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# On Modern Safeguard in the Field of Peaceful Application of Nuclear Energy

## I. Basic Considerations

## II. Preparational Considerations for a System Analysis

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## BASIC CONSIDERATIONS

## Part II
## PREPARATIONAL CONSIDERATIONS FOR A SYSTEM ANALYSIS

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## Part I
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1. It is useful to recall that one can distinguish between three fairly different phases of the development of the use of nuclear energy for power production. The first phase lasted from 1943 to 1955 and was by and large a military oriented phase. The Hanford and Windscale production plants had been built during this period. During that time keeping track of the power production was synonymous with keeping track of the availability of Plutonium. The second phase started obviously with President Eisenhower's programme, "Atoms for Peace", and the first Geneva Conference of 1955 can be considered as the milestone for that. Then it was research and development with a strong international exchange of information which characterized the style of the second phase. But by the same token, the reactor development was kind of a marginal scientific venture with no real commercial feed back. This changed immediately after the Oyster Creek event, that is in 1964, and this was the beginning of the third phase of reactor development. With some delay other countries have had their Oyster Creek event during these years too. The third phase is the phase of large scale industrial and commercial use of nuclear energy, including full scale industrial competition. It is also during this third phase that the commercial use of all steps of the fuel cycle, that is reprocessing, refabricating and possibly isotope separation, takes place, because a larger population of operating civilian power reactors requires it. And therefore, it is at the beginning of this

third phase that a properly designed safeguard system has to materialize. The design criteria for such a safeguard system must be oriented toward this third reactor phase in spite of the fact that already the second phase required some safeguard measures.

2. It is vital not to proliferate the use of nuclear energy into the military domain. If a nuclear weapons fabrication plant as such would easily be detectable, the safeguard system would then undoubtedly be directed towards the existence of such a plant, and in this case there would be no feed back to the peaceful use of nuclear energy whatsoever. Now it appears to be virtually impossible to detect nuclear weapons factories as such. Therefore, not a direct, but an indirect scheme of safeguard has to be employed.

3. This indirect scheme of safeguard concentrates on the supply of fissionable material, which would be necessary if the use of nuclear energy proliferates into the military domain. If one can ensure that all fissionable material does remain in the civil domain, such proliferation cannot take place. Therefore, it is the only and specific objective of a modern and properly designed safeguard system that all fissionable material, which is being used in the civil domain, remains there. But it is, logically, not the objective of a modern and properly designed safeguard system to control the peaceful application of nuclear energy as such. This creates undoubtedly a feed back from military concerns to the peaceful application of nuclear energy.

4. If the flow of fissionable material in the civil domain could be entirely and effectively contained in the civil domain, this would be the only necessary step. In this case it is irrelevant to know the amount and the quality of the fissionable material. Therefore, it must be the <u>first safeguard measure</u> of a modern safeguard system to make sure that such containment principle materializes where ever this is possible. A reactor building is by its very nature already a containment. The first safeguard measure would mean that one has to make sure that fuel elements for example enter the

reactor building only through one door and leave only through one door and that the building is tight for fuel elements otherwise. This may imply eventually, also more rigid gate controls for personnel and equipment in general.

Irradiated and unirradiated fuel elements (and fuel) require transportation. The first safeguard measure requires the extrapolation of the containment principle to transportation. This leads to the problem of safing and sealing. The reprocessing plant then has again a containment, again the gate controls must complete the containment requirements. Special interest must be given to the control of waste stream. The refabrication plant has basically similar features.

Therefore we conclude, that there is room for materializing the containment idea, that is, the first safeguard measure. The tightness of the physical containments and the fact that there is only one entrance and one exit for fuel can be verified once in the early stages of the construction of the principal nuclear facility in question. The tightness of the containment may be even more obvious, if there are certain established and mandatory ground rules for the general lay out and possibly the construction of the building of a principal nuclear facility. If the containment of the domain of civil application of nuclear energy would be 100 o/o tight, that is effective, no other safeguard measures would be necessary.

5. Concern has been expressed that the domain of civil application of nuclear energy remains a civil domain, inside it's tight containment. In other words, one can think of a situation, where in a principle nuclear facility a clandestine and military oriented loop hole for fissionable material can be installed. By pressing logics to it's extreme, one can argue that even in that case the clandestine military oriented product has finally to leave the containment and will be detected then, but admittedly this argument might not have too much practical importance. The more practical argument is that each principal nuclear facility has to be not too large and as specific as possible, for example it does not seem reasonable to make a whole nuclear complex, e.g. a whole national laboratory, a principle nuclear facility inside a tight containment. Some concern remains if the first safeguard measure remains the only one.

Similarly, reprocessing and even more so refabrication facilities handle fissionable material in the form of aqueous solutions or powders; there is a direct and continuous contact with the fissionable material. Therefore, there is a small but possibly finite possibility of diversion even if the first measure of safeguard (the containment) is rather thoroughly implemented. This is so, because the complete containment in the context of safeguards includes for example also gate controls, and these may be to some minor extent incomplete. In the case of reprocessing and refabrication facilities this is somewhat in contrast to the situation in the case of heterogeneous reactors, and this means virtually all existing reactors. In the case of these heterogeneous reactors the possibility of diversion is significantly smaller, as the fissionable material is contained there in a discrete and finite number of fuel subassemblies, and it is very difficult (if not impossible) to divert a whole fuel assembly if there is gate control. A small but possibly finite possibility of diversion may exist for the case of transportation too.

So it seems necessary to introduce besides of the first safeguard measure a second safeguard measure. This second safeguard measure is to safeguard the flow of fissionable materials throughout the whole fuel cycle. The relevant flow of fissionable material is comprised of bomb grade material only, that is plutonium and highly enriched uranium. If besides of such fissionable material, also low enriched uranium and source material is subject to the second safeguard measure, a feature of redundancy enters the picture.

6. This second measure, namely to safeguard the flow of fissionable material, can best be executed at certain strategic points. The first safeguard measure, that is the containment measure, provides for a kind of conservation of mass flow and it is therefore not necessary to follow the flow of fissionable material everywhere. An extended statistical analysis seems to be necessary to identify all of these strategic points and their relative importance. But if one follows the flow of fissionable material through the closed fuel cycle (reactor, reprocessing plant, refabrication plant and, if necessary, also the waste stream and the isotope separation plant), it is very likely that all entrances and exits of principal nuclear facilities

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are among such strategic points; additional strategic points inside and outside of the principal nuclear facilities may be necessary for assuring fully effective safeguards.

Perhaps one of the most important strategic points at all may be the chemical dissolver at the entrance of the reprocessing plant [1], [2]. In realizing this it becomes also apparent that it is no longer necessary to keep track of the power production scheme of a particular fuel element in a reactor in order to judge on the amount of produced plutonium. This is of particular importance, because to judge on the amount of the produced plutonium by keeping track of the reator power production is a cumbersome thing. If accuracies of 1 - 2 o/o are required, and this is certainly the case, it is not sufficient to know about the integral power production. In that case in addition to integral power measurement, the measurement of the power distribution, the management of fuel loading and control rod operation, the measurement of the spectrum and other information is necessary thus leading to the request, that virtually the last design and operational detail of a reactor, probably without the desired satisfactory ultimate safeguard results, shall be available. One should also bear in mind that power stretching of a given power reactor will be an affair of high commercial significance, and the scheme of power stretching has to be known, if the plutonium production shall be calculated from the power production. This is a dead end road. All what is necessary instead, fortunately, is to make sure that the irradiated fuel element reaches the dissolver of a reprocessing plant. This has been explained by other authors too [3], [4].
If the flow of fissionable material can be safeguarded at these strategic points, it is unnecessary for an inspector to go everywhere at any time. In that case the inner part of operating principal nuclear facilities remains untouched.

7. It is also undesireable that an inspector goes everywhere at any time and it is in turn very desireable indeed that the inner part of operating principle nuclear facilities remain untouched. This is particularly true for reprocessing and refabrication plants. It is namely the cheap fuel cycle which outweighs the higher capital costs of nuclear power plants as compared with conventional power plants and thus ultimately makes nuclear power superior to fossile power. This is demonstrated in table 1

where the contribution of the capital costs and fuel costs to the total energy generation costs are given for a typical coal fired power plant, a light water nuclear power station and a fast breeder power station (of the late 70's).

Table 1

Typical energy generation costs from different power plants (of the late 70's) in mills/kwh

| | coal fired plant | LWR nuclear power station | fast breeder power station |
|---|---|---|---|
| capital costs | 2,1 | 2,55 | 3,0 |
| fuel costs | 4,1 | 1,75 | 1,0 |
| total costs | 6,2 | 4,30 | 4,0 |

In order to have such a cheap fuel cycle, a cheap and effective reprocessing and refabrication with low specific costs must be obtained. This is particularly true for the forthcoming generation of fast breeder power stations, because these power stations are expected to have the highest capital costs. Fast breeder power stations will operate on the basis of the plutonium/uranium fuel cycle and the fuel elements will use plutonium/uranium fuel, beginning with the mixed oxides. Todays fuel fabrication technology is not yet sufficiently developed for meeting the required low specific production costs of something like 100 $/kg-fuel, industrial development still has to go some way. It is very likely that particularly the fuel fabrication technology will be an ever expanding and sophisticated one [5], [6], and that cheap specific fabrication costs will be an integral part for an over all competitivity, especially for fast breeder reactors.

