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A SECTORED ISOCHRONOUS RING FOR ACCELERATING HEAVY IONS

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An accelerator capable of accelerating heavy ions to energies around 7 MeV/nucleon is proposed. It consists of a pre-stripper, which may be either an electrostatic accelerator, a cyclotron, or a linac, and an isochronous cyclotron with separated magnets. Different factors relevant to the design of the isochronous ring and to the choice of the pre-stripper are discussed. It is concluded that the construction of such an accelerator does not present problems which go beyond the present state of cyclotron technology.

1. Introduction

Recently, nuclear reactions induced by heavy ions have received increased interest. For most nuclear physics and chemistry applications, projectiles with energies up to or slightly above the Coulomb barrier are required. When the lighter of two colliding particles is used as a projectile this energy is always lower than 7 MeV/nucleon. For some applications – especially in the field of transuranium elements – it would be very advantageous to have available beams of the heaviest natural elements.

Accelerators proposed for producing such particle beams are linear accelerators[1]), synchrotrons[2]) and electrostatic accelerators[3]). Cyclotrons, which have been used successfully for accelerating heavy ions especially by the Dubna group[4]), do not seem to be suitable for this application for reasons of cost. This is due to the fact that the radius of a cyclotron is – at constant particle velocity – inversely proportional to the charge-to-mass ratio:

$$r = (m/q)\,(v/B), \qquad (1)$$

(r, orbit radius; m, particle mass; q, particle charge; v, particle velocity; B, average magnetic field). The charge state of particles cannot be changed during acceleration in a cyclotron. As known, ion sources deliver ions of heavy elements at a very low charge-to-mass ratio (typically between 0.1 and 0.05 of the proton value) this results in very large and hence very expensive cyclotrons. Therefore, cyclotrons have only been proposed for accelerating ions up to argon[5]).

More than 15 years ago Tobias[6]) proposed to accelerate heavy ions by a linear accelerator, to increase their charge by stripping, and to inject the stripped particles into a cyclotron or synchrotron. It is evident that such a procedure would increase the range of application of cyclotrons considerably. A combination of a Wideroe linear accelerator and an isochronous cyclotron according to this scheme is being constructed at Orsay[7]). The difficulties of injection of the particles into the cyclotron can be overcome by placing the stripper foil into the cyclotron[8]). It is not certain, though, whether this injection scheme can be applied for ions of mass above 100, as recent measurements[9]) indicate, that the life time of solid stripping foils is very short for so heavy ions and as it is probably not possible to use a gas stripper inside a cyclotron for reasons of vacuum and available space.

It is the purpose of this paper to discuss the possibility of using an isochronous ring accelerator similar to the Zürich meson factory[10]), as a second stage of such an accelerator combination. This choice appears to offer the following advantages:

1. The particles can be injected into the ring with the charge after stripping. The stripper can therefore be placed somewhere between the linear accelerator and the cyclotron, which removes all limitations of space or vacuum.
2. A very high energy gain per turn can be achieved with moderate rf power by placing several rf structures between the separate magnets of the ring. This results in a high turn separation and consequently in a high extraction efficiency.
3. The high energy gain per turn also reduces the total length of the orbit and thereby diminishes the loss of particles by charge exchange collisions with the residual gas. Vacuum requirements are therefore smaller than in a conventional cyclotron.

A similar accelerator has recently been proposed by Dzelepov et al.[11]). They intend to use an electrostatic accelerator as the first stage and not to strip the ions before injection into the ring, thereby sacrificing one of the main advantages of the scheme. Their proposal has been incorporated into the proposed Indiana University Cyclotron[12]). Isochronous ring accelerators as an alternative to the Separated Orbit Cyclotron have recently been investigated in detail by Gordon[13]).

Many of his results apply also to heavy ion ring accelerators and will be mentioned in the following discussion.

2. General description of the accelerator

Before entering the discussion of detailed aspects of the accelerator, a general description is given in this section. The entire accelerator consists of a first stage ("pre-stripper") which accelerates the particles up to an energy between roughly 1 and 2 MeV/nucleon. Different factors relevant to the choice of this stage are considered in section 7. The main stage consists of a ring of N identically shaped magnets which form the guide field of an isochronous cyclotron. The field-free sections between the magnets provide the space for rf structures, injection, and extraction. Fig. 1 shows the schematic lay-out of an example with $N = 6$ and 4 rf structures.

