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von

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AN ANALYSIS OF ERRORS INVOLVED IN THE SUB-PROMPT CRITICAL TRANSIENT EXPERIMENTS IN SEFOR

I. A Proposed Analysis of the Sub-Prompt Critical Transient Experiments in SEFOR

In SEFOR part of the Doppler coefficient measurements will be carried out by means of subprompt critical excursions induced by a step-like reactivity insertion. The power transient curve allows a fairly direct determination of the Doppler coefficient as shown by one of the authors in KFK 153.[(1)] There were two approaches to the analysis as proposed in Ref. 1. The subject of this paper is to investigate the errors introduced into this analysis by the errors in the measurement of relative and absolute power and by the error in the delayed neutron data. The investigation of the sensitivity of the error in the Doppler coefficient to the size of the impressed transient answers the question of optimal transient size.

Analysis of the power transient curves results in the total energy coefficient, which contains both the Doppler coefficient and the fuel expansion coefficient. Possible errors coming from an uncertainty in the separation of these two coefficients are not discussed here, and it is assumed that the Doppler effect dominates the prompt reactivity feedback.

In the intended subprompt critical transients, the power rises rapidly to the prompt jump level, denoted here by ϕ_o. Following this, two slower processes, caused by the off-equilibrium value of the flux, become important. These are (1) the prompt negative energy coefficient, which tends to decrease the power level, and (2) the increasing delayed neutron source resulting from the prompt jump power level.

Both tendencies may be brought into near balance such that the flux remains constant for a certain period of time ($\sim$0.1 sec) following the prompt jump. For such a transient the (Doppler) energy coefficient (γ) is given simply by

$$\gamma = \frac{\overline{\lambda}}{\phi_o} \tag{1a}$$

where $\overline{\lambda}$ is the average decay constant of the delayed neutron precursors and ϕ_o the prompt jump power level.

Equation (1a) is a special case of the more general formula which holds for out of balance excursions:

$$\gamma = \frac{\overline{\lambda}}{\phi_o} - \frac{\phi_1 \, \Phi_o}{(\phi_o - \Phi_o) \, \phi_o^2} \tag{1b}$$

where ϕ_1 is the time slope of the power after the prompt jump and Φ_o denotes the initial power. The formulas (1a) and (1b) represent the first type of analysis proposed in Ref. 1.

The advantages of this type of analysis are obvious. The relationship between the Doppler coefficient and the experimental data is both simple and direct. However, since, at the most, only the prompt jump power level ϕ_o and the slope ϕ_1 are used from the total transient power curve, maximum advantage is not taken of the data available and inaccuracies and difficulties arise. In particular, the prompt jump power value and the power slope after the prompt jump may be difficult to determine for out-of-balance excursions. (see KFK 153[1]).

This can be improved by turning to the balanced excursion (1a). However, achieving the balance depends upon having an accurate and fine adjustment in the input reactivity, i.e., the FRED, device in SEFOR.

To reduce these inaccuracies a second type of experimental analysis was proposed in KFK 153. In this analysis the full power reading is used and inserted into the "inversed" kinetics equation, giving

$$R(t) = \Delta k(t) - \left[1 + \frac{\ell\dot{\phi}(t)-Q(t)}{\beta\phi(t)} \right] \tag{2}$$

where $R(t)$ = Doppler feedback reactivity

$\Delta k(t)$ = reactivity input (from FRED device in SEFOR)

ℓ = prompt neutron lifetime

$\phi(t)$ = power as a function of t

$Q(t)$ = power source of delayed neutrons

β = delayed neutron fraction

Equation (2) can be recognized as the familiar form

$$\Delta k_{Dop} = \Delta k_{Fred} - \Delta k_{Net}. \tag{3}$$

Considering equation (3), we see that for sub-prompt critical transient experiments the transient power levels are relatively low and the time available for analysis may be short. In this case the feedback reactivity would be only a fraction of Δk_{Fred}, and the desired quantity is the difference of two large, nearly equal measured quantities. Therefore, it is to be expected that the error in Δk_{Dop} will be large if a measured value of Δk_{Fred} is used. In KFK 153[1] this problem was circumvented by taking advantage of the fact that the Doppler coefficient will be essentially constant over the important time interval. (An examination of this assumption shows that for the worse case and a $1/T_f$ Doppler coefficient, the variation is only a few percent in 0.1 sec.) An analysis is proposed in which Δk_{Fred} is treated as a parameter, being adjusted until the curve of $\gamma(t)$ vs. time is a constant, where the function $\gamma(t)$ is given by

$$\gamma(t) = \frac{R(t)}{\int_0^t (\phi(t)-\Phi_o)dt} \tag{4}$$

and Φ_o is again initial steady state power level. This procedure appears to be advantageous in reducing the errors in the resulting Doppler coefficient, and the analyses described above have been studied to determine the nature of the errors involved and the requirements for achieving minimum error.

II. Statistical Error Analysis

The problem which has been centered upon here is that of assessing the importance of the balanced transient and whether or not the errors in the Doppler coefficient resulting from the proposed analyses will be a minimum for this transient. Obviously, if equation (1a) is used, the balanced transient must be achieved. While equations (2) and (4) apply for any transients, the question of minimizing the error still remains. The following error analysis deals with this question.

1. Simplification of Doppler Expression for Analysis of Errors

For the sake of this error analysis we introduce the same simplifications which were used in KFK 153 to derive the first method of analysis formulas (1a) and (1b) . These are setting $\ell\dot{\phi}(t) = 0$ and approximating $\Delta k_{Fred}(t)$ as an ideal step. Following these simplifications, equations (2) and (4) may be expanded in a Taylor series about the prompt jump time. Then $\gamma(t)$ becomes

$$\gamma(t) = \frac{1}{t} \cdot \frac{\Phi_o - \phi_o(1-\Delta k_{Fred})}{\phi_o(\phi_o-\Phi_o)\left[1+\sum_{\nu=1}^{\infty} a_\nu t^\nu\right]} + \left(\frac{\overline{\lambda}}{\phi_o} - \frac{\phi_1 \Phi_o}{\phi_o^2(\phi_o-\Phi_o)}\right) \tag{5}$$

with

$$\phi_o(\phi_o-\Phi_o) \left[1 + \sum_{v=1}^{\infty} a_v t^v \right] = \frac{1}{t} \phi(t) \int_o^t (\phi(t) - \bar{\Phi}_o)\, dt$$

Under the given assumptions terms of higher order in t do not appear. Additional higher order terms would appear in equation (5) if the experimental errors had been introduced in equation (4).

The first term of equation (5) is the term which is reduced to zero by choosing the appropriate value of Δk_{Fred} through a trial and error process. Thus the Doppler coefficient is given primarily by the constant term

$$\gamma = \frac{\bar{\lambda}}{\phi_o} - \frac{\phi_1 \Phi_o}{\phi_o^2 (\phi_o-\Phi_o)} \tag{6}$$

which will be used as a basis for the investigation of the propagation of errors. Note that this procedure reduces equation (5) to equation (1b). The question of whether or not a more realistic model will introduce larger errors is investigated in Section III.

2. Propagation of Errors

In order to analyze equation (6) in terms of statistical errors, two sources of errors in the SEFOR transient power measurements were recognized. First there is the error in the calibration of the initial power level, and, second, there is an error in the relative transient power levels. The first error is in reality systematic with respect to each of the transient experiments, but in determining the final Doppler energy coefficient measurement it introduces an additional statistical error. This calibration error can be expressed wholly in the initial power level, Φ_o, so setting

$$\psi_o = \frac{\phi_o}{\Phi_o}$$

and

$$\psi_1 = \frac{\phi_1}{\Phi_o}$$

makes equation (6) become

$$\gamma = \frac{1}{\Phi_o}\left[\frac{\overline{\lambda}}{\psi_o} - \frac{\psi_1}{\psi_o^2\,(\psi_o-1)}\right] = \frac{F}{\Phi_o} \tag{7}$$

and the factor F contains only errors in the relative power readings. The statistical error in the energy coefficient is then

$$\frac{\delta\gamma}{\gamma} = \sqrt{\left(\frac{\delta\Phi_o}{\Phi_o}\right)^2 + \left(\frac{\delta F}{F}\right)^2}\quad , \tag{8}$$

where the first term is the calibration error, and the second term must be generated from equation (7). This results in

$$\delta F = \left\{\left[\frac{\delta\overline{\lambda}}{\psi_o}\right]^2 + \left[\frac{\overline{\lambda}}{\psi_o^2} - \frac{\psi_1(3\psi_o^2-2\psi_o)}{\psi_o^4\,(\psi_o-1)^2}\right]^2 (\delta\psi_o)^2 + \frac{(\delta\psi_1)^2}{\psi_o^4\,(\psi_o-1)^2}\right\}^{1/2} \tag{9}$$

for a single transient experiment, where $\delta\psi_o$ is the error in the relative transient power measurement. The error in $\overline{\lambda}$ was evaluated to be about 4%, by propagating errors according to the equation for $\overline{\lambda}$,

$$\overline{\lambda} = \frac{\sum_m \sum_k S_m \, (\nu\beta_k)_m \, \lambda_{k,m}}{\sum_m \sum_k S_m \, (\nu\beta_k)_m}$$

where S_m is the fraction of power in fuel isotope m, $(\nu\beta_k)_m$ is the delayed neutron fraction per fission in group k for isotope m and $\lambda_{k,m}$ is the associated decay constant. The computation of $\delta\overline{\lambda}/\overline{\lambda}$ is described in Appendix A.

