## KERNFORSCHUNGSZENTRUM

## KARLSRUHE

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Institut für Reaktorentwicklung

Models, Methods and Digital Computer Programs for Analyses in
Reactor Dynamics with Emphasis on Fast Breeder Reactors
and Compressible Single-Phase Coolants
K. Hornyik


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December 1968<br>KFK 799<br>EUR 3964 d<br>Institut für Reaktorentwicklung<br>Models, Methods and Digital Computer Programs for Analyses in Reactor Dynamics with Emphasis on Fast Breeder Reactors and Compressible SinglePhase Coolants ※)

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## Preface

Obsoleteness is a common and often inherent feature found with digital computer codes. In spite of it a threefold purpose is seen in compiling a report such as this. One of its objectives is to serve as a manual for direct application of the codes described to the class of reactors and problems for which they were indended (see title). Secondly, a description of the model employed is to give sufficient information for potential applications and extensions to a range of cases wider than originally foreseen. Finally, some methods and techniques are presented which were developed during the course of this work and which are believed to have general significance in the field of reactor dynamics analysis.

The model and program described herein evolved from the FORE-code [-1,2_7. Therefore nearly all the model features of that code are incorporated as constituents of the basic model presented here. Abandoned parts are those which appeared either too restrictive (incompressible coolant) or overly simplifying (treatment of parallel channels). Many new features have been added; the most prominent ones concern the hydraulics model now able to cover any compressible single-phase coolant flowing through the reactor in two sections connected in series. Each of these sections itself may consist of an arbitrary number of different flow channels connected in parallel.

In this place the helpful assistance and advise is to be acknowledged of all those people who contributed to the completion of this work at various stages. The author is obliged especially to Ing. D. Janssen for valuable help with digital programming.

Karlsruhe, December 1968
Page
I. Introduction ..... 1
II. Thermohydraulic Model ..... 5
II. 1 General Lay-Out ..... 5
II. 2 Heat Source Distribution ..... 8
II. 3 Heat Flow and Temperatures ..... 10
II. 4 Coolant Hydraulics ..... 15
II. 5 Coolant Boundary Conditions ..... 15
III. Neutron Kinetics and Feedback Model ..... 17
IV. Program Description ..... 20
IV. 1 General Code Structure ..... 20
IV. 2 Input Description ..... 22
IV. 3 Output Description ..... 36
IV. 4 Storage- and COMMON-Requirements, Computing Time ..... 41
V. Comments, Suggestions for Further Development ..... 43
VI. Appendices:
A : Notation, Equations ..... 46
B : Decay Time Constants for Nodal Temperatures in Cylindrical Fuel Pin ..... 60
C : Predictor-Corrector-Schemes Applied in Solving the Hydraulic Problem ..... 63
D : Other Iterative Procedures Applied in Solving the Steady State Problem ..... 68
E : Stagnation and Reversal of Flow ..... 70
F : Integration of the Point Kinetics Equations ..... 73
G : Sample for Calculating Feedback Parameters from Results of Perturbation Calculations ..... 77
H : Evaluation of Integration Step Size, Numerical Stability of Integration of the Thermo-Hydraulic Equations ..... 80
84
I : Flow Diagrams
J : Subroutine REXION ..... 92
K : Functions COl and CPI ..... 95
L : Function RCBE ..... 96
M : Subroutines and Functions Concerning Coolant Properties: TPEl, HDV, FLAMI, ETAI ..... 101
N : Function ZEIT ..... 104
0 : Annotated Sample Problem ..... 105
VII. List of References ..... 107

## I. Introduction

Work reported here started as a part of the steam cooled fast breeder reactor project pursued by the GfK. Its early goal was to make available a tool for the analysis of the dynamic behavior of the type of reactors being considered at that time. Although a rather detailed analog model already existed [ 3 _ 7 it was found necessary to complement it with a digital code. Arguments concerning the choice of computational means are as numerous as they are inconclusive. Extensive experience gained during this work from comparisons with analog calculations have led to the conclusion that neither system is pre-eminently suited for the whole range of problems posed in reactor dynamics. It appears that reactor stability and control as well as systems circuit analysis are mainly a domain of analog studies whereas reactor accident analysis and certain aspects of reactor stability are suitably studied by using digital techniques. Storage capacity and computing speed of second generation machinery (IBM 7074/7094) together with considerations of numerical stability restricts the study of dynamics models to roughly the same amount of detail as can be accommodated by models designed for large analog computers ( 76 integrators, 34 multipliers, 360 pots) where number of components is the only restriction. Only third generation digital computers (IBM 360) give a lead to digital methods with respect to handing complex nonlinear problems encountered in accident analysis. However, the ease of making adjustments to the model which is afforded by the analog computer cannot be matched on digital machines, a situation which has remained essentially unaffected by the advent of even the latest digital equipment.

The model which has been adopted here is characterized by the following prominent features:
a) The model covers only components of the reactor proper, such as: core, blankets, bypasses within the pressure vessel, orifices, plenum chambers, etc. An external reactivity disturbance as well as one out of a choice of three boundary conditions for the coolant at reactor entry and exit must be given as functions of time ("driving functions"). ${ }^{+ \text {) }}$

[^0]b) The reactor is assumed to remain intact at all times. The only changes in reactor geometry áre those associated with certain feedback mechanisms. They are assumed to have negligible effects on the thermo-hydraulic characteristics of the reactor.
c) Fuel elements are assumed to have the shape of solid cylinders (pins) surrounded by annular cladding and cooled uniformly over the perimeter. Fuel melting can be handled in good approximation. The coefficient of heat transfer through the fuel cladding-gap is assumed to be invariant. Coolant flow within the fictitious annular channel is assumed to be entirely in the turbulent regime. Any structural components inside the core and the blankets are accounted for by introducing equivalent structure along the flow channel.
d) The hydraulic model permits rather accurate treatment of any single phase coolant (liquid or gaseous) for which the standard relations of state variables can be supplied. Pressure disturbances are assumed to propagate with infinite velocity throughout the coolant (extended version of the integral momentum model [4_7). A rather general formulation of the relation for the heat transfer between channel walls and coolant permits a wide range of applications. Coolant pressure drops in orifices and in the coolant channels are treated in the customary way and allow for integral treatment of turbulence promoters. An arbitrary number of different parallel flow channels individually orificed may be specified in both of the two main sections (radial blanket, core and axial blankets) which in turn are connected in series via an intermediary plenum chamber. Complete mixing and zero residence time are assumed for all of the three plenum chambers. Flow channels in the first section (radial blanket) may be treated as bypasses.
e) A special feature of the model is its ability to handle transient problems where flow stagnation or reversal occurs.
f) The time dependent behavior of the neutron density in the reactor core and blankets is described by the well known point reactor kinetics model. In order for this model to be consistent with the models for thermohydraulics and feedback it is consequent to assume time invariant functions for the spatial distribution of the thermal power density as well as for the reactivity importance throughout the reactor. The implications of these assumptions shall not be discussed in this report. Simplicity of this model was the guiding aspect in making this choice.
g) A versatile and comprehensive feedback model is incorporated which accounts for the Doppler effect, for various causes of density changes due to thermal expansion of components and for changes in the overall core geometry due to thermal expansion of certain components.
h) The reactor is assumed to be at an initial state of delayed criticality operating on a prescribed power level. The entire feedback of reactivity is thought to be balanced by external reactivity adjustments. (The relative value of this reactivity at different initial states can be used as a measure of the aperiodic stability of the reactor [5_7.) Furthermore, the thermohydraulic variables of the reactor also are assumed to be in a state of equilibrium at this power level. This equilibrium solution is obtained in a separate calculation but it is consistent with the set of equations describing the transient behavior.
i) The transient solution is obtained by integration of the pertinent system of differential equations in discrete steps using explicit first order backward difference formuli. The size of these steps may be determined from considerations of numerical stability and accuracy internal to the program.

In addition to these main features and functions of the basic model there are some contingent areas which have been given special attention as they are of considerable importance for many problem cases of a general class. Three of these are listed below.
j) The relations between state variables for superheated steam cannot be reduced to simple analytical formulations. Rather complex expressions on one hand and extensive tables on the other do not satisfy the need for compact and fast algorithms such as required by dynamics calculations in conjunction with multi-node models. Therefore, a set of polynomial expansions was generated by means of least squares fitting to the relations which are concerned and which were supplied accurately but inefficiently by other subprograms [-6_7. Since excessive accuracy is not required here these expansions have proved to be of all important convenience and this technique is recommended for all coolants where simple relations for the state variables do not exist in the range of interest.
$k$ ) Since many applications of this model relate to reactor accidents (excursions)-analyses a rather versatile model for SCRAM functions
(external reactivity vs. time, for emergency shut down) was worked out in several options to be described in this report.
I) The computer program based on this model (subroutine REXION) is designed to be integrated with another program describing the dynamic behaviour of the system external to the reactor. However, there are instances where such a program does not exist and/or may not be required. Examples for such cases are:

- stability and dynamics studies of the reactor alone
- analyses of rapid excursions with little feedback from the reactor on the coolant circuit behavior.

To facilitate such studies the program is provided with a control section (main program REX) which admits applications independent of any other program. Of course, in such cases time dependent boundary conditions for the coolant at reactor inlet and outlet must be supplied as input information to the control section.

Subsequently the model will be described in greater detail. Then a concise description of the computer program will be given complete with instructions for input preparation and output listings. The pertinent system of equations is compounded in APPENDIX A. Further appendices are dedicated to the treatment of several special topics regarding models, methods and codes. An annotated sample problem is included for illustration in the last appendix.

## II. Thermo-Hydraulic Model

## II.I General Lay-Out

The most general configurations that can be treated are sketched in Fig. l. The coolant passes the reactor in two sections which are connected in series. These sections are separated by the inermediary plenum chamber and terminated by the entry and exit plenum chamber, respectively. The spatial arrangement of plenum chambers and flow sections is arbitrary and may be of a kind as shown in Fig. 1. With respect to the sketch on the left it must be remarked that the length of both flow sections is assumed to be equal. Order and nomenclature of these components are associated with the initial direction of coolant flow which is from the entry to the exit plenum chamber as depicted in Fig. 1.

The plenum chambers are characterized by complete and instant mixing of all entering coolant streams, zero residence time and zero pressure drop of the coolant.


Fig. 1 Two Possible Configurations of Main Components

The first flow section is thought to represent the radial blanket. Its thermo-hydraulics are treated in a manner more crude than the one adopted for treating the second section. It does not contribute to the feedback of reactivity. It may be skipped entirely in the calculation. The coolant passes this first section through a number of parallel channels. Channels
with identical parameters are called a radial zone and are represented in the calculation by only one such channel. The number of radial zones is arbitrary ${ }^{+}$. Either one out of two different types of channels may be specified for any radial zone of the first section. The first type is to represent a typical radial breeder pin-channel-structure-combination consisting of a cylindrical pin with a uniform heat source, an equivalent annular flow channel which may be orificed at the entry, and an equivalent structural component (see Fig. 2). Neither axial subdivisions of the pin-channel-structure-unit nor annular subdivisions of the pin are provided since a rough model appears to be adequate in this place. The second type of channel is to represent a bypass and consists of two concentric tubes with coolant flowing in between. The inner tube may have an uniformly distributed heat source. Both tubes are assumed to be thermally insulated on their dry surfaces and may be orificed at the entrance. In particular, such a bypass may represent a single annular flow path outside the perimeter of the radial blanket.


Fig. 2 Channel Types of First Section

[^1]The second flow section is in many ways similar to the first one. It is to represent the core and the axial blankets of the reactor. Only one type of channel can be specified. Again, anyone such channel stands for a whole (radial-) zone of identical channels. The number of zones differing in geometry and/or thermo-hydraulic parameters is arbitrary. A channel consists of a cylindrical pin surrounded by an equivalent annular stream of coolant and an equivalent structural component. Flow may be orificed at the entrance. In axial direction (= direction of flow) the pin-channel-structure-unit is divided into the core section and one axial blanket section at both ends. Either one or both of these axial blanket sections may be omitted. The core section alone may be subdivided further into an arbitrary number of sections of equal height. In the core section the pin is composed of a cylindrical fuel body surrounded by annular cladding. For adequate description of the temperature distribution the fuel cylinder is subdivided by fictitious interfaces into an arbitrary number of concentric annuli of equal volume except for the outermost annulus and the center cylinder having half this volume.


Fig. 3 Pin-Channel-Structure-Unit of the Second Section

## II. 2 Heat Source Distribution

The reactor power determined by neutron kinetics in conjunction with external reactivity as well as feedback is released as thermal energy in various components of the reactor. The spatial distribution functions are assumed to be time invariant and are prescribed. Thus, a fixed fraction of the total power is released within the pins of each radial blanket zone and the walls of the inner tubes of each bypass zone. These fractions PB together with the dimensions of the components involved and in conjunction with the assumption of uniform heat source distribution determine the volumetric heat source strength required for the calculation of temperatures in the first flow section.

In the second flow section fixed fractions PBE and PBA of the total power are assumed to be released within the first and the second axial blanket, respectively. Distribution over radial zones and components of the axial blankets conform with those of the core. The remaining power ( = total (radial bl. zones + bypass zones) - l.ax. bl - 2.ax. bl) is distributed over the core in the following way (see also Fig. 4):
a) The axial distribution is specified by fractions corresponding to individual axial subsections. Note that:

- these fractions may be relative since normalization is carried out by the program;
- these fractions are proportional to the average linear rod power of each subsection as these subdivisions are of equal height;
- the same axial distribution applies to all parallel flow channels (rad. zones) in the second flow section.
b) The radial distribution is specified by fractions corresponding to the relative thermal power of each channel representing a radial zone. Note that:
- these fractions may be relative since normalization is carried out by the program;
- these fractions are proportional to the volumetric radial power distribution (averaged within each radial zone) only if the radial core zones have equal geometry;
- the radial distribution specified by these fractions applies to all axial core sections and to the axial blankets.
c) The power distribution over the core components is specified by fractions giving the relative volumetric thermal power density in each component (fuel, clad, coolant, structure). Note that:
- these fractions may be relative since normalization is carried out by the program;
- uniform power distribution is assumed in each of the four components;
- these fractions apply to all radial zones and axial sections of the core;
- in both, the radial and the axial blanket all power is assumedly released in the pins only.

a) Power Distribution: Core-Blankets-Bypasses

2. $a x . b l$.

core
1.ax.bl.

b) Radial and Axial Power Distribution in Core (and Axial Blankets)

c) Power Distribution in Core-Components

Fig. 4 Power Distribution

## II. 3 Heat Flow and Temperatures

For the calculation of the temperatures in the fuel pin the customary assumptions of

- uniform heat release within the entire fuel pin volume of any axial section and
- no heat conduction in axial direction of the fuel pin
are made. The cylindrical fuel body is subdivided by fictitious interfaces such as described before. The temperature distribution in each axial section of any such annulus is represented by a single temperature and heat transfer as well as heat storage is computed in terms of this temperature ("nodal model", "lumped model"). Under steady state conditions this temperature is equal to the volume average temperature of the annular cell; for the central cylinder it is the central (maximum) temperature and for the outermost annulus it is the fuel surface temperature. If the thermal conductivity of the fuel is a function of temperature these statements are no longer accurate since the program evaluates the conductivity for each annulus at the temperature of the adjacent outer annulus.

In addition to the thermal conductivity the specific heat of the fuel also may be a prescribed function of the fuel temperature.

Fuel melting is treated in an approximate form: if the representative temperature of any fuel node reaches the melting point upon rising, it is
kept constant until the net influx of heat into the volume equals the latent heat of fusion.

Recrystallization is treated in a similar manner upon a temperature drop through the temperature of recrystallization. Both, the temperature as well as the heat of fusion and recrystallization are taken to be equal. The program also admits the case that a part or all of the cells (nodes) in the fuel are molten at the initial equilibrium.

The nodal model which was selected here is one out of many possible choices. It is distinguished by yielding a correct steady state solution when the nodal temperatures are identified with the volumetric averages. This is significant especially with respect to calculating the reactivity feedback associated with the Doppler effect in the fuel. Regarding the transient solution one only can state that heat is conserved in the system of equations and that an increasing number of subdivisions will make the solution converge asymptotically to the correct solution. The results of a special study carried out for this purpose are described in APPENDIX B. and may serve as an aid in the selection of an adequate number of annular subdivisions for any particular case as a function of fuel parameters and accuracy requirements.

Heat transfer through the gap between fuel and cladding is assumed to be a function of the difference between the interface temperatures of fuel and clad only. The coefficient of heat transfer through the gap thus is a prescribed constant. More sophisticated models relating to this part may be introduced as they become available.

The annular cladding is treated as a thin slab and is subdivided by a fictitious interface into two annuli (slabs) of equal thickness. The nodal temperatures associated with these two annuli coincide with the temperatures at the inner and outer surface at steady state. In contrast to the fuel, the material parameters of the cladding are taken to be invariant. A heat source may be specified for the cladding and is assumed to be distributed evenly over the volume of each axial section.

In order to account for the effects brought about by the presence of structural components in core and blanket it is assumed that each channel also comprises a rectangular body of structural material extending over its full length (core + ax. blankets). This body is to be equivalent to the actual structure with respect to thermo-hydraulic characteristics. Three faces of
the prismatic body are assumed to be thermally insulated, the remaining face is in contact with the stream of coolant and permits exchange of heat. Only in the core region heat sources may be specified for the structure, which then are distributed uniformly over the entire volume of an axial section (the axial distribution being the same as in fuel, cladding, and coolant). The nodal temperature representative for the actual temperature distribution of each axial section coincides with the volume average temperature at steady state conditions.

Heat transfer between the coolant and either the cladding or the equivalent structure is computed following a slightly generalized form of the Nusselt equation as proposed by Sutherland [7]. It is based on the assumption of a coolant wholly in the turbulent flow regime ${ }^{+}$

$$
\begin{align*}
& N \mathrm{~F}=\mathrm{CH} \cdot \mathrm{Re}^{\mathrm{ALI} 1} \cdot \mathrm{Pr}^{\mathrm{AL2} 2} \cdot\left(\frac{\mathrm{~T} 3+273.16}{\mathrm{TW}+273.16}\right)^{\mathrm{AL} 3}  \tag{1}\\
& N \mathrm{NH}=\frac{(\mathrm{H} 3-\mathrm{AL} 4) \cdot \mathrm{DH}}{\text { FLAM }}  \tag{2}\\
& \mathrm{Re}=\frac{\mathrm{N} \cdot \mathrm{DH}}{E T A}  \tag{3}\\
& \operatorname{Pr}=\frac{\text { CP3 } \cdot \text { ETA }}{\text { FLAM }} \tag{4}
\end{align*}
$$

CH, ALI, AL2, AL3, AL4 ... prescribed parameters

Solving equation (1) for the film coefficient $H 3$ we get

$$
\begin{align*}
& \mathrm{H} 3=\mathrm{AL} 4+\frac{\mathrm{CH}}{\mathrm{DH}(1-\mathrm{ALI})} \cdot(\mathrm{N})^{\mathrm{ALI}} \cdot\left(\frac{\mathrm{FLAM}(1-\mathrm{AL} 2) \cdot \mathrm{CP} 3^{\mathrm{AL} 2}}{\mathrm{ETA}(\mathrm{ALI-AL2})}\right) \\
& \left(\frac{T 3+273.16}{T W+273.16}\right) \tag{5}
\end{align*}
$$

The additional constant AL4 is introduced in equation (2) for convenience of specifying an invariant film coefficient H3 = AL4 while at the same time setting ALI $=A L 2=A L 3=0$.

[^2]All of the intensive parameters in equation (5) refer to the coolant with exception of the wall temperature TW. These parameters are evaluated at the midpoint of each axial section. The state variables of the coolant are taken to vary linearily over the length of each section. The occurrence of TW in equation (5) necessitates an iterative procedure for determination of TW at the initial equilibrium. This is described in APPENDIX D.

The energy balance of the coolant is computed in terms of its enthalpy. Contributions of kinetic and potential energy are neglected. The coolant pressure is obtained from hydraulics calculations described in the next chapter. These two state variables are sufficient for obtaining all of the remaining coolant properties required in equation (5) by way of state relationships(i.e. temperature, specific heat, specific volume, viscosity and thermal conductivity). For the energy balance in the transient case it is assumed that heat storage in the coolant of each axial section is associated with the value of the exit enthalpy. This particular choice of a nodal model is not quite as accurate as the one employing the enthalpy averaged along the channel length of each individual section. However, the error in the transient may be kept small by increasing the number of axial sections. On the other hand, this nodal representation is particularily suited for handling the case of flow reversal during a transient, which is a major objective of this work. For further details concerning the treatment of flow stagnation and reversal see APPENDIX E.