3. Equilibrium orbits and orbit stability

Equilibrium orbits and orbit stability can easily be calculated if edge effects of the magnetic field are neglected (hard edge approximation). For simplicity we assume that the magnetic field inside the magnets is homogeneous. The orbits then consist of circles inside the magnetic field and straight lines outside. The azimuthal boundary lines of the magnets are assumed to be logarithmic spirals with angle δ, and α represents the angle occupied by one magnet. Fig. 2 shows a section of one of the magnets. Let r be the radius

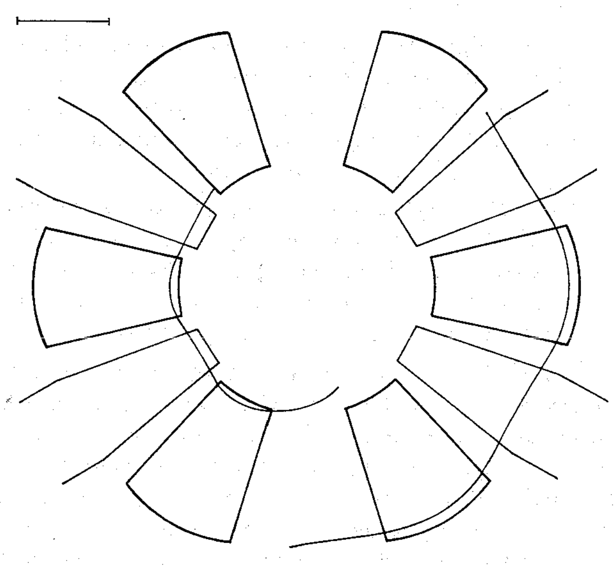

Fig. 1. Lay-out of an isochronous ring accelerator with 6 magnets and 4 rf structures. The scale is referred to in table 1.

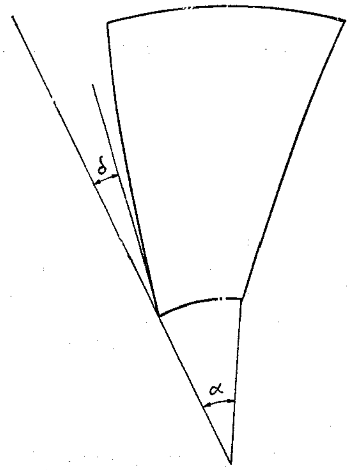

Fig. 2. Cross section of a single magnet.

of curvature of the orbit in the magnetic field and s be the mean radius, measured between the centre of the ring and the point of intersection of the orbit and the magnet boundary. For the equilibrium orbit it then follows

$$r \sin(\pi/N) = \sin \tfrac{1}{2}\alpha, \tag{2}$$

independent of δ.

Within the framework of the hard edge approximation it is also possible to evaluate the number of betatron oscillations per revolution, by using the matrix method[14]). The calculation is straight-forward, but somewhat lengthy and results in the following expressions:

$$\cos\sigma_r = \cos(2\pi/N) - \{1-\cos(2\pi/N)\}\{1-\cos[(2\pi/N)-\alpha]\} \times \{\cos[(2\pi/N)-\alpha] + \cos 2\delta\}^{-1}, \tag{3a}$$

$$\begin{aligned}\cos\sigma_z = 1 &- \{(\pi/N) + \sin(\pi/N)\sin[(\pi/N)-\tfrac{1}{2}\alpha](\sin\tfrac{1}{2}\alpha)^{-1}\} \\ &\times \{\mathrm{tg}[(\pi/N)-\tfrac{1}{2}\alpha+\delta] + \mathrm{tg}[(\pi/N)-\tfrac{1}{2}\alpha-\delta]\} + \\ &+ (2\pi/N)\sin(\pi/N)\sin[(\pi/N)-\tfrac{1}{2}\alpha](\sin\tfrac{1}{2}\alpha)^{-1} \times \\ &\times \mathrm{tg}[(\pi/N)-\tfrac{1}{2}\alpha+\delta]\,\mathrm{tg}[(\pi/N)-\tfrac{1}{2}\alpha-\delta],\end{aligned} \tag{3b}$$

$$\nu_{r,z} = (\tfrac{1}{2}N/\pi)\,\sigma_{r,z}. \tag{3c}$$

Details of these calculations will be published elsewhere[15]). Fig. 3 shows some typical results for $N = 6$ and different spiral angles δ. As can be seen from this figure it is not necessary to introduce a spiral angle when the magnet sector is chosen smaller