Equation (9) was evaluated for transients starting from 3 MW to 20 MW in SEFOR and for Doppler coefficients corresponding to computed values and approximately 1/2 of the computed values. In equation (9) the statistical error in the transient power reading is given by $\delta\psi_o$. There is an associated error, $\delta\psi_1$, in the slope of the transient power curve. In evaluating the magnitude of the error in F, a reasonable value for $\delta\psi_o/\psi_o$ can be estimated, since this is due primarily to noise in the instrumentation. The associated error $\delta\psi_1$, however, is not so easy to evaluate, so a rather pessimistic relationship between $\delta\psi_o$ and $\delta\psi_1$ was assumed. Consider that the error in ψ_o forms an error band $2\delta\psi_o$ wide about the correct values of $\psi(t)$ from $t \cong 0$ to $t = 0.1$ sec after the prompt jump. This is graphically presented in Fig. 1. A pessimistic estimation of the associated error in the slope would be given if the correct power curve $\psi(t)$, had a value of $\psi(t) - \delta\psi_o$, at $t \cong 0$, and $\psi(t) + \delta\psi_o$ at $t = 0.1$ sec, where $\psi(t)$ is the measured power curve. Assuming that the relative error is a linear function of time,

$$\begin{aligned} \psi(t) &= \psi(t) + \delta\psi(t) \\ &= \psi(t) \; [1 + \varepsilon(t)] \qquad (10) \\ &= \psi(t) \; [1 + \varepsilon_o + \varepsilon_1 \, t] \;, \end{aligned}$$

and

$$\psi_1(t) = \psi_1(t) + \delta\psi_1(t)$$

$$= \frac{d\psi(t)}{dt} \tag{11}$$

$$= [1 + \varepsilon_o + \varepsilon_1 t]\,\frac{d\psi(t)}{dt} + \psi(t)\,\frac{d\varepsilon(t)}{dt}\,.$$

Then

$$\delta\psi_1(t=0) = \delta\psi_1$$

$$= \psi_1\,\frac{\delta\psi_o}{\psi_o} - \frac{\psi_o\delta\psi_o}{.1}\left(\frac{1}{\psi_o} + \frac{1}{\psi(t=.1)}\right) \tag{12}$$

If $\psi_o(t)$ is assumed to be nearly constant from ≈ 0 to 0.1 sec, equation (12) gives

$$\delta\psi_1 = \psi_1\left[\frac{\delta\psi_o}{\psi_o}\right] -20\,\delta\psi_o. \tag{13}$$

For use in these evaluations it was possible to take the theoretical value of ψ_o,

$$\psi_o = \frac{1}{1 - \Delta k} \tag{14}$$

and to back out the value of ψ_1 from equation (7) since the value of γ which was used was known.

3. <u>Magnitude of Errors Vs. Transient Size</u>

The error in F was computed from equation (9) and the value of F from equation (7) for a variety of transient sizes, 2 initial power levels, and 2 magnitudes of Doppler coefficients. The initial power levels considered were 3 and 20 MW, the latter representing the maximum SEFOR steady state power level. The two Doppler coefficients correspond to ($T_f \frac{dk}{dT_f}$) values of .004 and .0083, where dk/dT_f is given by

$$\left(\frac{dk}{dT_f}\right)_{Dop} = - \frac{\text{Constant}}{T_f} . \tag{15}$$

Figures 2 through 5 show the values of dF/F obtained for these conditions. These errors characteristically go through a shallow minimum after dropping sharply from high values at smaller transients and rise to an asymptotic value at prompt criticality. The steep rise toward the lower transients is due primarily to the error in the delayed neutron parameters (first term of equation (9)*) above the point of minimum error, $\delta\overline{\lambda}$ does not contribute a significant amount to δF, nor does the error in the slope of the transient power, $\delta\psi_1$, based on the assumptions of equation (13). These points can be seen by substituting the expressions for ψ_1 and $\delta\psi_1$ from equations (7) and (13) into equation (9) and collecting into terms of a_n/ψ_o^n, for n = 0 to ∞. A good approximation to the curves in Figures 2 through 5 is obtained by dropping all terms of order n = 3 and greater, leaving δF as

$$\delta F = \left\{ \frac{\delta\overline{\lambda}^2}{\psi_o^2} + \left[10\,\gamma^2\Phi_o^2 + \frac{6\,\gamma^2\phi_o^2 - 14\,\overline{\lambda}\gamma\Phi_o}{\psi_o} + \frac{5\overline{\lambda}^2 - (10\overline{\lambda} - 40)\,\gamma\Phi_o + 7\gamma^2\Phi_o^2}{\psi_o^2} \right] \left(\frac{\delta\psi_o}{\psi_o}\right)^2 \right\}^{1/2} \tag{16}$$

Since the value of ψ_o increases toward ∞ as Δk approaches 1, the asymptotic value of $\frac{\delta F}{F}$ is simply

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* Compare also equations (32), (33) in KFK 153, where the same fact was discussed for a possible reduction of the influence of $\delta\overline{\lambda}$.

$$\left(\frac{\delta F}{F}\right)_{asmp} = \sqrt{10}\,\frac{\delta\psi_o}{\psi_o} \,. \tag{17}$$

The error in ψ_1 contributes only 1/10 of the factor 10 in this equation. In the term involving ψ_o^{-1} in equation (16) the error in ψ_1 enters into the factor 14, and then only 1/7 of this factor for the pessimistic assumptions made concerning the magnitude of $\delta\psi_1$. While $\delta\psi_1$ influences the ψ_o^{-2} term to a greater extent, this term itself does not become important unless the transient size is below the point of minimum error in each of Figures 2 through 5. Therefore if the transient size is held at or above the minimum error transient, the error curves are essentially independent of uncertainties due to the simplifying assumptions made to estimate the magnitude of $\delta\psi_1$.

From equation (16) it can also be seen that the error contribution from the delayed neutron parameters, $\delta\bar{\lambda}$, varies with $1/\psi_o^2$. Therefore, increasing the size of the transients decreases this contribution such that, depending upon the magnitudes of the statistical errors in the transient power measurement, an even larger uncertainty in the delayed neutron parameters may not have a large effect on the accuracy of the Doppler coefficient.

4. Minimum Error Transient

The minimum error transient may be determined from equation (16).

Minimizing δF^2 gives

$$\psi_{o,min} = \frac{2\left[\delta\bar{\lambda}^2 + C_2\left(\frac{\delta\psi_o}{\psi_o}\right)^2\right]}{-C_1\left(\frac{\delta\psi_o}{\psi_o}\right)^2} \tag{18}$$

where

$$C_1 = 6\gamma^2\Phi_o^2 - 14\bar{\lambda}\Phi_o$$

and

$$C_2 = 5\bar{\lambda}^2 - (10\bar{\lambda} - 40)\ \gamma\Phi + 7\gamma^2\Phi_o^2 \quad .$$

The magnitude of $(\delta F/F)_{min}$ can then be computed by substituting (18) into (16) and (7). Values of $\psi_{o,min}$ and $(\delta F/F)_{min}$ have been computed for the cases represented in Figures 2 through 5. These results are given in Table I along with asymptotic errors $(\delta F/F)_{asmp}$.

TABLE I

Minimum Error Transients

Case	Initial Power (MW)	$T_f \frac{dk}{dT_f}$	$\frac{\delta\psi_o}{\psi_o}$	$\psi_{o,min}$	$\left(\frac{\delta F}{F}\right)_{min}$	$\left(\frac{\delta F}{F}\right)_{asmp.}$
1	3	.004	.03	77	.055	.095
			.01	199	.027	.032
2	3	.0083	.03	42	.061	.095
			.01	101	.028	.032
			.003	775	----	.009
3	20	.004	.03	29	.067	.095
			.01	65	.028	.032
			.003	474	----	.009
4	20	.0083	.03	19	.076	.095
			.01	37	.029	.032
			.003	238	----	.009

A comparison of the values given in Table I with the curves in Figures 2 through 5 show quite good agreement, both with respect to $\psi_{o,min}$ and $(\delta F/F)_{min}$.

The values of $\psi_{o,min}$ in Table I have been plotted in Figure 6. These curves define a minimum error transient for the SEFOR sub-prompt critical transient experiments for the given conditions of the initial reactor power level, expected magnitudes of the Doppler coefficient and statistical error in the transient power reading. As long as the transients are above this minimum size, the error in the Doppler coefficient should not be greater than the values given in Fig. 7.

5. Minimum Permissible Transients

In cases where the error in the transient power measurement is small, (<.01), the error in $\bar{\lambda}$ dominates to high values of Δk. As seen from Figures 2 through 5, the minimum is then very broad and taking the curves in Fig. 6, which are generated for the minimum error transient, as the lowest permitted transient imposes unnecessary restrictions on the experiments. It is better therefore to define a minimum transient size on the steep slope to the left of the minimum error transient and approximate it from

$$\delta F \cong \frac{\delta\bar{\lambda}}{\psi_o} \qquad (19)$$

Then

$$\psi_{o,limit} = \frac{\delta\bar{\lambda}}{F} \times \frac{1}{\frac{\delta F}{F}} \qquad (20)$$

gives a minimum limit to the transient size for a given statistical error in the transient power measurement.

As an example, suppose that $\delta F/F = .05$ is the maximum allowable error in F. Then for the cases of Figures 2, 3 and 4, the values of $\psi_{o,limit}$ are those given in Table II. The corresponding errors in F are given also. These results show that

TABLE II

Minimum Permissible Transients and Associated Errors

Case	Initial Power	$T\frac{dk}{fdT_f}$	$\psi_{o,\text{limit}}$	$\delta F/F$: $\frac{\delta\psi_o}{\psi_o}$ = .03	.01	.003
1	3	.004	53.2	.053	.05	.05
2	3	.0083	25.6	.069	.052	.05
3	20	.004	15.4	.10	.058	.05

the approximate expressions (19) and (20) apply much better when $\delta\psi_o/\psi_o$ is small.

