
- 2) The energy release is calculated according to the reactivity input.

The first problem may be solved by using the FORE code or analogue computer techniques. A more refined technique for space dependent dynamics

is being investigated. We shall restrict our investigation to the second point.

Since the analysis of possible accident situation is widely based on unknown and hypothetical features, it is reasonable to study the effect of the following parameters.

1. Doppler coefficient (DC)
2. rate of reactivity ramp
3. primary flux
4. delayed neutrons

#### Bethe-Tait Model Containing Doppler Feedback

Some investigations [26-29] have been carried out in this field, but it is necessary to elaborate the model as we shall see below. The starting point of our analysis [30] is a modified Bethe-Tait method for a spherical symmetrical core. The separation into a time-dependent and a space-dependent function is established and perturbation theory is applied. Thus, the problem is reduced to a system of five non-linear, ordinary differential equations of the first order for the following variables:

1. energy
2. flux
3. delayed neutron precursor
4. feedback reactivity
5. first derivative of the feedback by expansion of the core

The integrals over the space variables, which occur in the differential equations, can be solved analytically because the energy dependence of the pressure and the radial dependence of flux and importance are approximated by polynomials. Results for vanishing DC are very easily understood. The destructive energy increases, if the rate of the reactivity ramp is augmented or the primary flux decreases. For the negative DC the flux is not monotonous during the excursion, since the Doppler feedback induces some oscillations. Now, the released energy depends very strongly upon the phase of such a cycle, at which the excursion is stopped by the expansion. Hence, a change of e.g. 5 % in the DC may involve a factor of 4 (fig. 3).

### The Relative Maximum of Released Energy

The values for the released energy of ref. [28] do not show any monotony, and for this reason the DC was switched off temporarily in ref. [29]. But the latter approximation induces a large overestimation for the released energy. Furthermore, the influence of delayed neutrons should be estimated.

Therefore, we have investigated the conditions for the occurrence of the relative maximum in the released energy [30]. A code named FAUN has been elaborated which incorporates this result. FAUN evaluates upper limits for the energy and the latter are also valid if the reactor parameters vary by some amount because of the uncertainty in the starting conditions.

With this code the influence of the parameters was checked numerically for a meltdown accident of a sodium cooled fast reactor (Na-1) for several ramp rates. The results in table 2 and fig. 3 show the features predicted by theory for a ramp rate of 100 \$/sec. There we listed the thermal energy above the boiling point, which is an upper limit for the destructive energy. The energy decreases with increasing DC. The dependency of the reactivity ramp and primary flux is very similar to the case of vanishing DC.

In this context the effect of delayed neutrons is very interesting. In any case, they reduce the energy, but there are two different reasons for vanishing DC on the one hand and for large negative coefficients on the other hand. In the first case the reduction arises from the time difference between delayed critical and prompt critical and is of the order of some ten per cent. In the latter case the delayed neutrons damp the oscillations mentioned above (fig. 4) and this effect leads to an essential reduction in energy (region below the intermediate line in table 2).

Now, this model is to be generalized to cylindrically geometry. This is important with respect to the investigation of the so called autocatalytic effect, which arises in cores with more than one zone.

### Equation of State

The equation for the saturation vapor pressure is valid in the temperature region below the critical point. From a temperature lying somewhat below the critical temperature onwards the pressure increases more rapidly due to the incompressibility of fluids [28]. In the code FAUN only the vapor pressure law is used up to now. Therefore, we get only upper limits for the energy release in some cases. These have been marked by asterisks in the table. FAUN also takes into account that a certain amount of fuel is vaporized so that the Doppler effect is reduced by this effect.

Up to now, the equation of state has been derived using the principle of corresponding states. An equation being more realistic will be deduced from a partition function that will be constructed according to the model of Eyring.