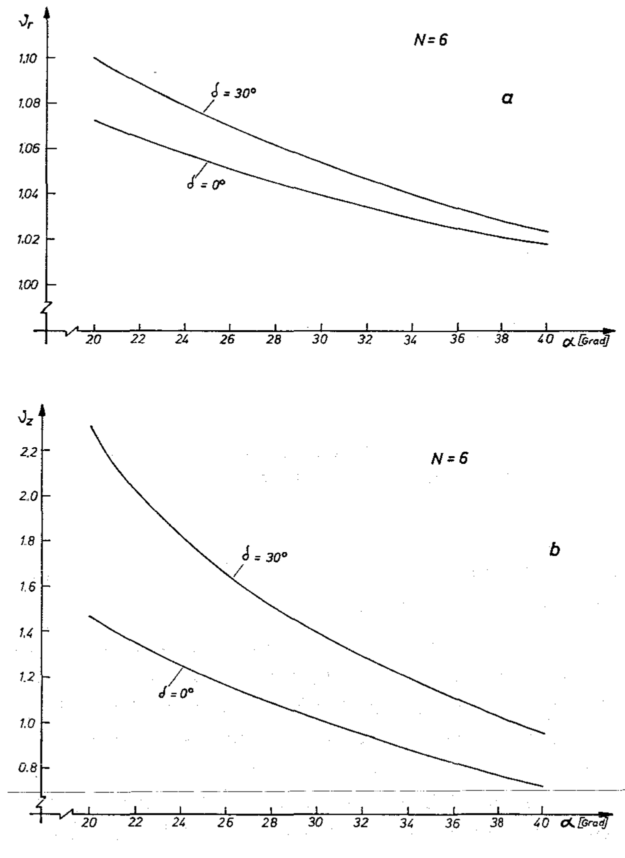

Fig. 3. Results of a hard edge calculation of (a) ν_r and (b) ν_z.

than the field-free sector ($\alpha < 30°$ in fig. 3). This result seems to hold equally well for other values of N.

Due to the low final energy of 7 MeV/nucleon the mass increase of the particles in the ring accelerator is less than 0.7%. It should therefore not be difficult to maintain isochronism. When the magnetic field is kept constant in the magnets, the angle α has to increase with increasing energy according to the formula[15]):

$$\gamma \{1+(N/\pi)\sin^2(\pi/N)\operatorname{ctg}\tfrac{1}{2}\alpha - (N/\pi)\sin(\pi/N)\cos(\pi/N)\} = \text{const.} \qquad (4)$$

Here γ is the ratio of total mass to rest mass ($1 < \gamma \leqslant 1.007$).

4. Magnet design

The magnets are proposed to be C-shaped magnets with the yokes at the periphery of the ring. Though this is not mandatory, homogeneous field magnets are preferred for reasons of ease of manufacture. The required precision of the magnetic field is determined by the maximum allowable phase excursion. This phase excursion increases in proportion to the number of turns n in the accelerator and to the harmonic number h of the radio frequency. This problem is discussed by Gordon[13]) who concludes that present day magnet technology permits the construction of accelerators with $hn < 650$. This limit appears somewhat conservative as the AEG cyclotrons at Karlsruhe and Jülich operate satisfactorily at $hn = 900$ and 1300, respectively.

5. Radiofrequency system

Orbit frequencies of heavy ion isochronous accelerators are considerably lower than those of corresponding proton accelerators. This probably excludes the use of rectangular cavities, which are used at the Zürich meson factory[10]), as rf structures for a heavy ion isochronous ring accelerator as this, would require either to use very large cavities or to operate on a very high harmonic of the orbit frequency, which is undesirable for reasons of magnetic field tolerances. As was pointed out in section 3 it is advantageous to choose the width of the magnets slightly smaller than the width of the field free sectors. The ratio, of the time spent in one field free sector to the revolution time, is then slightly larger than $\frac{1}{2}N$. It is therefore probably the best choice to use conventional dee structures of an angular extension of π/N and to operate on the N^{th} (or a slightly higher) harmonic.