Consider for example the fuel cycle costs of a typical Na-cooled 1000 MWe fast breeder reactor of the late 70's as they are foreseen today. The total fuel cycle costs $K_{total}$ can be roughly described as follows [7], [8]:

$$K_{total} = K_Z + K_A + K_F - K_{Pu} \qquad \text{(mills/kwh)}$$

where

$K_Z$ = interest charges for Pu

$K_A$ = reprocessing costs

$K_F$ = fabrication costs

$K_{Pu}$ = Pu-credit

Let us further consider an increase of 0.1 $\frac{mills}{kwh}$ for $K_{total}$. For the 30 years life-time of the 1000 MWe power station with a load factor of 0.8 this gives an over all increase of 21 mio. $, the present worth of this increase at the beginning of the plants life-time at 7 o/o interest rate, is 8.6 mio. $. This must be measured against the total capital costs for the new plant, which is something like 100 mio. $ (direct costs). One should further realize that the difference between competing bids usually cannot be larger than, say, 5 o/o, that is in our example 5 mio. $. Therefore, 0.1 mills difference in the fuel cycle may already be decisive for getting a reactor order in a competitive environment. One also must realize that a difference in power production costs of 0.2 - 0.3 $\frac{mills}{kwh}$ is usually considered to be a large enough incentive to develop a new reactor line, because a fraction of the savings in terms of present worth may make up for the development costs already. Also in this context therefore 0.1 $\frac{mills}{kwh}$ is not a small quantity.

An increase of 0.1 $\frac{mills}{kwh}$ in $K_{total}$ now appears, if, for example, the specific costs for fabrication increase from 100 $/kg to 130 $/kg. Whatever the assessment of importance to a difference of this order of magnitude might be, it is highly likely that the commercial fuel manufacturer and also the commercial reprocessor have to use the last trick in order to be competitive. Among other things also the possible hampering by highly redundant and therefore, not really necessary safeguarding of inspectors inside an operating fabrication plant must be avoided. A safeguard procedure which concentrates at certain strategic points will suit this situation best.

We also have to mention the problem of industrial proprietary information. It is true that going into a reactor does not reveal proprietary information too quickly, but this may be different already in case of a reprocessing plant and even more so in case of a fabrication plant. Sometimes the argument is put forward that the technology of todays TBP's reprocessing plants is principally known. But even in this case the operation details, like for example the reduction of the valences of Pu, are of commercial significance and further, at some date there might be a technological breakthrough, which changes the whole situation drastically. As mentioned before, the question of industrial proprietary information is more explicit in the Pu fuel fabrication plants of the next five years, and the problem becomes most obvious if one considers the case of an isotope separation plant. Also separation plants belong to the complex of the nuclear fuel cycle and here the application of highly confidential and possibly new technologies is most likely. Separation plants have to be subject to safeguard as it is the case with all the other principal nuclear facilities. Again effective safeguard and the integrity of industrial proprietary information can best be combined, if safeguard concentrates at the properly defined strategic points only.

8. The second safeguard measure, namely to measure the flow of fissionable material at certain strategic points, is effective in counterbalancing the possibly incomplete materialization of the containment principle, that is the first safeguard measure, as long as there is the flow of fissionable material, that is a throughput. But all principal nuclear facilities have an inventory, a hold up, which does not participate (per saldo) in the flow of fissionable material. With respect to a hold up of a principal nuclear facility only the first safeguard measure (containment) is efficient. The ratio of hold up to the integrated throughput in a given time period is characteristic for the degree with which the incompleteness of the first safeguard measure cannot be counterbalanced by the second safeguard measure. This ratio has to be as small as possible and therefore, this requirement determines at least qualitatively the distance between the strategic points. The smaller that distance is, the smaller is that above mentioned ratio and the more effective is the second safeguard measure. A decreasing hold up between strategic points usually is accompanied by increasing constancy of that hold up, and such constancy of the hold up is another desirable feature. A constant hold up namely allows to make firm conclusions about possible

diversions by only measuring the flow of fissionable material past the adjacent strategic points. This will be more explicitly explained in a later chapter.

It is important to realize that the distance between two strategic points with its associated hold up is the free parameter which allows for the adjustment of the efficiency of the safeguard system in question to a required and quantified level. In designing such particular safeguard system it should be the intention however to start with the entrances and exits of the involved principal nuclear facilities as the only strategic points and to increase the number of additional strategic points only to the extent which is necessary for meeting the required and quantified level of systems efficiency.

For that, the required level of systems efficiency has indeed to be quantified. A criterion of the following type must be given by the safeguard authorities:
"The requirements of safeguard are met if with x o/o confidence level the material balance is closed within y o/o."
For the sole purpose of illustrating this statement it shall be mentioned that x o/o may be something like 95 o/o and y o/o may be something like 2 o/o. The exact figures have to come from a particular and detailed system analysis and they may be reviewed from time to time in the light of technological improvement and operational experience. Such quantification is also necessary for the unavoidable and forthcoming cost benefit analysis of such safeguard systems. Remarkably enough, none of the existing safeguard systems has established such a quantified criterion. And all the existing safeguard systems are open ended therefore and that is the ultimate point of concern. Establishing the above mentioned quantified safeguard criterion means that this open endedness is cut and safeguard becomes a rational venture. The later chapters of this paper will deal with the mathematical aspect of all this in greater detail.

In paragraph 5 the concern about clandestine loop holes inside a principal nuclear facility has been mentioned. Here too the freely adjustable distance between strategical points helps to make these loop holes such a remote possibility that the quantified criterion is met.

More generally, one can always think of mechanisms which are extremely remote and which exist more or less only by logical argument but have no practical significance. And here it is useful to recall that the objective of safeguards is not to make diversion impossible rather than to make it more improbable. Meaningful safeguard has limited objectives with limited efforts.

In paragraph 4 it has been mentioned that it is desirable to have certain established and mandatory ground rules for the general lay out and possibly the construction of the building of a principal nuclear facility in order to make the containment function of the building obvious. The same is true for making the strategic points effective and obvious. The dissolver of a chemical reprocessing plant for example can locally be somewhat separated from the main plant and would be accessible therefore for safeguard inspectors. Or the internal storage area of a fuel fabrication plant may, by proper design of the building, be separated from the manufactoring area and in-between there would be a strategic point, where the day by day amounts of fissionable material have to pass by. One has to realize that the majority of chemical reprocessing and fuel fabrication plants are still to come and therefore there is room and time for establishing these ground rules. To a lesser extent this is also true for power reactors, but the whole safeguard problem is much easier there anyway (at least for heterogeneous reactors as mentioned above). Only so far as these ground rules for the general lay out of the building are concerned, that the search for design details is relevant. In advance of the construction of the building of a principal nuclear facility the compliance with these ground rules shall be verified. According to the content of the three safeguard measures there is no reason to require other design details than those mentioned above.

9. Measuring the flow of fissionable material at the strategic points can only be done with a certain accuracy. Integrating the flow over a certain time intervall leads to certain absolute inaccuracies which beyond a certain threshold cannot be accepted. Therefore kind of readjusting the scale is necessary from time to time and this can be done by inventory taking. This may also be necessary if unforeseen events happen, for example a sufficiently large discrepancy in the material balance which cannot be explained.

Therefore a third safeguard measure has to be established, namely the inventory taking.

Inventory taking should be considered only as the third line of defense and it should be the intention to have that as rarely as possible. The assessment of accuracies of measuring the flow of fissionable material, of the distance between strategic points and of other safety system parameters shall be such that such inventory taking possibly coincides with the routine wash outs of the principal nuclear facilities. A reprocessing plant for example envisages such wash outs about twice every year anyway. Inventory taking in itself has certain inaccuracies and can be done by different methods. One method is the wash out. By that the inventory is temporarily transformed into a flow, the inventory leaves the plant past the strategic point at the exit. Therefore it is again not necessary for the inspector to touch the inner parts of the plant. If however the operator of the plant prefers an in plant inventory taking this might be done then. In any event, the mode of inventory taking should be at the discretion of the operator, provided that the envisaged mode of inventory taking meets the actual requirements for accuracy.

10. The three safeguard measures have now been described. The leading idea is to concentrate safeguard action at the strategic points to make it more efficient and less intrusive. Therefore the identification of these strategic points and the evaluation of their efficiency is the first task of the designer of a safeguard system. It should be possible to accomplish that task within a year or two.

If safeguard concentrates at these strategic points of the flow of fissionable material, one naturally looks for the possible use of instruments to measure this flow. This may be also of special importance if one looks for the rapid expansion of the production of nuclear energy. Germany alone expects 20 - 30 000 MWe or so by 1980, the corresponding figures for the common market are 50 - 75 000 MWe, for the whole world 200 000 MWe or more [9]. To safeguard such a population of reactors and associated fuel cycle facilities in all likelihood instruments are needed in order to keep up with this dynamic expansion. The development of these instruments is the second task.

Contrary to the first task this may require more time, maybe something like four or five years. After a successful development of a tamper proof version of these instruments safeguard inspectors shall gradually be replaced by these instruments to the largest possible extent. A proper automatic data processing system is expected to handle and evaluate all these instrument readings.