## 5. Long-Time Behaviour of Fast Power Reactors

### General Remarks

The investigations of the long-time behaviour of fast power reactors include the overall time behaviour caused by the fuel burn-up and the fuel management. In this connection we distinguish two sets of problems:

1. the time behaviour during one reloading cycle or only a few cycles, which is usually the object of fuel burn-up studies, taking into consideration  $\Delta k_{\text{eff}}$ , power distribution, utilization of the fuel etc.
2. the time behaviour of the isotopic composition of the fuel during a very great number of reloading cycles caused by recycling the Pu (externally closed fuel-cycle).

### Cyclic Loading Schema of the Fuel Elements

The time behaviour for a burn-up cycle, i.e. between two successive changes of fuel elements, is studied on the basis of a cyclic schema for single groups of fuel elements in the core [31]. These groups are defined as follows:

- 1) n fuel elements of the same radial region r, which are not necessarily adjacent, are combined to a group.
- 2) In every reloading operation only one element of a group is changed.
- 3) The difference between the burn-up of the single elements of a group is 1/n of the maximal flux time, (these states should be realized or approximated by appropriate fuel management before the steady operation of the reactor).
- 4) The burn-up of the n elements of a group after discharging one element and restoring the "fresh" state of a group (i.e. the averaged burn-up of the group is a minimum) is approximately  $A_1=0, A_2 = \frac{1}{n} A_{\max}, \dots, A_n = \frac{n-1}{n} A_{\max}$ , ( $A_{\max}$  = maximum fuel burn-up in MWd/t). Then the averaged burn-up of a group is given by

$$\bar{A}_{\min} = A_{\max} \frac{n-1}{2n}$$

- 5) The burn-up of the elements of a group before changing an element (the group is "burnt out", i.e. the averaged burn-up is a maximum) is approximately  $A_1 = \frac{1}{n} A_{\max}, A_2 = \frac{2}{n} A_{\max}, \dots, A_n = A_{\max}$ . Then the averaged burn-up of a group is

$$\bar{A}_{\max} = A_{\max} \frac{n+1}{2n}$$

- 6) In a reloading operation always this element must be exchanged which has just reached the maximum burn-up  $A_{\max}$  or which would exceed  $A_{\max}$  during the next burn-up cycle.

The burn-up cycle of the groups is shown in fig. 3 schematically for n=3. (The circles symbolize the fuel elements of a group, the numbers denote the burn-up of the elements immediately after and before changing an element respectively.)

The real burn-up state of the core varies between the two states  $\bar{A}_{\min}$  and  $\bar{A}_{\max}$ . For calculating  $\Delta k_{\text{eff}}$  between the two extreme states, only the difference

$$\Delta \bar{A} = \frac{1}{n} A_{\max}$$

is important. Thus, by establishing groups of fuel elements in the specified manner the influence of the burn-up is reduced to 1/n of the maximum burn-up of the single fuel elements.

The maximum  $\Delta k_{\text{eff}}$  and the reloading cycle  $\Delta t$  (corresponding to  $A_{\text{max}}$  and the power of the reactor) are determined by the choice of  $n$ .  $\Delta t$  is the reloading cycle for the central group at the position of the maximum neutron flux and hence for all groups in the whole core.  $\Delta t$  is given by  $T/n$ ,  $T$  = life-time of the fuel element in the centre of the core.

The utilization of the fuel can be optimized by establishing several radial reloading-zones with various values for  $n$ . The influence of these space-dependent  $n$  on  $\Delta k_{\text{eff}}$  and the utilization of the fuel is shown in table 3 for two reloading zones.

The results given in this table are calculated for the sodium cooled fast reactor study Na-1 with the Russian group set. The establishment of a second zone with  $n = 4$  has no influence on  $\Delta k_{\text{eff}}$ , but improves the utilization of the fuel from 79 % up to 85 %.

The previous investigations of the time behaviour for one or a few burn-up cycles are carried on and completed using a one-dimensional burn-up code [32] taking into consideration the internal fuel management of the core and the blanket. In this code the z-dependent burn-up is regarded by an appropriate approximation. The purpose of further investigations is to find out appropriate design burn-up and operation data which optimize the behaviour of fast reactors.