6. Turn separation, injection and extraction

The question of turn separation has been discussed extensively by Gordon[13]) to whom we therefore refer for the details. He obtains the following expression for the turn separation ds,

$$\mathrm{d}s = \Delta s\,\{1 - ng(\phi)\} - \Delta_i s\,, \qquad (5)$$

where Δs is the maximum radius gain per turn, $\Delta_i s$ is the initial radial width of the beam, n the number of turns, and $g(\phi)$ a function of the rf phase, which depends on the rf wave form. As we may neglect the slight dependence of α on the particle energy, Δs is given by the following formula:

$$\Delta s/s = \Delta r/r = \tfrac{1}{2}\,\Delta E/E\,, \qquad (6)$$

where ΔE is the peak energy gain per turn and E the particle energy. Assuming a charge state of ζ and a mass of A of the ion and putting $E = A\varepsilon$ we get from this formula

$$\mathrm{d}s/s = \tfrac{1}{2}\,\zeta e U/(A\varepsilon)\,. \qquad (7)$$

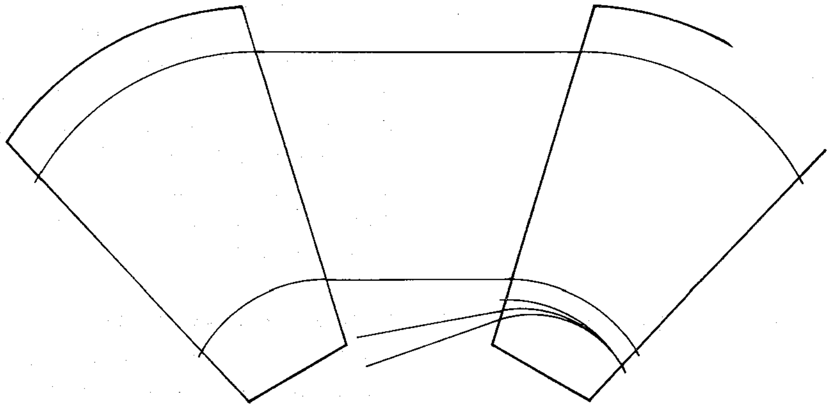

Fig. 4. Proposed injection scheme using one electrostatic septum in the magnetic field. The three initial orbits correspond to field strengths of 0, 50 and 100 kV/cm. The overall size of the magnet is that of the smaller accelerator in table 1.

Here e is the elementary charge and U the sum of the peak voltages of all rf cavities.

When a minimum value of ds is assumed in eq. (5) the function $g(\phi)$ (which vanishes at $\phi = 0$) limits either the number of turns n, or the maximum phase ϕ_m. For a sinusoidal waveform $g(\phi) = 1 - \cos\phi$. By superimposing a third harmonic on the accelerating rf (which can be accomplished by exciting one of the cavities at the third harmonic frequency) the size of $g(\phi)$ can be reduced considerably at the expense of a slight increase of the accelerating voltage. It is then possible to increase the number of turns and/or the phase width of the accelerated beam. The required amplitude of the third harmonic has been calculated by Gordon[13]), to whom the reader is referred.

Injection and extraction can be achieved by use of electrostatic septa and/or magnetic channels inside the magnets. An example of the injection scheme is shown in fig. 4. As can be seen from this figure, injection can be achieved with a single electrostatic septum operated at not more than 100 kV/cm.

7. Choice of the pre-stripper accelerator

Potential pre-stripper accelerators are electrostatic accelerators, cyclotrons, and linear accelerators.

Single-stage electrostatic accelerators have the substantial disadvantage of placing the ion source at the high voltage terminal. Tandem accelerators which avoid this drawback are limited in intensity and versatility. An appreciable loss in intensity results from the additional stripper at the high voltage terminal as each stripper divides the beam into several components of different charge and only one of these components is accepted by the following stages of the accelerator. Versatility is reduced by the fact that not all elements form negative ions readily.