But it should be recalled that this instrumentation and automatic data processing is only the second stage of establishing a modern safeguard system, the first stage, which can be implemented much more readily, is the identification and installation of the strategic points.

11. After these considerations one arrives at the following scheme for a modern safeguard system:

a) The objective of a modern safeguard system is to reduce significantly the possibility of diversion of fissionable material from the domain of peaceful use of nuclear energy.

b) It is the fissionable material in the domain of peaceful use of nuclear energy and not the peaceful use of atomic energy as such that must be subject to safeguard, which is in view of the ultimate purpose of such safeguard, namely to prevent the illegal manufacturing of nuclear weapons, an indirect approach.

c) The design of a modern safeguard system is governed by a quantified criterion of the following type:

"The requirements of safeguards are met, if with x o/o confidence level the material balance is closed within y o/o."

By such a quantified criterion, to be spelled out by the safeguard authorities, the up to now existing open endedness of safeguard is closed and safeguard becomes a rational venture.

d) The first safeguard measure is to materialize the principle of containing the fissionable material to the greatest possible extent. Therefore this first safeguard measure covers among other things: real containments (buildings) of principal nuclear facilities, gate controls, waste control, safing and sealing, in particular in the case of transportation.

e) The second safeguard measure is to measure the flow of fissionable material at a finite number of strategic points. The assessment of strategic points, their distance and therefore the hold up between two of these strategic points and their required accuracy of flow measurement shall be such, that the quantified criterion c) is met. In particular it will be the amount and the constancy of the hold up between two strategic points which has to be taken into account when this assessment is made.

f) The third safeguard measure is inventory taking, intentionally a rare event for readjusting the scale of flow measurement, which should coincide to the largest possible extent with the anyway expected regular wash outs. The type of inventory taking shall be at the discretion of the operator of a principal nuclear facility, provided that the accuracy of the choosen type of inventory taking is in conformity with the purpose of that inventory taking.

g) Inspectors shall not be allowed to interfere with the operation of a principal nuclear facility and shall have access only to the strategic points.

If in the course of safeguard experience it can be demonstrated that also another area of a principal nuclear facility has to be touched, this other area shall be identified as another strategic point by proper agreements between the involved parties or authorities.

h) Design details of a principal nuclear facility are of relevance for safeguard purposes only insofar, as certain ground rules for the general lay out of the building must be implemented. These ground rules are there to make the containment function of the building obvious and to identify in advance the strategic points and enhance their efficiency.

i) On a somewhat larger time scale tamper proof instruments for measuring the flow of fissionable material at the strategic points shall be developed and their readings shall be processed by an suitable automatic data processing system. As these instruments come up, they shall gradually replace the safeguard inspectors.

This concludes the basic considerations, that is part I of this paper. Part II will deal now with more specific aspects of the mathematical analysis of the safeguard problem.

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# ON MODERN SAFEGUARD
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## Part II

## PREPARATIONAL CONSIDERATIONS FOR A SYSTEM ANALYSIS

by

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### 1. The unit cell of modern safeguard

Part II of this paper shall outline a number of mathematical considerations which prepare for a more rigid and complete systems analysis. In so doing the considerations will concentrate on a single principal nuclear facility the character of which does not have to be specified in detail, but one may think of a fuel fabrication plant. This principal nuclear facility shall have one entrance and one exit only and these two points shall be the strategic points. A fuel fabrication plant has also exits for wastes but in the context of this more abstract consideration we can include these side exits in the above mentioned one exit.

The principal nuclear facility (from here on we will refer to it simply as facility) shall have an inventory, that is a hold up. Throughout our considerations we will be driving towards situations where the hold up can be made as constant as possible because that allows for quick and fairly clean safeguard conclusions. In part I of our paper we have outlined the approach where a facility may be subdivided into a number of units between additional strategic points in order to decrease the hold up and to increase the constancy of the hold up of such units [1] . In the course of the following considerations however we will not make the assumption, that the hold up is necessarily constant. The goal of these now following considerations is to arrive at a number of general terms which make future discussions on safeguard easier and may help to prepare a system analysis. In short: We are considering the unit cell of a modern safeguard system:

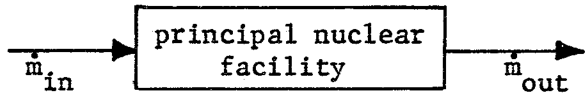


Fig. 1

In Fig. 1, $\dot{m}_{in}$ denotes the rate of input, $\dot{m}_{out}$ the rate of output.

2. The invoking of the third safeguard measure.

The second safeguard measure of measuring $\dot{m}_{in}$ and $\dot{m}_{out}$ envisages the difficulty, that these measurements have a certain inaccuracy. These inaccuracies sum up and after a certain time they will lead to a certain total uncertainty of indicating the hold up. We now assume that an inventory taking has to take place when the integrated inaccuracies have passed a certain threshold, that means, the third safeguard measure shall be invoked then.

As a matter of fact, there are possibilities for other criteria to invoke the third safeguard measure. For example: The inventory as calculated from input and output has passed a certain level, or the averaged time derivative of the hold up as calculated from input and output has passed a certain level, or a combination of both. Or one can think of certain tracer techniques which lead to the establishment of a criterion to invoke the third safeguard measure. Or further, considerations for the incompleteness of the first safeguard measure may lead to the invoking of the third safeguard measure.

Here we will assume however, that the third safeguard measure takes place when the integrated inaccuracies of the flow measurements have passed a certain level.

We further assume here that the operator is aware of the results and characteristics of the inspectors measurements. This is not necessarily the case, the opposite may happen, but with the considerations of this paper we are driving towards the definition of rather general terms and therefore this assumption will not have too strong consequences here. A more specific system analysis however,may arrive at quite different detailed conclusions indeed, if the assumption is made that the operator is not aware of the inspector's measurements.

3. The three statements of statistical nature.

The inventory of the facility as calculated by the flow measurements of the inspector at time t shall be the following:

$$J\ (t) = \int_0^t \dot{m}_{in}\ dt' - \int_0^t \dot{m}_{out}\ dt' + J\ (0) \qquad (1)$$

As mentioned before, $\dot{m}_{in}$ and $\dot{m}_{out}$ are the input and output rates as measured by the inspector and J (0) is the inventory at time t = 0, one can think of a situation, where t = 0 has been the last inventory taking. We are going to determine the time $t_0$ at which the total uncertainty of $J(t_0)$ has reached an absolute value, say $U_0$, for exemple lo kg or so, according to the following equation for the propagation of the respective error σ :

$$\sigma_J^2\ (t) = \int_0^t \frac{d(\sigma^2)in}{dt}\ dt + \int_0^t \frac{d(\sigma^2)out}{dt}\ dt + \sigma_0^2 \qquad (1a)$$

At that time a direct measurement of the inventory is being required by the safeguard authority. The directly measured inventory shall give the result I $(t_0)$. If J (0) makes use of the information coming from the last inventory taking we have

$$J(0) = I(0)$$

Let us recall: We made the assumption that the operator is aware of J(t) and I $(t_0)$.

Also in the following considerations we have constantly to be aware of this quasi partnership of operator and inspector and we have to distinguish sharply between statements of the operator and statements of the inspector.

There will be three main classes of statements:

α) The probability of diversion $P_D$ $(d_0)$
This will be principally a statement of the inspector, it is his assessment of the probability that an amount $d \geq d_0$ has been diverted with a probability $P_D$.

β) The risk of detection $R_D$ $(d_0, m_0)$
This will be principally a statement of the operator. If the operator intends to divert a certain quantity $m_0$ he is able to calculate the risk $R_D$ that the inspector makes a statement $P_D$ $(d_0)$.

γ) The probability of proofing $P_P$ $(\xi_0, m_0)$
This will be principally a statement of the safeguard system designer. The system designer considers the diversion of $m_0$ by the operator and calculates the probability $P_P$ that the inspector can make a statement $P_D$ about the diversion of the fraction $\xi$ of $m_0$.

4. The Gaussian distribution

The considerations of this chapter do not refer necessarily to Gaussian distributed errors of a measurement. But it will be of help if general relations are more specifically spelled out in terms of the Gaussian distribution. We therefore introduce the parameters of the Gaussian distributions here.