#### Long-Time Behaviour of the Fuel Composition - The Steady State of Pu-Composition

The long-time variations of the isotopic composition of the fuel in fast reactors with externally closed fuel cycle are caused by the burn-up of the fuel and the external fuel management. In the course of time the isotopic composition of the fuel becomes stationary as is shown in [33]. The relative isotopic composition of the plutonium is the so-called  ${}^{\infty}\text{Pu}$ .

For the external fuel management, the following cases are considered:

Case1: Common reprocessing of the core and blanket elements and drawing off the breeding gain after reprocessing (Case C in [33]),

Case 2: separate reprocessing of the core and blanket elements and drawing off the breeding gain out of the blanket (Case B in [33]).

For both cases there are different values for the  $P_{\infty}$ . Table 4 shows the  $P_{\infty}$  for a sodium-cooled fast reactor and a steam-cooled reactor of 1000 MWe [34].

It was found that  $P_{\infty}$  values are not very sensitive to the burn-up and the loading scheme [35].

The study of the long-time behaviour of the fuel-composition and all the problems connected with it is continued by considering the details of the external fuel management and the blanket management, which may be important for the variation of the fuel composition [36]. In order to restrict the numerical efforts to a reasonable scale, which is necessary to study these problems, time-averaged spectra for core and blanket regions are taken as a basis. The criticality of the reactor is approximately ensured by appropriate conditions for determining the new fuel composition after reprocessing. Also the breeding gain (positive or negative) is determined by these conditions.

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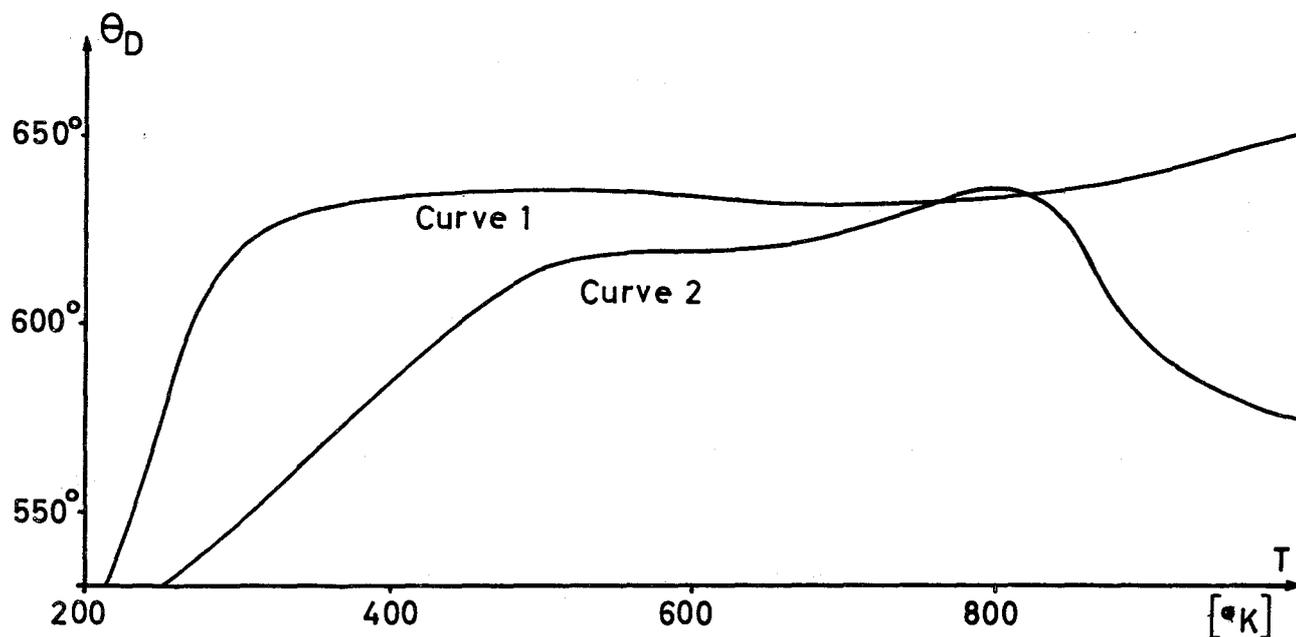