A cyclotron pre-stripper would considerably facilitate the problem of matching the two accelerator stages. The ratio of magnetic fields, as well as that of frequencies in the two stages, could be kept fixed. It would therefore be easy to change the output energy continuously over a wide range. For reasons of cost, however, a cyclotron does not seem to be feasible for accelerating very heavy ions to an energy as low as 1 MeV/nucleon. Uranium ions, e.g., can probably be obtained at a charge of 11+ from ion sources[16]). The diameter of a cyclotron for accelerating U^{11+} to 1 MeV/nucleon would have to be larger than 4 m at an average field of 15 kG.

Linear accelerators operate at fixed rf frequency and deliver particles at the same velocity irrespective of their mass. The two accelerator stages could easily be matched if the ring accelerator is operated at the same rf frequency (or a harmonic). On the other hand, the constant velocity at the end of the linear accelerator results in some difficulties when a continuous variation of final energy of the accelerator is wanted. This problem is discussed in more detail in section 9.

8. Variation of particles

When all particles are injected into the ring at the same velocity (as would be the case with a linear accelerator pre-stripper) the equilibrium orbits remain the same when the magnetic field B is changed such that the quantity

$$b = B\,\zeta/A,$$

remains constant. Consequently the revolution time is not altered and no change of the rf frequency is required. The upper limit of B due to saturation results in a lower limit of the specific charge ζ/A.

When the velocity of the injected particles is not constant, the rf frequency has to be changed, in addition to the magnetic field.

9. Variation of final energy

When the injection and extraction radii are kept fixed, the ratio of initial to final velocities is constant in the ring accelerator. A continuous variation of output energy can therefore be achieved when the input velocity is changed continuously. This would of course require to alter the rf frequency in proportion to the input velocity, which is quite feasible with the rf structures proposed. This mode of operation is possible with an electrostatic accelerator or a cyclotron pre-stripper. The problem is much more difficult when a linear accelerator pre-stripper is used which delivers particles at fixed velocity. The following three procedures seem feasible:

a. The extraction radius can be changed. This method which has been proposed by Gordon[13]) is probably only feasible for a small range of energies. Let us, e.g., consider a 2 to 8 MeV/nucleon machine for which the ratio of final to initial radius is 2:1. When the output energy were to be changed by a factor of 2 the new ratio of extraction to injection radii would have to be 1.4. This means that the radius of extraction would have to be changed over 60% of the radial extension of the machine. It appears to be difficult to achieve extraction at so small radii.

b. The injection radius can be changed by a factor $(h \pm 1)/h$. The harmonic number is then changed from h to $h \pm 1$. Dzelepov et al.[11]) have pointed out that it is then unnecessary to alter the radiofrequency as the phases of the different cavities can be adjusted independently to the new revolution frequency. The particles then pass the accelerating gaps before or after the maximum of the rf voltage, and the energy gain per turn is reduced by the factor

$$\sin\{\tfrac{1}{2}\pi(h \pm 1)/h\}.$$

This factor equals 0.97 and 0.98 for $h = 6$ and 8, respectively. The output energy is then changed by a factor of $(h \pm 1)^2/h^2$ which equals 1.36 and 0.69 for $h = 6$ and 1.29 and 0.77 for $h = 8$. A continuous variation of energy can of course not be obtained by this procedure.

c. The same change of harmonic can be achieved by accelerating or decelerating the particles between linear accelerator and ring by a fixed amount and leaving the radius of injection unchanged. The energy has again to be changed by a factor of $(h \pm 1)^2/h^2$. As this acceleration or deceleration can take place behind the stripper at the higher charge it can be achieved with a single rf cavity.

A combination of procedure (a) with either (b) or (c) would allow a continuous variation of final energy over a considerable range also with a linear accelerator injector.

10. Vacuum requirements

Heavy ions may change their charge in collisions with atoms of the residual gas in the accelerator. As only ions of one charge state can be accelerated in a cyclotron, the pressure in the vacuum tank should be kept low enough, to render the loss due to charge exchange collisions negligible. Little is known about the relevant cross sections in the energy range of interest. We therefore base our discussion on the extrapolation developed in [2]), where a cross section σ is assumed which is inversely proportional to the particle velocity such that

$$\sigma\beta = 10^{-17}\ [\mathrm{cm}^2],$$

where β is the velocity in terms of the velocity of light. The total loss of particles due to charge exchange then only depends on the time T spent during acceleration. The ratio of output current I to input current I_0 is given by

$$I/I_0 \approx \exp\{-10^{10}pT\},$$

where p is the pressure in Torr and T is measured in sec. The time T is easily evaluated as the revolution time τ is constant

$$T = n\tau = \{(E_f - E_i)/\Delta E\} = A(\varepsilon_f - \varepsilon_i)/(\zeta e U).$$

Here $E_f = A\varepsilon_f$ and $E_i = A\varepsilon_i$ are final and initial energy, respectively.