III. Errors in Inversion-Type Analysis in "Paper-Experiments".

The analysis described above was based entirely on the constant term of equation (5). The first term can be adjusted to zero by varying the value of Δk_{Fred}. But it was assumed that higher order terms vanish or are at least unimportant for the error of this analysis. If one introduces a statistical error into the transient power measurement it might be possible that these terms will substantially[2)] add to the expected errors generated above for the constant term only. More important, they could make the error in the Doppler energy coefficient more sensitive to the transient size and thus require fine adjustment of FRED. Because these terms are rather complicated functions of ψ_o, γ, Φ_o, $\overline{\lambda}$ and the higher derivatives of ψ_o, it was not practical to attempt a statistical error computation of them. A technique of using paper experiments with appropriate errors introduced into the transient power curves was adopted instead. This technique has the additional advantage of including the effects of the $\ell\dot{\phi}(t)$ term, which was ignored in the above analysis.

1. Procedure for Paper Experiments

The procedure used for these paper experiments is the following:

(1) The curve of SEFOR power vs. time is computed for a given set of reactor conditions using a point kinetics code.

(2) The resulting power curve is modified to represent an experimental curve including a specific type of error.

(3) The modified power curve is analyzed, using the inverse kinetics equation, to determine the value of the Doppler coefficient.

(4) The resulting Doppler coefficient is compared with the "correct" value which was used in step 1 in determining the transient curve in order to determine the amount of error resulting from the modification in step 2.

(5) Steps 1 through 4 are repeated for various modifications and reactor conditions.

The procedure outlined above is not useful for computing the absolute magnitudes of the statistical errors because a truly statistical error cannot be expressed in the form of specific modifications to the power curve. It is useful, however, as a supplement to the analysis of the constant term discussed in Section II since it must include, by its nature, errors which have possibly been excluded in the previous treatment due to simplifying assumptions. If these paper experiments show that for a variety of modifications to the power the error in the Doppler coefficient is no more sensitive to the transient size than it was for the constant term error, the effects of time terms can be assumed to be small.

2. Results of Paper Experiments

The unmodified power curves were computed from the kinetics equations assuming a step in reactivity of less than 1 dollar. Seven delayed neutron groups were used, with the normal sixth groups for U-238 and Pu-239 separated because of a substantial difference in the decay constants. The data of Keepin[3] were adapted to the seven groups.

Modifications to the power curve took the form

$$\phi_{mod}(t) = \phi_{unmod}(t) \left[A_o + A_1(t-t_o) \right] + A_2. \qquad (20)$$

Two separate modifications have been tested. These are

Modification 1 $\quad \phi_{mod}(t) = \phi_{unmod}(t) \; \left[1.03\right]$

Modification 2 $\quad \phi_{mod}(t) = \phi_{unmod}(t) \left[1.03-0.6t \right] +.3(MW)$

The analysis in step 3 of the procedure is in the form of a trial and error approach, using different values of Δk_{Fred}. The objective is to find the value of Δk_{Fred} which makes a plot of the function $\gamma(t)$, constant with time, i.e., the Doppler coefficient γ. Fig. 8 shows such a plot for a transient starting from 3 MW, a Δk_{Fred} value of 0.9852 dollars, assuming $T_f \frac{dk}{dT_f} = .004$, and an error in the power curve given by modification 1, above. From Fig. 8 it can be seen that using a value of about 0.9855 for Δk_{Fred} in the kinetics equation gives a relatively constant value of 2.595×10^{-3} for γ. The transient used in Fig. 8 had a Δk_{Fred} value of 0.9852, so modification of the power caused a 0.03 cent change in the value of Δk which makes the first term of equation (5) go to zero. Since the correct value of γ is 2.57×10^{-3} the modification for this transient caused a 1 percent error in the Doppler coefficient.

Curves similar to Fig. 8 were generated for other sized transients and the correspondig errors in γ evaluated. Fig. 9 shows the resulting errors plotted as a function of the transient size. Curves 3 and 4, generated with the paper experiment, are for the two modifications to the transient power and neither contains any error in $\overline{\lambda}$. The two curves (1 and 2) from the constant term analysis are for $\delta\psi_o/\psi_o$ values of 3 and 0.3 percent. Since the modifications made in the paper experiments were only samples out of a statistical assembly of errors in the power measurement, $\delta\psi_o/\psi_o$, the magnitudes of the curves from the paper experiments cannot be compared with the magnitudes of the curves representirg the average statistical error. However, comparisons of the shapes of the 4 curves can reveal similarities and differences and provide an indication of the sensitivity of the errors in γ to transient size.

First, since the paper experiments do not consider an error in $\bar{\lambda}$, curves 1 and 2 of Fig. 9 rise more quickly for ψ_o values below 50. Second, modification 2 approximates the assumptions made in the analysis of the constant term for $\delta\psi_o/\psi_o$ = 3% with regard to the relationship of $\delta\psi_1$ to $\delta\psi_o$. From Fig. 1 it can be seen that the indicated error in the measured curve nearly corresponds to modification 2. Therefore it is to be expected that the shapes of curves 1 and 4 of Fig. 9 will agree well where the error in ψ_1 dominates. From equation 5 it can be shown that $\delta\psi_1$ dominates the ψ_o^{-2} term and thus curves 1 and 4 should be more similar at low values of ψ_o.

On the other hand, modification 1, which produces curve 3, has very little error in ψ_1, so it tends to be flatter at low values of ψ_o. In addition, since curves 3 and 4 do not appear to turn up at $\psi_o \cong 80$ as curve 1 does, the modifidations to the transient power curves in the paper experiments must be more representative of a .003 to .01 statistical error in ψ_o. With these observations in mind, it may be concluded that the shapes of the curves of $\delta\gamma/\gamma$ vs. transient size obtained from the paper experiments are not unlike those obtained from the error analysis of the constant term, only. Fig. 8 likewise shows, for a specific case, that the higher order terms in equation (5) do not have a large influence on the constant value of $\gamma(t)$.

3. Conclusions about the Error as a Function of the Transient Size

From the above analyses it may be concluded that equation (18) can be used to determine the best transient size to obtain the minimum error in the Doppler energy coefficient. The error then will be less than the asymptotic error, given by equations (17) and (8), providing the transient is larger than that given by equation (18). For cases where the statistical error in the relative transient power measurement is of the order of 1 percent, equation (18) may impose unnecessary restrictions on the transient size. In these cases equation (20) provides a more practical lower limit of the transient size.

IV. Conclusions

An investigation of the error in the energy (Doppler) coefficient (γ, \$/MW-sec) resulting from an analysis of subprompt critical transients revealed three basic types of errors. These are the following:

(1) Error in the overall power calibration $d\Phi_0/\Phi_0$;

(2) Errors in the delayed neutron data appearing essentially in the forms of β (equal to 1 \$) and $\bar{\lambda}$, the average decay constant; and

(3) Error in the relative power measurement, ψ.

Errors in the power calibration and in β affect the accuracy of the absolute energy coefficient through the determination of the unit of coefficient, 1 \$/MW-sec. They affect the accuracy of all power measurements by an equal amount, thereby having no effect on relative measurements, such as the measurement of temperature dependence of the Doppler effect, or relative power curves. Only the error in power calibration contributes a small amount to the error in the absolute Doppler energy coefficient.

The average statistical error in $\bar{\lambda}$ was computed to be about 4 %, depending slightly upon the assumption made concerning the degree of correlation between the partial errors from different fissionable isotopes. The error in γ introduced by $\bar{\lambda}$ increases strongly with decreasing transient size, thereby determining a "minimum" transient size, Δk_{min}. Above this "minimum" transient the error in γ is relatively insensitive to the transient size and is dominated by the error in the relative power measurement. For a single measurement of the transient power curve, ψ, the error in γ is given approximately by

$$\frac{\delta\gamma}{\gamma} \cong 3 \frac{\delta\psi}{\psi},$$

i.e. three times the error of the relative power measurement. The "minimum" transient size can be calculated by defining the maximum permitted error in γ, $(\frac{\delta\gamma}{\gamma})_{max}$, and computing

$$\Delta k_{min}(\$) = 1 - \frac{\gamma\Phi_o}{\delta\bar{\lambda}} \left(\frac{\delta\gamma}{\gamma}\right)_{max} ,$$

using an estimated value of γ and providing that the desired $\delta\gamma/\gamma$ is not less than 3 $\frac{\delta\psi}{\psi}$ (an important consideration when the error in the relative power measurement is large).

The adjustment of the reactivity worth of FRED should be such that it is possible to place its worth within a specified portion of the range between δk_{min} and 1 \$. Taking a set of SEFOR conditions which give a high value of Δk_{min}; that is, γ is assumed to be low at 2.5×10^{-3} \$/MW-sec and Φ_o is low at 3 MW; and using realistic values of $\delta\bar{\lambda} = 0.04 \times 0.9 = 0.024$ sec^{-1}, and $\delta\gamma/\gamma$ to be 3 $\frac{\delta\psi}{\psi}$ with $\delta\psi/\psi = 0.03$, Δk_{min} is 0.97. This "upper limit" of Δk_{min} still provides a 3c range for the worth of the FRED device, and a precision in the worth of FRED on the order of slightly less than 1c appears to be all that is required.

APPENDIX A. Statistical Error in the Average Delayed Neutron Precursor Decay Constant, $\delta\bar{\lambda}$.

The error in the delayed neutron precursor decay constant, $\delta\bar{\lambda}$, enters directly into the error analysis of the constant terms in Section II. Since the power in SEFOR is not all produced by one fuel isotope and since the delayed neutron parameters are not the same for all fuel isotopes, $\bar{\lambda}$ is best determined by taking

$$\left.\begin{aligned}\bar{\lambda} &= \frac{\sum_{m=1}^{3} \nu_m S_m \sum_{i=1}^{6} \beta_{i,m} \lambda_{i,m}}{\sum_{m=1}^{3} \nu_m S_m \sum_{i=1}^{6} \beta_{i,m}} \\ &= \frac{\sum_{m=1}^{3} S_m \sum_{i=1}^{6} b_{i,m} \lambda_{i,m}}{\sum_{m=1}^{3} S_m \sum_{i=1}^{6} b_{i,m}}\end{aligned}\right\} \tag{A1}$$

where S_m = fraction of power in isotope m

ν_m = number of neutrons per fission in isotope m

and $b_{i,m} = \nu_m \beta_{i,m}$ = number of delayed neutrons per fission in group i from isotope m.