Much of what is said above also applies to the calculation of temperatures in the radial and axial blankets as well as in the bypass(es). However, there the calculation is simplified by prescribing invariants for all material properties as well as for the film coefficient of heat transfer to the coolant and by lumping the pin temperature distribution of each section into one node. In the case of the blanket pin the nodal temperature coincides with the maximum temperature on the center axis for steady state conditions, when the coefficient of heat transfer is specified so as to include also heat transfer through cladding gap and breeder pin, which are not accounted for separately ${ }^{+}$. For the nodes associated with equivalent structure and/or bypass walls the nodal temperatures agree with the

[^3]volume averaged temperatures under the same conditions as before. It is recalled that the model does not provide axial subdivisions for any of the blankets and/or bypasses. Fig. 5 sketches the nodal representation of the reactor, on which the thermo-hydraulic model is based.

In solving the ordinary differential equations resulting for this nodal model (see APPENDIX A) a single explicit backward difference scheme is applied. It is known that in this case the step size for integration over time must comply with certain criteria in order to ensure numerical stability of this method. Since these criteria are prohibitively complex in the underlying case approximate criteria were developed, which yield conservative estimates of the maximum step size for stable integration of the thermo-hydraulic equations. Details on this subject are compiled in APPENDIX H.


Fig. 5 Nodal Representation of the Reactor for Thermo-Hydraulic Model

## II. 4 Coolant Hydraulics

Three kinds of pressure losses are distinguished in each flow channel.
a) Pressure loss at the channel entrance:

$$
\begin{equation*}
\Delta \mathrm{p}_{\mathrm{E}}=1.5 \cdot \frac{\mathrm{~N}^{2} \cdot \mathrm{v}}{2 \cdot \mathrm{~g} \cdot 10^{4}} \quad L^{-} \mathrm{at} 7 \tag{6}
\end{equation*}
$$

It is assumed that no pressure recovery occurs at the channel exit. Note that this pressure loss will occur at the other end of the channel in case of flow reversal.
b) Pressure drop through orifice:

$$
\begin{equation*}
\Delta \mathrm{p}_{\mathrm{T}}=\mathrm{FT} \cdot \frac{\mathrm{~N}^{2} \cdot \mathrm{v}}{2 \cdot g \cdot 10^{4}} \quad\left[\mathrm{at}_{-}\right] \tag{7}
\end{equation*}
$$

The coefficient $F T$ may be chosen different for each radial zone and is both purpose and means of controlling the distribution of coolant flow through the various flow paths in parallel.
c) Pressure loss due to friction along the wetted surface of the channels (cladding + equivalent structure) and to acceleration:

$$
\begin{equation*}
\Delta \underline{p}_{R}=\frac{N^{2}}{g \cdot 10^{4}}\left[\frac{\mathrm{FR} \cdot \Delta H}{2 \cdot D H} \cdot V+(V A-V E)\right] \quad<_{-7} \tag{8}
\end{equation*}
$$

In the present model pressure loss associated with flow of coolant through the plenum chambers is neglected. The condition determining the distribution of coolant mass flow through parallel channels is that of equal pressure drop in all of these channels. For the initial steady state an iterative procedure is used to achieve this. In the transient calculation a scheme of delayed correction is applied to satisfy the criterion of equal pressure drops without need for iterations. APPENDIX C gives a rather detailed account of the pertinent procedures. APPENDIX E is referredto for more information about the calculation of pressure drops in the case of flow stagnation or reversal.

## II. 5 Coolant Boundary Conditions

There are three links by which this model representing the reactor is coupled to the remaining system. These are:
a) state of coolant at the entrance
b) " " " " " exit
c) disturbances of external excess reactivity (control rods - control system)

Items a) and b) are not entirely independent of each other. In any case it appears practical to prescribe the enthalpy of the coolant as it enters the reactor. The exit enthalpy then is determined directly by the balance of energy. Regarding the hydraulic boundary conditions there is no such unique choice. Therefore, the model was made to accommodate three different choices:

```
B.C.l Prescribe: - pressure at entry, as a function of time;
    - coolant mass flow rate through reactor, at initial
        steady state;
    - pressure at the exit relative to the initial value,
                        as a function of time.
    Obtain: - pressure at the exit, at initial steady state;
    - coolant mass flow rate, as a function of time.
```

This particular set of boundary conditions is advantageous in connection with the integration of this model with a corresponding model for the external coolant circuit.

```
B.C.2 Prescribe: -pressure at exit, as a function of time;
    -coolant mass flow rate, as a function of time;
    Obtain: -pressure at entry, as a function of time.
```

B.C. 3 Prescribe: - pressure at entry, as a function of time;
- coolant mass flow rate, as a function of time;
Obtain: - pressure at exit, as a function of time.

For B.C. 2 an iterative procedure is required to find the initial steady state solution since in all cases the calculation starts at the entrance where coolant pressure is unknown in this particular instance. For the transient calculation of the same case a scheme of delayed correction is applied thus avoiding time consuming iterative procedures (see APPENDIX C).

## III. Neutron Kinetics and Feedback Model

For the description of the transient reactor power the well known point reactor kinetics model is employed. Its solution requires integration of a coupled system of ordinary linear differential equations with time dependent coefficients. This integration is carried out using a semianalytic method taken over in main parts from the FORE-code [ $\underline{I}_{1} 7$ and described in APPENDIX $F$. The basic scheme of integration outlined in the appendix is augmented by two other schemes. One of them is applied in the vicinity of prompt criticality since the basic scheme is singular at this point. The other scheme corresponds to the prompt jump approximation and is a more efficient scheme of integration which applies when the system is well below prompt critical. Decisions about selecting the proper scheme out of these three are based on the results of certain tests carried out at each step of the step by step-integration. Among other criteria the size of such a step is limited by the relative change of the reactor power during a step, for which upper and lower limit may be prescribed. The step size will be decreased or increased respectively until the actual value of the relative change falls within these limits. A correlation between both, accuracy and stability of this method of integration and this criterion for selection of step size could not be established in the framework of this effort. On the other hand, practical experience gained from its successful application as well as successful application of the same criterion in other methods of integration of point kinetics equations have given enough confidence to validate this approach.

The feedback model, too, draws main features from the FORE-code but it is implemented with a number of significant additions and modifications. In spite of the already great amount of details included in this feedback model it is felt that much more refinement might be required in certain cases. However, at this point such refinement must be left with further development efforts.

Presently, all feedback of reactivity accounted for is associated with temperature and/or density changes in the core only. The reactivity feedback from the Doppler effect is taken to be proportional to the weighted average over the fuel volume in the core of the natural logarithm of the fuel temperature in ${ }^{\circ} K$. Both, axial and radial weights may be specified. These weights may be relative as normal-
ization is carried out in the program. The normalization is such that an isothermal temperature rise $\Delta T=T_{1}-T_{0}$ throughout the fuel in the core will result in a Doppler reactivity change of $\Delta k_{D}=\operatorname{RCD} \cdot \ln \left(T_{1} / T_{0}\right)$, where RCD is the Doppler constant specified in the input. The basic formula for calculating the Doppler feedback reactivity is

$$
\begin{gather*}
\Delta \mathrm{k}_{\mathrm{D}}=\mathrm{RCD} \frac{\int \mathrm{ARD} \cdot \mathrm{AXD} \cdot \ln \left((\overline{\mathrm{TF}}+273 \cdot 16) /\left(\overline{\mathrm{TF}}_{\mathrm{o}}+273 \cdot 16\right) \cdot \mathrm{d} \mathrm{~V}_{\mathrm{F}}\right.}{\int \mathrm{ARD} \cdot \mathrm{AXD} \cdot \mathrm{dV}_{\mathrm{F}}}  \tag{9}\\
\overline{\mathrm{TF}}=\frac{1}{\mathrm{~V}_{\mathrm{z}}} \cdot \int \mathrm{TF} d V_{\mathrm{z}} \ldots \text { fuel temperature averaged over the pin } \\
\quad \begin{array}{l}
\text { volume } V_{\mathrm{z}} \text { of an axial section }
\end{array}  \tag{10}\\
\mathrm{V}_{\mathrm{z}}=\mathrm{RI}^{2} \cdot \pi \cdot \Delta \mathrm{H}
\end{gather*}
$$

The effects on reactivity of changes in the density of any core component are treated correspondingly as weighted averages over the component volume in the core of the respective temperatures multiplied by their linear thermal expansion coefficient. As in the case of the Doppler feedback, the weights are assumed to be separable into functions of the axial and radial coordinate (index) respectively (normalization not required). Thus, the product of feedback coefficient times expansion coefficient of each component is equal to the total feedback from this component if a uniform temperature change by $1{ }^{\circ} \mathrm{C}$ is imposed throughout the core. Here the basic formula is

$$
\begin{equation*}
k_{i}=-R C_{i} \frac{\int A R_{i} A X_{i} \cdot T_{i} \cdot A E_{i} \cdot d V_{i}}{\int A R_{i} \cdot A X_{i} \cdot d V_{i}} \tag{11}
\end{equation*}
$$

where the index i refers to the three solid components: fuel (F), clading (C), structure (M). For the coolant density changes can be computed directly as the specific volume $V$ is available:

$$
\begin{equation*}
k_{s}=\operatorname{RCS} \cdot \frac{\int \operatorname{ARS} \cdot \operatorname{AXS} \cdot(1 / V) \cdot d V_{s}}{\int A R S \cdot A X S \cdot d V_{S}} \tag{12}
\end{equation*}
$$

The effects of changes in the overall core geometry (height and radius) are also comprised in the feedback model. Increases in the core height are associated with thermal expansion of the fuel and radial increases with lateral expansion of the structural components of the core. Weighting functions have not been incorporated in this part of the model as yet. The
pertinent formuli are:

$$
\begin{align*}
& \mathrm{k}_{\mathrm{H}}= \mathrm{RCH} \cdot \frac{\int \mathrm{AEF} \cdot \overline{\mathrm{TF}} \cdot d V_{\mathbf{F}}}{\int d V_{\mathbf{F}}}  \tag{13}\\
& \mathrm{k}_{\mathrm{R}}= \mathrm{RCR} \cdot \frac{\int \mathrm{AEM} \cdot \overline{\mathrm{TM}} \cdot d V_{M}^{\prime}}{\int d V_{M}^{\prime}}  \tag{14}\\
& V_{M}^{\prime}=\Delta H \cdot A N \tag{15}
\end{align*}
$$

With the general formulations used for the temperature and density feedbacks on reactivity (equation (11), (12)) it is possible to include mechanisms other than the change of smear density of a component caused by a change in its temperature. For instance, one may account for the effect of coolant displacement by cladding expansion and for effective density changes of all components caused by dimensional changes in the supporting core structure due to thermal expansion or contraction. In such cases, the reactivity coefficients ( $R C_{i}$ ), the weights ( $\left.A R_{i}, A X_{i}\right)$ and the expansion coefficients ( $A E_{i}$ ) appearing in equations (11) and (12) must be determined by superposition of the various effects to be included. This is illustrated by a typical example presented in APPENDIX G.

## IV. 1 General Code Structure

Flexibility in operation and application of any complex program is furthered by employing modular programming techniques. This principle was followed here to a large extent. One of the most prominent exceptions is the feedback calculation, which is attached to the thermohydraulic calculations. The reason for this is a gain in computational speed and reduction in storage requirements since all contributions to the feedback originate from changes in thermo-hydraulic variables and may be computed efficiently along with these variables. Another exception is the calculation of the film coefficient of heat transfer to the coolant in the core. Here, too, the calculation is tied in strongly with thermo-hydraulics and an integrated treatment appears advantageous.

The programming language used in all parts is FORTRAN IV compatible with FORTRAN II if all READ and WRITE and COMMON statements are adjusted to comply with FORTRAN II rules. The program has been tested out successfully on the following computers: IBM 7074, 7094, 360/65. The version for IBM 7074 computers requires OVERLAY structure.

Fig. 6 lists all routines required by this program together with their major functional characteristics. Fig. 7 shows a logic flow diagram depicting the interconnections between these routines.

The main program REX is intended for independent application of the model, i.e. for cases where the boundary conditions of the coolant are known functions of time. Provisions are made for linking the present program with a compatible digital program describing the dynamics of the external circuit. In such a case the program may be used without the main program REX as a subroutine called by the name REXION (level 2 in Fig. 7). Guide lines and examples for such applications are given in APPENDIX J. Calls of this subroutine with different values for the last argument (IX) in the list causes execution of various operations such as: reading of input, calculation of initial steady state solution, execution of one step of the transient integration, output of variables. Some of the remaining arguments serve for input of coolant boundary conditions to the subroutine, others return information concerning: actual transient time after last step of integration, size of last step, power, energy and the compliment of coolant boundary conditions upon completion of the last step. Problem termination is also a

| Name | Type |  | Called Subprogram(s) |
| :---: | :---: | :---: | :---: |
| REX | MAIN | controlling program for indep. applications, input and evaluation of coolant boundary conditions, output frequency control | REXIめN, ZEIT |
| REXIØN | SUBROUTINE | controlling program, input of parameters for and execution of step size- and termination control | PREP, C $\varnothing 1, C P 1$, P $\varnothing W E R, S T S$, TRS, STEPC, $\varnothing U T P U T$ |
| $\overline{\text { PREP }}$ |  | input of parameters concerning geometry, thermohydr., feedback, iteration, output volume control, preparation of const. coefficients |  |
| ST'S | - " - | solution of initial steady state problem | TPEl, HDV, FLAM1, ETAl, C $¢ 1$ |
| TRS | - " - | " " transient problem | TPE1, HDV, FLAMI, ETAI, Cめ1, CP1 |
| STEPC | - " - | calculation of max. step size for stable integration of thermo-hydr. |  |
| $\overline{\text { ¢UTPUT }}$ | - " - | output of thermo-hydraulic variables and feedback reactivities |  |
| PØWER | - " - | input of point kinetics parameters, integration of point kinetics | RCBE |
| $\overline{\text { RCBE }}$ | FUNCTION | external (controlled) reactivity disturbance, input of pertinent parameters |  |
| TPE1 | SUBROUTINE | temperature and specific heat of coolant from enthalpy and pressure |  |
| HDV | FUNCTION | specific volume of coolant from temperature and pressure |  |
| $\overline{\text { FLAMI }}$ | - $11-$ | thermal conductivity of coolant from temperature and pressure |  |
| ETAI | - 11 | dynamic viscosity of coolant from spec. vol. and temperature |  |
| $\overline{C \emptyset 1}$ | - $11-$ | temp. dep. thermal conductivity of fuel, input of pert. parameters |  |
| CP1 | - " - | temp. dep. specific heat of fuel, input of pertinent parameters |  |
| 2EIT | - ${ }^{11}$ - | computer clock |  |

Fig. 6 List and Major Functions of all Routines Comprised by the Complete Program
function of this routine and is signalled to the calling program via the argument IX.


Fig. 7 Logic Fiow Diagram

Instead of giving a detailed description or listing of all the routines involved it is considered to be both sufficient and helpful to include flow diagrams of these routines (APPENDIX I). The subprograms of level 4 should be considered non-standard since certain applications may demand individually supplied routines. Guide lines for the construction of such routines are given in the APPENDICES $K, L, M, N$ together with descriptions of the particular versions of those routines which were developed in conjunction with studies for the steam cooled fast breeder project $1-8 / 7$. For the sake of completeness the input required by these specific program versions also is included in the subsequent input description.

## IV. 2 Input Description

Data input to the program is arranged in blocks. Each block is headed by a card carrying the block number IP (column 3) and a commentary (columns 4 75) which will be reproduced in the output. There are nine blocks of data as follows.

| IP Block Significance | Input for |  |
| :--- | :--- | :--- |
| 1 | system data | REXION |
| 2 | coolant enthalpy at entry | REX |
| 3 | boundary condition parameter | $"$ |
| 4 | pressure | " |
| 5 | initial coolant mass flow rate and pressure |  |
| 6 | coolant mass flow rate |  |
| 7 | initial reactor power | $"$ |
| 8 | output control- and clock termination | $"$ |
| 9 | parameter of job-card |  |

Data blocks 2, 4, 5, 6, 7 contain the information concerning all the external conditions (boundary conditions) required for independent application. The only other external condition needed is the external reactivity disturbance, which is specified as part of the system data in block $I P=1$. Besides this block 1 contains all the characteristics of the reactor as well as control parameters concerning the computation, output volume, and termination. For convenience block $l$ therefore is subdivided into subblocks, each of these headed by a card carrying the subblock number IPT (column 7) and a commentary (columns 8-79) which will be reproduced in the output. There are eight subblocks as follows:

## IPT Subblock - Significance

1 geometry and thermo-hydraulic properties
2 temp. dep. fuel properties
3 point kinetics parameters and external reactivity
4 feedback parameters
5 iteration- and output volume control parameters
6 step- and termination-control parameters
7 (not used at present)
8 end of subblock-card

Any particular set of boundary conditions out of the choice of three is specified by the boundary condition parameter IBC (block 3) which may assume one of the values $1,2,3$ accordingly. In either case the three subsequent blocks have to be used in the following way:

| IBC = | 1 | 2 | 3 |
| :--- | :--- | :---: | :---: |
| block 4 | entry pressure | exit pressure | entry pressure |
| block 5 | initial mass flow <br> rate, exit pressure <br> change | omit | omit |
| block 6 | omit | coolant mass <br> flow rate | coolant mass <br> flow rate |

A job may consist of several transient problems to be solved successively. For the first problem of any job a full set of data for blocks l-8 must be supplied in the order indicated by the block numbers. In any further problem only those blocks and/or subblocks need to re-occur in which new data are to replace old values. As an exception to this, the data block 8 must be the last block in the complete set of data for anyone problem. After the data set for the last problem in a job block 9 must appear. An example for block sequence pertaining to a feedback parameter study is given below (subblock numbers in parentheses).
$1(1,2,3,4,5,6,7,8), 2,3,4,6,7,8,,1(4,8), 8,,1(4,8), 8,$,


Note, that in each case subblock 8 must appear last in block 1 and block 8 must appear last in each problem.