This formula shows the influence of a high accelerating voltage U on the loss of particles.

11. Design examples

In this section two design examples of 2 to 7 MeV/nucleon accelerators are given which differ only in the minimum specific charge ζ/A of the particles. The heaviest ions which may be accelerated in the two rings are ^{132}Xe and ^{238}U, respectively. For the charge of the stripped ions we rely on the extrapolations by Schmelzer et al.[16]) and assume a gas stripper. The extrapolated charge-to-mass ratios for Xe and U

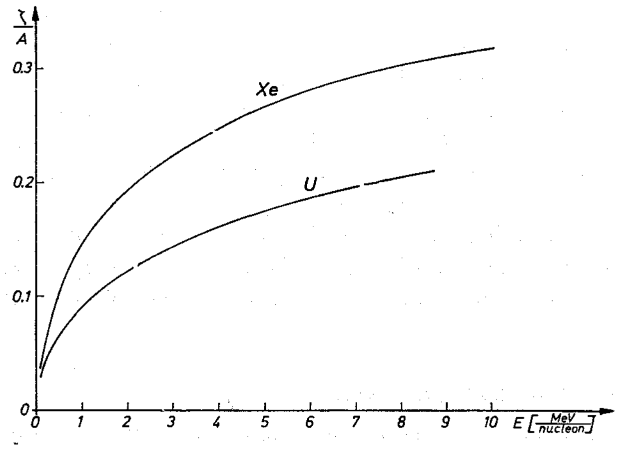

Fig. 5. Charge-to-mass ratio ζ/A of Xe and U ions behind a gas stripper. The curves are based on an extrapolation performed by Schmelzer et al. [16]).

are shown in fig. 5. The general lay-out of these accelerators is that shown in fig. 1. A maximum magnetic field of $B = 17$ kG and a total accelerating voltage of $U = 1$ MV have been assumed. The latter is the value on wich the design of the Zürich meson factory is based[10]). It would require a peak voltage of 125 kV when four dees are used. Table 1 summarizes the dimensions of the accelerator.

12. Conclusion

It is concluded that an isochronous ring accelerator combined with a pre-accelerator is technically feasible for accelerating heavy ions to the energies required for nuclear physics and chemistry experiments. As the same is true for other types of accelerators[1-3]) the decision between the different types depends heavily on a comparison of construction and operating costs rather than on technical features. It is therefore unfortunate that no reliable cost estimate can be given at present. A cost estimate is also required for determining the optimum stripping energy for each type of pre-stripper. It should be pointed out, however, that the design of such an accelerator does not present problems which go beyond the present state of cyclotron technology.

TABLE 1

Number of magnets N	6	
Maximum magnetic field B_{max} [kG]	17	
Total accelerating voltage U [MV]	1	
Harmonic number h	6	
Magnet angle α [°]	26	
Spiral angle δ [°]	0	
ν_r	1.05	
ν_z	1.16	
Initial energy ε_i [MeV/nucleon]	2	
Final energy ε_f [MeV/nucleon]	7	
Minimum charge to mass ratio ζ/A	0.184	0.122
Heaviest ion to be accelerated	$^{132}Xe^{25+}$	$^{238}U^{29+}$
Mean radius at injection s_i [m]	1.41	2.19
Mean radius at extraction s_f [m]	2.64	4.10
Orbit frequency [MHz]	2.19	1.41
Radiofrequency [MHz]	13.14	8.46
Peak radius gain at injection Δs_i [cm]	6.7	6.7
Peak radius gain at extraction Δs_i [cm]	3.6	3.6
Loss due to charge exchange at 10^{-7} Torr [%]	<1.3	<3.0
Maximum number of turns n	27	41
hn	162	246
Length of scale in fig. 1 [m]	1.0	1.5
Approximate weight of one magnet [t]	80	250

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