For the most part, at least, $\bar{\lambda}$ will be evaluated by equation (A1) using the independent quantities S_m, $b_{i,m}$, and $\lambda_{i,m}$, which were taken from measurements which are not directly related to the transient experiments. Thus it is meaningful to compute the error in $\bar{\lambda}$ by itself. In the first case we will assume that the errors in these quantities are completely uncorrelated. This results in

$$\delta\bar{\lambda} = \left[\frac{\sum_{m=1}^{3} S_m^2 \sum_{i=1}^{6} (b_{i,m}\delta\lambda_{i,m})^2 + \sum_{m=1}^{3} S_m^2 \sum_{i=1}^{6} (\lambda_{i,m}-\bar{\lambda})^2 \, \delta b_{i,m}^2}{\left[\sum_{m=1}^{3} S_m \sum_{i=1}^{6} b_{i,m}\right]^2}\right]^{1/2} \tag{A2}$$

where the errors in S_m are considered to be systematic and thus not included in the "statistical" formula (A2).

Equations (A1) and (A2) were used to compute the values of $\overline{\lambda}$ and $\delta\overline{\lambda}/\overline{\lambda}$ presented under case 1 in Table A1. It can be seen that the statistical error in $\overline{\lambda}$ is small, less than 4 percent, and this error should not present a problem in the analysis of either the subprompt critical experiments themselves or the error analysis under section II.

Values of the total delayed neutron fraction, β, and its fractional error are also presented in Table A1. These are given by

$$\beta = \sum_{i=1}^{6} \beta_i = \sum_{i=1}^{6} \frac{\sum_{m=1}^{3} S_m \, b_{i,m}}{\sum_{m=1}^{3} S_m \, \nu_m} \tag{A3}$$

and

$$\delta\beta = \sqrt{\sum_{i=1}^{6} \delta\beta_i^2} = \sqrt{\sum_{i=1}^{6} \sum_{m=1}^{3} \left[\frac{S_m \quad \delta b_{i,m}}{\sum_{m=1}^{3} S_m \quad \nu_m}\right]^2} \tag{A4}$$

respectively.

Case 1 considers the $b_{i,m}$ and $\lambda_{i,m}$ values to be totally independent and, therefore, uncorrelated in the errors. There is cause to believe, however, that there is some dependence in the value of $\lambda_{i,m}$ and their errors may not be totally uncorrelated. This belief is based in the fact that each of the delayed neutron groups is representative of a pseudo-fission product which may be composed of the delayed neutron contributions from a single or number of real fission product isotopes. Because the combination of isotopes for a given pseudo-fission product is not the same for fissions from different fuel isotopes, the composite decay constants do not agree exactly between fuel isotopes. However, on the isotopic fission product level, the value of λ would, indeed, be the same for all fuel isotopes and the corresponding errors would be fully correlated. In order to assess

TABLE AI

Average Delayed Neutron Parameters and their Statistical Error

Case	β	$\frac{\delta\beta}{\beta}$	$\bar{\lambda}$ (sec^{-1})	$\frac{\delta\bar{\lambda}}{\bar{\lambda}}$
1. Fully uncorrelated errors for SEFOR fuel	.00355	.0237	.570	.0360
2. Correlated errors in $\lambda_{i,m}$ values between materials for SEFOR fuel	.00355	.0237	.570	.0431
3. Individual fuel isotopes				
U 238	.01526	.0334	.785	.0447
Pu 239	.00215	.0339	.389	.0612
Pu 240	.00293	.0651	.447	.1103
4. Error with modification of power partition				
+ 10 % U 238 fissions	.00369	.0236	.581	.0359
- 10 % U 238 fissions	.00341	.0238	.558	.0362
5. Seven group set	.00355	.0237	.549	.0470

the effect of such a correlation, the errors in $\lambda_{i,m}$ were assumed to be correlated for case 2, Table A1. In this case the value of $\overline{\lambda}$ is again given by equation (A2), but the errors in $\overline{\lambda}_i$ must be independent of fuel isotope. Thus $\delta\overline{\lambda}$ is now given by

$$\delta\overline{\lambda} = \left[\frac{\sum_{i=1}^{6} \delta\overline{\lambda}_i^2 \left(\sum_{m=1}^{3} S_m b_{i,m} \right)^2 + \sum_{m=1}^{3} S_m^2 \sum_{i=1}^{6} \left(\lambda_{i,m} - \overline{\lambda} \right)^2 \delta b_{i,m}^2}{\left[\sum_{m=1}^{3} S_m \sum_{i=1}^{6} b_{i,m} \right]^2} \right]^{1/2} \tag{A5}$$

where $\delta\overline{\lambda}_i$ is the average error over fuel isotopes, obtained from

$$\delta\overline{\lambda}_i = \frac{\sum_{m=1}^{3} S_m b_{i,m} \delta\lambda_{i,m}}{\sum_{m=1}^{3} S_m b_{i,m}} \tag{A6}$$

From Table AI it can be seen that this increases the fractional error in $\overline{\lambda}$ to well over 4 percent, but still not enough to be of concern in near prompt critical transient experim... . Since correlation of the errors in $\lambda_{i,m}$ does not affect the values of β and $\delta\beta$, they are the same as in case 1.

In order to understand the reasons for obtaining such low values of $\delta\overline{\lambda}/\overline{\lambda}$, similar computations were performed for the individual fuel isotopes. These results are shown in case 3 of Tabel AI. It is seen that each of the individual fuel isotopes has a larger error in $\overline{\lambda}$ than that obtained for case 1 with the power combination in SEFOR.

The fact that the error of the combined $\overline{\lambda}$ is smaller than the individual error of its components shows that a subdivision of an error into components allows for compensation in the combined error. The degree of compensation depends on the detailed values of the λ_i's and on the power partition. Fortunately the compensation effect is fairly large for the power partition realized in SEFOR as discussed in the following paragraph.

The U-238 has a much higher yield than Pu-239 of delayed neutron precursors in the groups with large values of $\lambda_{i,m}$ (groups 5 and 6). These are the groups which contribute the most to the average decay constant $\overline{\lambda}$. Consequently, U-238 groups 5 and 6 each contribute about 25 percent of the total value of $\overline{\lambda}$ while Pu-239 groups 5 and 6 contribute a total of 23 percent in spite of the fact that U-238 produces only 0.13 as many fissions (see equation (A1). In computing the error in $\overline{\lambda}$ however, the contribution from each fuel isotope is weighted by S_m^2 (equation (A2)), so the contribution of U-238 relative to the contribution of Pu-239 is reduced from 0.13 to about .017. In Table AI we see that the fractional error in U-238 and Pu-239 are .0447 and .0612, respectively. Because the value of $\overline{\lambda}$ for Pu-239 is so much smaller than that for U-238, conversion of these fractional errors to absolute errors reverses the order and the U-238 $\delta\overline{\lambda}$ value becomes .0352 while that for Pu-239 is only .0238. The quantity $\delta\overline{\lambda}$ is that which is used in equation (A2).

Thus we see that U-238 has relatively high values of both $\overline{\lambda}$ and $\delta\overline{\lambda}$ by comparison with Pu-239. Since the weighting of U-238 relative to Pu-239 is higher when computing the mixed value of $\overline{\lambda}$ than when computing $\delta\overline{\lambda}$, the value of $\overline{\lambda}$ tends toward the higher values of U-238 while the value of $\delta\overline{\lambda}$ tends toward the lower Pu-239 values. This results in a value of $\delta\overline{\lambda}/\overline{\lambda}$ for the mixture which is remarkably lower than that of any individual isotope.

In Cases 1 and 2 of Table AI the isotopic power partition S_m, was held constant and did not enter into the error analysis. This power partition can be obtained in SEFOR from static fission foil measurements. There would be a non-negligible experimental error involved in these measurements, and at first consideration, since U-238 contributes so much to the value of $\overline{\lambda}$, one might suspect that the error in $\overline{\lambda}$ is very sensitive to the error in power partition. To measure this sensitivity, the quantity $\delta\overline{\lambda}/\overline{\lambda}$ was recomputed according to equation (A2) but with the fraction of power in U-238 first increased by 10 percent and then decreased by 10 percent, in each case modifying the S_m value of Pu-239 to take up the difference.

Case 4 of Table AI shows the results, As one would expect, the higher yields for U-238 cause a direct correlation between the values of β and $\overline{\lambda}$ and the U-238 fraction of fissions. A 10 percent increase in S_m of U-238 causes a 4 percent increase in β and a 2 percent increase in $\overline{\lambda}$.

The fractional errors in β and $\bar{\lambda}$ due to delayed neutron parameters are essentially insensitive to changes in the fission partition of about 1% and remain at about 2.4 and 3.6 percent, respectively. From these results it appears that the average delayed neutron parameters and their errors are not greatly affected by errors in the fission partitions used. However, in order to obtain a more realistic evaluation it is necessary to relate the 10 percent variation in S_m for U-238 that was used in the above results to the actual expected error in these partitions.

Steady state foil measurements should give ratios of fission rates in the fuel isotopes to considerably better accuracy than 10 percent. Thus we see that the ratios

$$R_{U-238} = \frac{S_{U-238}}{S_{Pu-239}} \cong .130$$

$$R_{Pu-240} = \frac{S_{Pu-240}}{S_{Pu-239}} = .0137$$

will have errors less than .013 and .0014, respectively. In the case of U-238 this corresponds to an error in S_m of less than 0.0113, which is nearly the same as the value (.0114) by which the two cases under number 4 in Table AI were modified. Therefore, it may be concluded that the statistical errors in the fission partitions contribute much less to the errors in $\bar{\lambda}$ than to the errors in the delayed neutron parameters, and one is justified in omitting these errors, as is done in Cases 1 and 2 of Table AI*.