In an effort to make input as well as output consistent with respect to dimensional units all quantities are measured in terms of the following basic units:

| mass: | kg | pressure: | at |
| :--- | :--- | :--- | :--- |
| length: | m | energy (therm.) : | kcal |
| time: | sec | reactivity: | $\not \subset$ |
| temperature: | ${ }^{\circ} \mathrm{C}$ | power: | MW |

Power could be measured in units of kcal/sec, however, this is not at all a customary unit for reactor power and MW are used for this variable instead. The same holds true for pressure which is measured in at ( $\mathrm{kp} / \mathrm{cm}^{2}$ ) instead of $\mathrm{kp} / \mathrm{m}^{2}$. It is possible to apply an entirely different set of basic units for input and output (as long as the set is selfconsistent) after adjusting four conversion factors which are required and defined in subroutine PREP (Cl, C3, C4) and subroutine RCBE (C6). They have the following significance:

$$
\begin{array}{rl}
\mathrm{Cl} & =238.89(\mathrm{kcal} / \mathrm{MWsec}) \\
\mathrm{C} 3 & =9.81 \cdot 10^{4}(\mathrm{~kg} \mathrm{~m} / \mathrm{kp} \mathrm{sec} \\
& 2 \\
\mathrm{C} 4 & =1296 \cdot\left(\mathrm{kp} / \mathrm{m}^{2} \mathrm{Ct}\right) \\
\mathrm{C} 3 & \left(\mathrm{~kg} \mathrm{~m} / \mathrm{kp} \mathrm{~h}^{2}\right) \\
\mathrm{C} & 9.81\left(\mathrm{~kg} \mathrm{~m} / \mathrm{kp} \mathrm{sec}{ }^{2}\right)
\end{array}
$$

Uniform formats are used for all input data:
integer : IP, output volume control parameters... I3
all others ...................................... I7
real : ...................................................E 11.4

Restrictions listed in parentheses are imposed by DIMENSIONS specified in the program and may be altered if necessary. In the present version the following values are used:

$$
N R B D=4, N R D=5, N X D=10, N N D=10, N D D=10, N Y D=50
$$

| card | variable | format | units | significance | restrictions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| System data (block 1) |  |  |  |  |  |
| 1 | IP | I3 | - | block identification, columns 4-75 free for commentary | $=1$ |

Geometry and Thermo-Hydraulics (subblock 1)

| 2 | IPT | I7 | - | subblock identification, columns 8-79 free for commentary | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | NRB <br> NBE <br> NBA <br> NR <br> NX <br> NN | I7 <br> " <br> " <br> 11 <br> 11 <br> 11 |  | ```number of radial zones (incl. bypass zones) in first flow section (= 0 ... no first section) first axial blanket (entrance) (= 0 ... no first axial blanket) second axial blanket (exit) (= 0 ... no second axial blanket) number of radial zones in second flow section (core+ax.bl.) number of axial sections in core number of annular zones in fuel pins``` | $0<N R B<(N R B D)$ 0,1 0,1 $0<N R \ll(N R D)$ $0<N X<(N X D)$ $2<N N \ll(N N D)$ |
| 4 | HC <br> HBE <br> PBE <br> HBA <br> PBA | $\begin{gathered} \text { E } 11.4 \\ " \\ " \\ " \\ " \end{gathered}$ | $\begin{gathered} \mathrm{m} \\ \mathrm{~m} \\ \% / 100 \\ \mathrm{~m} \\ \% / 100 \end{gathered}$ | height of core <br> height of first axial blanket <br> fraction of total reactor power released in first ax. bl. <br> height of second axial blanket <br> fraction of total reactor power released in second ax. bl. |  |

For a type 1 channel of the first flow section card 5 is as follows:

| $5 a$ | BP | $" 1$ | - | indicator for channel type |
| :---: | :---: | :---: | :---: | :--- | :--- |
|  | R2 | $"$ | $m$ | breeder pin radius |
|  | R3 | $"$ | $m$ | outer radius of equivalent annular flow channel |
|  | D4 | $"$ | $m$ | thickness of equivalent structure component |
|  | WP | $"$ | wetted perimeter of equivalent structure component |  |
|  | AN | - | total number of identical pins having these specifications |  |

For a type 2-channel of the first flow section card 5 has an alternate meaning as follows

| $5 b$ | BP <br> R2 <br> R3 <br> D4 <br> WP <br> AN | $\begin{gathered} \text { E } 11.4 \\ " 1 \\ " \\ " \\ " \\ " \end{gathered}$ | $\begin{gathered} \mathrm{m} \\ \mathrm{~m} \\ \mathrm{~m} \\ \mathrm{~m} \\ - \end{gathered}$ | ```indicator for channel type inner radius of annular flow channel outer radius of " " " thickness of outer wall " " inner " number of identical channels having these specifications``` | $=1$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | PB <br> HA <br> CV <br> FR <br> FT |  | $\begin{gathered} \% / 100 \\ \mathrm{kcal} / \mathrm{m}^{2} \mathrm{sec}^{\circ} \mathrm{C} \\ \mathrm{kcal} / \mathrm{m}^{30} \mathrm{C} \end{gathered}$ | ```fraction of total reactor power released in this radial zone equivalent film coefficient of heat transfer to coolant equivalent volumetric specific heat friction factor for channel walls orificing factor``` |  |

Cards 5 and 6 have to be repeated NRB times for each radial zone and/or bypass zone of the first flow section. 1 If $\mathrm{NRB}=0$ cards 5 and 6 must be omitted.

| 7 | R1 <br> R2 <br> R3 <br> D4 <br> WP <br> AN | " | m <br> m <br> m <br> m <br> m | radius of fuel (inner radius of clad) <br> radius of pin (outer radius of clad) <br> outer radius of equivalent annular flow channel <br> thickness of equivalent structural component <br> wetted perimeter of equivalent structural component <br> number of identical channels having these specifications |
| :---: | :---: | :---: | :---: | :---: |
| 8 | $\begin{aligned} & \mathrm{R} \varnothing \mathrm{~F} \\ & \mathrm{R} \varnothing \mathrm{C} \\ & \mathrm{R} \varnothing \mathrm{M} \\ & \mathrm{AEF} \end{aligned}$ | " | $\begin{aligned} & \mathrm{kg} / \mathrm{m}^{3} \\ & \mathrm{~kg} / \mathrm{m}^{3} \\ & \mathrm{~kg} / \mathrm{m}^{3} \\ & \mathrm{o}_{\mathrm{C}}-1 \end{aligned}$ | ```density of fuel density of clad density of structure linear coefficient of thermal expansion of fuel``` |


|  | $\begin{aligned} & \text { AEC } \\ & \text { AEM } \end{aligned}$ | E 11.4 11 | $\begin{aligned} & { }^{\circ} \mathrm{C}^{-1} \\ & { }^{0} \mathrm{C}^{-1} \end{aligned}$ | linear coefficient of thermal expansion of clad     <br> $"$ $"$ $"$ $"$ $"$ |
| :---: | :---: | :---: | :---: | :---: |
| 9 | C $\varnothing \mathrm{C}$ <br> CØM <br> CPC <br> CPM <br> CHC <br> CHM |  | $\begin{gathered} \mathrm{kcal} / \mathrm{m} \sec ^{\circ} \mathrm{C} \\ \mathrm{ncal} / \mathrm{kg}{ }^{\circ} \mathrm{C} \\ " \end{gathered}$ | ```thermal conductivity of clad " " " structure specific heat of clad " " " structure coefficient for heat transfer relation: clad-coolant " " " " " : structure-coolant``` |
| 10 | HGP <br> TM <br> WM <br> FR <br> FT | > 11 > 11 > 11 > 11 | $\begin{gathered} \mathrm{kcal} / \mathrm{m}^{2} \mathrm{sec}^{\circ} \mathrm{C} \\ { }^{\circ} \mathrm{C} \\ \mathrm{kcal} / \mathrm{m}^{3} \end{gathered}$ | ```film coefficient for heat transfer through fuel-clad gap melting point of fuel latent volumetric heat of fusion in fuel friction factor for channel walls orificing factor``` |
| 11 | AQR <br> AQF <br> AQC <br> AQS <br> AQM | 11 <br> 11 <br> 11 <br> 17 |  | relative channel power for this radial zone relative volumetric power density in fuel " " " " |
| 12 | ARD <br> ARF <br> ARC <br> ARS <br> ARM | 11 <br> il <br> 11 <br> 11 <br> 11 |  | relative weight of this radial zone for Doppler reactivity effects <br> relative weight of this radial zone for reactivity effects from fuel density <br> relative weight of this radial zone for reactivity effects from clad density <br> relative weight of this radial zone for reactivity effects from coolant density <br> relative weight of this radial zone for reactivity effects from structure density |


| 13 | HFS <br> CVF <br> HMS <br> CVM | "1 | $\begin{array}{r} \mathrm{kcal} / \mathrm{m}^{2} \mathrm{sec}^{\circ} \mathrm{C} \\ \mathrm{kcal} / \mathrm{m}^{3}{ }^{\circ} \mathrm{C} \\ \mathrm{kcal} / \mathrm{m}^{2} \mathrm{sec}^{\circ} \mathrm{C} \\ \mathrm{kcal} / \mathrm{m}^{3}{ }^{\circ} \mathrm{C} \end{array}$ | first axial blanket, equivalent film coefficient of heat transfer pin-coolant <br> first axial blanket, equivalent volumetric specific heat of pin <br> first axial blanket, equivalent film coefficient of heat transfèr structure coolant <br> first axial blanket, equivalent volumetric specific heat of structure. |
| :---: | :---: | :---: | :---: | :---: |
| 14 | HFS <br> CVF <br> HMS <br> CVM | "1 | 11 <br> 11 <br> II <br> 11 | $\text { second axial blanket }\}$ |

Card 13 is omitted if $N B E=0$. , card 14 is omitted if $N B A=0$. Cards 7-14 are repeated NR times for each radial zone of the second flow section (core).

| 15 | AQX <br> AXD <br> AXF <br> AXC <br> AXS <br> AXM | $E 11.4$ <br> " <br> " <br> " <br> " <br> " | - - - - - - | relative power density of this axial section <br> relative weight of this axial section for reactivity effects from Doppler effect in fuel <br> relative weight of this axial section for reactivity effects from fuel density changes <br> relative weight of this axial section for reactivity effects from clad density changes <br> relative weight of this axial section for reactivity effects from coolant density changes <br> relative weight of this axial section for reactivity effects from structure density changes |
| :---: | :---: | :---: | :---: | :---: |

Card 15 is repeated $N X$ times for each axial section of the core.

| 16 | ALI <br> AL2 <br> AL3 <br> AL4 | $\begin{gathered} \text { E } 11.4 \\ " 1 \\ " \\ " \end{gathered}$ | $\mathrm{kcal} / \mathrm{m}^{2} \mathrm{sec}^{\circ} \mathrm{C}$ | exponents in heat transfer relation (see eq. (I)) <br> constant in heat transfer relation (see eq. (2)) |
| :---: | :---: | :---: | :---: | :---: |

Parameters Pertaining to the Temperature Dependent Fuel Properties (subblock 2)

| 17 | IPT | I7 |  | subblock identification, columns 8-79 free for commentary | $=2$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | CKI <br> CK2 <br> CK 3 | $\begin{gathered} \text { E } 11.4 \\ " \\ " \end{gathered}$ | $\begin{aligned} & \mathrm{BTU} / \mathrm{ft} \mathrm{hr}{ }^{\circ}{ }_{\mathrm{F}} \\ & \mathrm{BTU} / \mathrm{ft} \mathrm{hr} \mathrm{~F}^{\mathrm{F}} \\ & \mathrm{BTU} / \mathrm{ft} \mathrm{hr}{ }^{\circ} \mathrm{F}^{3} \end{aligned}$ | coefficients for polynomial representation of thermal conductivity of fuel (APPENDIX K) |  |
| 19 | $\begin{aligned} & \mathrm{CCl} \\ & \mathrm{CC} 2 \\ & \mathrm{CC} 3 \end{aligned}$ |  |  | $\left\{\begin{array}{l} \text { coefficients for polynomial representation of } \\ \text { specific heat of fuel (see APPENDIX K) } \end{array}\right.$ |  |

Cards 18 and 19 refer to the specific form of subroutines COl and CPl described in APPENDIX K.
Parameters for Point Kinetics and External Reactivity Function (subblock 3)

| 20 | IPT | I7 | - | subblock identification, columns 8-79 free for commentary | $=3$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | $\begin{aligned} & \not \subset \mathrm{M} \\ & \mathrm{ND} \end{aligned}$ | $\begin{gathered} \text { E } 11.4 \\ \text { I7 } \end{gathered}$ | $\sec ^{-1}$ | $B / A$ <br> number of precursor groups | ₹ (NDD) |
| 22 | $\begin{aligned} & \text { FY } \\ & \text { LA } \\ & \text { FY } \\ & \text { LA } \\ & \text { FY } \\ & \text { LA } \end{aligned}$ | $\begin{gathered} \text { E } 11.4 \\ " \\ " \\ " \\ " \\ " \end{gathered}$ | $\sec ^{-1}$ <br> $\sec ^{-1}$ $\sec ^{-1}$ | $\left.\begin{array}{l}B_{1} / B \\ \lambda_{1} \\ B_{2} / B \\ \lambda_{2} \\ B_{3} / B \\ \lambda_{3}\end{array}\right\}$relative fractional yield and decay constant for <br> precursor groups $1,2,3$ |  |

Card 22 is to be repeated as necessary to accommodate all parameters for the ND precursor groups.

| 23 |  |  | This card (plus eventual following cards) contains param- <br> eters required as input to subprogram RCBE defining the <br> external reactivity function. A set of such cards required <br> for the particular version described in APPENDIX L is <br> described in that place. |
| :---: | :---: | :---: | :--- | :--- |

Feedback Parameters (subblock 4)


Parameters for Controlling Iteration in Finding Initial Steady State Solution and for Output Volume Control (subblock 5)


| 29 | IRBN <br> IRBX <br> IRBD | $\begin{aligned} & \text { I3 } \\ & " \\ & " \end{aligned}$ | - | $\left.\begin{array}{l} \text { minimum } \\ \text { maximum } \\ \text { increment } \end{array}\right\} \begin{aligned} & \text { of loop parameter for output of thermo-hydr. } \\ & \text { variables, concerning radial zones and/or } \\ & \text { bypasses of first flow section } \end{aligned}$ | < NRB |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | IRN <br> IRX <br> IRD | " | - | ```- " - ..... , concerning radial zones of second flow section (core + ax.bl.)``` | ₹ NR |
|  | IXN <br> IXX <br> IXD | 1 | - | - "- ...., concerning axial sections of core | < NX |
|  | IRIN <br> IRIX <br> IR1D |  | - | minimum$\left.\begin{array}{l}\text { maximum } \\ \text { increment }\end{array}\right\} \quad$of loop parameter for output of fuel temperatures <br> concerning radial zones of second flow <br> section (core + ax.bl. $)$ | §NR |
|  | $\begin{aligned} & \text { IXIN } \\ & \text { IXIX } \\ & \text { IXID } \end{aligned}$ | " <br> " | - | - " concerning axial sections of core | < NX |
|  | INN <br> INX <br> TND | " | - | ```concerning annular subdivisions of fuel pellet``` | < NN |

Parameters for Step- and Termination Control (subblock 6)

| 30 | IPT | I7 | - | subblock identification, columns 8-79 free for commentary |
| :--- | :---: | :---: | :---: | :--- | :--- |
| 31 | NTE | $"$ | - | number of steps after which approx. calculation of max. <br> stable step size is repeated |


| 32 | XMA <br> XIN <br> XIX <br> PIN <br> PIX | $\begin{gathered} \text { E } 11.4 \\ " \\ " \\ " \\ " \end{gathered}$ | sec <br> sec <br> sec | ```(reactor-) time limit for integration (start at time X = 0.) minimum step size admitted for integration. maximum " " " " " minimum relative change in power during step, propose doubling of step size if actual change is less. maximum relative change in power during step, halve step size if actual change is higher.``` |
| :---: | :---: | :---: | :---: | :---: |

Parameter space available for purposes as yet undefined (subblock 7)

| 33 | IPT | I7 | - | subblock identification, columns 8-79 free for commentary |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

If no such parameters are needed card 33 may be omitted. If parameters are specified in accordance with alterations in subroutine REXION they have to be given on cards inserted after card 33

| End Card for block 1 (subblock 8) |
| :--- |
| 34 IPT I7 - subblock identification, columns 8-79 free for comments, <br> this card indicates the end of the system data |

## Inlet Enthalpy (block 2)

| 35 | IP | I3 | - | block identification, columns 4-75 free for comments | $=2$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 36 | NY | I7 | - | number of points given to define coolant inlet enthalpy (Y) as function of time (X) | ₹ (NYD) |
| 37 | $X$ $Y$ $X$ Y X Y | $\begin{gathered} \text { E } 11.4 \\ " \\ " \\ " \\ " \\ " \\ " \end{gathered}$ | $\begin{gathered} \mathrm{sec} \\ \mathrm{kcal} / \mathrm{kg} \\ \mathrm{sec} \\ \mathrm{kcal} / \mathrm{kg} \\ \mathrm{sec} \\ \mathrm{kcal} / \mathrm{kg} \end{gathered}$ | $\left\{\begin{array}{l} \} \text { coordinates of first point } \\ \{" \end{array} \begin{array}{cc} " \text { second } " \\ " & \text { " third } " \end{array}\right.$ | $=0$. |

[^4]
## Coolant Boundary Condition (block 3)

| 38 | IP | I3 | - | block identification, columns 4-75 free for comments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 39 | IBC | I7 | - | indicator for choice of set of boundary conditions |

Pressure (block 4)

| 40 | IP | I3 | - | block identification, columns 4-75 free for comments | $=4$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 41 | NY3 | I7 | - | number of points given to define pressure (Y) as a function of time (X) | ₹ (NYD) |
| 42 | $\begin{aligned} & \mathrm{X} \\ & \mathrm{Y} \\ & \mathrm{X} \\ & \mathrm{Y} \\ & \mathrm{X} \\ & \mathrm{Y} \end{aligned}$ | $\begin{gathered} \text { E } 11.4 \\ " \\ " \\ " \\ " \\ " \\ " \end{gathered}$ | sec <br> at sec at sec at | $\left.\begin{array}{l} \} \text { coordinates of first point } \\ \} \end{array}\right\}$ | $=0$. |

Card 42 has to be repeated as to accommodate the coordinates of all NY points.

## Initial Coolant Mass Flow Rate and Deviation of Exit Pressure from Steady State Value as Function of Time (block 5)

| 43 | IP | I3 | - | block identification, columns 4-75 free for comments | $=5$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 44 | GO NY | $\begin{gathered} \mathrm{E} 11.4 \\ \mathrm{I7} \end{gathered}$ | $\mathrm{kg} / \mathrm{sec}$ | ```coolant mass flow rate at initial steady state number of points given to define exit pressure deviation (Y) as function of time (X)``` | $\begin{aligned} & >0 \\ & <(N Y D) \end{aligned}$ |
| 45 | $\begin{aligned} & X \\ & Y \end{aligned}$ $\mathrm{X}$ Y $\mathrm{X}$ $Y$ | $\begin{gathered} \text { E } 11.4 \\ " \\ " \\ " \\ " \\ " \end{gathered}$ | sec <br> at <br> sec <br> at <br> sec <br> at |  | $=0$. |

Card 45 has to be repeated as to accommodate the coordinates of all NY points. The coordinate $Y$ of the first point may be different from zero when step changes in the exit pressure are to be studied.
Cards 43-45 are omitted if IBC $\neq 1$

| 46 | IP | I3 | - | block identification, columns 4-75 free for commentary | $=6$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 47 | NY | I7 | - | number of points given to define coolant mass flow rate ( $Y$ ) ás function of time ( $X$ ) | < (NYD) |
| 48 | $\begin{gathered} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{X} \\ \mathrm{Y} \\ \mathrm{X} \\ \mathrm{Y} \end{gathered}$ | $\begin{gathered} \text { E } 11.4 \\ " \\ " \\ " \\ " \\ " \end{gathered}$ | $\begin{gathered} \mathrm{sec} \\ \mathrm{~kg} / \mathrm{sec} \\ \mathrm{sec} \\ \mathrm{~kg} / \mathrm{sec} \\ \mathrm{sec} \\ \mathrm{~kg} / \mathrm{sec} \end{gathered}$ | $\}$ coordinates of first point $\} \quad " \quad$ " second " $\}$ | $\begin{aligned} & =0 . \\ & >0 . \end{aligned}$ |

Card 48 has to be repeated as to accommodate the coordinates of all NY points. Cards $46-48$ are omitted if $I B C=1$

Initial Reactor Power (block 7)

| 49 | IP | I3 | - | block identification, columns 4-75 free for comments | $=7$ |
| :--- | :--- | :---: | :---: | :--- | :--- |
| 50 | PO | E I1.4 | MW | initial reactor power |  |

Output Frequency Control and Clock Termination Control (block 8)

| 51 | IP | I3 | - | block identification, columns $4-75$ free for comments, indicates start of computation | $=8$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 52 | NPR <br> NCL <br> CLM | $\begin{gathered} \text { I7 } \\ \text { I7 } \\ \mathrm{E} \\ 11.4 \end{gathered}$ | - - - | number of steps between printed output <br> " " " interrogation of computer clock <br> maximum computer running time of this problem, in units of computer clock |  |
| End of Job Card (block 9) |  |  |  |  |  |
| 53 | IP | I3 | - | block identification, columns $4-75$ free for comments, indicates end of job | $=9$ |

## IV. 3 Output Description

The first part of the output is a complete and annotated listing of all input data as well as all eventual comments given on block and subblock identification cards.

Next the steady state feedback is printed out (in $\$$ units) which needs to be balanced by external reactivity in order to make the reactor critical at its initial state. The absolute value of this number has no real significance but rather its value relative to values at states with other initial conditions.

The following part of the output gives the time dependent behavior of the most important system variable. A set of values of these variables is printed out every NPR (card 52) steps of the integration, the amount of variables in the output may be controlled by the parameters specified on card 29. Units of all varialbes in the output are consistent with the input and are the same as listed previously. The first set of values always refers to the initial steady state at time $X=0$. Only in this first set the absolute values of all contributions to the feedback reactivity are given. ${ }^{+}$) In all further steps the feedback contributions printed out are the deviations from the initial values printed out in the first set referring to time $X=0$. Values of the state variables of the coolant at the reactor boundaries (entrance, exit) are printed both, in the first and in the last line of each set. Whenever these may be different, the values in the last line refer to time $X$ given with this set whereas the corresponding value in the first line refers to the time at the beginning of this step. (Note, that $X$ marks the time at the end of a completed step of integration.)

Notations used in this part of the output are listed in the subsequent table. It may be convenient to refer to Fig. 8 for locating most of these variables.