The Gaussian distribution $\phi_\sigma$ $(x|\mu)$ is given as follows

$$\phi_\sigma (x|\mu) = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad (2)$$

$\phi$ is probability density
$x$ is the value of a measurement
$\mu$ is the mean value of the measurements
$\sigma$ is the standard deviation, $\sigma^2$ is the variance

The probability, that the measurement x falls between certain limits A and B is then given by the following expression

$$P\ (A \leq x \leq B) = \int_A^B \phi_\sigma\ (x'|\mu)\ dx' \qquad (3)$$

We have

$$\int_{-\infty}^{+\infty} \phi_\sigma\ (x'|\mu)\ dx' = 1 \qquad (4)$$

For practical numerical purposes one has to realize that

$$\int_{\mu-4\sigma}^{\mu+4\sigma} \phi_\sigma\ (x'|\mu)\ dx' = 0{,}9999366 = 1 - 6{,}3 \cdot 10^{-5} \approx 1 \qquad (5)$$

One therefore can call the range $(\mu - 4\sigma,\ \mu + 4\sigma)$ of the width $8\sigma$ the range of uncertainty U

$$U = 8\sigma \qquad (6)$$

The relative uncertainty shall be given by the following expression:

$$u = \frac{U}{\mu} \qquad (7)$$

The following figure illustrates these relations:

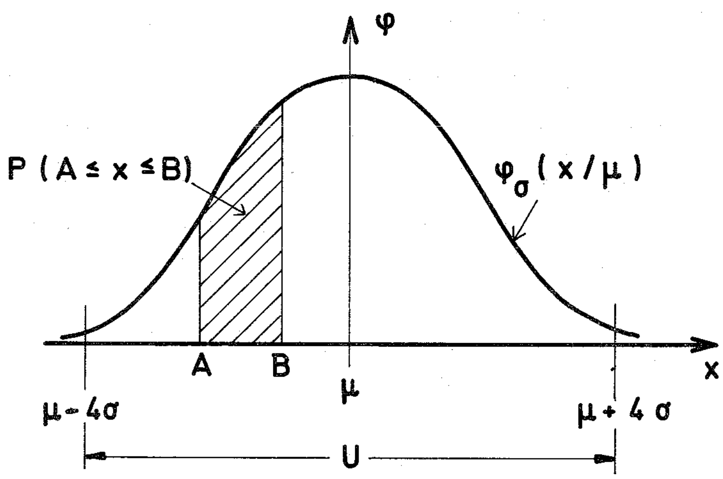


Fig. 2

One should recall that in the theory of diffusion we have (see also (1a) )

$$\sigma^2 \sim t$$

5. The comparison of different inventory measurements

The measurement of the input and output rates, that is the flow, may have resulted at time $t_0$ in a certain value for the inventory $J = J(t_0)$ according to (1). The mechanism of equation (1) results in only one value. But for a rigid statistical assessment we have to have at least two flow measurements, therefore we do have to make the assumption that all flow measurements are performed at least twice, either by repeating each flow measurement or better by installing a second channel for flow measurement. As a matter of fact, one should not discard the possibility that this second channel for flow measurement could be the operators channel. This has to some extent been considered more recently[2]. Such a procedure will result in a situation where the inspector (and according to our assumption also the operator) is aware of a mean value of $J(t_0)$ and the connected standard deviation $\sigma_J$. We denote $J_0$ to be that mean value. The true value of the hold up will then be somewhere in the intervall $(J_0 - 4\sigma_J, J_0 + 4\sigma_J)$, (see (5) and (6) ). According to our assumptions of paragraph 2.) $t_0$ will have been choosen by the inspector in such a way, that at $t_0$ the absolute range of uncertainty $U(t_0)$ has passed a certain threshold and the inspector invokes the third safeguard measure, the inventory taking. This inventory taking results in an actual value Ia.
Please note: All third safeguard measurements result in I, second safeguard measurements in J. We now distinguish between three different cases:

a) The inventory measurement is exact. This means that Ia is the true value.

b) There exists an a priori knowledge of the standard deviation $\sigma_I$ of taking the inventory. Further, the inventory taking shall be executed n times, therefore there exists a mean value $I_0$ (if $n > 1$) and the standard deviation $\frac{\sigma_I}{\sqrt{n}}$ for the mean value of the n measurements. The true value will then lie in the interval

$$I_0 - 4\frac{\sigma_I}{\sqrt{n}}, \quad I_0 + 4\frac{\sigma_I}{\sqrt{n}}$$

c) There exists only the a priori knowledge of the standard deviation $\sigma_I$ of taking the inventory, but the inventory taking shall be executed only once, no mean value exists therefore.

5a) We now consider the case a) in more detail. The result of the inspectors flow measurement is distributed as follows

$$\phi_{\sigma_J}(x|J_0) = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma_J} \cdot e^{-\frac{1}{2}\left(\frac{x - J_0}{\sigma_J}\right)^2} \qquad (8)$$

According to our assumptions the result of the actual inventory taking Ia is the true value. We therefore can illustrate the situation as follows:

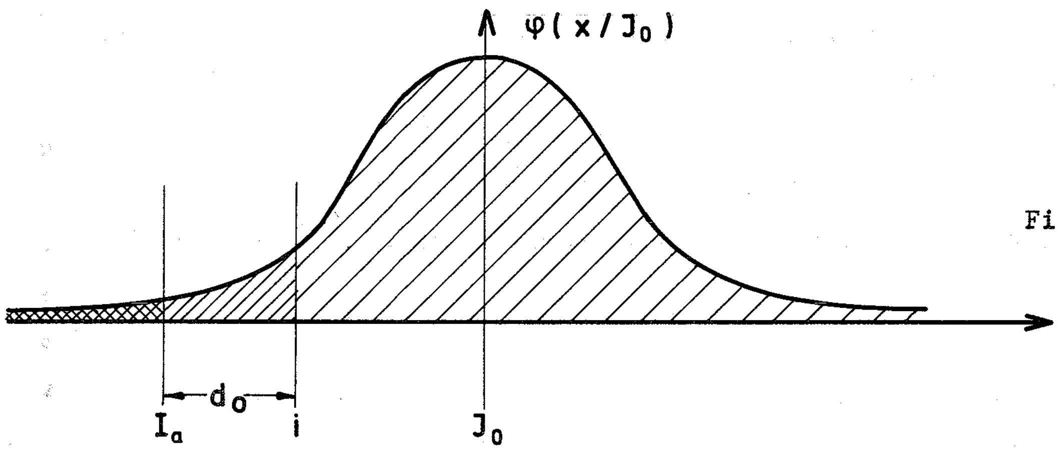


Fig. 3

In the general case we have to assume $Ia \neq J_0$. The difference $(J_0 - Ia) > 0$ indicates that there might have been a diversion.
In that case of a diversion the missing amount is either still in the containment or may have been brought out of the containment by making use of the incompleteness of the containment. The inspector now cannot conclude that the diverted amount was $J_0$ - Ia. All what he can say is the following set of statements:

α) The true value of J is within the range ($i \leq J \leq \infty$) with the following probability:

$$P_{i,\infty} = P\ (i \leq J \leq \infty) = \int_{i}^{\infty} \phi_{\sigma_J}\ (x'|\ J_0)\ dx' \qquad (9)$$

$i = Ia + d_0$

Therefore a quantity $d \geq d_0$ has been diverted with a probability $P_{i,\infty}$

β) The true value of J is within the range ($Ia \leq J \leq i$) with the following probability:

$$P_{Ia,i} = P\ (Ia \leq J \leq i) = \int_{Ia}^{i} \phi_{\sigma_J}\ (x'|J_0)\ dx' \qquad (1o)$$

Therefore a qantity $0 \leq d \leq d_0$ has been diverted with a probability $P_{Ia,i}$

γ) The true value of J is within the range ($-\infty \leq J \leq Ia$) with the following probability:

$$P_{-\infty,\ Ia} = P\ (-\infty \leq J \leq Ia) = \int_{-\infty}^{Ia} \phi_{\sigma_J}\ (x'|J_0)\ dx' \qquad (11)$$

Therefore a quantity $d \geq 0$ has been added (!) with a probability $P_{-\infty,\ Ia}$

The statement α) can be used to assess the probability of diversion of an amount $\geq d_0$. It should be realized however that such an assessment only makes sense in the context of the statements β) and γ) , in particular γ). This can best be demonstrated, if $Ia = J_0$. In that case an amount $\geq d_0$ has been diverted with the probability $P_{d_0,\infty}$, but it is also true that the same amount $\geq d_0$ has been added with the same probability (!)
With this reservation we define

$$P_D\ (d_0) = \int_{Ia\ +\ d_0}^{\infty} \phi_{\sigma_J}\ (x'|J_0)\ dx' \qquad (12)$$

for the statement, that an amount $d \geq d_0$ has been diverted with the probability $P_D(d_0)$.
Because of the above mentioned reservation $P_D$ as such is only then intuitivly indicative for diversion, if the probability for having added something is sufficiently small, or in other words, if $P_D$ is sufficiently large, say above 0,85 or so. If that is not the case, the statement α) has always to be

accompanied by statements β) and γ).