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* The same conclusion can be arrived at more directly by expanding equation (A2) to include terms due to the statistical errors in the S_m values. Applying this expanded equation to Case 1 of Table AI gives

$$\delta\bar{\lambda} = \left[.000421 + .75\,(\delta R_{U-238})^2 + .0919\,(\delta R_{Pu-240})^2 \right]^{1/2}$$

for the statistical error in $\bar{\lambda}$. If then, δR_{U-238} and δR_{Pu-240} are given 10 percent statistical errors, $\delta\bar{\lambda}$ increases only from 0.0205 without any errors in R_{U-238} and R_{Pu-240}, to 0.0234 with the 10 percent errors.

The error analysis of $\bar{\lambda}$ would be improved if more delayed groups were used. The extreme case would be, of course, one group for each isotopic precursor, in which case the errors in $\lambda_{i,m}$ would be fully correlated with respect to material. Unfortunately, delayed neutron measurements do not permit such a distinction, and such a large number of groups would be unwieldy in the kinetics equations, anyway. It is sometimes desirable, as in the paper experiments performed for this study, to reduce the total number of effective groups used in the kinetics equations to around 6 or 7, instead of the 18 groups indicated by equation (A1).

A basic 7 group set for the three fuel isotopes of U-238, Pu-239 and Pu-240 was generated for the paper experiments in Section III of this study. In the first 5 groups there is quite good agreement between the $\lambda_{i,m}$ values for U-238 and Pu-239. Thus in these groups the $\lambda_{i,m}$ values of Pu-239 were chosen, as well as the $\delta\lambda_{i,m}$ values. In the sixth group the values of $\lambda_{i,m}$ for the different fuel isotopes do not agree well. Thus group 6 was taken to be Pu-239 only, with the values of $\lambda_{i,m}$ and $\delta\lambda_{i,m}$ taken from Pu-239. Group 7 includes only U-238 amd Pu-240 and has the $\lambda_{i,m}$ and $\delta\lambda_{i,m}$ values of U-238. For the seven group set the values of $b_{i,m}$ must be modified a little. Normal values apply to the first 5 groups, but for U-238 and Pu-240 in group 6 and for Pu-239 in group 7, the values must be zero. Since the $\lambda_{i,m}$'s are not dependent upon the fuel isotope, equation (A5) (for 7 groups) must be used for $\delta\bar{\lambda}$, but with $\delta\bar{\lambda}_i$ taking the assigned value. The average decay constant, itself, is given by

$$\bar{\lambda} = \frac{\sum_{i=1}^{7} \lambda_i \sum_{m=1}^{3} S_m \, b_{i,m}}{\sum_{m=1}^{3} S_m \sum_{i=1}^{7} b_{i,m}} \qquad \text{(A7)}$$

Equations (A3) and (A4) again apply for β and $\delta\beta$.

From Table AI it can be seen that the 7 group set (Case 5) results in a slightly lower value of $\bar{\lambda}$ and somewhat higher value of $\delta\bar{\lambda}/\bar{\lambda}$, primarily due to the fact that the $\lambda_{i,m}$ values are independent of fuel isotope and the errors fully correlated. Again the error in $\bar{\lambda}$ is small enough to be of little concern, and the seven group set of delayed neutrons canbe considered to be quite accurate for the paper experiment study of Section III.

Acknowledgement

The authors thank Dr. L. Caldarola for stimulating discussions.

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3. G. R. Keepin, Delayed Neutron Parameters for Fast Fission in U-238, Pu-239 and Pu 240, as reported in ANL-5800.

Fig. 1

Assumed Relationship Between Error in Ψ_1 and Error in Ψ_0

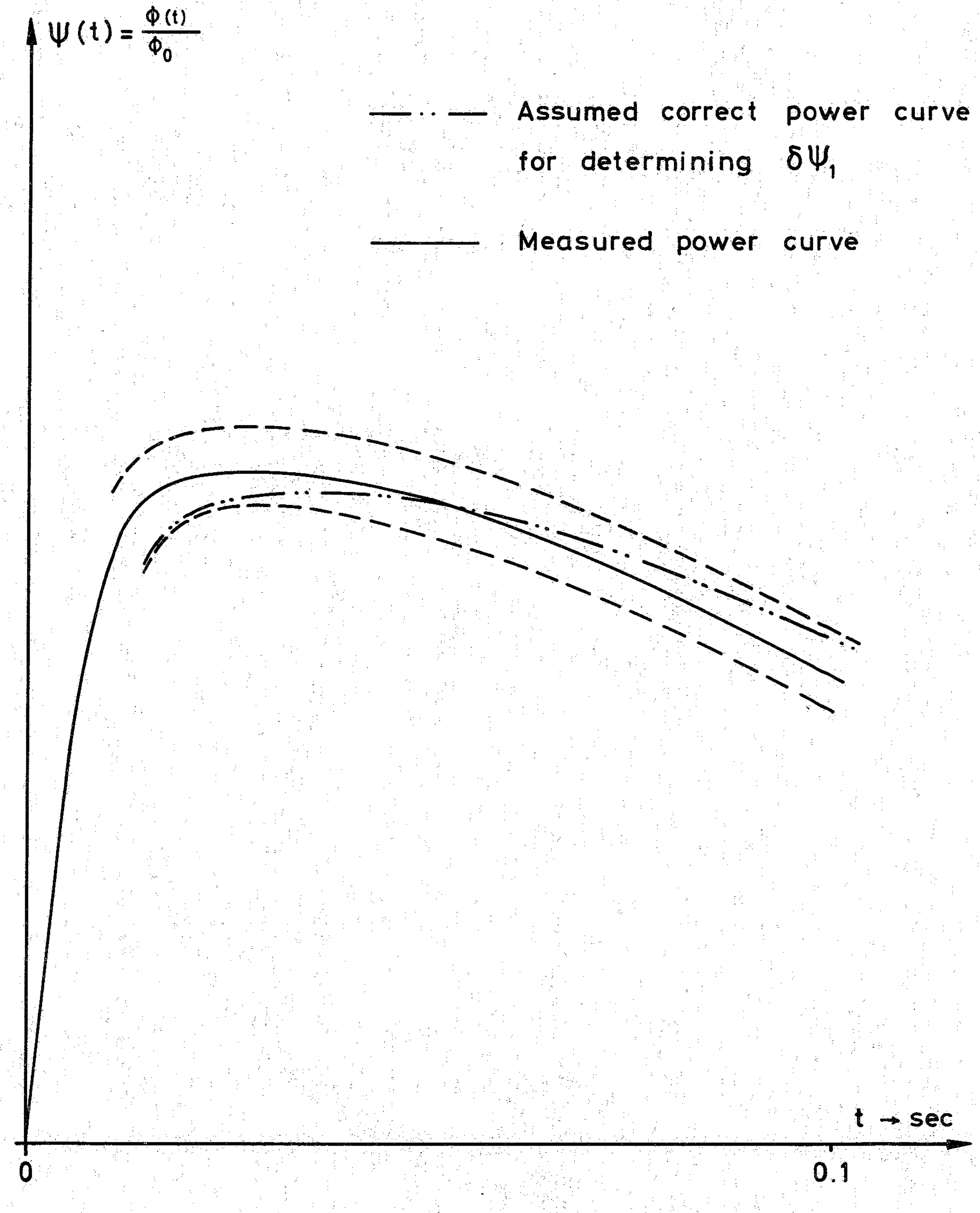


Fig. 2 Fractional Error in F-Factor vs. Transient Size for 3MW Initial Power and $T_f \left(\frac{dk}{dT}\right)_f = .0040$