[^5]

axial core section

Fig. 8 Schematic Sketch of the Reactor and a Core Channel Section

| Variable | Units | Other <br> Notation | Significance |
| :---: | :---: | :---: | :---: |
| X | sec | t | time (transient calculation starts at time = 0.) |
| I | sec | $\Delta t$ | time increment, as used for last step completed. Initially ( $t=0$.) : I = increment computed and proposed for first step to be taken. |
| P | MW |  | total reactor power |
| $E$ | MWsec | $={ }_{0} \mathrm{P} \cdot \mathrm{dt}$ | total energy released since beginning of transient |
| T | \$8 | $\Delta \mathrm{k}_{\mathrm{T}}$ | external (controlled) excess reactivity |
| PEX | at |  | coolant pressure at reactor entrance (first plenum chamber) |
| PAX | at |  | " " ." " exit (third " " ) |
| WEX | $\mathrm{kcal} / \mathrm{kg}$ |  | " enthalpy " " entrance (first " " ) |
| WAX | $\mathrm{kcal} / \mathrm{kg}$ |  | " " " " exit (third " " ) |
| G | $\mathrm{kg} / \mathrm{sec}$ |  | coolant mass flow rate. $\mathrm{C}_{\text {c }}^{\text {c }}$ |
| K | \% | $\Delta \mathrm{k}_{\mathrm{K}}$ | total feedback reactivity |
| D | \$ | $\Delta k_{D}$ | feedback reactivity from Doppler effect |
| H | \$ | $\Delta \mathrm{k}_{\mathrm{H}}$ | " " change in core height |
| R | 8 | $\Delta \mathrm{k}_{\mathrm{R}}$ | " " " " " radius |
| F | \% | $\Delta \mathrm{k}_{\mathrm{F}}$ | $" \quad " \quad " \quad \text { " fuel temp. } \quad \text { ( } \quad \text { steady state }(x=0 .): \text { ab- }$ |
| C | \$ | $\Delta \mathrm{k}_{\mathrm{C}}$ | " " " " " clad temp. $\quad$ transient ( $\mathrm{X} \neq 0$. ) relative |
| S | \$ | $\Delta \mathrm{k}_{S}$ | " " " " " coolant density values |
| M | \% | $\Delta \mathrm{k}_{\mathrm{M}}$ | " " " " structure temp. |
| IRB | - | - | index for radial zone and/or bypass in first flow section |
| IR | - | - | " " " " in second flow section (core + ax.bl.) |
| IX | - | - | " " axial section of core |
| TEB | ${ }^{0} \mathrm{C}$ |  | coolant temp. at entry to radial zone and/or bypass, after orifice |
| TAB | ${ }^{\circ} \mathrm{C}$ |  | " " " exit from " " " |


| WEB | kcal/kg |  | coolant enthalpy at entry to radial zone and/or bypass |
| :---: | :---: | :---: | :---: |
| WAB | kcal/kg |  | " " " exit from " " " |
| PEB | at |  | " pressure " entry to " " " after orifice |
| PAB | at |  | " " " exit from " " " |
| TFB | ${ }^{\circ} \mathrm{C}$ |  | central temperature of pin in rad. zone or inner wall of bypass, resp. |
| TMB | ${ }^{\circ} \mathrm{C}$ |  | temperature of structure " " " outer " " " |
| N | $\mathrm{kg} /\left(\mathrm{sec} \mathrm{m}^{2}\right)$ |  | area flow rate of coolant " " " " bypass |
| TZ | ${ }^{\circ} \mathrm{C}$ |  | temperature |
| PZ | at |  | pressure $\}$ of coolant in intermediary (second) plenum chamber |
| WZ | $\mathrm{kcal} / \mathrm{kg}$ |  | enthalpy |
| TE1 | ${ }^{\circ} \mathrm{C}$ |  | coolant temp. at entry to first axial blanket, after orifice |
| TAI | ${ }^{\circ} \mathrm{C}$ |  | " " " " " " " section of core |
| TA2 | ${ }^{\circ} \mathrm{C}$ |  | " " " exit from second axial blanket |
| PE1 | at |  | " pressure at entry to first axial blanket, after orifice |
| PAI | at |  | " " " " " " section of core |
| PA2 | at |  | " " " exit from second axial blanket |
| TFl | ${ }^{\circ} \mathrm{C}$ |  | central temperature of pin in first axial blanket |
| TF2 | ${ }^{\circ} \mathrm{C}$ |  | " " " " " second " " |
| N | $\mathrm{kg} /\left(\sec \mathrm{m}^{2}\right)$ |  | area flow rate of coolant in this radial zone of the core |
| TFO | ${ }^{\circ} \mathrm{C}$ |  | central fuel temperature |
| TFN | ${ }^{\circ} \mathrm{C}$ |  | fuel edge temperature |
| TCl | ${ }^{\circ} \mathrm{C}$ |  | clad temperature, inner edge $\}$ taken at midplane of axial section IX |
| TC2 | ${ }^{\circ} \mathrm{C}$ |  | " " , outer " |
| TM | ${ }^{\circ} \mathrm{C}$ |  | structure temperature |
| TA | ${ }^{\circ} \mathrm{C}$ |  | coolant temperature.... |
| WA | kcal/kg |  | " enthalpy |
| PA | at |  | " pressure $\{$ taken at exit of axial section IX |
| V | $\mathrm{m}^{3} / \mathrm{kg}$ |  | " spec. volume |



At the end of each problem the total number of steps and the (reactor-) time at the end of the transient are printed in the output.

## Other messages:

A certain check on input data is carried out by way of the block- and subblock identification parameters IP and IPT, respectively. If the block (subblock) identification card required at the beginning of any block (subblock) is missing or if the parameter IP (IPT) is incorrect (out of permissible range) a message: INPUT ERROR is printed out together with the incorrect value of IP (IPT). After this the job is terminated.

If the step size required by the criteria for numerical stability andor limited relative power changes is smaller than the specified lower limit XIN a message appears: STEP SIZE TOO SMALL, together with the last value of the time variable $X$. After this the job is terminated.

If the reading of a computer clock is made available through the function ZEIT (see APPENDIX N) a message appears (COMPUTING TIME CONSUMED) at the end of the output only if termination of the problem was caused by computation time exceeding the specified value CLM. After this computation continues with the next problem (if any).

If the specific version of function RCBE is used, which is described in APPENDIX L, a message will appear (SCRAM AT $X=\ldots$ ) whenever a scram situation is encountered; the current value of the time parameter is printed after the equal sign. A scram situation arises when at any step the actual reactor power exceeds for the first time the scram level PSC = PO • SCR. The message is printed at this event and comes before any other eventual output with this step.

## IV. 4 Storage- and COMMON-Requirements, Computing Time

COMMON-storage is required by five of the subprograms on level 3. These are PREP; STS; TRS; STEPC; ØUTPUT. In the case of integrating the program as subroutine REXION with another program already having its own COMMON requirements it is suggested to use block-common-statements whenever FORTRAN IV is available. Otherwise collation of the COMMON-requirements of both program sections is necessary.

Storage restrictions of the IBM 7074 require $\varnothing$ VERLAY structure of the program. The following structure is suggested (structure A):

RØUTSEGMENT: REX, REXIØN, PØWER, RCBE, STEPC, ØUTPUT, ZEIT, TPE1, HDV, FLAMI, ETAI, CØ1, CPI

ØRIGIN A : PREP
$\emptyset$ RIGIN A : STS
ØRIGIN A : TRS

In case $\emptyset V E R L A Y$ procedures do not permit back-references to the next higher level, a different structure should be used (structure B):

RØUTSEGMENT: REX, REXIØN, PØWER, RCBE, STEPC, ØUTPUT, ZEIT, CØ1, CP1

ØRIGIN A : PREP
$\emptyset R I G I N A: S T S$, TPEI, HDV, FLAMI, ETAI, C $\varnothing 1$
$\emptyset R I G I N A: T R S, ~ T P E l, H D V, F L A M 1, E T A 1, C \varnothing 1, C P 1$

The present version of the program has the following approximate total storage requirements:

10 K with $\varnothing$ VERLAY (structure B)
21 K without $\varnothing \mathrm{VERLAY}$

Computing times of individual problems depend on the number of nodes, output volume and frequency, complexity of subprograms on level 4 , step size and range of integration. The following example (see also APPENDIX $\varnothing$ ) should give an indication of typical computing times.

```
Problem: see APPENDIX \(\varnothing\) (subprograms as in APPENDICES K, L, M)
Number of nodes: \(N R B=2, N R=2, N B E=N B A=1, N X=5, N N=4\)
Output frequency: \(N P R=5,10,20\)
Stable step size: \(0,5-5,0\) milliseconds
computation time per step (average): \(0,9 \mathrm{sec}\) IBM 7074
                                    0,2 sec IBM 7094
                                    \(0,07 \mathrm{sec}\) IBM 360/65 (FØRTRAN
                                    \(\emptyset P T=1\) )
```


## V. Comments, Suggestions for Further Development

The models, methods and codes described in this report have proved to be useful and efficient tools for fast breeder analysis. Even though a relatively short time has elapsed since completion of the program a number of applications to accident analysis has provided significant results. Among these are:

- calculation of power reactivity coefficients for various reactor states of equilibrium;
- calculation of transient reactor behavior in blow down accidents (rupture of main steam pipes);
- parametric studies relating to scram characteristics, applied to blow down accidents.

Aside from these applications to specific systems many other studies can be envisaged including also investigations of a more general nature. The detailed and realistic features of the hydraulic model could - for example serve as the basis for studies concerning both, design and analysis of reactors.

This report shall be not complete without pointing out weaknesses and areas which will require further development in order to improve efficiency and applicability of this analytical tool. The remainder of this section is dedicated to a discussion of the more important ones of these items.

## The Model

The main deficiency of the model is the fact that it does not cover the coolant circuit and other pertinent components external to the reactor. Although certain analyses can be carried out with such a restricted model (for example: rapid excursions extending over a few seconds only) most practical cases will require an integral treatment of the whole reactor system. Therefore, the development of a consistent dynamics model and code for the external circuit is an essential implementation needed and is an immediate goal for further efforts in this area.

Another shortcoming of the model is seen in the treatment of the feedback. The great variety encountered in reactor core designs entails an almost equal variety in feedback mechanisms and virtually precludes the establishing of a general feedback model covering all possible cases. Feedback
mechanisms which cannot be accommodated by the ones already included in the present model will have to be added in individual cases, preferrably in the form of subprograms.

A third part of the model potentially requiring additional development is channel orificing. The orificing coefficients FT are determined by criteria related to channel temperature rise, channel power, channel pressure drops, e.t.c., all of these being characteristics of the static reactor design. Since most of the static design calculations do not yield directly the coefficients FT trial and error procedures have to be used with the present program which turns out to be quite time consuming. This could be avoided by developing a program consistent with the underlying model and code, which would calculate these coefficients on the basis of the pertinent criteria mentioned above.

Further areas for development of the model are identified with heat transfer through the fuel cladding-gap and with the plenum chambers. Provisions have already been made for a more sophisticated treatment of the gap coefficient as it might become available.

## Methods

The approximate nature of the criteria determining the step size for integration may require special caution in certain cases such as

- during rapid changes in the coolant boundary conditions;
- reactor power near prompt criticality.

Similarily, the nodal representation of the fuel and the treatment of fuel melting need consideration when temperature dependence of thermal conductivity of the fuel and fuel melting are of importance. In all of these cases recourse may be taken to either one or both of the following means:

- increasing the number of nodes;
- limiting the step size to values considerably below the stability limit.

In either case the associated penalty in computation time has to be considered and may be severe. To overcome these problems different methods of treating fuel melting, thermo-hydraulics and point kinetics will have to be devised. Some available methods [9,10_7 which feature good numerical
stability as well as high accuracy were discarded because of the complexities introduced by them. However, there is sufficient incentive to develop new and efficient methods for treating the problems mentioned above:

- fuel heat transfer;
- fuel melting;
- coolant thermo-hydraulics;
- integration of point kinetics;
- determination of max. step size for numerically stable integration.

At this point it cannot be clear to what extent any such innovation might be incorporated in the present program.

## Program

Experience gained up to date has indicated that computation time will not be a limiting factor with normal cases. The calculation of a typical transient arising with a reactor accident as described in APPENDIX $\varnothing$ requires about one minute computer time on an IBM $360 / 65$ system to cover a range of interest of about 10 seconds of reactor time. Systems with a greater number of nodes will increase computation time accordingly as will be the case when using slower computers (IBM 7094: ca. 3 minutes; IBM 7074 ca. 12 minutes for the same problem). Computer storage practically does not impose limitations; COMMON and DIMENSION statements are formulated in such a way as to facilitate adjustments in special cases.

APPENDIX A : Notation Equations

The following list of notations is complete only in conjunction with the lists for input and output given in sections IV. 2 and IV. 3.

| $A E_{i}$ | linear coefficient of thermal expansion |  |
| :---: | :---: | :---: |
| ${ }^{A R}{ }_{i}$ | heat source density, coefficient for rad. zonedistribution | $\left\{\begin{array}{l} \text { component i } \\ (=F, C, M) \end{array}\right.$ |
| $\mathrm{AX}_{\mathrm{i}}$ | heat source density, coefficient for axial sectiondistribution |  |
| $\mathrm{C}_{\text {i }}$ | precursor concentrations ( $i=1,2, \ldots$ ND) |  |
| Cl | $=238,89 \mathrm{kcal} / \mathrm{MWsec}$ |  |
| $C \emptyset 1$. | thermal conductivity of fuel (function) |  |
| CP | specific heat (at constant pressure) |  |
| CP1 | specific heat of fuel (function) |  |
| CP3 | specific heat of coolant, average over axial section |  |
| CPA | " " " " , exit of axial section |  |
| CPE | " " " , entry of axial section |  |

hydraulic diameter, wetted surface includes cladding + structure
" " , wetted surface of cladding only
" " , wetted surface of structure only
DP coolant pressure drop in axial section
DX step size, time increment ( $\equiv \mathrm{I}$ )
$\left.\begin{array}{l}\text { ETA }\end{array}\right\}$ dynamic viscosity of coolant $\left[\mathrm{kps} / \mathrm{m}^{2} \overline{7}\right.$
FC factor, indicator for flow direction
$\left.\begin{array}{l}\text { FLAM } \\ \text { FLAML }\end{array}\right\}$ thermal conductivity of coolant $/ \mathrm{Fcal}_{\mathrm{kc}}^{\mathrm{m}} \mathrm{h}{ }^{\circ} \mathrm{C}_{-7}$
FQ net area for coolant flow in a radial zone
$g \quad 9.81 \mathrm{~m} / \sec ^{2}$, acceleration of gravity

```
net flow rate per unit area of coolant in channel ( \equivN)
height of axial section
film coefficient of heat transfer between coolant and channel walls
```

" " " " " $"$ "
" " 11
total core height
index, for radial zones
index, for axial sections
argument, mode control of subroutine REXION
argument, " " " function RCBE
excess reactivity
" " , external, ( $\equiv$ T)
" " , feedback (total), (
" " , Doppler effect, ( $\equiv \mathrm{D}$ )
" ", change in core height, $(\equiv \mathrm{H})$
" " , change in core radius, ( $\equiv \mathrm{R}$ )
" ", change in fuel density, ( $\equiv \mathrm{F}$ )
" " $"$ " " clad $"$, (
" " , " " coolant ", ( 1 S )
" $"$, " " structure density, ( $\equiv \mathrm{M}$ )
total steady state feedback (relative value, output)
generalized Nusselt's number
dimension, maximum value for $N R B$

| $"$ | , | $"$ | $"$ | $"$ | $N R$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $"$ | , | $"$ | $"$ | $"$ | $N X$ |
| $"$ | , | $"$ | $"$ | $"$ | $N N$ |
| $"$ | , | $"$ | $"$ | $"$ | $N D$ |
| $"$ | , | $"$ | $"$ | $"$ | $N Y$ |

    fraction of total reactor power released in core
    coolant pressure at entry of axial section
    $V_{i} \quad$ volume of component $i(=F, C, M)$
$V_{F} \quad$ total volume of fuel in core
$V_{S} \quad$ total volume of coolant in core

WA enthalpy of coolant, exit of axial section
WE " " " entry of axial section
WI " " " average over axial section
WA enthalpy of coolant at channel exit

2
$\alpha \quad$ reciprocal of generalized friction factor
B total fraction of delayed neutrons
$\lambda \quad$ thermal conductivity of coolant $\leq \mathrm{kcal} / \mathrm{m} \mathrm{h}{ }^{\circ} \mathrm{C}_{\mathbf{Z}} 7$
$\Lambda$ prompt neutron generation time
$\rho$ density of coolant (= 1./V)

## Equations

Balance of total power:

$$
\begin{aligned}
& \mathrm{PC}+\mathrm{PBE}+\mathrm{PBA}+\mathrm{PB}=1.0 \\
& \mathrm{~PB}=\sum_{\mathbb{R B}}(\mathrm{PB})
\end{aligned}
$$

Heat source density in various components:

```
radial blanket: pin }\begin{array}{l}{\mathrm{ bypass in inner wall }}\end{array}}\quadR2M={\begin{array}{l}{R2}\\{2.0.WP}
```

$q_{R B}=\frac{P \cdot C I \cdot P B}{A N \cdot P I \cdot R 2 \cdot R 2 M \cdot H T}$
$\left.\begin{array}{ll}\text { radial blanket: } & \text { structure } \\ \text { coolant } \\ \text { bypass: } & \text { outer wall }\end{array}\right\}$ no source
axial blanket: pin
first blanket

$$
q_{B 1}=\frac{P \cdot C 1 \cdot P B E}{P I \cdot H B E} \cdot \frac{A Q R}{\sum_{I R}(A Q R \cdot A N)} \cdot \frac{1}{R 2^{2}}
$$

second blanket

$$
q_{B 2}=\frac{P \cdot C I \cdot P B A}{P I \cdot H B A} \cdot \frac{A Q R}{\sum_{\mathbb{R}}(A Q R \cdot A N)} \cdot \frac{1}{R 2^{2}}
$$

axial blanket: structure coolant

Core:

$$
\begin{aligned}
& q_{E}=\frac{p \cdot C I \cdot P C}{H C / N X} \cdot \frac{1}{S A Q \cdot \sum_{\mathbb{X}}(A Q X) \cdot \sum_{\mathbb{R}}(A Q R \cdot A N)} \\
& S A Q=A Q F+A Q C+A Q S+A Q M
\end{aligned}
$$

fuel:

$$
q_{F}=q_{E} \frac{A Q F \cdot A Q X \cdot A Q R}{R I^{2} \cdot P I}
$$

clad:

$$
q_{C}=q_{E} \frac{A Q C \cdot A Q X \cdot A Q R}{\left(R 2^{2}-R I^{2}\right) \cdot P I}
$$

coolant:

$$
q_{S}=q_{E} \cdot \frac{A Q S \cdot A Q X \cdot A Q R}{\left(R 3^{2}-R 2^{2}\right) \cdot P I}
$$

structure:

$$
q_{M}=q_{E} \cdot \frac{A Q M \cdot A Q X \cdot A Q R}{W P \cdot D 4}
$$

## Steady State Temperatures

## core:

fuel: $\quad T F_{n-1}=T F_{n}+q_{F} \cdot \frac{R I^{2}}{4 \cdot N N \cdot C D I\left(T F_{n}\right)} \quad n=1, \ldots N$

$$
T F N=T F_{N}=T C l+q_{F} \frac{R I}{2 \cdot H G P}
$$

cladding: $T C I=T C 2+q_{F} \frac{R 1}{C \varnothing 2} \cdot \frac{R 2-R I}{R 2+R I}+q_{C} \frac{(R 2-R I)^{2}}{2 \cdot C \varnothing 2}$

$$
\mathrm{TC} 2=\mathrm{T} 3+q_{F} \cdot \frac{R 1^{2}}{2 \cdot R 2 \cdot H 23}+q_{C} \frac{\left(R 2^{2}-R 1^{2}\right)}{2 \cdot R 2 \cdot H 23}
$$

structure: $T M=T 3+q_{M} \cdot \frac{D 4}{H 34^{\prime}}$ where $H 34^{\prime}=H 34 \frac{1}{1+\frac{D 4 \cdot H 34}{3 \cdot C \varnothing 4}}$
coolant: $\quad W A=W E+\frac{H C / N X}{G F} \cdot\left(q_{S}+q_{F} \frac{R 1^{2}}{\left(R 3^{2}-R 2^{2}\right)}+q_{C} \frac{R 2^{2}-R 1^{2}}{R 3^{2}-R 2^{2}}+q_{M} \frac{W P \cdot D 4}{\left(R 3^{2}-R 2^{2}\right) \cdot P I}\right)$
$W I=(W A+W E) \cdot 0,5$
$T A=f_{T}(P A, W A)$.
$T 3=(T A+T E) \cdot 0,5$
first axial blanket (second ax.bl. correspondingly):
pin: $\quad T F 1=T 3+q_{B 1} \frac{R 2}{2 \cdot H F S}$
structure: $T M 1=T 3$
coolant: $\quad W A=W E+q_{B 1} \cdot \frac{H B E \cdot R 2^{2}}{\left(R 3^{2}-R 2^{2}\right) \cdot G F}$
$W I=(W A+W E) \cdot 0,5$
$T A=f_{T}(P A, W A)$
$T 3=(T A+T E) \cdot 0,5$
$\left.\begin{array}{l}\text { radial blanket } \\ \text { bypass }\end{array}\right\} \quad R 2 M=\left\{\begin{array}{l}R 2 \\ 2 \cdot W P\end{array}\right.$
$\left.\begin{array}{l}\text { pin: } \\ \text { inner wall: }\end{array}\right\} \mathrm{TFB}=\mathrm{T} 3+\mathrm{a}_{\mathrm{RB}} \cdot \frac{\mathrm{R} 2 \mathrm{M}}{2 \cdot \mathrm{HA}}$
$\left.\begin{array}{l}\text { structure: } \\ \text { outer wall: }\end{array}\right\} T S B=T 3$
coolant:

$$
\begin{aligned}
& W A=W E+q_{R B} \frac{H T \cdot R 2 \cdot R 2 M}{\left(R 3^{2}-R 2^{2}\right) \cdot G F} \\
& W I=(W A+W E) \cdot 0,5 \\
& T A=f_{T}(P A, W A) \\
& T 3=(T A+T E) \cdot 0,5
\end{aligned}
$$