5b) We now take into account the more realistic case b)
The inspector has in that case besides of (8) the distribution for the inventory taking. For the measurement I we get:

$$\phi_{\sigma_I/\sqrt{n}}(I|I_0) = \frac{1}{\sqrt{2\pi}} \cdot \frac{\sqrt{n}}{\sigma_I} \cdot e^{-\frac{1}{2}\left(\frac{I - I_0}{\sigma_I/\sqrt{n}}\right)^2} \qquad (13)$$

We therefore obtain the following situation:

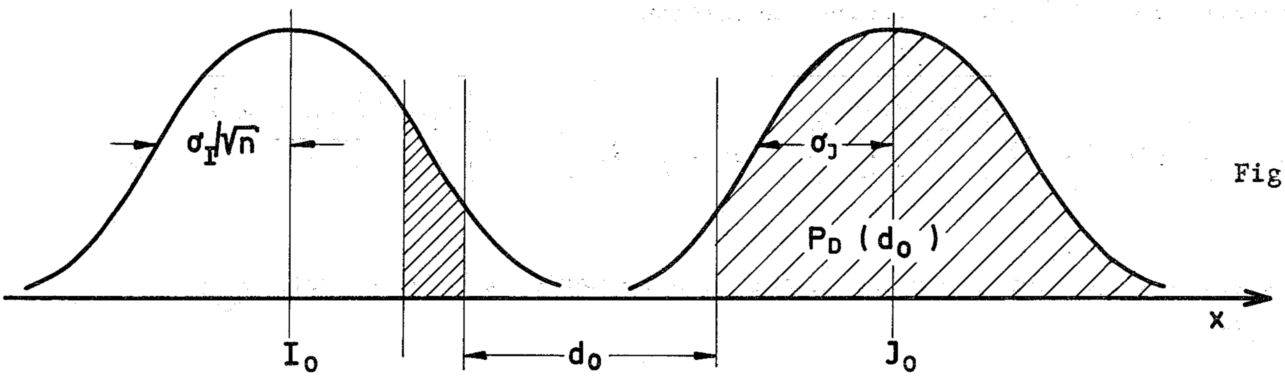


Fig. 4

The true value of the inventory as taken from the third safeguard measure is with the probability $\phi_{\sigma_I/\sqrt{n}}(I|I_0)\, dI$ at I. According to 5a) this gives the following contribution $d\, P_D(d_0)$

$$d\, P_D\, (d_0) = \phi_{\sigma_I/\sqrt{n}}(I|I_0)\, d\, I \cdot \int_{I + d_0}^{\infty} \phi_{\sigma_J}(x'|J_0)\, dx' \qquad (14)$$

I may vary from $-\infty$ to $+\infty$ and we have to make the following integration in order to arrive at $P_D\, (d_0)$:

$$P_D\, (d_0) = \int_{-\infty}^{+\infty} \phi_{\sigma_I/\sqrt{n}}(I|I_0)\, dI \cdot \int_{I + d_0}^{\infty} \phi_{\sigma_J}(x'|J_0)\, dx' \qquad (15)$$

This is a double integral and we will be able to execute one of these two integrations. For that we consider the range of integration:

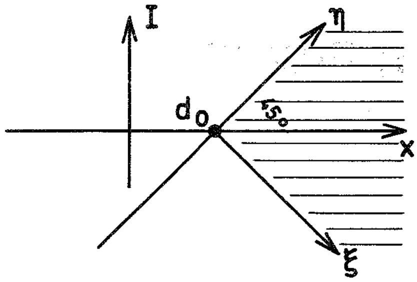


Fig. 5

With the transformation

$$x = \frac{1}{\sqrt{2}}(\xi + \eta) + d_0 \qquad \text{or} \qquad \xi = \frac{1}{\sqrt{2}}(I - (x - d_0))$$

$$I = \frac{1}{\sqrt{2}}(\eta - \xi) \qquad\qquad \eta = \frac{1}{\sqrt{2}}(I + (x - d_0))$$

and the Jacobian

$$\frac{\delta\ (x, I)}{\delta\ (\xi, \eta)} = 1$$

we obtain from (15) the following expression:

$$P_D(d_0) = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \frac{\sqrt{n}}{\sigma_I} e^{-\frac{1}{2}\left[\frac{\frac{1}{\sqrt{2}}(\eta-\xi)-I_0}{\sigma_J/\sqrt{n}}\right]^2} d\eta \cdot \int_0^{\infty} \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_J} e^{-\frac{1}{2}\left[\frac{\frac{1}{\sqrt{2}}(\xi+\eta)+d_0 - J_0}{\sigma_J}\right]^2} d\xi$$

The first integration along the $\eta$ axis can be performed and we obtain the following

$$P_D(d_0) = \int_{I_0+d_0}^{\infty} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\frac{\sigma_I}{n} + \sigma_J^2}} e^{-\frac{1}{2}\left[\frac{Z' - J_0}{\sqrt{\frac{\sigma_I^2}{n} + \sigma_J^2}}\right]^2} dZ' \qquad (17)$$

where $$Z = \xi + d_0 + I_0 \qquad (18)$$

If one takes (8) and (12) one realizes, that (18) is of the same nature as (12) . $I_0$ is the mean value and the standard deviation has been enlarged, $\sigma_J$ has been replaced by $\sqrt{\frac{\sigma_I^2}{n} + \sigma_J^2}$ . One can arrive immediately at (17) with the following reasoning: In case of no diversion the mean value $I_0$ of n inventory takings is expected to be distributed with the distribution $\phi_{\sigma_J}(x|J_0)$, that means, that

$$\phi_{ges} = \int_{-\infty}^{+\infty} \phi_{\sigma_J}(I_0' \,|J_0)\, \phi_{\sigma_I/\sqrt{n}}(I|I_0')\, dI_0' \qquad (19)$$

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is the distribution, against which the actually measured mean value of n inventory measurements has to be balanced in the same sense as in case a) that is (12), if a statement on possible diversion is to be obtained. According to the theorem of propagation of errors we have

$$\phi_{ges} = \phi_{\sqrt{\sigma_J^2 + \sigma_I^2/n}}(x|J_0) \qquad (20)$$

This situation is illustrated in the next figure

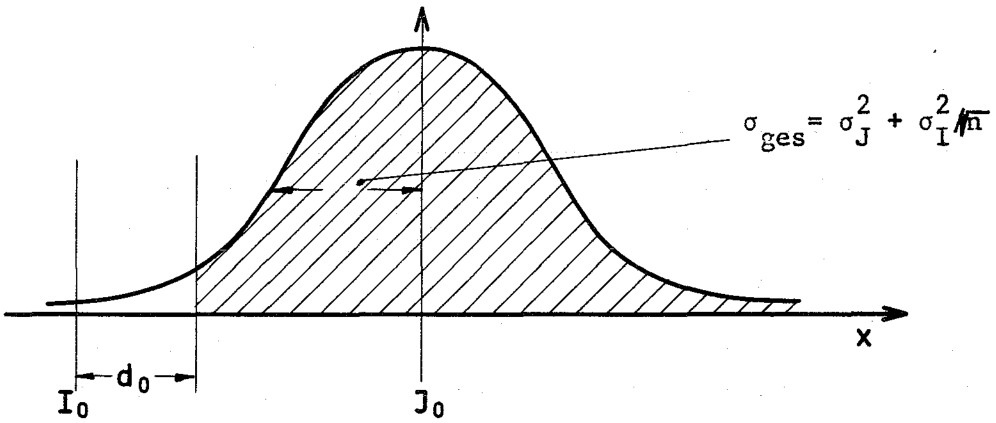



Fig. 6

The situation of Fig. 4 and Fig. 6 is identical according to the equality of (15) and (17).

5c) Now it is easy to deal with case c). Here again $\sigma_I$ is known a priori according to our assumption, but only one inventory taking is executed. Therefore we apply (17) with n = 1 and $I_0$ = Ia, if Ia is that one actually measured value of inventory taking.

## 6. Numerical examples:

Let us introduce

$$\varepsilon = \frac{\sigma_I}{\sigma_J} \qquad (21)$$

Diagram 1 shows equation (17) for the following ε values

$$\varepsilon = 0, \; \varepsilon = 1, \; \varepsilon = 2$$

The plot has been normalized as follows

$$\nu = \frac{d_0}{8\sigma_J} \quad \text{(in the diagram's notation } U_0 = 8\sigma_J\text{)}$$

and $$\alpha = \frac{J_0 - I_a}{8\sigma_J}$$

7. The risk of diversion $R_D$

The risk of diversion is a statement of an operator who is contemplating about the question whether he can risk a diversion or not. This operator has one more information than the inspector, namely he knows the amount $m_0$ which he intends to divert. In the following considerations we make the assumption, that this quantity $m_0$ is known exactly, in case it is known only with a certain accuracy, all relevant standard deviations $\sigma^2$ have to be enlarged according to $\sigma^2_{tot} = \sigma^2 + \sigma^2_{m_0}$.

Let us recall that the operator is aware of the flow measurements of the inspector at the time of his contemplation about the diversion of $m_0$, he therefore knows $\phi_{\sigma_J}(x|J_0)$. This distribution is the basis for expecting the results of the inventory measurements I immediately before the inventory taking. If the operator diverts $m_0$ he has to consider a different distribution, namely $\phi_{\sigma_J}(x|J_0 - m_0)$ as the basis for his expectation of the I's. Let us assume for the moment, that the inventory taking will be exact, that $\sigma_I = 0$, an actual inventory taking gives therefore the exact value of inventory. This situation is illustrated in the following figure:

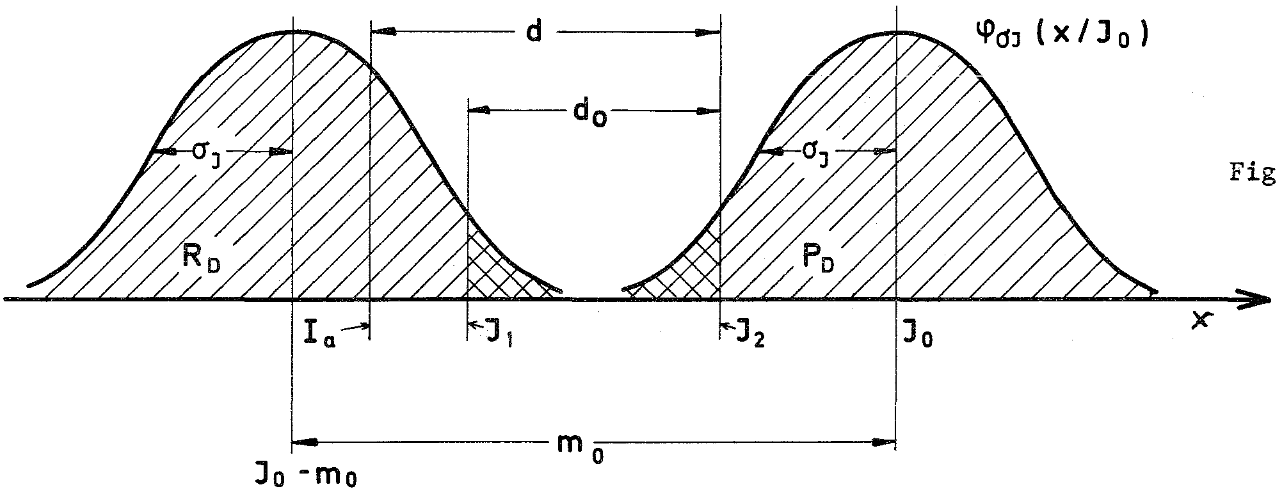



Fig. 7

Now the operator argues about the situation of this actual inventory taking. With the probability

$$P_{-\infty,\, J_1} = P\,(-\infty \leq I \leq J_1) = \int_{-\infty}^{J_1} \phi_{\sigma_J}\, (x'|J_0 - m_0)dx' \qquad (22)$$

this actual inventory taking is within the range $(-\infty, J_1)$, say at Ia. In that case the operator has to expect a statement $P_D$ from the inspector as follows: With the probability

$$P_D\,(d) = \int_{Ia+d}^{+\infty} \phi_{\sigma_J}\, (x'|J_0)\, dx' \qquad (23)$$

an amount of material $\geq$ d has been diverted. With the probability $P_{-\infty,\, J_1}$ the value of d in (23) is $\geq d_0$ and $P_{-\infty,\, J_1}$ is the risk that $Ia \leq J_1$. We therefore define $P_{-\infty,\, J_1}$ to be the risk $R_D^1\, (d_0, m_0)$.

The operator associates the risk

$$R_D\,(d_0, m_0) = \int_{-\infty}^{J_1} \phi_{\sigma_J}\, (x'|J_0 - m_0)\, dx' \qquad (24)$$

to a statement of the inspector that with the probability $P_D$

$$P_D = \int_{J_2}^{\infty} \phi_{\sigma_J}\, (x'|J_0)\, dx' \qquad (25)$$

an amount of material $\geq$ d, where $d \geq d_0$, has been diverted. Before we interprete (24) and (25) further, we will introduce the case of inaccurate inventory taking. In that case the probability of having an actual inventory taking within the range $(-\infty, J_1)$ is

$$P_{-\infty,\, J_1} = P(-\infty \leq I \leq J_1) = \int_{-\infty}^{J_1} \phi_{\sqrt{\sigma_J^2 + \sigma_I^2}}\, (x'|J_0 - m_0)\, dx' \qquad (22a)$$

instead of (22), because the standard deviation $\sigma_I$ is broadering the former distribution $\phi_{\sigma_J}\, (x'|J_0 - m_0)$, the inventory taking contains its own error. Now the operator expects a statement of the inspector after having taken the inventory. The inspector is not aware of the former considerations of the operator and makes a straight forward statement taking into account $\sigma_J$ and $\sigma_I$, according to (17) with $n = 1$. Therefore we have:

$$P_{d(d)} = \int_{Ia+d}^{\infty} \phi_{\sqrt{\sigma_J^2 + \sigma_I^2}} (x' | J_0) \, dx' \qquad (23a)$$

We therefore have instead of (24) and (25) the following

$$R_D (d_0) = \int_{-\infty}^{J_1} \phi_{\sqrt{\sigma_J^2 + \sigma_I^2}} (x' | J_0 - m_0) \, dx' \qquad (24a)$$

and $$P_D = \int_{J_2}^{\infty} \phi_{\sqrt{\sigma_J^2 + \sigma_I^2}} (x' | J_0) \, dx' \qquad (25a)$$

As a matter of fact, if parallel to our previous considerations the inventory taking takes places n times, only $\frac{\sigma_I}{\sqrt{n}}$ is to be taken into account for the mean value of that inventory taking. We now discuss (24a) and (25a). The mathematical form of $R_D$ $(d_0)$ is simple but the important thing is to realize the interconnections of the parameters of (24a) and (25a). In order to do this, we make clear that the operator starts the line of arguments with a reflection on the value of $P_D$ which he is ready to face. Therefore this value $P_D$ is known to him, say 0,85 for instance. By that the operator arrives at $J_2$. Now the operator continues to reflect on the lower limit of the d's, which he is ready to face, that is $d_0$. By that he obtains $J_1 = J_2 - d_0$, the upper limit of (24a). This approach for defining $R_D$ becomes fully obvious, if there is an established threshold of alarm, say $P_D = (P_D)_{AL}$ and $d_0 = (d_0)_{AL}$. In that case the operator will probably consider these values for $P_D$ and $d_0$ as a starting point for his assessment of the risk $R_D(d_0, m_0)$. The diagrams 2 and 3 have been plotted for this letter case. The following thresholds of alarm have been considered. In case of diagram 2 we have

$$(d_0)_{AL} \geq 0,1 \ U_J$$
$$(P_D)_{AL} = 0,7; \ 0,90; \ 0,99$$

In case of diagram 3 we have

$$(d_0)_{AL} \geq 0,2 \ U_J$$
$$(P_D)_{AL} = 0,7; \ 0,90; \ 0,99$$

The other parameters are as follows

$$\varepsilon = \frac{\sigma_I}{\sigma_J}$$

$$\mu = \frac{m_0}{8\sigma_J} \qquad ( 8\sigma_J = U_0 \text{ in Diagram 3 } )$$

$\mu$ therefore is the normalized amount of material which the operator intends to divert.

8. The probability of proofing $P_P$

We recall that this probability of proofing $P_P$ is a statement of the safeguard system designer.

We assume for the moment that the inventory taking is accurate, an actual inventory taking Ia gives the true inventory.

The inspector of the safeguard system which the designer is considering, is aware of the distribution $\phi_{\sigma_J}$ $(x|J_0)$ of the flow measurements. Now the system designer assumes that the operator diverts the quantity $m_0$. In the sequence of the arguments of the designer he only can expect that an actual inventory taking will take place, it does however not take place yet in reality. The designer therefore can only attribute a certain probability for obtaining a certain result of inventory taking. This leads the designer to the distribution $\phi_{\sigma_J}$ $(x|J_0 - m_0)$ for the prediction of the actual value Ia of inventory taking. The fact, that this inventory does not take place, is one of the differences between the statement of the inspector $P_D$ and that of the system designer $P_P$. In the system designer's line of argument, an inspector has the chance to detect an amount $d \geq d_0$ with a value of $P_D > 0$ whatever it may be:

$$A\ (d_0) = \int_{-\infty}^{+\infty} \phi_{\sigma_J}\ (x'|J_0 - m_0)\ dx' \cdot \int_{x'+d_0}^{\infty} \phi_{\sigma_J}\ (x''|J_0)\ dx'' \qquad (26)$$

The mathematical form of (26) is the same as in (15), we therefore can write immediately

$$A\ (d_0) = \int_{d_0}^{\infty} \frac{1}{2\pi} \frac{1}{\sigma_J\sqrt{2}}\ e^{-\frac{1}{2}\left[\frac{\rho - m_0}{\sqrt{2}\,\sigma_J}\right]^2} d\rho \qquad (27)$$

$m_0$ is a known quantity for the system designer. The proofing probability asks for the probability that a fraction $\geq \zeta$ of $m_0$ can be detected. We therefore introduce the quantity $\zeta' = \frac{\rho}{m_0}$ in the integral and arrive at the following expression

$$A\ (d_0) = \int_{\zeta_0}^{\infty} \frac{1}{2\pi} \frac{m_0}{\sigma_J\sqrt{2}}\ e^{-\frac{1}{2}\left[\frac{\zeta' - 1}{\sqrt{2}\,\sigma_J}\right]^2 \cdot m_0^2} d\zeta' \qquad (28)$$

This is already the proofing probability $P_P\ (\zeta_0, m_0)$ in case of an exact inventory taking. According to 5c) the procedure in case of an inaccurate inventory taking is exactly parallel to (15), all what has to be changed is the standard deviation. We therefore obtain finally

$$P_P\ (\zeta_0, m_0) = \int_{\zeta_0}^{\infty} \phi_{\sigma_{tot}}\ (\zeta'|1)\ d\zeta' \qquad (29)$$

where

$$\sigma_{tot} = \frac{\sqrt{2\sigma_J^2 + 2\ \sigma_I^2}}{m_0} = \frac{\sigma_J\sqrt{2 + 2\varepsilon^2}}{m_0} \qquad (3o)$$

The designer now makes the statement, that his safeguard system is capable to detect a fraction $\zeta \geq \zeta_0$ of a diversion $m_0$ with the probability $P_P$. Note: the larger $m_0$ is the smaller is the standard deviation $\sigma_{tot}$ of the distribution $\phi_{\sigma_{tot}}$ $(\zeta'|1)$. In diagram 4 equation (29) is evaluated. In this diagram the abzissa is

$$\mu = \frac{m_0}{8\sigma_J}$$

The parameters $\varepsilon$ and $\zeta$are as follows

$$\zeta = \frac{d_0}{m_0}$$

and

$$\varepsilon = \frac{\sigma_J}{\sigma_I}$$

The considerations of part II of this paper shall help to prepare a more rigid system analysis. They shall help to focus the attention to the fact that all safeguard systems are necessarily not ideal and that there is room for making quantitative statements. In distinguishing between the statements of the inspector, the operator and the system designer one can see that the evaluation of safeguard systems is not a straight forward thing.