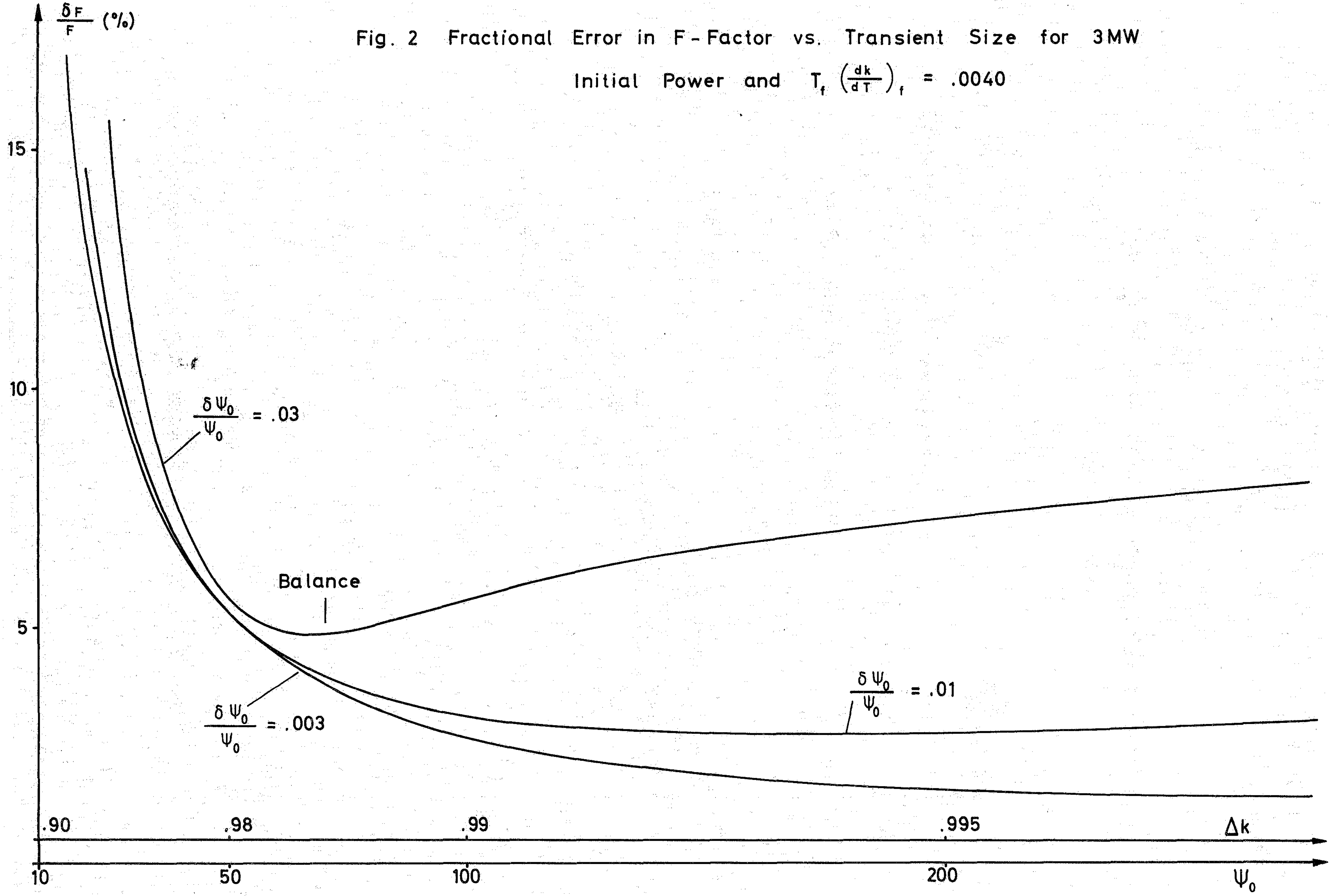


Fig. 3 Fractional Error in F-Factor vs. Transient Size for 3 MW Initial Power and $T_f\left(\frac{dk}{dT}\right)_f = .0083$

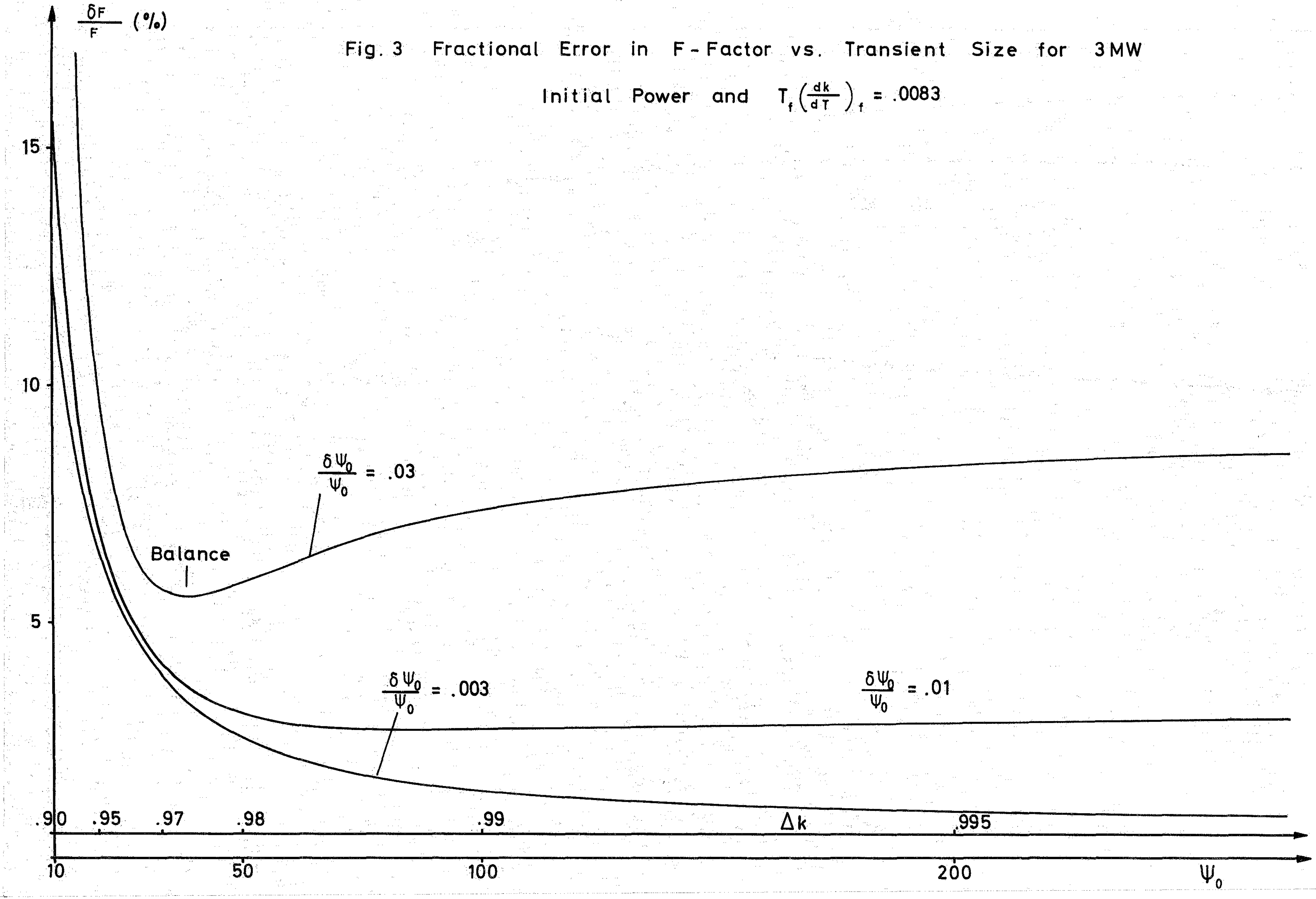


Fig. 4 Fractional Error in F-Factor vs. Transient Size for 20 MW Initial Power and $T_f \left(\frac{dk}{dT}\right)_f = .004$

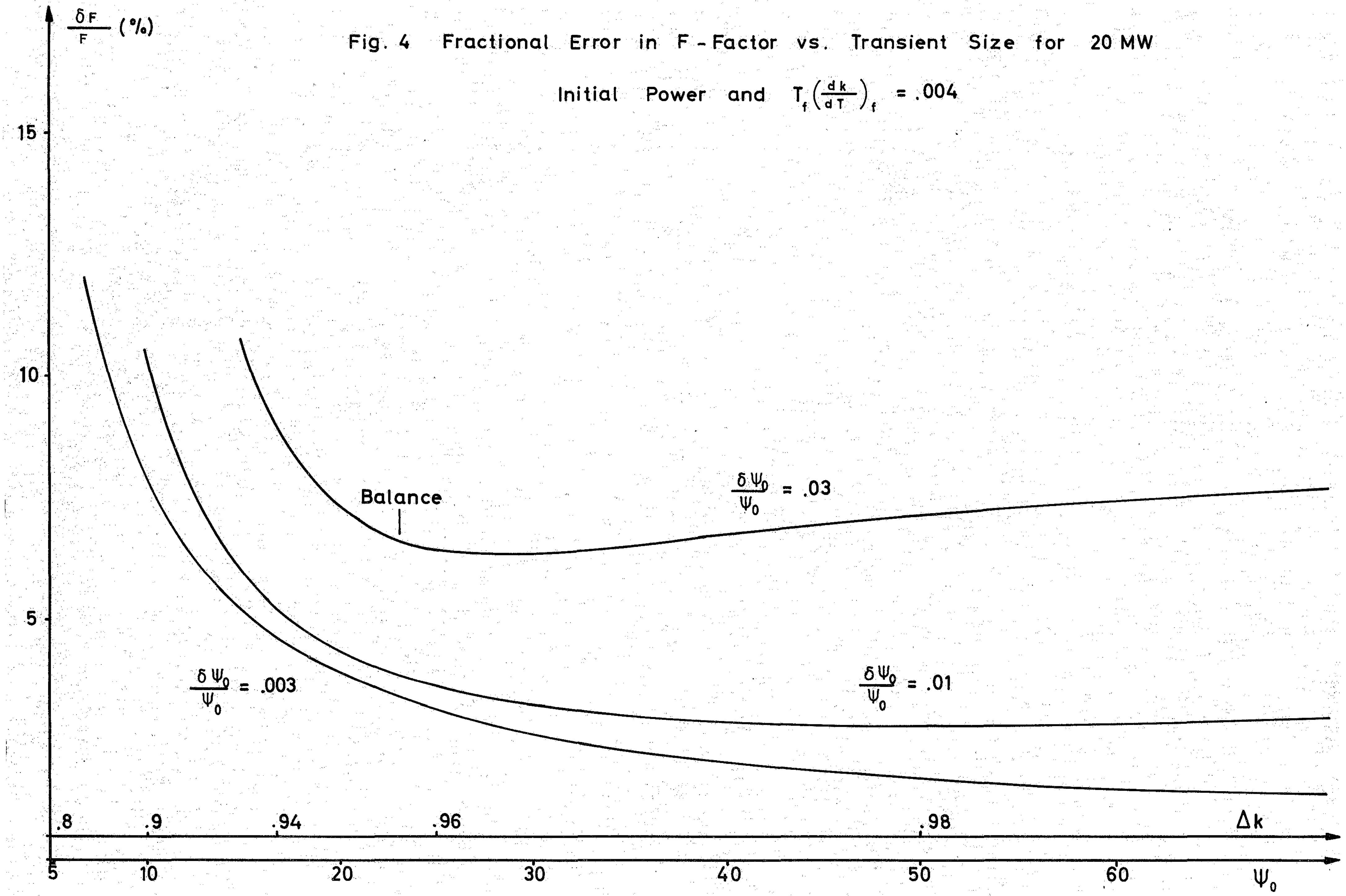


Fig. 5 Fractional Error in F-Factor vs. Transient Size for 20 MW Initial Power and $T_f\left(\frac{dk}{dT}\right)_f = .0083$