## Transient Temperatures

Core:
fuel:
cladding: $\quad T C 1=T C 1+\frac{D X}{R \phi 2 \cdot C P 2}\left\{\mathrm{q}_{\mathrm{C}}+\frac{4}{\mathrm{R}^{2}-\mathrm{RI}{ }^{2}}[\mathrm{RI} \cdot \mathrm{HGP} \cdot(\mathrm{TFN}-\mathrm{TCl})-\right.$

$$
\left.\left.-(T C 1-T C 2) \cdot \frac{R 2+R 1}{2 \cdot(R 2-R 1)} \cdot C \varnothing 2\right]\right\}
$$

$$
T C 2=T C 2+\frac{D X}{R \emptyset 2 \cdot C P 2}\left\{q_{C}+\frac{4}{R 2^{2}-R I^{2}} \cdot\left[\frac{R 2+R 1}{2 \cdot(R 2-R 1)}\right.\right.
$$

$$
\cdot(\mathrm{TC} 1-\mathrm{TC} 2) \cdot \mathrm{C} \varnothing 2-(\mathrm{TC} 2-\mathrm{T} 3) \cdot \mathrm{R} 2 \cdot \mathrm{H} 23]\}
$$

$$
\begin{aligned}
& T F O=T F O+\frac{D X}{R \varnothing 1 \cdot C P I(T F O)} \cdot\left\{q_{F}+\frac{4 \cdot N N}{R 1^{2}} \cdot[ \right. \\
& \left.\left.-\left(T F O-T F_{1}\right) \cdot C \varnothing 1\left(T F_{1}\right)\right]\right\} \\
& T F_{n}=T F_{n}+\frac{D X}{R \varnothing I \cdot C P I\left(T F_{n}\right)} \cdot\left\{q_{F}+\frac{4 \cdot N N}{R I^{2}} \cdot\left[(n-0.5) \cdot\left(T F_{n-1}-T F_{n}\right) \cdot\right.\right. \\
& \text { - } \left.\left.C \varnothing 1\left(T F_{n}\right)-(n+0.5)\left(T F_{n}-T F_{n+1}\right) \cdot C \not \|_{1}\left(T F_{n+1}\right)\right]\right\} \\
& \mathrm{n}=\mathrm{l}, \ldots \mathrm{~N}-\mathrm{I} \\
& T F N=T F N+\frac{D X}{R \varnothing 1 \cdot C P 1(T F N)} \cdot\left\{\begin{array}{l}
q_{F}+\frac{4 \cdot N N}{R 1^{2}}[(2 \cdot N N-1) .
\end{array}\right. \\
& \left.\left.\left(\mathrm{TF}_{\mathrm{N}-1}-\mathrm{TFN}\right) \cdot \mathrm{C} 1(\mathrm{TFN})-(\mathrm{TFN}-\mathrm{TCl}) \cdot \mathrm{RI} \cdot \mathrm{HGP} \quad\right]\right\}
\end{aligned}
$$

```
structure: \(\quad T M=T M+\frac{D X}{R \varnothing 4 \cdot C P 4} \cdot\left\{q_{M}-\frac{H 34^{\prime}}{D 4}(T M-T 3)\right\}\)
coolant: \(\quad W A=W A+D X \cdot V \cdot\left\{q_{S}+\frac{2 \cdot H 23 \cdot R 2}{R 3^{2}-R 2^{2}}(T C 2-T 3)+\frac{H 34 \cdot W P}{P I \cdot\left(R 3^{2}-R 2^{2}\right)}\right.\).
\(\left.\cdot(T M-T 3)+\frac{G F}{H C / N X}(W E-W A)\right\}\)
\(W I=(W A+W E) \cdot 0,5\)
\(T A=f_{T}(P A, W A)\)
\(T 3=(T A+T E) \cdot 0,5\)
```

first axial blanket (second ax.bl. accordingly):
$\begin{array}{ll}\text { pin: } & T F I=T F 1+\frac{D X}{C V F}\left\{q_{\mathrm{BI}}-\frac{2 \cdot \mathrm{HFS}}{\mathrm{R} 2} \cdot(\mathrm{TFI}-\mathrm{T} 3)\right\} \\ \text { structure }: & T M I=T M 1+\frac{D X}{C V M}\left\{-\frac{H M S}{D 4} \cdot(T M 1-T 3)\right\} \\ \text { coolant: } & W A=W A+D X \cdot V\left\{\frac{2 \cdot H F S \cdot R 2}{R 3^{2}-R 2^{2}}(T F 1-T 3)+\frac{H M S \cdot W P}{P I \cdot\left(R 3^{2}-R 2^{2}\right)} .\right.\end{array}$

- $\left.(T M 1-T 3)+\frac{G F}{H B E}(W E-W A)\right\}$
$W I=(W A+W E) \cdot 0,5$
$T A=f_{T}(P A, W A)$
$T 3=(T A+T E) \cdot 0,5$
$\left.\begin{array}{l}\text { radial blanket } \\ \text { bypass }\end{array}\right\} \quad W P M=\left\{\begin{array}{l}W P \\ 2 \cdot R 3 \cdot P I\end{array}\right.$
$\left.\begin{array}{l}\operatorname{pin} \\ \text { inner wall }\end{array}\right\}: T F B=T F B+\frac{D X}{C V}\left\{q_{R B}-\frac{2 \cdot H A}{R 2}(T F B-T 3)\right\}$
$\left.\begin{array}{l}\text { structure } \\ \text { outer wall }\end{array}\right\}: T M B=T M B+\frac{D X}{C V}\left\{-\frac{H A}{D 4}(T M B-T 3)\right\}$
coolant:

$$
\begin{gathered}
W A=W A+D X \cdot V\left\{\frac{2 \cdot H A \cdot R 2}{R 3^{2}-R 2^{2}}(T F B-T 3)+\frac{H A \cdot W P M}{P I \cdot\left(R 3^{2}-R 2^{2}\right)} \cdot\right. \\
\left.\cdot(T M B-T 3)+\frac{G F}{H T}(W E-W A)\right\}
\end{gathered}
$$

$$
\begin{aligned}
& W I=(W A+W E) \cdot 0,5 \\
& T A=f_{T}(P A, W A) \\
& T 3=(T A+T E) \cdot 0,5
\end{aligned}
$$

Orifices: conservation of enthalpy when passing orifice is assumed.

## Plenum Chambers:

```
zero pressure loss of coolant
zero residence time of coolant in chamber
instantaneous mixing of all entering streams of coolant:
\(W I=\frac{\sum_{\mathbb{R}}\left(F Q \cdot G F \cdot W A_{L}\right)}{\sum_{\mathbb{R}}(F Q \cdot G F)}\)
where: \(F Q=P I \cdot\left(R 3^{2}-R 2^{2}\right) \cdot A N\)
```


## Fuel Melting

The basic model for treating fuel melting and recrystallization is the same as in [1]. However, the computational technique differs and admits all possible cases (fuel partially or entirely molten at initial steady state, partial and/or total melting and/or recrystallization during transient). Melting point and recrystallization point are assumed to be identical (TM) as are the values of heat of fusion and heat of recrystallization (UM, [ kcal/m $\mathrm{m}^{3}$ 7).

In the calculation a variable $U$ is associated with each fuel node, which is a record of the latent heat of fusion of this node:

```
U = O. ........ solid
U = UM ........ molten
O<U<UM ...... partially molten
```

If in the steady state calculation the temperature of any fuel node TF is such that

$$
\begin{array}{ll}
T F>T M & U=U M \\
T F<T M & U=0 .
\end{array}
$$

During the transient calculation fuel temperatures first are calculated according to the equations given before. Then a hypothetical enthalpy
change is calculated:

$$
\mathrm{UH}=(\mathrm{TF}-\mathrm{TM}) \cdot \mathrm{R} \emptyset 1 \cdot \mathrm{CPI}(\mathrm{TM})
$$

which may be $\geqslant 0$ depending on $T F \geqslant T M$. This enthalpy change if added to U yields a new quantity UN

$$
\mathrm{UN}=\mathrm{U}+\mathrm{UH}
$$

which is subject to decisions on further procedures:
$U N<0\{U=0 \ldots$ the node was solid and remained solid, set $T F=T F$
$U>0 .$. the node just solidified entirely, $T F=T F+U /(R \varnothing l \cdot C P I(T M))$ $\mathrm{U}=0$ 。
$U N>0\{U N<U M \ldots$ fuel is partially molten, set $T F=T M$

$$
\mathrm{U}=\mathrm{UN}
$$

$U N>U M \ldots\left\{\begin{array}{l}U=U M \text { the node was molten, set } T F=T F \\ U<U M \text { the node just melted entirely, set }\end{array}\right.$

$$
\begin{aligned}
T F & =T F-(U M-U) /(R \varnothing 1 \cdot C P I(T M)) \\
U & =U M
\end{aligned}
$$

Fig. Al should help to illustrate this scheme.


Fig. Al Flow Diagram for Calculations Regarding Fuel Melting

Film Coefficient of Heat Transfer to Coolant:
from cladding:

$$
\begin{gathered}
\mathrm{H} 23=\mathrm{AL} 4+\frac{\mathrm{CH} 2}{\mathrm{DH} 2(\mathrm{ALI})} \cdot(\mathrm{GF})^{\mathrm{ALI}} \cdot\left(\frac{\mathrm{FLAM}^{(1-\mathrm{AL} 2)} \cdot \mathrm{CP}^{\mathrm{AL} 2}}{\mathrm{ETA}(\mathrm{AL1}-\mathrm{AL2})}\right) \cdot \\
\cdot\left(\frac{\mathrm{T} 3+273.16}{\mathrm{TC} 2+273.16}\right)^{\mathrm{AL} 3}
\end{gathered}
$$

from structure:

$$
\mathrm{H} 34=\mathrm{AL} 4+\frac{\mathrm{CH} 4}{\mathrm{DH} 4^{(1-\mathrm{ALI})}} \cdot(\mathrm{GF})^{\mathrm{ALI}} \cdot\left(\frac{\mathrm{FLAM}^{(1-\mathrm{AL2} 2)} \cdot \mathrm{CP}^{\mathrm{AL} 2}}{\mathrm{ETA}}\right) \cdot
$$

- $\left(\frac{T 3+273.16}{\mathrm{TM}+273.16}\right)^{\mathrm{AL} 3}$

$$
\text { with } \quad \begin{aligned}
\text { DH2 } & =\frac{2 \cdot\left(R 3^{2}-R 2^{2}\right)}{R 2} \quad \mathrm{DH} 4=\frac{4 \cdot P I \cdot\left(R 3^{2}-R 2^{2}\right)}{W P} \\
\text { FLAM } & =f_{\lambda}(P A, T 3) \\
\text { ETA } & =f_{\eta}(V, T 3) \\
\text { CPA } & =f_{C P}(P A, T A) \\
\text { CP3 } & =(C P A+C P E) \cdot 0,5
\end{aligned}
$$

## Hydraulic Model

Pressure Losses:
pressure loss at channel entrance ( $\rho=1 / \mathrm{V}$ ):

$$
\begin{gathered}
\Delta \mathrm{p}_{\mathrm{E}}=\left[\left(\frac{\mathrm{v}_{1}}{\varphi}\right)^{2}-\left(\mathrm{v}_{0}\right)^{2}\right] \frac{\rho}{2 g} \\
\text { with: } \quad \mathrm{v}_{0}=0 . \\
\mathrm{v}_{1} \cdot \rho=\frac{\dot{\mathrm{m}}}{\mathrm{~F}_{\mathrm{q}}}=\mathrm{GF} \\
\\
\frac{1}{\varphi^{2}}=1,5 \\
\left.\Delta \mathrm{p}_{\mathrm{E}}=1,5 \cdot \frac{\mathrm{GF}^{2}}{2 \cdot g \cdot 10^{4}} \cdot \mathrm{v} I^{-} a t_{-}\right]
\end{gathered}
$$

pressure loss through orifice:

$$
\Delta p_{T}=F T \cdot \frac{G F^{2}}{2 g \cdot 10^{4}} \cdot V \quad\left[a t_{-}\right]
$$

pressure loss in channel due to friction and acceleration

hydraulic diameter:

$$
\mathrm{DH}=\frac{4 \cdot \mathrm{PI} \cdot\left(\mathrm{R} 3^{2}-\mathrm{R} 2^{2}\right)}{\mathrm{WPM}+2 \cdot \mathrm{PI} \cdot \mathrm{R} 2}
$$

wetted perimeter:

$$
\begin{aligned}
& W P M=\left\{\begin{array}{l}
W P \ldots \text {.... radial blanket } \\
2 \cdot R 3 \cdot P I \ldots \text { bypass } \\
W P \ldots . . . \text { first, second ax.bl., core } \\
\Delta p_{R}=\frac{G F^{2}}{g \cdot 10^{4}} \cdot\left[\frac{\mathrm{FR} \cdot \Delta H}{2 \cdot D H} \cdot V+(V A-V E)\right] \text { Lat_7 } \\
V A=f_{V}(P A, T A) \\
V
\end{array}\right. \\
& =(V A+V E) \cdot 0,5
\end{aligned}
$$

Feedback Equations

$$
\text { subscript } x=\left\{\begin{array}{l}
0 \ldots \text { steady state value } \\
T \ldots \text { transient value }
\end{array}\right.
$$

Doppler feedback:

$$
\begin{aligned}
D_{x}= & R C D \cdot \frac{\sum_{\mid x} \cdot \sum_{i R}\left(A D X \cdot A D R \cdot A N \cdot R I^{2} \cdot \ln \left(\overline{T F}_{x}+273 \cdot 16\right)\right)}{\sum_{i x} \cdot(A D X) \cdot \sum_{\mathbb{R}}\left(A D R \cdot A N \cdot R I^{2}\right)} \\
& \overline{T F}_{x}=\frac{1}{N N} \cdot \sum_{i N}\left(T F_{x}\right)
\end{aligned}
$$

core height feedback:

$$
H_{x}=R C H \cdot \frac{\sum_{X} \cdot \sum_{R R}\left(A N \cdot R l^{2} \cdot A E F \cdot \overline{T F}_{x}\right)}{N N \cdot \sum_{\mathbb{R}}\left(A N \cdot R l^{2}\right)}
$$

core radius feedback:

$$
R_{x}=R C R \cdot \frac{\sum_{X} \cdot \sum_{\mathbb{R}}\left(A N \cdot A E M \cdot T M_{x}\right)}{N N \cdot \sum_{\mathbb{R}}(A N)}
$$

fuel density feedback:

$$
F_{x}=-R C F \frac{\sum_{X} \cdot \sum_{R}\left(A X F \cdot A R F \cdot A N \cdot R I^{2} \cdot A E F \cdot \overline{T_{x}}\right)}{\sum_{\mathrm{L}} \cdot(A X F) \cdot \sum_{\mathbb{R}}\left(A R F \cdot A N \cdot R I^{2}\right)}
$$

cladding density feedback:

$$
\begin{gathered}
C_{x}=-R C C \frac{\sum_{\sum_{X}} \cdot \sum_{i R}\left(A X C \cdot A R C \cdot A N \cdot\left(R 2^{2}-R I^{2}\right) \cdot A E C \cdot \overline{T C}_{x}\right)}{\sum_{i x}(A X C) \cdot \sum_{\mathbb{R}}\left(A R C \cdot A N \cdot\left(R 2^{2}-R I^{2}\right)\right)} \\
\overline{T C}_{x}=\left(T C l_{x}+T C C_{x}\right) \cdot 0,5
\end{gathered}
$$

structure density feedback:

$$
M_{x}=-R C M \frac{\sum_{\sum_{X}} \cdot \sum_{\mathbb{R}}\left(A X M \cdot A R M \cdot A N \cdot W P \cdot D 4 \cdot A E M \cdot T M_{X}\right)}{\sum_{1 X}(A X M) \cdot \sum_{\mathbb{R}}(A R M \cdot A N \cdot W P \cdot D 4)}
$$

coolant density feedback:

$$
S_{x}=-R C S \frac{\sum_{\mid X} \cdot \sum_{i R}\left(A X S \cdot A R S \cdot A N \cdot\left(R 3^{2}-R 2^{2}\right) / V_{x}\right)}{\sum_{i x}(A X S) \cdot \sum_{\mathbb{R}}\left(A R S \cdot A N \cdot\left(R 3^{2}-R 2^{2}\right)\right)}
$$

Total steady state feedback:

$$
K_{0}=\left(D_{0}+H_{0}+R_{0}+F_{0}+C_{0}+S_{0}+M_{0}\right)
$$

This total steady state feedback is assumed to be compensated by external reactivity.

Transient feedback
$D=D_{x}-D_{0}$
$F=F_{x}-{ }_{0}$
$M=M_{x}-M_{0}$
$\mathrm{H}=\mathrm{H}_{\mathrm{x}}-\mathrm{H}_{\mathrm{o}}$
$C=C_{x}{ }^{-C}$
$R=R_{x}-R_{0}$
$S=S_{x}-S_{0}$

## Total transient feedback

## APPENDIX B : Decay Time Constants for Nodal Temperatures in Cylindrical Fuel Pin

Taking fuel temperatures relative to the temperature TCl of the inner edge of the cladding we can write the system of equations for the nodal fuel temperatures as follows:

$$
\begin{array}{r}
R \emptyset 1 \cdot C P 1 \cdot \frac{d T F O}{d x}=q_{F}+\frac{4 \cdot C \emptyset 1 \cdot N N}{R I^{2}}[ \\
R \emptyset 1 \cdot C P 1 \cdot \frac{d T F}{d x} n=q_{F}+\frac{4 \cdot C \emptyset 1 \cdot N N}{R I^{2}}\left[(n-0.5) \cdot\left(T F_{n-1}-T F_{n}\right)-(n+0 \cdot 5) \cdot\left(T F_{n}-T F_{n+1}\right)\right] \\
n=1, \ldots, N-1
\end{array}
$$

$$
R \varnothing 1 \cdot C P I \cdot \frac{d T F N}{d \bar{x}}=q_{F}+\frac{4 \cdot C \varnothing I \cdot N N}{R I}\left[(2 \cdot N N-I) \cdot\left(T F_{N-I}-T F N\right)-\frac{R I \cdot H G P}{C \varnothing I} \cdot T F N\right]
$$

All parameters except for volumetric heat source $q_{F}$ are assumed to be invariants for this consideration. Fictitious subdivisions separate the cylindrical fuel pin into NN-l concentric annuli of equal volume, as well as a central cylinder and an outer most annulus having only half of this volume. In the case of steady state the temperatures $\mathrm{TF}_{1}, \mathrm{TF}_{2}, \ldots, \mathrm{TF} \mathrm{N}_{\mathrm{N}}$, represent the volume average temperature of each annulus, respectively. TFO and TFN represent the temperature at the center axis and on the surface edge, respectively.