Fig. 1 Debye temperature  $\Theta$  as a function of temperature

Curve 1: Fictitious Debye temperature used for calculating a temperature - depending correction factor for DC without solid-state properties to get DC with solid-state properties .

Curve 2 : Debye temperature which describes the specific heat.

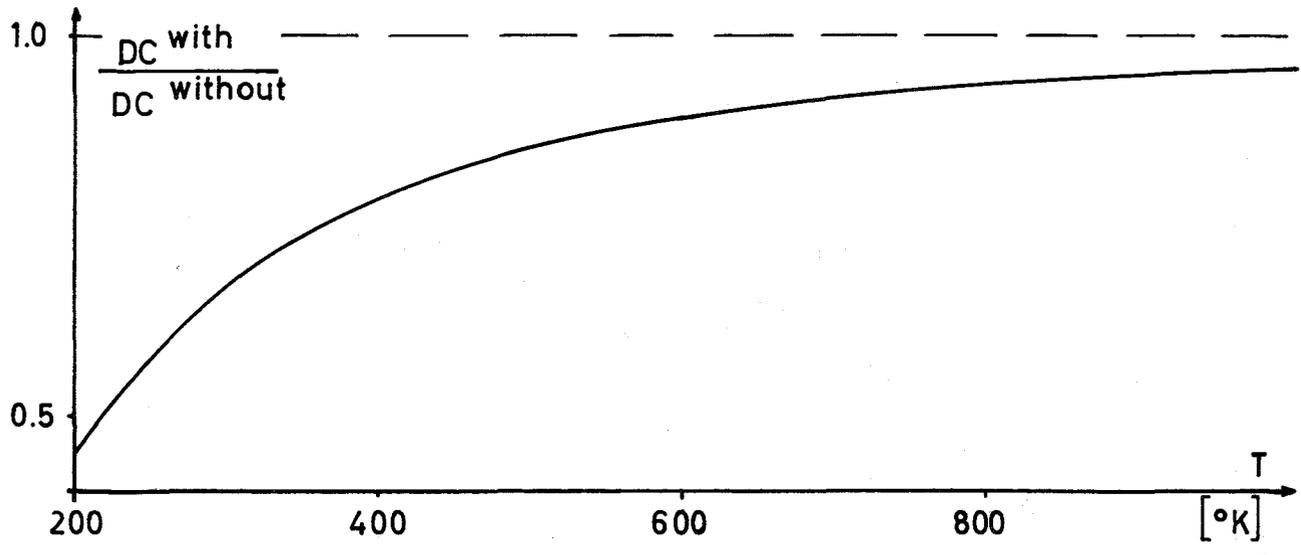
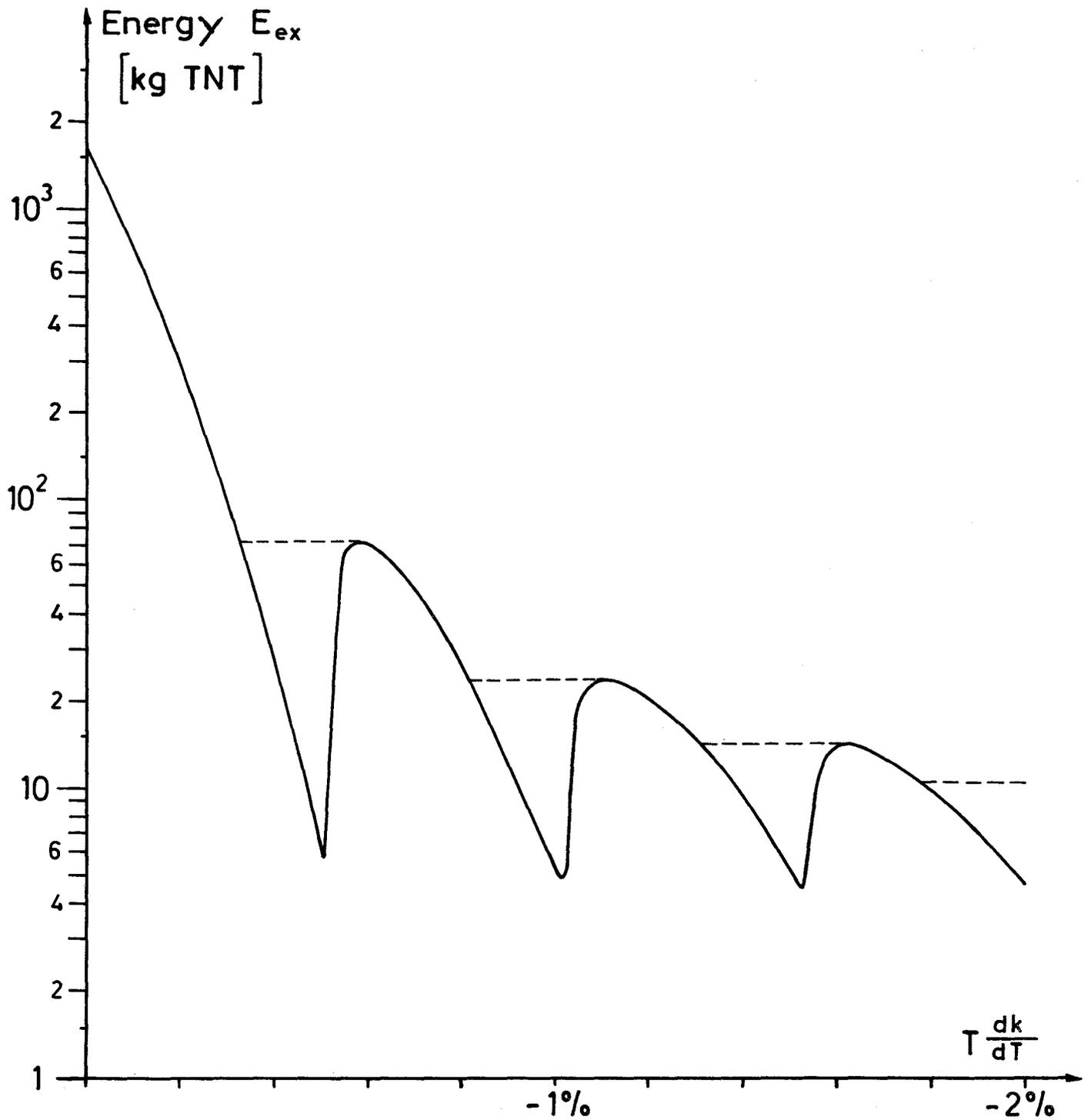


Fig. 2 Solid state correction factor for the DC



**Fig. 3** Destructive energy in kg TNT for excursions with reactivity input  $100 \text{ } \$/\text{sec}$ ,  $\Phi_0 = 10^{10} \text{ sec}^{-1} \text{ cm}^{-2}$ . Delayed neutrons suppressed.