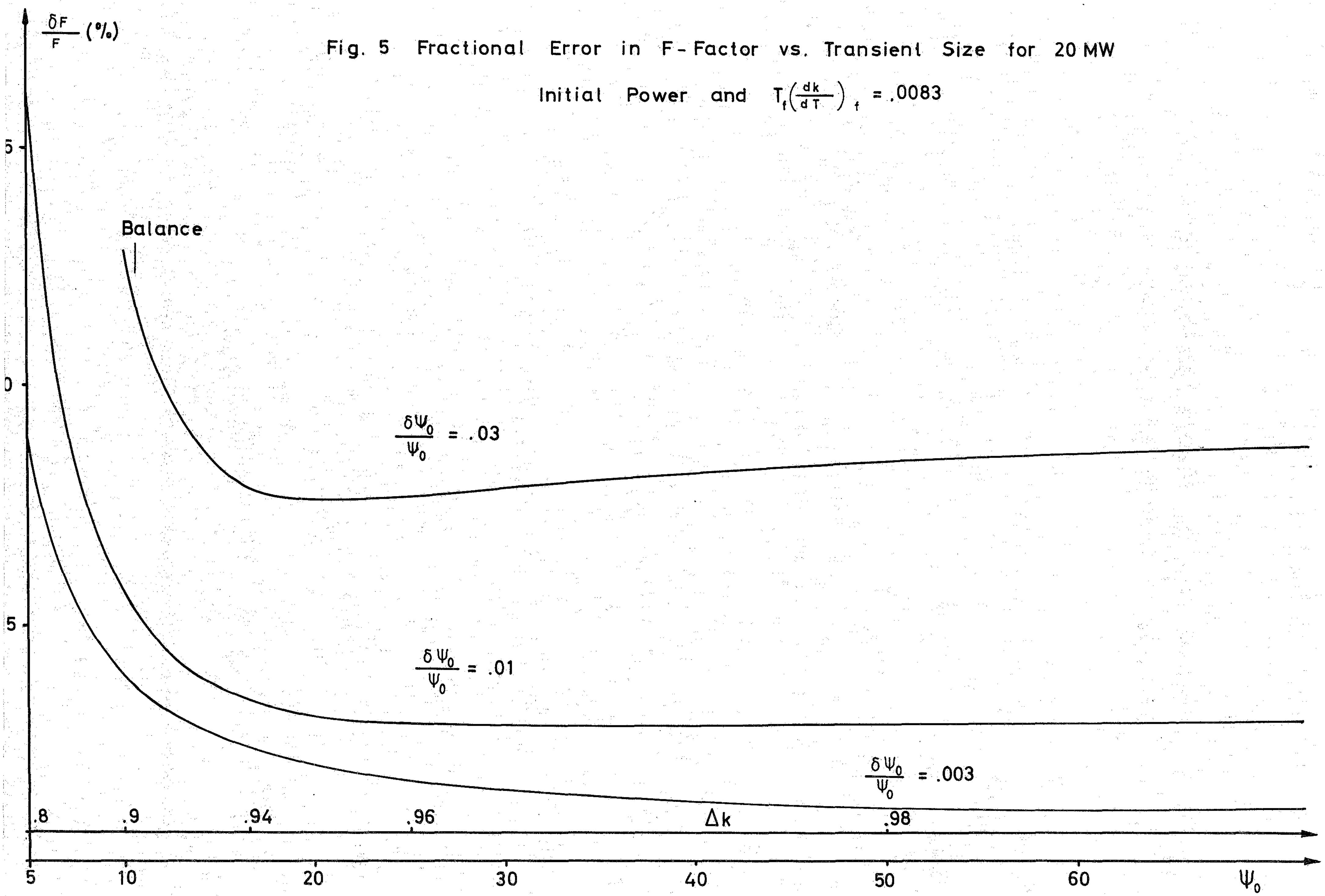


Fig. 6 Transient Size for Minimum Statistical Error as Functions of Error in Relative Power Measurements

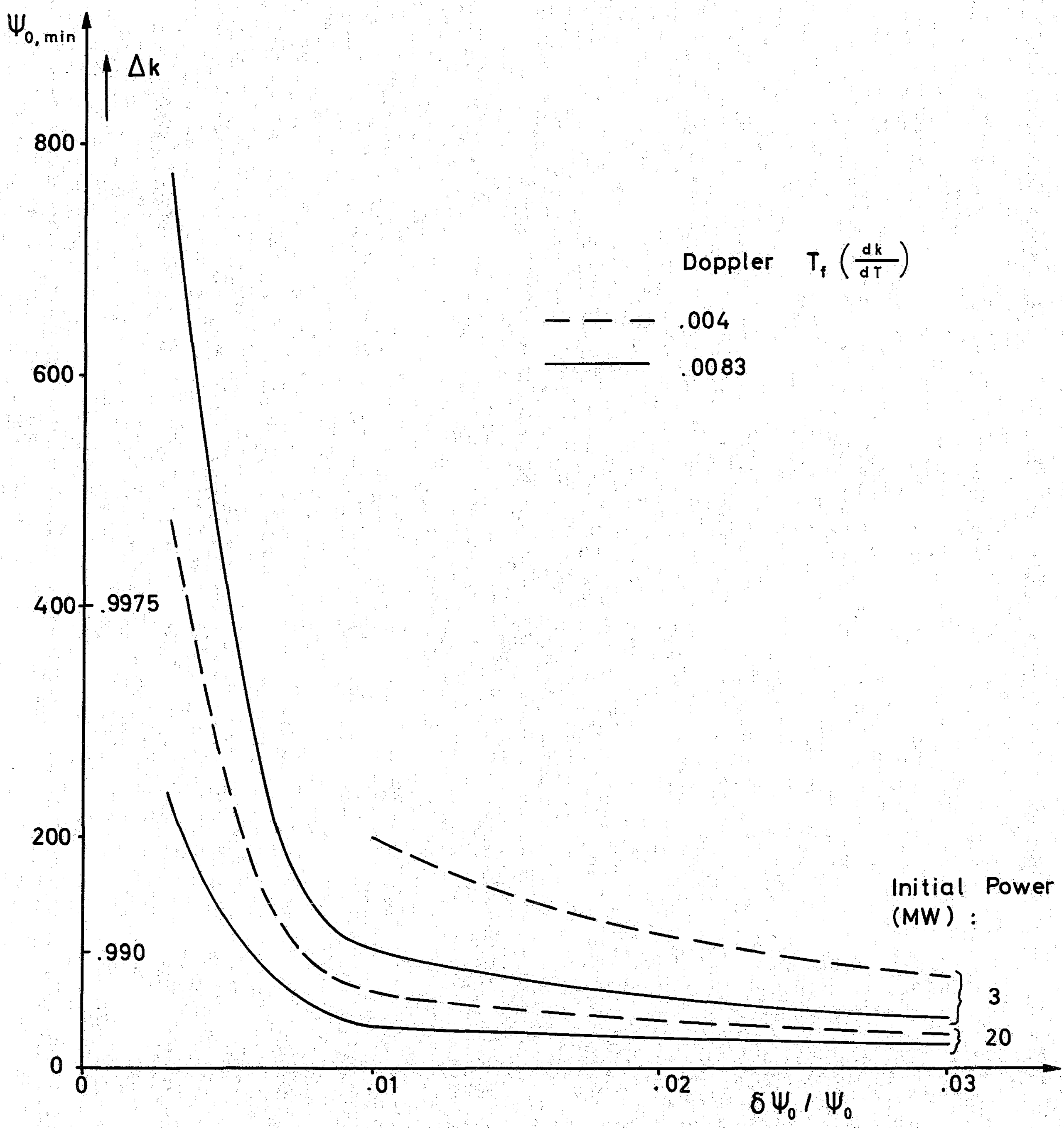


Fig. 7 Minimum Error in Doppler Coefficient as Function of Fractional Errors in Power Calibration $\left(\frac{\delta\phi_0}{\phi_0}\right)$ and Transient Power Measurement $\left(\frac{\delta\psi_0}{\psi_0}\right)$