We assume that starting from a steady state equilibrium the heat source $q_{F}$ vanishes at time $x=0$. Consequently, all temperatures eventually will drop to the reference level ( $\mathrm{TFO}=T \mathrm{~F}_{\mathrm{n}}=\mathrm{TFN} \Rightarrow 0$.) . The asymptotic solution of the temperature transients may be found by means of Laplace transformation as follows:

This is a linear and homogeneous system of equations for the temperatures $T F_{n}$ with all negative Eigenvalues $s_{n}$. The asymptotic behavior is characterised

$$
\begin{aligned}
& \frac{\mathrm{RI}^{2}}{4 \cdot \mathrm{NN}} \cdot \frac{\mathrm{R} \varnothing 1 \cdot \mathrm{CP} 1}{\mathrm{C} \varnothing 1} \cdot \mathrm{~s} \cdot \widetilde{\mathrm{TFO}}= \\
& \frac{R_{1}^{2}}{4 \cdot N N} \cdot \frac{R \emptyset 1 \cdot C P 1}{C \emptyset 1} \cdot s \cdot \widetilde{T F}_{n}=(n-0.5) \cdot\left(\widetilde{T F}_{n-1}-\widetilde{T F}_{n}\right)-(n+0.5) \cdot\left(\widetilde{T F}_{n}-\widetilde{T F}_{n+1}\right) \\
& \mathrm{n}=1,2 \ldots \mathrm{~N}-1
\end{aligned}
$$

by the Eigenvalue $s_{N}$ with the smallest magnitude. It is evident that this Eigenvalue as all others is a function only of three parameters:

$$
s_{N}=s_{N}\left(N N, \frac{R I \cdot H G P}{C \varnothing I}, R I^{2} \cdot \frac{R \varnothing 1 \cdot C P I}{C \varnothing I}\right)
$$

where the last parameter appears as a scaling factor only.
On the other hand the solution of this particular problem may be obtained analytically without discretization [-11_7. The Eigenvalue with the smallest magnitude is given by:

$$
s_{\infty}=\left(\frac{\mathrm{RI}}{\alpha}\right)^{2} \cdot \frac{\mathrm{R} \varnothing \mathrm{I} \cdot \mathrm{CPI}}{\mathrm{C} \varnothing I}
$$

where $\alpha$ is the smallest value satisfying the transcendental equation

$$
\alpha \cdot \frac{J_{1}(\alpha)}{J_{0}(\alpha)}=\frac{\mathrm{R1} \cdot \mathrm{HGP}}{\mathrm{C} \varnothing 1}
$$

In much the same way as above the Eigenvalue $s_{\infty}$ is a function of two parameters:

$$
s_{\infty}=s_{\infty}\left(\frac{R I \cdot H G P}{C \varnothing 1}, R 1{ }^{2} \cdot \frac{R \varnothing 1 \cdot C P I}{C \varnothing I}\right)
$$

Fig. Bl is a plot of the function $s_{N}$ and $s$ (multiplied by the scaling factor $\left.R 12 \cdot \frac{R \varnothing 1 . C P 1}{C \varnothing 1}\right)$ versus $\frac{R I \cdot H G P}{C \varnothing 1}$. These were obtained numerically using a separate program written for this purpose.

As may be expected, the function $s$ is the asymptotic function to $s_{N}$, as NN goes to infinity. Thus, for any given set of values for the characteristic parameters one may obtain from this plot
a) the asymptotic behavior of the correct solution,
b) the asymptotic behavior of the approximate solution resulting for $N N+1$ nodes,
c) the required number of nodes to obtain a predetermined accuracy in the asymptotic solution for this case

Although vanishing of the heat source in conjunction with the simplifying assumptions made constitute a rather specific case of transient behavior, it may be considered typical for fast transients. The simplicity of the diagram in Fig. Bl renders this a useful tool in selecting a proper mesh for a variety of pertinent problems.


Fig. Bl Asymptotic Time Constants for Temperature Decay in Cylindrical Fuel Pin

APPENDIX C : Predictor-Corrector-Schemes Applied in Solving the Hydraulics Problem.

The general expression for the pressure drop in a flow channel (index i) is

$$
\begin{aligned}
\Delta p_{i} & =\Delta p_{E i}+\Delta p_{T i}+\Delta p_{R i}+\Delta p_{A i} \\
& =\frac{1}{g} G F_{i}^{2} \cdot\left\{\frac{1 \cdot 5+F T_{i}}{2.0} V E+\frac{F R_{i}}{2 \cdot D H_{i}} \Sigma \Delta H_{j} V_{i j}-\left(V A_{i}-V E_{i}\right)\right\}
\end{aligned}
$$

The first term in parantheses represents entry and orifice losses, the second term frictional losses and the last term acceleration loss. If one assumes the specific volume to be constant throughout the channel and moreover to be the same in all (parallel) channels

$$
V_{E}=V_{i j}=V A_{i}=\bar{V}
$$

one may write the total pressure drop in a simpler form:

$$
\Delta \mathrm{p}_{i}=\frac{1}{g} \mathrm{GF}_{i}^{2}\left\{\frac{1.5+\mathrm{FT}_{i}}{2.0}+\frac{\mathrm{FR}_{i} \cdot \mathrm{HT}}{2 \cdot \cdot \mathrm{DH}_{i}}\right\} \cdot \overline{\mathrm{V}}=\frac{1}{g} \mathrm{GF}_{\mathrm{i}}^{2} \cdot \frac{1}{\alpha_{i}^{2}} \cdot \overline{\mathrm{~V}}
$$

where

$$
\frac{1}{\alpha_{i}^{2}}=\left\{\frac{1.5+\mathrm{FT}_{i}}{2.0}+\frac{\mathrm{FR}_{i} \cdot \mathrm{HT}}{2 . \mathrm{DH}_{i}}\right\}
$$

is a generalized coefficient of friction associated with each channel. The second relevant equation is obtained from the mass balance of the coolant:

$$
\sum_{i}\left[\left(G F_{i}\right) \cdot\left(A N_{i} \cdot\left(R 3_{i}^{2}-R 2_{i}^{2}\right) \cdot P I\right)\right]=\sum_{i}\left[\left(G F_{i}\right) \cdot\left(F Q_{i}\right)\right]=G
$$

where

$$
F Q_{i}=A N_{i} \cdot\left(R 3_{i}^{2}-R 2_{i}^{2}\right) \cdot P I
$$

a) Steady State Equilibrium

Calculation of flow distribution in parallel channels:

The condition that all $\Delta p_{i}$ be equal in all parallel channels, together with the approximate expression for $\Delta p_{i}$ and the mass balance yields the
predictor formula:

$$
G F_{i}=G \cdot \frac{\alpha_{i}}{\sum_{i}\left(F Q_{i} \cdot \alpha_{i}\right)}
$$

With these estimates for the area net flow rates $G F_{i}$ the associated pressure drops $\Delta p_{i}$ may be calculated. A correction formula is obtained by assuming that the correct pressure $d r o p \Delta p^{*}$ is related to the correct net flow rates $G F_{i}^{*}$ by

$$
\Delta p_{i}^{*}=\Delta p_{i} \cdot\left(\frac{\mathrm{GF}_{i}^{*}}{\mathrm{GF}_{i}}\right)^{2}
$$

Combination of this relation with the mass balance yields the correction formula:

$$
G F_{i}^{*}=G F_{i} \frac{G / \sqrt{\Delta p_{i}}}{\sum_{i}\left(F Q_{i} \cdot G F_{i} / \sqrt{\Delta p_{i}}\right)}
$$

This correction is applied iteratively until the maximum difference between any two values $\Delta p_{i}$ is less than a specified number $D P$. When this condition is satisfied one may compute a pressure drop $\overline{\Delta p}$ from the relation:

$$
G=\sum_{i}\left(G F_{i} \cdot F Q_{i}\right)=\sum_{i}\left(G F_{i}^{*} \cdot F Q_{i}\right)=\sum_{i}\left(G F_{i} \cdot \sqrt{\frac{\Delta p}{\Delta p_{i}}} \cdot F Q_{i}\right)
$$

as $\overline{\Delta p}=\left[\frac{\sum_{i}\left(G F_{i} \cdot F Q_{i}\right)}{\sum_{i}\left(G F_{i} \cdot F Q_{i} / \sqrt{\Delta p_{i}}\right)}\right]^{2}$

This pressure drop $\overline{\Delta p}$ will be a good approximation of the actual pressure drop between the two plenum chambers.

## Calculation of the entry pressure (B.C.2):

From the predictor formula and the approximation for the pressure drop $\Delta p_{i}$, both given previously in this appendix, we can derive:

$$
\Delta p_{i}=\Delta p=\frac{l}{g} \cdot\left[\frac{G}{\sum_{i}\left(F Q_{i} \alpha_{i}\right)}\right]^{2} \cdot \bar{V}
$$

For the case of two flow sections connected in series (radial blanket, axial blankets + core) the corresponding formula is:

$$
\Delta p=\frac{I}{g}\left[\frac{G}{\left(\sum_{i}\left(F Q_{i} \alpha_{i}\right)\right)_{R B}+\left(\sum_{i}\left(F Q_{i} \alpha_{i}\right)\right)_{A B+C}}\right]^{2} \cdot \bar{V}
$$

A good estimate for $\bar{V}$ is obtained by taking the average of VE and VA which in turn are determined by enthalpy and pressure at entrance and exit. Since the entrance pressure is not known an iterative procedure might be applied. This is not done here in the light of the fact that the expression for $\Delta p$ is itself only an approximation. This pressure drop $\Delta p$ and the given exit pressure $p_{\text {out }}^{m}$ are combined to give the entry pressure $p_{\text {in }}$ :

$$
p_{\text {in }}=p_{\text {out }}^{*}+\Delta p
$$

Using this entry pressure we proceed to compute pressure drops as indicated. Finally, we obtain an exit pressure $p_{\text {out }}$, in general, will differ from the given value $p_{\text {out }}^{*}$. The difference is used directly and iteratively for a correction of the entry pressure:

$$
p_{\text {in }}^{*}=p_{\text {in }}-\left(p_{\text {out }}-p_{\text {out }}^{*}\right)
$$

until the magnitude of the difference becomes less than a specified number DP.
b) Transient Calculations

The dominating requirement of short computing times together with the experience obtained from applications of this model to many practical cases have prompted the scheme of deferred correction as regarding the calculation of transient hydraulics. It implies that all necessary corrections are deferred to the consecutive step of the integration. The accuracy of this scheme shall not be discussed here but has been found satisfactory in all cases.

The correction is carried out in two consecutive parts. The first correction consists in a proportional change of all area net flow rates $\mathrm{GF}_{i}$ so as to affect the total flow rate of coolant $G$ but not the relative distribution through parallel channels. This correction is applied in order to meet the boundary conditions across the reactor (prescribed pressure drop, or prescribed total coolant flow rate). Thus, for the latter case of prescribed total coolant flow rate the correction obvious-
ly is:

$$
G F_{i}^{Y}=G F_{i} \cdot\left(\frac{G}{\sum_{i}\left(G F_{i} \cdot F Q_{i}\right)}\right)
$$

whereas in the case of prescribed pressure drop $\Delta p^{\#}$ the corresponding formula is:

$$
G F_{i}^{F}=G F_{i} \cdot\left(\sqrt{\frac{\Delta p^{W}}{\Delta p}}\right)
$$

where

$$
\overline{\Delta \mathrm{p}}=\left[\frac{\sum_{i}\left(G F_{i} \cdot F Q_{i}\right)}{\sum_{i}\left(G F_{i} \cdot F Q_{i} / \sqrt{\Delta p_{i}}\right.}\right]^{2}
$$

in analogy to the corresponding equation derived for the steady state calculations. For the case of two flow sections in series the pressure drop $\overline{\Delta p}$ is computed for either section from this formula and added to give

$$
\overline{\Delta \mathrm{p}}={\overline{\Delta p_{1}}}_{1}+\overline{\Delta \mathrm{p}}_{2}
$$

The second correction is such that the channel net flow rates $G F$ are altered while the total coolant flow rate $G$ remains constant. This correction is applied in order to achieve equality of pressure drops in parallel flow paths. It is derived directly from the mass balance equation and the relation for the correct pressure drop $\Delta p^{*}$ given earlier:

$$
G F_{i}^{* *}=G F_{i}^{*} \cdot \sqrt{\frac{\Delta p}{\Delta p_{i}}}
$$

where

$$
\overline{\Delta p}=\left[\frac{\sum_{i}\left(G F_{i}^{*} \cdot F Q_{i}\right)}{\sum_{i}\left(G F_{i}^{F} \cdot F Q_{i} / \sqrt{\Delta p_{i}}\right.}\right]^{2}
$$

The logic flow diagrams presented in Fig. Cl and Fig. C2 illustrate the procedures described in this appendix.


Fig. Cl Flow Diagran for Steady State Hydraulics Calculations


Fig. C2 Flow Diagram for Transient Hydraulics Calculations

APPENDIX D : Other Iterative Procedures Applied in Solving the Steady State Problem
a) Pressure Drop

Calculating the pressure drop from the given formuli requires knowledge of the specific volume of the coolant at the exit of the section under concern. Since this is a function of pressure (as well as enthalpy) the following iteration is devised for obtaining the steady state solution:

$$
\begin{aligned}
& \mathrm{DP}=\frac{(\mathrm{GF})^{2}}{2 \cdot g} \cdot\left\{\frac{\mathrm{FR} \cdot \Delta \mathrm{H}}{2 \cdot \mathrm{DH}}\left(1,+0.5 \cdot \varepsilon_{v}\right)+\varepsilon_{v}\right\} \mathrm{VE} \\
& P A=P E-D P \\
& T A=f_{T}(P A, W A) \\
& V A=f_{V}(P A, T A) \\
& \varepsilon_{V}=\frac{V A-V E}{V E}
\end{aligned}
$$

The starting guess for $\varepsilon_{V}$ is zero; the termination criterion is on the magnitude of the difference between two consecutive values of $\varepsilon_{v}$ being smaller than a prescribed quantity $E V$. In the transient calculation the specific volume at the exit of the section is taken from the preceeding time step.
b) Heat Transfer and Channel Wall Temperatures

Determining the channel wall temperatures $T C 2$ (cladding) and TM (structure) from the equations given in APPENDIX A requires another iterative procedure since the film coefficient itself is a function of the wall temperature. For the temperature TM of the structure we have:

$$
\begin{aligned}
& \mathrm{H} 34=\widehat{\mathrm{H} 34} \cdot\left(\frac{1}{1+\varepsilon_{T}}\right)^{\mathrm{AL3}}+\mathrm{AL4} \\
& \mathrm{H} 34^{\prime}=\mathrm{H} 34 \cdot \frac{1}{1+\frac{\mathrm{D} 4 \cdot \mathrm{H} 34}{3 \cdot \mathrm{C} 4^{\prime}}} \\
& \mathrm{TM}=\mathrm{T3}+\mathrm{q}_{\mathrm{M}} \cdot \frac{\mathrm{D} 4}{\mathrm{H} 34^{\prime}} \\
& \varepsilon_{T}=\frac{\mathrm{TM}-\mathrm{T} 3}{\mathrm{~T} 3+273.16}
\end{aligned}
$$

Initialization and termination are done similarily as in case a) using the prescribed quantities ETC and ETM respectively for the wall temperature of cladding and structure. For the transient calculation the values of these two temperatures from the preceeding step are taken to calculate the film coefficients.

APPENDIX E : Stagnation and Reversal of Flow

Although there is only one minor modification required to adjust the model for accommodating flow reversal the numerical methods are affected extensively. The adjustment in the model concerns the shift in location of the entry pressure loss with flow reversal. This applies to both the radial blanket flow section as well as the core flow section. Thus, reversed flow experiences pressure losses from entry, friction and orificing, in this order.

Many of the numerical procedures used in similar context elsewhere are not suited for the treatment of flow reversal as they are numerically unstable at reversed flow direction. Also, it appears desirable to maintain the original sequence in which the calculation computes enthalpies and pressures, the sequence thus being independent of the direction of flow. This sequence was chosen to coincide with the initial direction of flow which was also used as a basis for the nomenclature and sequence of the various flow sections in series (entrance plenum chambers - rad. bl. - intermediary plenum chamber - first ax. bl. - core - second ax. bl. - exit plenum chamber). For both of these reasons - stability and simplicity - the method selected here is well suited.
a) Hydraulics

Aside from the adjustment for shift in location of entry losses there is no significant change required in the hydraulics-model and -calculation in the program. Reversed flow is indicated by a negative coolant flow rate. All other pressure losses (friction, acceleration) may be computed by the original formuli if the change in sign is accounted for by a factor $F C= \pm 1$ as in the following example.

$$
\Delta p_{i}=p_{i+1}-p_{i}=F C \cdot \frac{(G F)^{2}}{2 g} \cdot\left[\frac{F R \cdot \Delta H}{2 \cdot D H} \cdot \bar{V}_{i}+\left(V_{i+1}-V_{i}\right)\right]
$$

where

$$
F C= \begin{cases}+1 & \text { for normal flow } \\ -1 & \text { for reversed flow }\end{cases}
$$

and $i, i+l$ refers to two consecutive axial nodes along the flow path the order of which does not change with change of flow direction. In case of stagnation of flow $\left(G=G F_{i}=0\right)$ all pressure drops $\Delta p_{i}$ would also vanish. Hence, a fictitious and very small flow rate must be maintained
$\left(10^{-8} \mathrm{~kg} / \mathrm{sec}\right)$ in order to avoid deviding by zero in the correction formuli given in APPENDIX C.
b) Thermodynamics

Flow reversal does not affect the equations for enthalpy balance in the coolant if WE and WA stands for the coolant enthalpy entering and leaving the node, respectively, and irrespective of flow direction. The schematic diagram in Fig. El shows how the nodal enthalpies are interrelated. A node entered by one arrow only indicates, that this enthalpy is transferred directly from one node to the other. If such a connection occurs between two nodes at different times $\left(t_{j}\right.$ and $t_{j+1}$ ) this is equivalent to a transport delay of the length of the time increment I. It can be seen from this figure that there is in effect such a transport delay applied between all neighbouring nodes, including the nodes representing the plenum chambers ${ }^{+}$). In as much as the step size of the integration approaches the transport time $\Delta t_{T}$ this method will account for the actual transport effects (see APPENDIX H).
c) Boundary Conditions

When using program REX, flow reversal is achieved by appropriate choice of the boundary conditions. With B.C.l. flow reversal occurs when the specified exit pressure exceeds the entry pressure, which is also specified. In the case of B.C. 2 and B.C. 3 flow reversal occurs when the total coolant flow rate $G$ becomes negative. The present version of the main program REX is such that for all three cases the enthalpy of the coolant entering the reactor under conditions of reversed flow is constant and equal to the value at this point (exit plenum chamber) just before flow reversal. Thus, the enthalpy values specified in data block 2 become irrelevant in case of flow reversal.
+) To be quite exact at this point the prior assumption of zero residence time of the coolant in the plenum chambers must be corrected to state that a delay of a length $I$ is included, which in general however will be small and may be neglected relative to actual residence times.