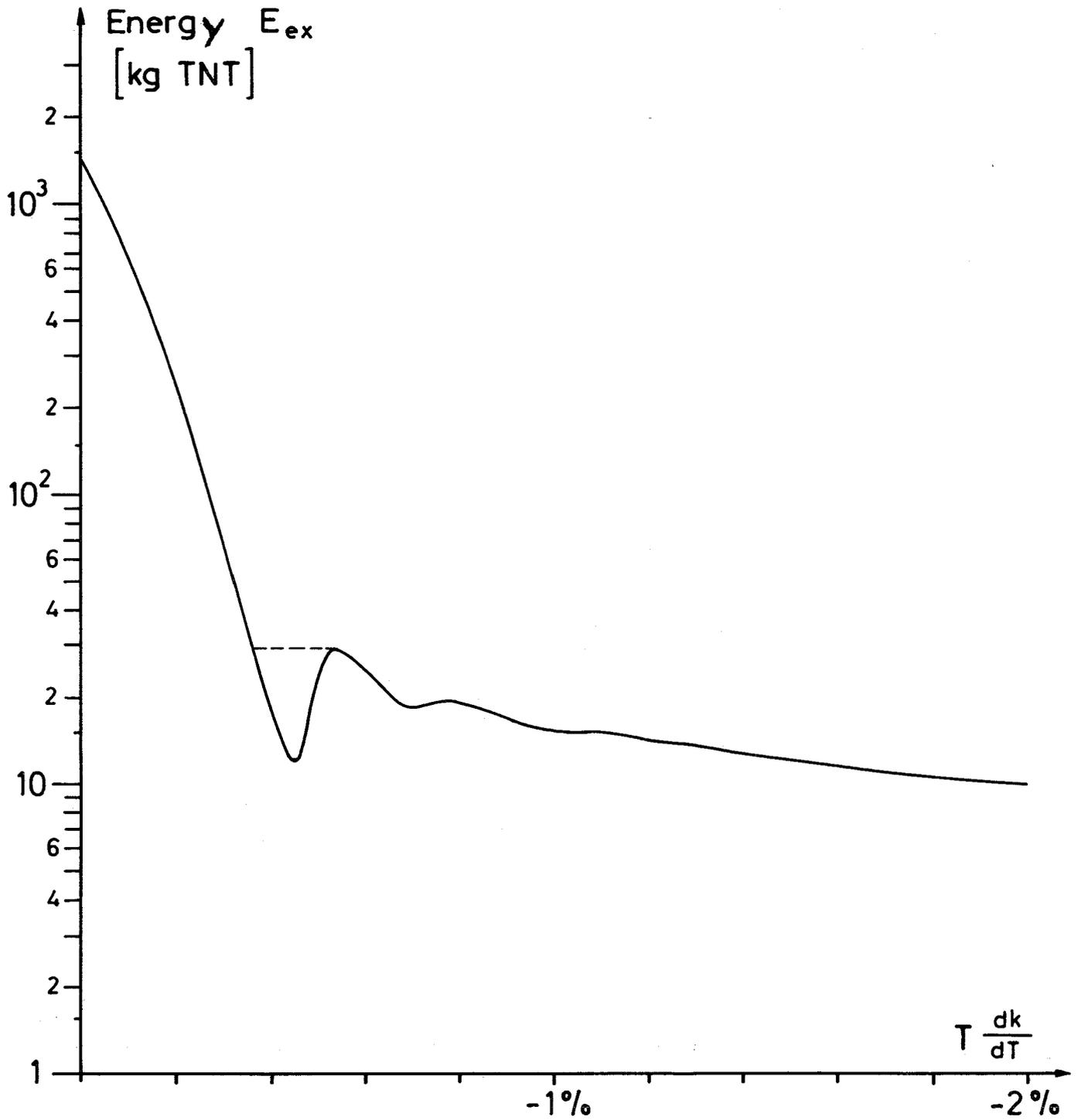


Fig. 4 Destructive energy in kg TNT for excursions with reactivity input  $100 \text{ } \$/\text{sec}$ ,  $\phi_0 = 10^{10} \text{ sec}^{-1} \text{ cm}^{-2}$ . Delayed neutrons included.