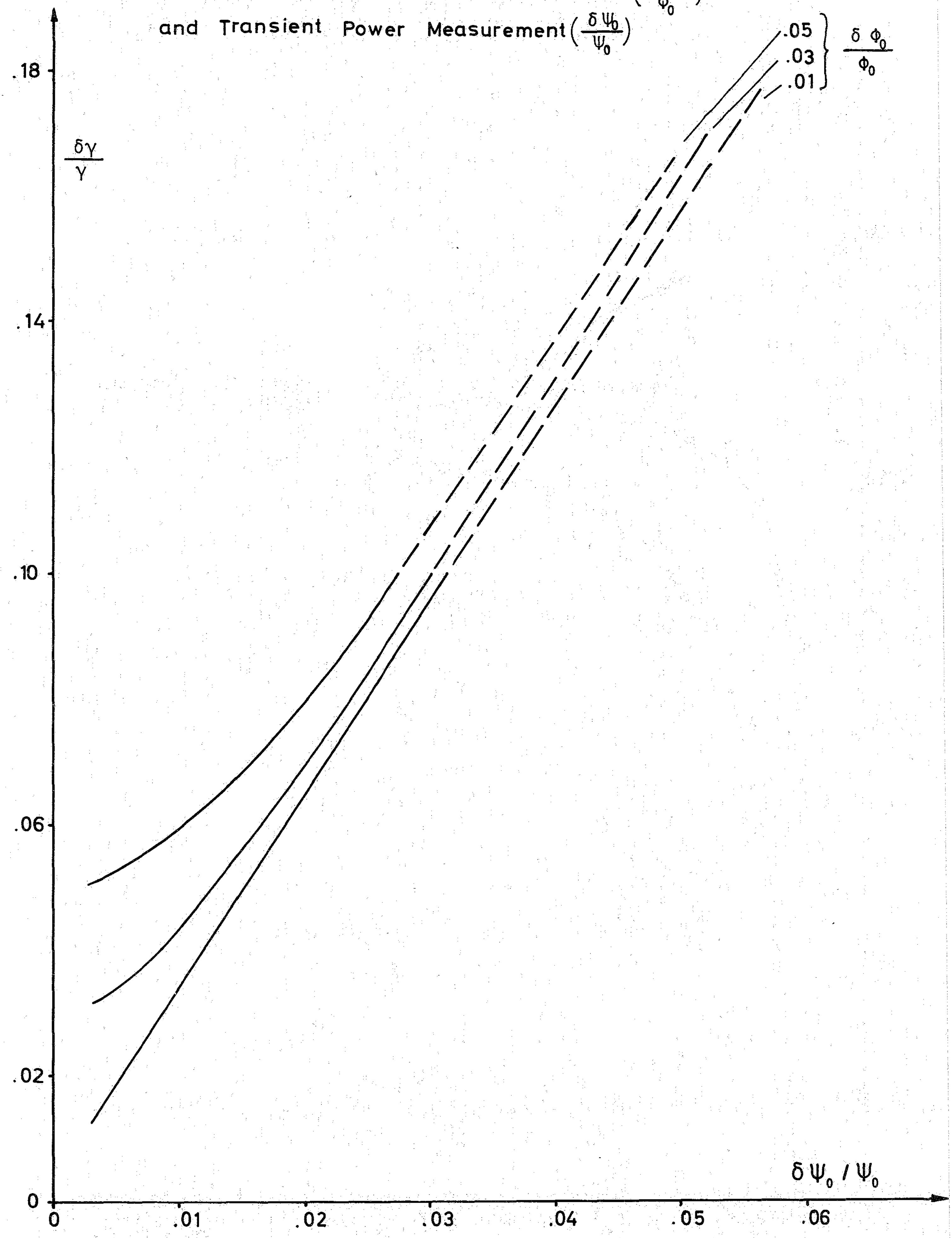
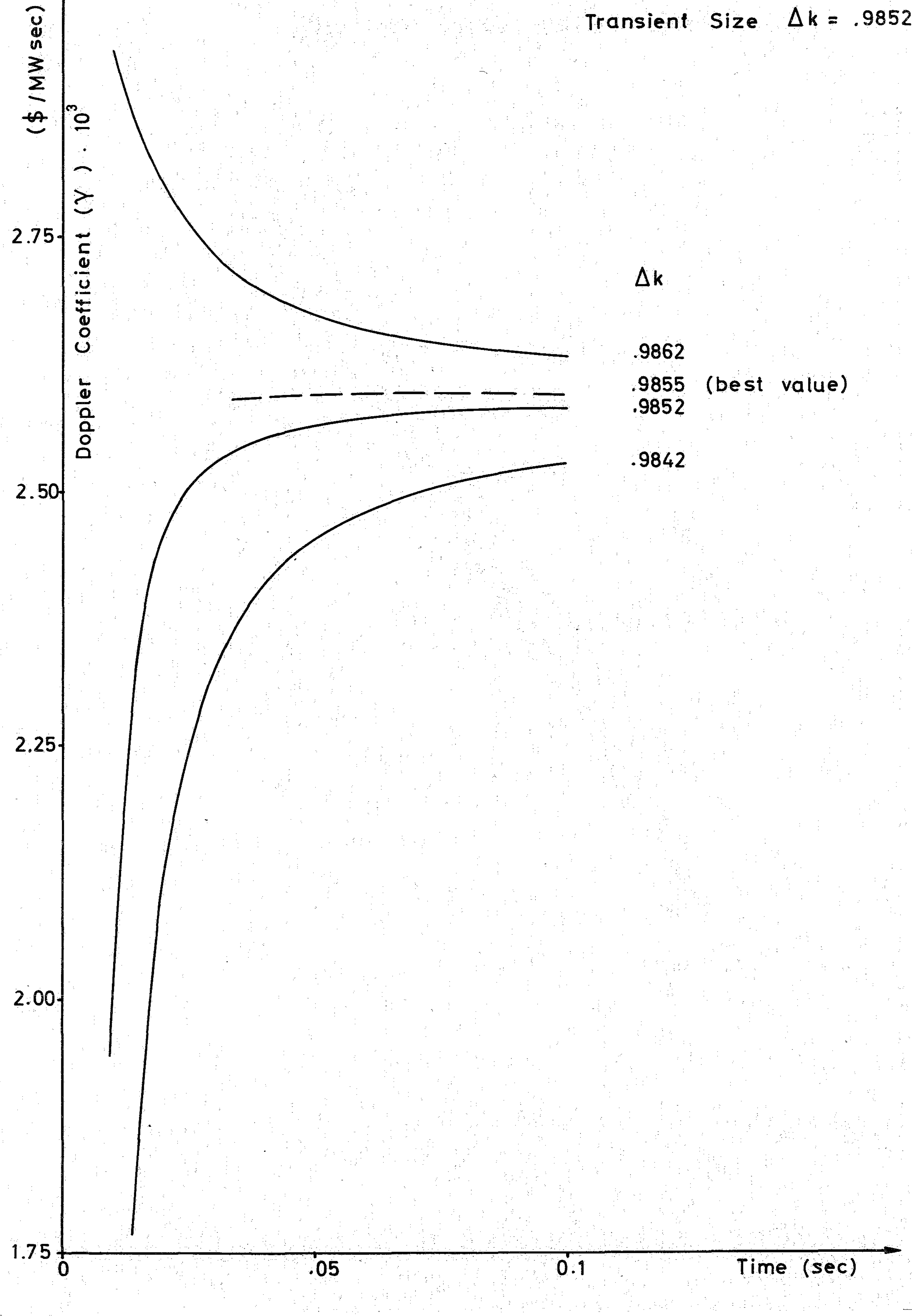
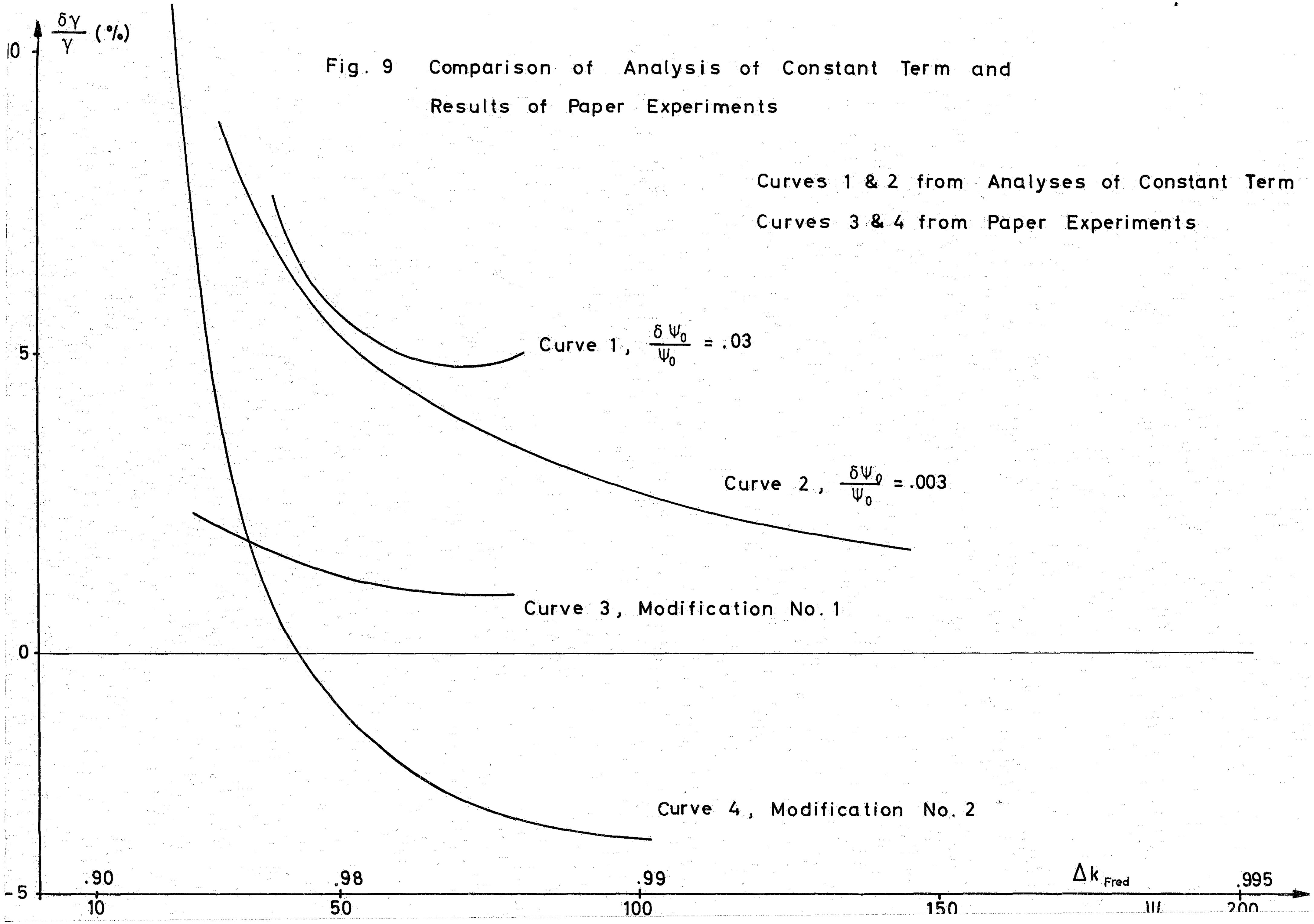


Fig. 8 Trial and Error Curves for Determining the Correct Doppler Coefficient

Transient Size $\Delta k = .9852$





$\frac{\delta\gamma}{\gamma}$ (%)
10
5
0
-5
Fig. 9 Comparison of Analysis of Constant Term and Results of Paper Experiments
Curves 1 & 2 from Analyses of Constant Term
Curves 3 & 4 from Paper Experiments
Curve 1, $\frac{\delta\Psi_0}{\Psi_0}$ = .03
Curve 2, $\frac{\delta\Psi_0}{\Psi_0}$ = .003
Curve 3, Modification No. 1
Curve 4, Modification No. 2
.90
.98
.99
.995
Δk_{Fred}
10
50
100
150
W
200