APPENDIX $F$ : Integration of the Point Kinetics Equations

Equations:

$$
\begin{aligned}
& \frac{d P}{d t}=\frac{1}{\Lambda} \cdot[\beta \cdot \Delta k-\beta] \cdot P+\sum_{i} \lambda_{i} C_{i} \\
& \frac{d C_{i}}{d t}=\frac{1}{\Lambda} \cdot \beta_{i} \cdot P-\lambda_{i} C_{i} \quad i=1,2, \ldots N D \\
& \Delta k=\Delta k_{T}+\Delta k_{k} \quad[\$ \text {-units } \mid
\end{aligned}
$$

Substitutions:

$$
\begin{aligned}
& Y_{i}=\frac{\Lambda \lambda_{i}}{\beta_{i}} \cdot C_{i} \\
& f_{i}=\frac{\beta_{i}}{\beta} \\
& \omega=\frac{\beta}{\Lambda}
\end{aligned}
$$

Yield:

$$
\begin{aligned}
& \frac{d P}{d t}=\omega \cdot\left[(\Delta k-1) \cdot P+\sum_{i} f_{i} \cdot Y_{i}\right] \\
& \frac{d Y_{i}}{d t}=\lambda_{i} \cdot\left[P-Y_{i}\right]
\end{aligned}
$$

The basic assumption facilitating efficient integration by a semi-analytic method is that changes in the variables $Y_{i}$ may be neglected relative to the change in $P$ during a short time interval. Furthermore, the reactivity also is to remain constant during this interval. We introduce the following two quantities

$$
\begin{aligned}
& \frac{1}{\tau_{0}}=\omega \cdot\left(\Delta k_{0}-1\right) \\
& S_{0}=\frac{1}{\Delta k_{0}-1} \cdot \sum_{i} f_{i} Y_{i 0}
\end{aligned}
$$

where the subscript 0 refers to (known) values at the beginning of an interval $\Delta t$ over which we want to integrate. With this we now can write:

$$
\frac{d P}{d t}=\frac{1}{\tau_{0}}\left[P+S_{0}\right]
$$

which under the assumption made above can be integrated to give

$$
P=\left(P_{0}+S_{0}\right) \cdot e^{\frac{\Delta t}{\tau_{0}}}-S_{0}
$$

Integrating again yields the energy. release:

$$
\Delta E=\int^{\Delta t} P d t=\left(P_{0}+S_{0}\right) \cdot \tau_{0} \cdot\left(e^{\frac{\Delta t}{\tau_{0}}}-1\right)-S_{0} \cdot \Delta t
$$

and

$$
E=E_{0}+\Delta E
$$

The variables $Y_{i}$ corresponding to the concentration of the precursors are obtained from integration of the corresponding equations:

$$
Y_{i}=Y_{i_{0}}+\lambda_{i}\left[\Delta E-Y_{i_{0}} \cdot \Delta t\right]
$$

Assuming equilibrium at the power level $P_{0}$ as the initial condition, the starting values of all variables have to be:

$$
\begin{array}{ll}
\rho_{0}=0 & S_{0}=-P_{0} \\
\frac{1}{\tau_{0}}=-\omega & Y_{i_{0}}=P_{0} \\
E_{0}=0
\end{array}
$$

Although the scheme is complete at this point two special cases shall be considered:
a) Prompt Criticality

## Condition:

$$
\left|\frac{\Delta t}{\tau_{0}}\right|<\varepsilon_{1} \ll 1
$$

If this inequality is satisfied for an appropriate value of $\varepsilon_{1}$, it is acceptable to approximate the exponential function by the first two terms of its Taylor expansion:

$$
P=\left(P_{0}+S_{0}\right) \cdot\left(1+\frac{\Delta t}{\tau_{0}}\right)-S_{0}=P_{0} \cdot\left(1+\frac{\Delta t}{\tau_{0}}\right)+S_{0} \cdot \frac{\Delta t}{\tau_{0}}
$$

For sufficiently small values of $\varepsilon_{1}\left(=10^{-6}\right)$ we may simplify further and write:

$$
P=P_{0}+S_{0} \frac{\Delta t}{\tau_{0}}
$$

For the energy increment $\Delta E$ we get in the same manner:

$$
\Delta E=\left(P_{0}+S_{0}\right) \cdot \tau_{0} \cdot\left(\frac{\Delta t}{\tau_{0}}+\left(\frac{\Delta t}{\tau_{0}}\right)^{2} \cdot \frac{1}{2}\right)-S_{0} \cdot \Delta t=\left(P_{0}+S_{0} \cdot \frac{1}{2} \cdot \frac{\Delta t}{\tau_{0}}\right) \cdot \Delta t
$$

The inequality above may be rewritten as:

$$
\left|\Delta k_{0}-1\right|<\frac{10^{-6}}{\Delta t \cdot \omega}
$$

which normally will be satisfied only when the excess reactivity $\Delta k_{o}$ is very nearly equal to unity (one dollar) (prompt criticality).
b) Prompt Jump Approximation

Condition:

$$
\frac{\Delta t}{\tau_{0}}<\varepsilon_{2}<0
$$

If this condition is satis fied for sufficiently large values of $\varepsilon_{2}$ (e.g.-18.4) it is acceptable to neglect the experimental function relative to unity. Thus we get:

$$
\begin{aligned}
P & =-S_{0} \\
\Delta E & =-\left(P_{0}+S_{0}\right) \cdot \tau_{0}-S_{0} \cdot \Delta t=P_{0} \cdot \tau_{0}+S_{0} \cdot\left(\tau_{0}-\Delta t\right) \\
& =\left[-P_{0}-S_{0} \cdot\left(1+\frac{\Delta t}{\tau_{0}}\right)\right] \cdot \tau_{0}
\end{aligned}
$$

The inequality above may be rewritten as:
$\Delta K_{0}<1-\frac{18.4}{\omega \cdot \Delta t}$
which is satisfied whenever the reactor is sufficiently below prompt critical. The approximate solution obtained in this case is equivalent to the well-known prompt jump approximation.

Note, that both inequalities given under a) and b) depend on the step size $\Delta t$ of the integration. The flow chart given in Fig. Fl illustrates the integration scheme described above.


Fig. FI Flow Diagram of the Integration Scheme for Point Kinetics

APPENDIX G : Sample for Calculating Feedback Parameters from Results of Perturbation Calculations

The example given below is based on feedback models employed by the FOREcode [ ${ }^{-1} / 7$ and also by [12_7. These models are slightly modified and restricted to space dependence in axial direction only. Consequently, the radial distribution coefficients ARD, ARF, ARC, ARS, ARM are insignificant and may be assigned arbitrary values ( $\neq 0$ ). Axial subdivisions are introduced at equal intervals along the vertical axis of the core.

A set of basic reactivity coefficients is assumed to be available, for instance, from one dimensional perturbation calculations for the particular case of interest:

| $\left(\frac{\partial k}{k} / \frac{\partial \rho_{n}}{\rho_{n}}\right)_{1 x}$ | rel. change in reactivity resulting from a relative change in density of material $n(=F, C, S, M)$ in axial section IX, |
| :---: | :---: |
| $\left(\frac{O K}{K} / \frac{O H C}{H C}\right)$ | relative change in reactivity resulting from a relative change in core height HC, |
| $\left(\frac{\partial k}{k} / \frac{\partial R}{R}\right)$ | relative change in reactivity resulting from a relative change in core radius $R$, |
| $(o k / \partial T F)_{I X}$ | change in reactivity resulting from a change in fuel temperature TF by $1{ }^{\circ} \mathrm{K}$ in axial section IX due to the Doppler effect (Doppler coefficient) (fuel reference temperature $900^{\circ} \mathrm{K}$ ) |
| $\beta$ | fraction of delayed neutrons |

Additional parameters appearing in the following equations are
$\alpha_{S} \ldots .$. volume fraction of coolant per axial channel section

$$
=\frac{R 3^{2}-R 2^{2}}{R 3^{2}+W P \cdot D 4 / P I}
$$

$B_{1} \ldots$. ...volume fraction of spacers per axial channel section $B_{2} \ldots \ldots$ ". " $"$ subassembly cans per axial channel section

Before presenting the formuli special mention shall be made of some of the mechanisms and associated assumptions (indexing code refers to subsequent paragraphs):
$a 3), b 1), d 1):$ the effective density of each of the components: fuel, clad and structure is not changed by the radial expansion of the respective component;
d 2), $d$ 3), $d$ 4) : radial expansion of the core in any point is assumed to be a linear function of the local structural temperature alone.
a) Fuel Temperature Effects
a 1) Doppler effect:

$$
\begin{aligned}
A \times D=(\partial k / \partial T F)_{\mathbb{X}} & \ldots \ldots \text { axial distribution coefficients } \\
& \text { (unnormalized) } \\
R C D=\frac{900}{\beta} \sum_{\mathbb{1}}(A X D) & \ldots . \text { Doppler constant, }, \phi \text {-units }
\end{aligned}
$$

a 2) Change in core height due to axial expansion of fuel:

$$
R C H=\frac{1}{\beta}\left(\frac{\partial k}{k} / \frac{\partial H C}{H C}\right)
$$

a 3) Change in effective fuel density due to axial expansion of fuel:

$$
\begin{aligned}
& A X F=\left(\frac{\partial k}{k} / \frac{\partial \rho_{F}}{S_{F}}\right)_{1 x} \\
& R C F=\frac{1}{\beta} \cdot \sum_{1 x}(A X F)
\end{aligned}
$$

b) Cladding Temperature Effects
b l) Change in effective cladding density due to axial expansion of cladding:
$A X C_{1}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{c}}{\rho_{c}}\right)_{1 x}$
b 2) Change in effective coolant density due to radial expansion of cladding:

$$
A \times C_{2}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{s}}{\rho_{s}}\right) \cdot \frac{\frac{d \rho_{s}}{\rho_{s}} / d T C}{\frac{d \rho_{s}}{\rho_{c}} / d T C}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{s}}{\rho_{s}}\right) \cdot \frac{-2 \cdot A E F \cdot \frac{1-\alpha_{s}-\left(B_{1}+B_{2}\right)}{\alpha_{s}}}{-A E C}
$$

Total:

$$
\begin{aligned}
& A X C=A X C_{1}+A X C_{2} \\
& R C C=\frac{1}{B} \cdot \sum_{1 \times}(A X C)
\end{aligned}
$$

c) Coolant Density Effect:
$A \times S=\left(\frac{\partial k}{k} / \frac{\partial \rho_{s}}{\rho_{s}}\right) \cdot \frac{1}{\rho_{s}}$
$R C S=\frac{1}{\beta} \cdot \sum_{\mid x}(A X S)$
d) Structure Temperature Effects
d 1) Change in effective structure density due to axial expansion of structure:
$A X M_{1}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{M}}{\rho_{M}}\right)$
d 2) Change in effective fuel density due to radial expansion of (supporting) structure
$A \times M_{2}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{F}}{\rho_{F}}\right) \cdot \frac{\frac{d \rho_{F}}{\rho_{F}} / d T M}{\frac{d \rho_{M}}{\rho_{M}} / d T M}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{F}}{\rho_{F}}\right) \cdot \frac{-2 \cdot A E M}{-A E M}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{F}}{\rho_{F}}\right) \cdot 2$
d 3) Change in effective cladding density due to radial expansion of (supporting) structure
$A \times M_{3}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{c}}{\rho_{c}}\right) \cdot \frac{\frac{d \rho_{c}}{\rho_{c}} / d T M}{\frac{d \rho_{M}}{\rho_{M}} / d T M}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{c}}{\rho_{c}}\right) \cdot \frac{-2 \cdot A E M}{-A E M}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{c}}{\rho_{c}}\right) \cdot 2$
d 4) Change in effective coolant density due to radial expansion of (supporting) structure
$A \times M_{4}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{s}}{\rho_{s}} \cdot \frac{\frac{d \rho_{s}}{\rho_{s}} / d T M}{\frac{d \rho_{M}}{\rho_{m}} / d T M}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{s}}{\rho_{s}} \cdot \frac{2 \cdot A E M \frac{1-\alpha_{s}}{\alpha_{s}}}{-A E M}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{s}}{\rho_{s}}\right) \cdot\left(-\frac{2 \cdot\left(1-\alpha_{s}\right)}{\alpha_{s}}\right)\right.\right.$
d 5) Change in effective coolant density due to expansion of structure (spacers and subassembly cans)

$$
A X M_{5}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{s}}{\rho_{s}}\right) \frac{\frac{d \rho_{s}}{\rho_{s}} / d T M}{\frac{d \rho_{M}}{\rho_{M}} / d T M}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{s}}{\rho_{s}}\right) \cdot \frac{-A E M \frac{3 B_{1}+2 B_{2}}{\alpha_{s}}}{-A E M}=\left(\frac{\partial k}{k} / \frac{\partial \rho_{s}}{\rho_{s}}\right) \frac{3 \cdot B_{1}+2 \cdot B_{3}}{\alpha_{s}}
$$

Total: $A X M=\sum_{i=1}^{5} A X M_{i}$

$$
R C M=\frac{1}{\beta} \sum_{1 X} A X M
$$

d 6) Change in core radius due to radial expansion of structure $R C R=\frac{1}{\beta}\left(\frac{\partial k}{k} / \frac{\partial R}{R}\right)$

# APPENDIX H : Determination of Integration Step Size, Numerical Stability of Integration of the Thermo-Hydraulic Equations 

a) Estimation of the Maximum Stable Step Size for Integration of the Thermo-Hydraulic Equations

Numerical instabilities occurring during integration by nodal approximation of problems of heat transfer and heat transport impose a dominating limitation on the step size of integration. With the nodes fixed at their spatial coordinates there is a maximum stable step size for the integration which depends on both the physical parameters involved as well as the particular method of discretization and mesh spacing. A classic approach to determine the maximum stable step size is the one of small perturbations which in general leads to an Eigenvalue problem. Computer storage requirements and considerations of computing speed preclude such an approach in most practical cases. To achieve the same purpose a much simpler technique is employed here which yields a conservative (too small) maximum step size granting numerical stability of the integration. A certain amount of flexibility is maintained by multiplying this step size estimate by a given constant coefficient EST before applying it to the integration. Furthermore, re-evaluation of the maximum stable step size may be restricted to every NTE'th step during the integration and keeping it unaltered for all the steps in between. This facilitates savings in computing time for step size determination in case of slowly varying transients. If the maximum stable step size (MSS) comes out to be smaller than the current step size, the latter will be halved until it in turn is smaller than MSS. If MSS is larger than twice the current step size the latter may be doubled for the following step of the integration if other tests are satisfied. These tests are listed under b) in this appendix.

The perturbation method consists of introducing small perturbances $\varepsilon_{i}$ in all variables of the pertinent system of equations. These must not grow for any of the variables from any one step to the next in order for the system to be stable. Since we require the true solution to hold along with the perturbation solution we may subtract the (unpertubed) original system of equations from the perturbed one. In the case of a linear system - as we are dealing with - the remaining equations on the right hand side are linear and homogeneous in the perturbations $\varepsilon_{i}$ which were introduced at the beginning of the step. The components of the left hand vector are the perturbations $\varepsilon_{i}^{*}$ at the end of the step.

Application of this scheme to the pertinent equations for transient temperatures in all components of one axial section (see APPENDIX A) we get the following system of equations:

$$
\left.\begin{array}{l}
\varepsilon_{1}^{*}=\varepsilon_{1} \cdot\left(1-\Delta t \cdot \alpha_{11}\right)+\varepsilon_{2} \cdot \Delta t \cdot \alpha_{12} \\
\varepsilon_{2}^{*}=\varepsilon_{1} \cdot \Delta t \cdot \alpha_{21}+\varepsilon_{2} \cdot\left(1-\Delta t \cdot \alpha_{22}\right)+\varepsilon_{3} \cdot \Delta t \cdot \alpha_{23} \\
\vdots \\
\varepsilon_{j}^{*}= \\
\vdots \\
\varepsilon_{J}^{*}=
\end{array} \varepsilon_{j-1} \cdot \Delta t \alpha_{j, j-1}+\varepsilon_{j} \cdot\left(1-\Delta t \cdot \alpha_{j j}\right)+\varepsilon_{j+1} \cdot \Delta t \cdot \alpha_{j, j+1}\right)
$$

where all of the coefficients $\alpha_{i, j}$ are positive. Note, that some of these coefficients are functions of time as they contain temperature dependent fuel properties, coolant net flow rate and film coefficients.

Instead of solving the Eigenvalue-problem associated with the requirement of decreasing perturbations we apply a criterion of the form 1 13_7:

$$
\sum_{j=1}^{J}\left|A_{j i}\right|<1 \quad j=1,2, \ldots J
$$

where $A_{j i}$ are the elements of the coefficient matrix in the system of equations above. Guarding against numerical instabilities of oscillatory nature this criterion yields a conservative estimate for the maximum stable step size in the following manner:

$$
\begin{aligned}
& \sum_{i=1}^{J}\left|A_{j i}\right|=\Delta t_{j} \alpha_{j, j-1}+\left(-1+\Delta t_{j} \alpha_{j j}\right)+\Delta t_{j} \cdot \alpha_{j, j+1}=1 \\
& \Delta t_{j}=\frac{2}{\alpha_{j, j-1}+\alpha_{j j}+\alpha_{j, j+1}} \\
& \Delta t_{\max }=\operatorname{MIN}\left(\Delta t_{j}\right)
\end{aligned}
$$

In view of the last equation not all $\Delta t_{j}$ need to be evaluated since certain ones always are larger than certain other ones by nature of the expressions from which they are computed. For instance, this is the case for the fuel nodes in any particular axial section, where only the surface node (TFN) needs to be considered here.

With regard to the equations for the coolant it is convenient to replace the enthalpy perturbations by temperature perturbations using the re-
lations:

$$
\begin{aligned}
& \varepsilon_{\mathrm{WA}}=\mathrm{CPA} \cdot \varepsilon_{\mathrm{TA}} \\
& \varepsilon_{\mathrm{T} 3}=0.5 \cdot \varepsilon_{\mathrm{TA}}
\end{aligned}
$$

The last equation follows from the assumption of a linear profile for the coolant temperature over anyone axial section.

Applying the equation for $\Delta t_{j}$ to our particular system we get the following set of equations (core):
fuel edge:

$$
\Delta t_{1}=\frac{R \not \subset 1 \cdot \mathrm{CPI} \cdot \mathrm{RI}^{2}}{4 \cdot N N \cdot[(2 \cdot N N-1) \cdot C \emptyset 1+\mathrm{RI} \cdot \mathrm{HGP}]}
$$

cladding, inner edge:

$$
\Delta t_{2}=\frac{R \phi 2 \cdot C P 2 \cdot\left(R_{2}^{2}-R I^{2}\right)}{4 \cdot\left[R 1 \cdot H G P+\frac{(R 2+R 1)}{2 \cdot(R 2-R 1)} \cdot C \phi 2\right]}
$$

cladding, outer edge:

$$
\Delta t_{3}=\frac{\mathrm{R} \varnothing 2 \cdot \mathrm{CP} 2 \cdot\left(\mathrm{R} 2^{2}-\mathrm{R1} 1^{2}\right)}{4 \cdot\left[\frac{(\mathrm{R} 2+\mathrm{R} 1)}{2 \cdot(\mathrm{R} 2-\mathrm{RI})} \cdot \mathrm{C} \varnothing 2+\mathrm{R} 2 \cdot \mathrm{H} 23\right]}
$$

coolant:

$$
\Delta t_{4}=\frac{2}{\left[\frac{2 \cdot \mathrm{H} 23 \cdot \mathrm{R} 2}{\mathrm{CP} 3 \cdot\left(\mathrm{R} 3^{2}-\mathrm{R} 2^{2}\right)}+\frac{\mathrm{H} 34^{\prime} \cdot \mathrm{WP}}{\mathrm{CP} 3 \cdot \mathrm{PI} \cdot\left(\mathrm{R3}^{2}-R 2^{2}\right)}+\frac{\mathrm{QH}}{\Delta H}\right] \cdot V}
$$

structure:

$$
\Delta t_{5}=\frac{\mathrm{R} \varnothing 4 \cdot \mathrm{CP} 4 \cdot \mathrm{D} 4}{\mathrm{H} 34^{\prime}}
$$

Corresponding equations are obtained for blankets and bypasses. The proposed step size then is obtained from

$$
\Delta t=E S T \cdot M I N\left(\Delta t_{j}\right)
$$

The last term in parentheses of the denominator in the expression for $\Delta t_{4}$ is the reciprocal of the transport time $\Delta t_{T}$ of the coolant through the distance $\Delta H$. In cases where this term dominates the maximum stable
step size is approximately twice the transport time $\Delta t_{T}$ :

$$
\Delta t_{M}=2 \cdot \Delta t_{T}
$$

b) Step Size Limitations from Relative Changes of Total Power

Another criterion for the selection of the step size is applied in conjunction with the relative shange in total power:

$$
\Delta P=\frac{P_{n+1}-P_{n}}{P_{n}}
$$

where the subscript refers to successive instances in time.

If $\Delta P>\Delta P_{\text {Max }}$ the step size is halved and integration of the point kinetics equations is repeated until $\Delta \mathrm{P}<\Delta \mathrm{P}_{\mathrm{Max}}$.

If $\Delta \mathrm{P}<\Delta \mathrm{P}_{\text {Min }}$ the step size may be doubled for the consecutive step.
The values for $\Delta \mathrm{P}_{\mathrm{Max}}$ and $\Delta \mathrm{P}_{\text {Min }}$ are chosen from experience to be in the order of about 0.1 to 0.01 .
c) Further Limitations

Finally, arbitrary limits may be set for the maximum and minimum value of the step size (XIX, XIN). If a step size smaller than XIN is required by either criterion given under a) and b) the computation is terminated. If a step size larger than XIX is permissible, the computation will continue using the most recent step size just below this upper limit. If, for the first step of integration, the stability criterion results in a step size larger than the upper limit XIX, then this value (XIX) will be taken as the size of the initial step of integration.

## APPENDIX I: Flow Diagrams

The subsequent flow diagrams cover all routines on levels 1,2 , and 3 as well as the reactivity function RCBE. Conventional symbols are employed to a large extent. Unconventional symbols are explained as Follows:


WRITE statement


READ \& WRITE statement

if: $\quad \mathrm{X} \quad 0 \ldots \ldots$ go to $\mathrm{A} \quad$ IF statement
X $0 \ldots .$. go to $B$

if: $\quad I B=1 \ldots$ go to $A$ computed GO TO statement $I B=2 \ldots$ go to $B$ $I B=3 \ldots$ go to $C$


> DO statement; loop index goes from 1 to $N$. For simplicity, the loop index is omitted inside the loop. Examples: $T(,, N N)$ corresponds to $T$ (IX,IR,NN) WI $\varnothing(I X 1$,$) corresponds to WI \emptyset(I X 1, I R)$ $S$ corresponds to $S(I X, I R)$

Variable names in the flow diagrams are the same as in the actual routines and may differ from the names used elsewhere in this report.

