KERNFORSCHUNGSZENTRUM

KARLSRUHE

April 1967 KFK 559

Institut für Experimentelle Kernphysik

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II. Physikalisches Institut der Universität Heidelberg
und Institut für Experimentelle Kernphysik
auf dem Kernforschungszentrum Karlsruhe

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Abstract:

It is pointed out that the tables of electron radial wave functions by Bhalla and Rose are incorrect as far as positons are concerned.

We infer from recent papers [1,2] that there is considerable interest in accurate electron radial wave functions (ERWFs) for positon beta decays. In particular, allowed Fermi transitions give, if they are superallowed, important information on the universal Fermi interaction [1] or, if they are isospin forbidden, on the isospin impurity of the nuclear states [2]. In both of these applications one needs rather accurate ERWFs in order to arrive at reliable conclusions. For these reasons it seems now necessary to point out that the tables of ERWFs by Bhalla and Rose [3-5], which are widely used, do not agree with our (unpublished) results as far as positons are concerned. In this note we give an example of our results in order to demonstrate the disagreement in a quantitative way. Also we suggest the reason why the results of Bhalla and Rose might be incorrect, and discuss the evidence we have for the reliability of our results.

The ERWFs under consideration correspond to a uniformly extended nuclear charge distribution of radius ρ , which is not screened by atomic electrons. Thus we are concerned with a well defined problem, and results obtained by different authors should be identical. In this connection it is necessary to know that Bhalla and Rose [3-5], although they give the value 0.4285 for the radius constant (which relates the nuclear radius ρ to the mass number A), actually [6] used a slightly smaller value of approximately 0.4276. If this fact is allowed for, our results agree with the tables of Bhalla and Rose in the case of negatons. For positons, however, the wave functions f and g deviate as is demonstrated in table 1, whereas the ratios f/g and the phase shifts Δ still agree. Therefore it is the normalization which is incorrect.

It might be surprising that the normalization can be wrong for positons while it is correct for negatons, since formally the only difference between these two cases is the sign of the nuclear charge Z. However, Bhalla and Rose need sign conventions in order to derive their [3] eqs. (18b) and (19). Equation (18b) implies the choice

$$\exp[i(\eta-\bar{\eta})] = \mathrm{Sign}(Z)\,(y+i\gamma)/|\sqrt{\gamma^2+y^2}|$$

which is in accordance with their explicit statements concerning the phases of η and $\bar{\eta}$. But eq.(19) implies the choice

$$\exp[i(\eta-\bar{\eta})] = (y+i\gamma)/|\sqrt{\gamma^2+y^2}|$$

which is contradictory in the case of positons corresponding to $Z < 0$. This might give a plausible explanation for the observed discrepancy.

In our calculation the normalization is accomplished in a different way which does not depend on sign conventions nor explizitly on the sign of Z. Thus, since we obtain correct results in the case of negatons, our results for positons are also expected to be correct. But we have even stronger evidence that our results are reliable. Using a drasticly different method, namely the general method developed for the screened field [7], we obtain identical results both in the case of negatons and of positons.

ERWFs for specific cases can be supplied on request.

The calculations were performed on the Siemens 2002 computer at Heidelberg.

Z (A)	p	Ref.	f_1	g_1	f_1/g_1	$\tan\Delta_1$	f_{-1}	g_{-1}	f_{-1}/g_{-1}	$\tan\Delta_{-1}$
-26 (56)	1.0	B+R	380783	253264	150350	100844	668681	897678	744901	114650
		Bü	380765	253248	150353	100844	668518	897453	744906	114647
	6.0	B+R	429430	205286	209186	272140	282437	506407	557727	228038
		Bü	428518	204834	209203	272132	281592	504914	557703	227981
-90 (228)	1.0	B+R	275241	712205	386463	588269	138055	507689	271928	202384
		Bü	274496	710278	386463	588278	136820	503143	271931	202392
	6.0	B+R	340055	766965	443377	279122	942027	394814	238600	607019
		Bü	323908	730532	443386	279126	883843	370433	238597	607008

Table 1

Comparison of some results from Bhalla and Rose (B+R) and from the present work (Bü). The notation of B+R is used, signs and powers of ten are omitted. The very small deviations of f/g and tanΔ are probably due to the fact that our radius constant 0.4276 is not quite exactly the same as that of B+R. (Also, since some parts of the calculation by B+R are performed to an accuracy of 10^{-6} only, their final results, in particular tanΔ, are not always expected to be accurate to six digits). The values of f and of g, however, show a significant discrepancy.

References

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