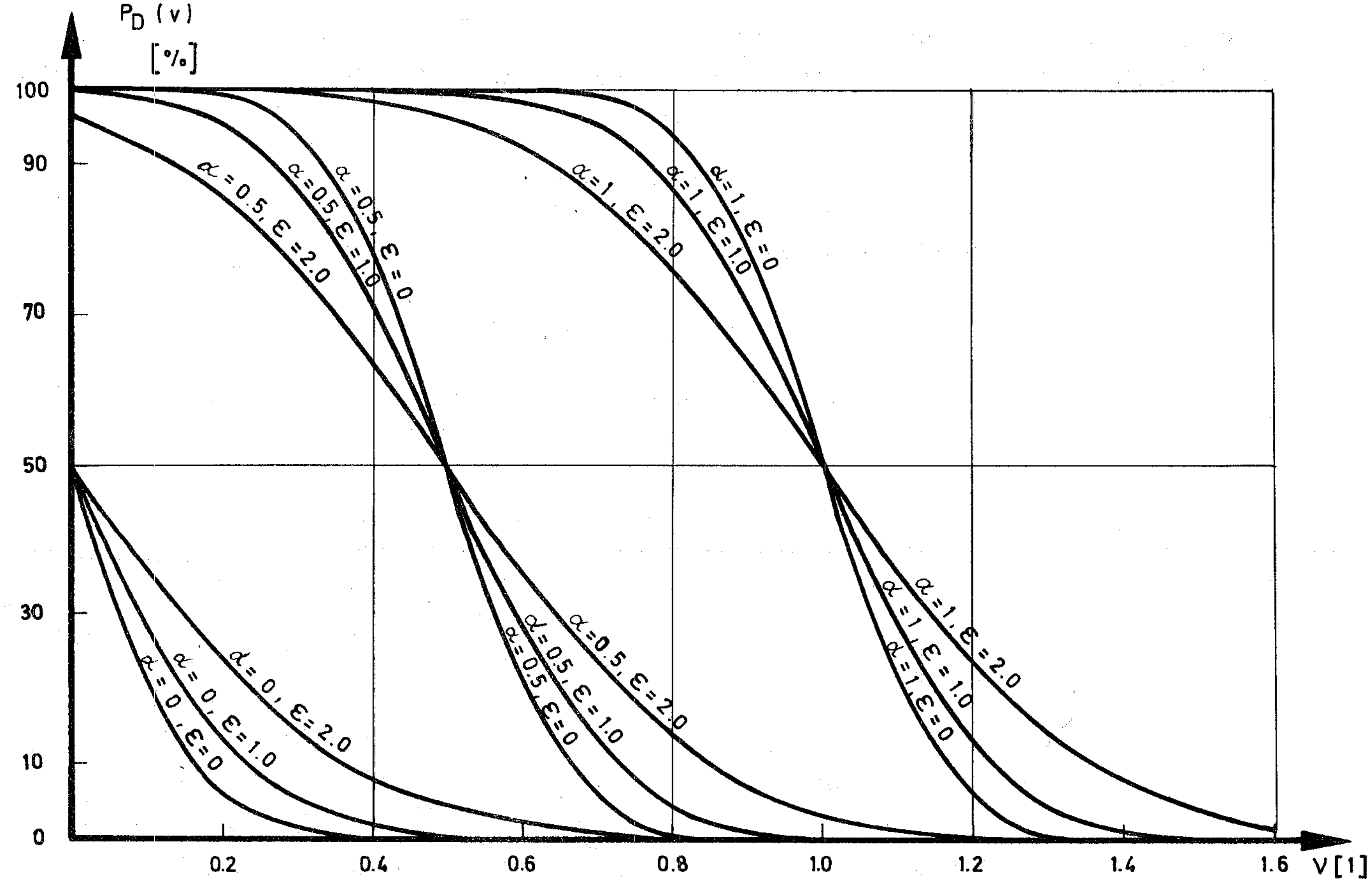
## Literature (Part II)


/¯1_7 Baeckmann, A.v.; Gmelin, W.; Gupta, D.; Häfele, W. — Fissile Material Flow Control at Strategic Points in a Reprocessing Plant

KFK 8o1 (1968)

/¯2_7 Gmelin, W.; Gupta, D.; Häfele, W. — Use of Statistical Analysis for the Establishment of Material Balance in a Reprocessing Plant

KFK 8o2 (1968)



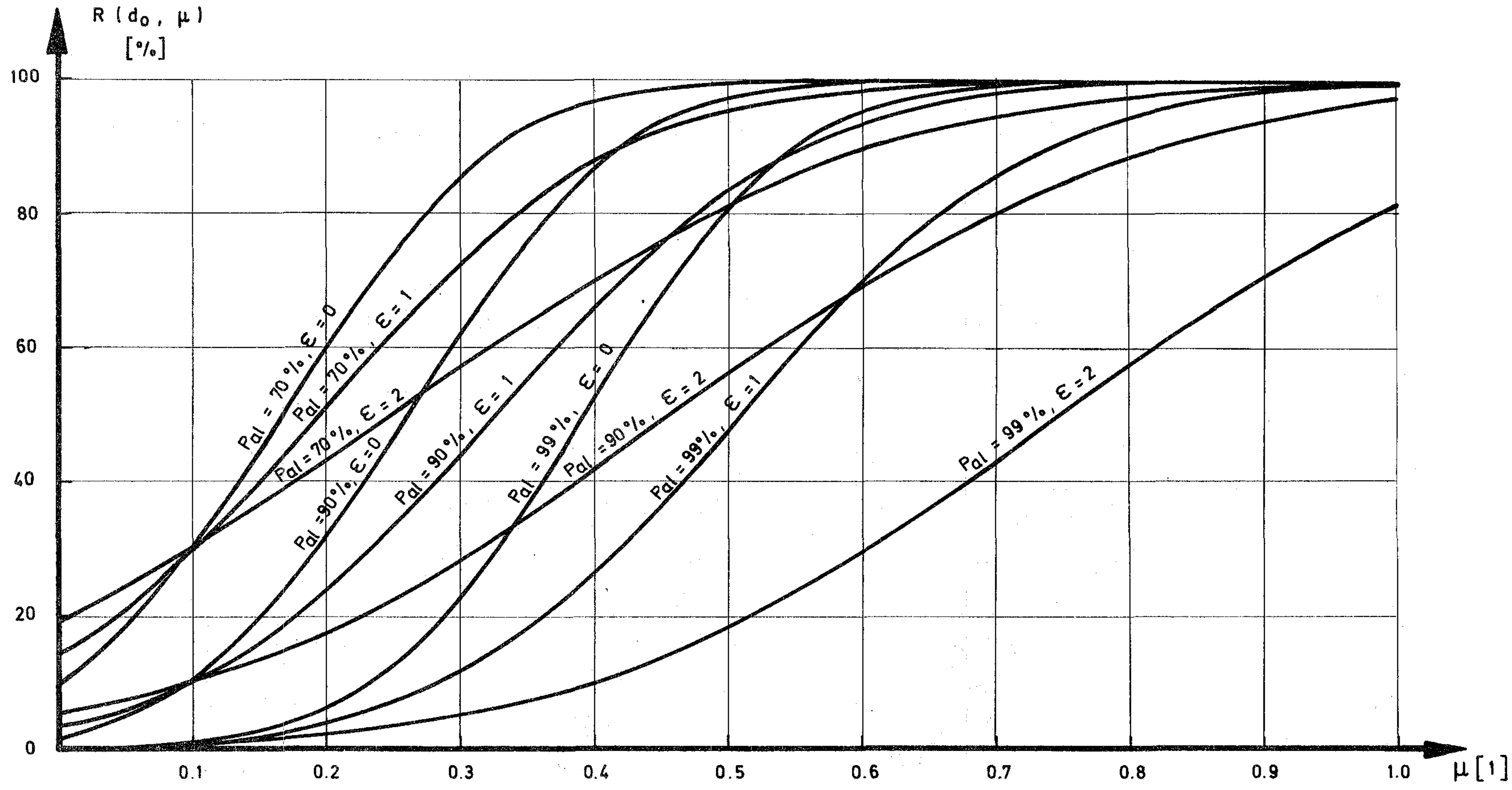

where :

$$V = \frac{d_o}{U_o}$$

$$\alpha = \frac{(J_o - I_a)}{U_o}$$ The difference between the mean values of throughput and inventory measurements normalized to $U_0$

$$\varepsilon = \frac{\sigma_I}{\sigma_J}$$ The ratio of standard deviations of throughput ($\sigma_J$) and inventory ($\sigma_I$) measurements