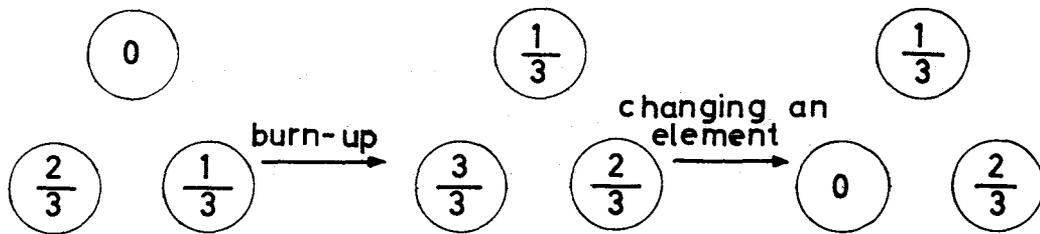


Fig. 5 Burn-up cycle for a group of three fuel elements

Table 1 Influence of two different weighting spectra in core and blanket.

	core weighting spectrum only	different core and blanket spectra
$k_{eff}$	$0.9996 \pm 1 \cdot 10^{-4}$	$0.9994 \pm 1 \cdot 10^{-4}$
$\Delta k_{Na-Void}$ (50% voided)	$0.0158 \pm 2 \cdot 10^{-4}$	$0.0156 \pm 2 \cdot 10^{-4}$
$BR_{int}$	1.073	1.071
$BR_{ext}$	0.394	0.395
$BR_{tot}$	1.467	1.466

**Table 2:** Destructive energy in kg TNT for excursions with a reactivity input of 100 \$/sec. Effects of delayed neutrons included.

$\beta \frac{dk}{dT}$ %	$\phi_0 \text{   cm}^{-2} \text{ s}^{-1} \text{  }$			
	$10^7$	$10^{10}$	$10^{13}$	$10^{16}$
0	1730 <sup>M</sup>	1560 <sup>M</sup>	1300 <sup>M</sup>	990 <sup>M</sup>
-0.1	880 <sup>M</sup>	730	560	350
-0.25	240	170	98	33
-0.5	72	71	74	30
-0.75	60	37	26	23
-1.	22	24	21	18
-2.	11	11	6	7

**Table 3:** The dependence of  $\Delta k_{eff}$  and the utilization of the fuel on various reloading zones. n=3 means a variation of fuel burn-up between 33 000 and 66 000 MWd/to.

n	$\Delta k_{eff}(\%)$		Utilization of the fuel (%)	Reloading cycle $\Delta t$ [d]
	without fission products	with fission products		
Zone I Zone II n=3	0,76	-	79	196
Zone I: n=3 Zone II: n=4	0,74	2,5	85	

**Table 4:** Stationary Pu-Composition  $Pu^{\infty}$  (atom %) for 1000 MWe Breeders

Reactor	Na-Cooling		H <sub>2</sub> O-Steam Cooling	
	1	2	1	2
Pu 239	76,8	60,8	74,0	63,7
Pu 240	20,3	32,2	22,7	30,5
Pu 241	2,4	4,8	2,3	3,4
Pu 242	0,5	2,2	1,0	2,4

Table 5: Properties of investigated Systems No. 8 and 9 of |6|

	Core		Blanket
	No. 8	No. 9	No. 8 and 9
Material	Concentration in Atoms/cc ( $\times 10^{-24}$ )		
Fe	0.00806	0.00806	0.00806
Cr	0.00211	0.00211	0.00211
Ni	0.00083	0.00083	0.00083
Na	0.0123	0.0123	0.0069
O	0.0144	0.0144	0.024
U238	0.00648	0.0063	0.012
Pu239	0.00072	0.0009	0.00012
Pu240	0.00036	0.00045	0.
Fiss.Prod.	0.00072	0.00072	0.
$R_{\text{Core}}^{\text{crit}}$  cm	233.7	97.76	45. (thick)
Group Set	KFK 26-10	KFK 26-10	KFK 26-10