DIAGRAM : 1 PROBABILITY OF DIVERSION · $P_D$ ($d_o$) AS A FUNCTION OF ν FOR GAUSSIAN DISTRIBUTIONS



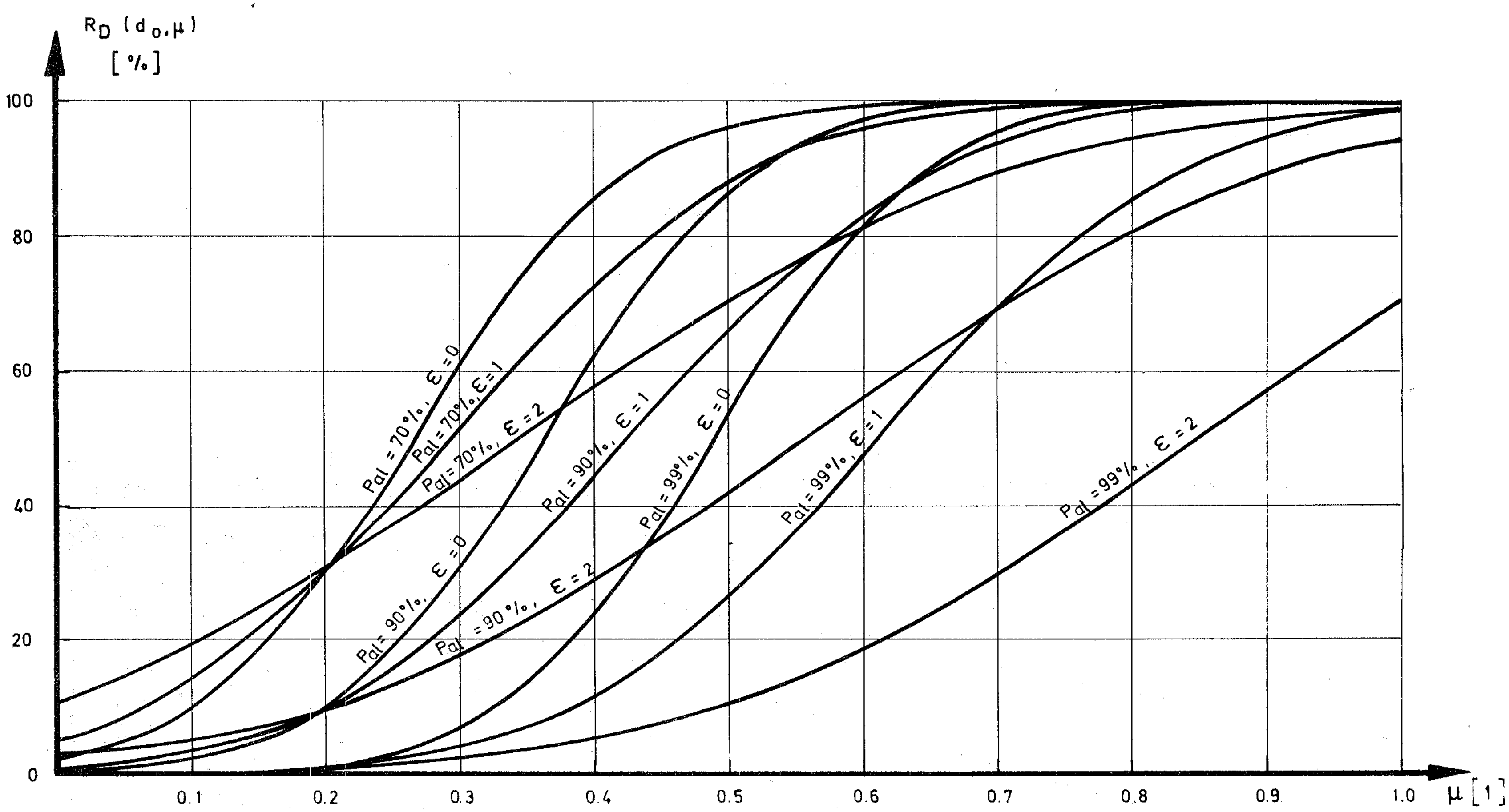

where : $P_{al}$ = The probability $(P_D)_{AL}$

$\mu = \frac{m_o}{U_o}$

$m_o$ = The amount the operator schedules to divert

$\varepsilon = \frac{\sigma_I}{\sigma_J}$ see DIAGRAM 1

DIAGRAM : 2 RISK OF DIVERSION $R_D(d_o, m_o)$ AS A FUNCTION OF $\mu$ WITH $(d_o)_{AL} = 0.1\ U_o$ FOR GAUSSIAN DISTRIBUTIONS



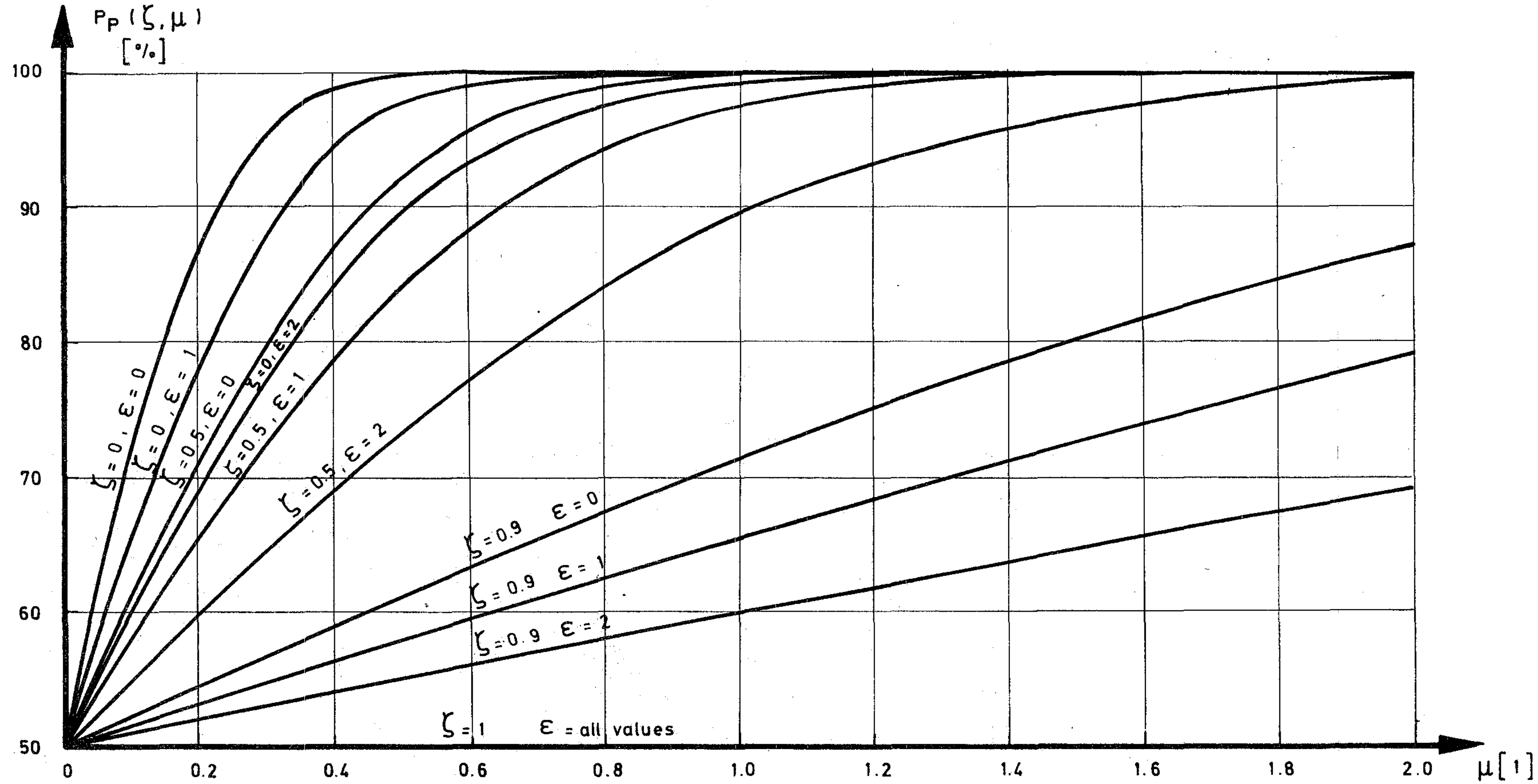

where: $P_{al}$ = The probability $(P_D)_{AL}$

$\mu = \frac{m_o}{U_o}$

$m_o$ = see DIAGRAM 2

$\varepsilon = \frac{\sigma_I}{\sigma_J}$ see DIAGRAM 1

DIAGRAM: 3 RISK OF DIVERSION $R_D$ $(d_o, m_o)$ AS A FUNCTION OF $\mu$ WITH $(d_o)_{AL} = 0.2\ U_o$ FOR GAUSSIAN DISTRIBUTIONS




where: $\mu = \dfrac{m_o}{U_o}$

$m_o$ = see DIAGRAM 2

$\zeta = \dfrac{d_o}{m_o}$ , $\varepsilon = \dfrac{\sigma_I}{\sigma_J}$ see DIAGRAM 1

DIAGRAM : 4 THE PROBABILITY $P_P(\zeta, m_o)$ FOR GAUSSIAN DISTRIBUTIONS AS A FUNCTION OF $\mu$