Vertical dashed lines appearing in the flow diagram of subroutine STS signify iterative procedures as described in APPENDICES C and D.



Fig.I2: Flow Diagram for Subroutine REXION


Fig.I3: Flow Diagram for Subroutine PREP


Fig. I4: Flow Diagram for Subroutine STS



Fig.I6: Flow Diagram for Subroutine STEPC


Fig. I7: Flow Diagram for Subroutine output


Fig.IS: Elow Diagram for Subroutine POWER


APPENDIX J : Subroutine REXION: Description and Instructions for Use The program described in this report can be used without the mainprogram REX by simply calling the subroutine REXION. The calling program must provide information specifying time dependent coolant boundary conditions et.c. The following rules should be observed:

The subroutine is called by:

CALL REXION (X, DX, P, E, G, PEX, PAX, WEX, WAX, IX)

The last argument in the list $I X$ is used both as a parameter controlling the mode of operation of the subroutine as well as an error indicator. Calling the subroutine with different values for IX indicates the following operations:
$I X=1 \ldots$ reading of all data required by the subroutine; these data are the ones designated SYSTEMS DATA constituting subblocks $1-8$ as described in Chapter IV.2. A replica of this input is printed as first part of the output.

IX $=2$... calculation of the initial steady state solution, boundary condition B.C.I

IX $=3 .$. calculation of the initial steady state solution, boundary condition B.C. 2

IX $=4 \ldots$ calculation of the initial steady state solution, boundary condition B.C. 3

IX $=5$... output of all system variables, pertaining to time $X$, as described in Chapter IV. 3

IX $=6 \ldots$ integration of the transient problem over one step of length DX. The step length is determined internally from considerations presented in APPENDIX $H$.

After calling the subroutine with $I X=6$, one out of three possible returns is made to the calling program:

IX $=6 \ldots$ (value unchanged) successful completion of the step of integration

IX $=0 . .$. successful completion of the transient problem (internal termination criterion satisfied)
$I X=-1 \ldots$ integration not successful, step size required for stable and accurate integration is smaller than permitted minimum value XIN.

The remaining arguments in the list of the subroutine are explained in the input description, Chapter IV.2, and the list of notations, APPENDIX A. Which of these are input (I) and which are output ( $\varnothing$ ) depends on the operational mode and is described in the following table:

| IX | X | DX | P | E | G | PEX | PAX | WEX | WAX | Comment |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | $\varnothing^{(1)}$ | $\phi^{(3)}$ | I | $\varnothing^{(1)}$ | I | I | $\emptyset$ | I | $\emptyset$ | B.C.1 |  |
| 3 | $\varnothing^{(1)}$ | $\varnothing^{(3)}$ | I | $\phi^{(1)}$ | I | $\emptyset$ | I | I | $\varnothing$ | $\text { B.C. } 2$ | steady |
| 4 | $\varnothing^{(1)}$ | $\phi^{(3)}$ | I | $\varnothing^{(1)}$ | I | I |  | I | $\phi$ | B.C. 3 | tate |
| 6(2) | $\emptyset^{(2)}$ | $\varnothing^{(4)}$ | $\varnothing$ | $\varnothing$ | $\varnothing$ | I | $I / \varnothing^{(5)}$ | I | $\varnothing$ | B.C.I |  |
| (3) | $\phi^{(2)}$ | $\emptyset^{(4)}$ | $\varnothing$ | $\varnothing$ | I | $\varnothing$ | I | I | $\varnothing$ | B.C. 2$\}$ | transient |
| (4) | $\phi^{(2)}$ | $\emptyset^{(4)}$ | $\varnothing$ | $\emptyset$ | I | I | $\varnothing$ | I | $\varnothing$ | B.C.3 |  |

(1) ... quantity is set equal to zero
(2) ... time at the end of last step of integration
(3) ... is set equal to the maximum step size for stable integration as calculated from initial steady state conditions - or to the maximum permissible step size XIX - which ever is smaller
(4) ... step size used for the last step of integration
(5) ... when calling the subroutine this argument has to be equal to the difference $\Delta p$ of the current exit pressure and its initial value: $\Delta p=P_{e x}-P_{e x, 0}$. Upon return this argument will be set equal to the acutal current exit pressure PAX.

None of the parameters listed with $\varnothing$ in the last three lines ( $1 X=6$ ) must be altered in the calling program during integration of anyone transient problem. Also, it should be noted that all arguments listed with I in the last three lines are evaluated at time $X_{j-1}$ at the beginning of the step whereas the remaining arguments listed with $\varnothing$ refer to the time $X_{j}=X_{j-I}+D X$.

The flow diagram in Fig. 31 gives an example for the integrated use of subroutine REXION. This flow diagram is essentially identical with the one of the main routine REX.


Fig. Jl Sample Flow Diagram for Application of Subroutine REXION

APPENDIX K : Functions CØ1 and CP1

The functions CØ1 and CPl are to compute the temperature dependent thermal conductivity and specific heat of the fuel. Both of these two functions have three arguments:

```
C\emptyset1 (T, IR, IC)
CP1 (T, IR, IC)
```

where $T$... temperature of the fuel
IR .. index of the radial of the second flow section
IC .. parameter for mode control: $\left\{\begin{array}{l}=1 \ldots \text { input, preparation } \\ =2 \ldots \text { calculation }\end{array}\right.$

If input data are required for either of the two functions, they may be supplied in subblock 2 of the SYSTEM DATA. If data card 17 is inserted then each of the two functions is called just once, in the order C $\varnothing$ and CPl, with $I C=1$. Thus, the required data cards must follow directly after card 17, such as card 18 and 19 shown in Chapter IV.2. For the routine calculation of the functional values the functions are called with $I C=2$. If neither function $C \not \subset 1$ nor $C P 1$ require input the corresponding data cards 17, 18, 19 may be omitted and the parameter IC may be ignored.

The second argument IR may be significant in the case that fuel with different thermal properties in different radial core zones is specified.

Second order polynomials are used at present for both functions:

$$
\begin{aligned}
& T^{*}=a \cdot T-b \\
& C \not \subset 1=\left(C K 1+T^{*} \cdot\left(C K 2+T^{*} \cdot C K 3\right)\right) \cdot C K O \\
& C P 1=\left(C C 1+T^{*} \cdot\left(C C 2+T^{*} \cdot C C 3\right)\right) \cdot C C O
\end{aligned}
$$

With $a=1.8$ and $b=38.0$ the transformation from $T$ to $T^{\#}$ corresponds to a transformation from ${ }^{\circ} \mathrm{C}$ to ${ }^{\circ} \mathrm{F}$ with a reference temperature of $70{ }^{\circ} \mathrm{F}$ ( $\cong 21^{\circ} \mathrm{C}$ ). The coefficients $C K O$ and $C C O$ may be used for converting to different units (see [-]_7).

APPENDIX L : Function RCBE

The function $R C B E$ is to compute the external excess reactivity as a function of time and the total reactor power. The latter variable is included for providing the possibility of scram functions initiated by power level signals. If other system variables are to be used in controlling the reactor by external excess reactivity appropriate adjustments must be made in the present version of the program (CALL-statements).

The function is called by:

$$
\operatorname{RCBE}(X, P, I C)
$$

where

$$
\begin{aligned}
& \text { X ... time (sec) } \\
& \text { P ... total reactor power (MW) } \\
& \text { IC .. mode control parameter }\left\{\begin{array}{l}
=1 \ldots \text { input } \\
=2 \ldots \text { steady state } \\
=3 \ldots \text { transient }
\end{array}\right.
\end{aligned}
$$

If input data to this routine are required they may be supplied in subblock 3 of SYSTEM DATA (when the function RCBE is called once with IC = I) directly after reading of the delayed neutron parameters. Thus, eventual data cards (cards numbered 23) must follow right after the last card numbered 22.

For calculating the steady state solution the function RCBE is called once with $I C=2$ and with the second argument $P$ equal to the initial power level PO. Preparatory calculations involving PO can be carried out during this step (e.g. setting of a scram level relative to the initial power). Upon returning the function should be set equal to zero, as the initial excess reactivity is equal to zero as required by the equilibrium condition.

Calls of RCBE with IC $=3$ are the standard type in the transient case and are to give external excess reactivity as a function of the arguments: time (X) and power (P). It is essential for this mode of the routine that it may be called with decreasing values for $X$. This is the case when the step size has to be halved as a result of failing the test concerning maximum relative power change.

Description of the present version of RCBE

The present version of RCBE is intended specifically to supply excess reactivity as a function of time in case of a reactor scram. Three different
ways of specifying such functions are offered. One of them employs linear interpolation between points in the $\Delta k_{\text {ext }} / t-p l a n e$ and thus it also is suited to represent quite general functions not necessarily relating to an emergency shut down.
a) Mode 1:

The external excess reactivity remains zero at all times
b) Mode 2:

The external excess reactivity as a function of rod position is described by straight lines interconnecting points in the $\Delta \mathrm{k}_{\mathrm{T}} / \mathrm{Z}-\mathrm{pl}$ ane. This function must be given in a normalized form $\left(\Delta k_{T}^{*} / Z^{*}\right)$ where

$$
\begin{aligned}
\Delta \mathrm{k}_{\mathrm{T}}^{*} & =\Delta \mathrm{k}_{\mathrm{T}} / \mathrm{RT} \\
\mathrm{z}^{*} & =\mathrm{Z} / \mathrm{HR}
\end{aligned}
$$

Thus, the last point in the $\Delta \mathrm{k}_{\mathrm{T}}^{\mathrm{K}} / \mathrm{Z}^{\mathrm{K}}$-plane should be ( 1,1 ) which corresponds to the total external excess reactivity $R T$ being inserted at the end position $H R$ of the rods. The first point should have the coordinates ( 0,0 ) since zero excess reactivity is required for the initial equilibrium.

Scram action is triggered by the power reaching the excess level PS given by

$$
P S=P O \cdot S C R
$$

After this instant in time (XS) rod motion sets in with a time delay TD. Assuming constant acceleration by a multiple AF of the earths gravity $g$ the rod position $Z$ is obtained from:

$$
Z=X M^{2} \cdot(g \cdot A F)
$$

where:

$$
X M=X-(X S+T D)
$$

Furthermore, it may be assumed that a fraction $F B$ of all rods starts its motion from the edge of the reactor ( $Z=0$.) whereas the remaining rods (1. -FB ) start at $Z=H B$ (in particular: edge of core of axial blanket). The total reactivity is divided proportionately; the initial value of $\Delta k$ ext being zero the value at the final rod position becomes

$$
\Delta k_{T, f i n a l}=R T-R T \cdot(1-F B) \cdot \Delta k_{T}(Z=H B)
$$

c) Mode 3

Scram triggering and rod delay are treated the same way as in mode 2. However, in this mode the external excess reactivity may be specified directly as a function of time after start of the rod motion:

$$
\Delta \mathrm{k}_{\mathrm{T}}=\Delta \mathrm{k}_{\mathrm{T}}(\mathrm{XM})
$$

d) Mode 4

Scram triggering as well as rod delay and rod motion are treated the same way as in mode 2. However, in this mode the spatial distribution of the external excess reactivity over the axial direction $Z$ of the reactor (height $H T$ ) follows a $\cos ^{2}$-law. Note that the distance of total rod travel $H R$ must be less than or equal to HT. Thus, the unnormalized function $\Delta \mathrm{k}_{\mathrm{T}}$ is obtained by direct integration:

$$
\Delta k_{T}=\frac{1}{2}\left[\frac{2 \cdot \mathrm{Z}}{H T}-\frac{1}{P I} \cdot \sin \left(P I \frac{2 \cdot Z}{H T}\right)\right]
$$

If a fraction (I. - FB) of all rods starts from a point $Z=H B$ the excess reactivity at the end position will no longer be RT but will be

$$
\left.\Delta \mathrm{k}_{\mathrm{T}, \text { final }}=\mathrm{RT}-\mathrm{RT} \cdot(1 \cdot 0-\mathrm{FB}) \cdot \Delta \mathrm{k}_{\mathrm{T}(\mathrm{Z}}=\mathrm{HB}\right)
$$

| card | name | format | units | comments | restrictions |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 23/1 | IV | I7 | - | mode parameter | 1, 2, 3, 4 |

No other card is required if IV $=1$. If IV $=2$ or $I V=4$ the next card is $23 / 2$ :

| 23/2 | HR <br> HB <br> RT <br> FB <br> TD <br> AF | $\begin{gathered} \mathrm{E} 11.4 \\ " \\ " \\ " \\ " \\ " \end{gathered}$ | $\begin{gathered} \mathrm{m} \\ \mathrm{~m} \\ \not \phi \\ \text { - } \\ \text { sec } \\ - \end{gathered}$ | total distance of travel for rod fraction $F B$ <br> $Z=H B$ is starting position for rod fraction (1.0-FB) <br> total reactivity inserted when all rods at end position (only if $F B=1.0$ ) <br> fraction of rods starting from $Z=0$. (remainder from $Z=H B$ ) time delay between power level trip and start of rod motion. AF•g is acceleration for rod motion |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 23/3 | $\begin{gathered} \mathrm{SCR} \\ \mathrm{~N} \end{gathered}$ | $\begin{gathered} \mathrm{E} \\ \mathrm{IL} .4 \\ \mathrm{I} 7 \end{gathered}$ |  | PS $=P O \cdot S C R$ is power level at which scram is triggered number of points in $\left\{\begin{array}{l}\Delta k^{*} / Z^{*}-p l a n e(I V=2) \\ \Delta k / X M-p l a n e(I V=3)\end{array}\right.$ | $<$ (NYD) |
| 23/4 | X <br> Y <br> X <br> Y <br> X <br> Y | $\begin{gathered} \mathrm{E} \quad 11.4 \\ " \\ " \\ " \\ " \\ " \end{gathered}$ |  | $\left.\left.\left\{\begin{array}{l}\text { coordinates of first point } \\ \text { " " second " " " third " }\end{array}\right\} \begin{array}{l}\text { "his card is to be repeated } \\ \text { as required by parameter } N\end{array}\right\} \begin{array}{l}\text { on card 23/3 (maximum } \\ \text { number of cards is 17) }\end{array}\right\}$\begin{tabular}{l}
\end{tabular} | $=0$. |
| 23/5 | $\begin{array}{r} \mathrm{SCR} \\ \mathrm{HT} \end{array}$ | E 11.4 | $\begin{aligned} & - \\ & \text { m } \end{aligned}$ | PS $=P O \cdot S C R$ is power level at which scram is triggered length of half wave for $\cos ^{2}$-distribution of reactivity over $Z$-coordinate (in particular: total reactor height) |  |

The input cards must be arranged according to the following table.


APPENDIX M : Subroutines and Functions Concerning Coolant Properties: TPEI, HDV, FLAMI, ETAI

Calculations concerning heat transfer to the coolant as well as coolant pressure drop require certain variables, the functional relationships of which are derived from the equations of state of the coolant medium:

$$
\begin{aligned}
& T=\mathrm{f}_{\mathrm{T}}(\mathrm{P} 3, W I) \ldots \text { temperature } \quad{ }^{-0} \mathrm{C}_{-} 7 \\
& \left.C P=f_{C P}(P 3, W I) \ldots \text { specific heat } \quad \text { kcal/kg }{ }^{\circ} \mathrm{C}\right] \\
& V=f_{V}(P 3, T) \quad \ldots \text { specific volume } \quad\left[\mathrm{m}^{3} / \mathrm{kg} 7\right. \\
& \lambda=f_{\lambda}(P 3, T) \quad \ldots . \text { thermal conductivity } L \mathrm{kcal} / \mathrm{mh}{ }^{\circ} \mathrm{C}_{-} 7 \\
& \eta=f_{\eta}(V, T) \quad \ldots . . \text { dynamic viscosity } \quad\left[\mathrm{kps} / \mathrm{m}^{2} \bar{Z}\right.
\end{aligned}
$$

It is convenient to combine the computation of $T$ and $C P$ since

$$
C P=\left.\frac{\partial W I}{\partial T}\right|_{P Z=\text { const }}
$$

With this the following subprograms are introduced

| SUBROUTINE | TPEI (P3, WI, T, CP) |
| :--- | :--- |
| FUNCTION | HDV $(P 3, T)$ |
| FUNCTION | FLAMI $(P 3, T)$ |
| FUNCTION | ETAI $(V, T)$ |

A particular set of these functions is described below which pertains to superheated steam. In other cases where the coolant may be approximated by an ideal gas the pertinent relations are quite simple:

$$
\begin{aligned}
& T=a+b \cdot W I \\
& C P=1 / b \\
& V=c \cdot \frac{(T-273.16)}{P 3}
\end{aligned}
$$

In the case of an incompressible coolant (e.g. Iiqu. sodium) the last equation can be simplified to:

$$
V=\text { const. }
$$

If - as in the case of liquid sodium - the film coefficient of heat transfer to the coolant is approximately an invariant the values of $\lambda$ and $\eta$ are irrelevant and the corresponding functions may consist of simple statements such as

$$
\begin{aligned}
& \text { FLAMI }=1.0 \\
& \text { ETAI }=1.0
\end{aligned}
$$

## Functions for Superheated Steam

The present analysis puts a premium on computational speed. In contrast to this the development of subprograms for state variables of superheated steam has been guided up to now by considerations of accuracy and range of applicability of a single expression. Using such conventional subprograms / 6_7 it turned out that about $75 \%$ of the computer time needed for a typical transient problem was used by these subprograms alone. Consequently, a new set of subprograms was developed using simpler and faster methods yet having sufficient accuracy for transient problems in concern. The most severe restriction lies with the range of applicability. Using techniques indicated below one may choose position and size of this range with accuracy being a function primarily of the latter. In order to guarantee accuracies better than $1-3 \%$ it is necessary to limit the range to pressures between 50 and 200 at.

The technique adopted for generating efficient representations of the functions $f_{T}, f_{V}, f_{\lambda}$ was least squares fitting of polynomials of varying order in two variables. The functions of $[7$ were used as reference. In this manner the following expressions were used for the approximation of $f_{T}$ :

$$
\begin{aligned}
& T=a_{N} \cdot W I^{N}+a_{N+1} \cdot W I^{N+1}+\ldots \quad a_{N+M} \cdot{ }_{N I}{ }^{N+M} \\
& a_{N+i}=a_{N, K} \cdot P 3^{K}+a_{N, K+I} \cdot P 3^{K+1}+\ldots a_{N, K+L} \cdot P 3^{K+L}
\end{aligned}
$$

where

$$
\begin{aligned}
& i=0,1,2, \ldots \ldots M \\
& \mathrm{~N}, \mathrm{~K}=-3,-2, \ldots \ldots+2,+3 \\
& M, L=0,1, \ldots \ldots 5,6
\end{aligned}
$$

The choice of a suitable set of parameters $N$, $M, K$, $L$ for the range and order of the polynomials was based on the magnitude of the residue as well as on the magnitude of $M$ and $L$, i.e. if two approximations had residues of comparable magnitude the favored choice was the polynomial of lower order. The fitting procedure was carried out in two successive steps, first fitting the function at discrete values of one argument (e.g. pressure P3). The coefficients $a_{n}(P 3)$ which are obtained from these fits are functions of the pressure $P 3$ known only at a number of discrete points and may be fitted in turn by polynomials in this variable. The specific heat CP can be obtained analytically by differentiation of the polynomial with respect to enthalpy WI and taking the reciprocal.

Corresponding procedures were employed for fitting the other two functions $f_{V}$ and $f_{\lambda}$.

Extension of the range of applicability to higher enthalpies and temperatures (within the same range of pressure: 50-200 at) is possible by taking the following approach (ideal gas):

$$
\begin{aligned}
& \text { for } W I>900 \mathrm{kcal} / \mathrm{kg} \\
& \text { use } T=T(W I=900)+\frac{W I-900}{C P}(W I=900)
\end{aligned}
$$

The corresponding relations for specific volume and thermal conductivity are:

$$
\begin{aligned}
& \text { for } T>700 \\
& \text { use } V=V_{(T=700)^{+}} \frac{T-700}{P 3} \cdot 0.0051 \\
& \text { and FLAM }=\text { FLAM }_{(T=700)^{+}}+(T-700) \cdot 1 \cdot 2 \cdot 10^{-4}
\end{aligned}
$$

No least square fitting procedure was necessary with the function $f_{\eta}$ since an analytic expression was already available [6_7for approximation of this function in a range very nearly identical with the one required in this program.

APPENDIX $N$ : Function ZEIT

With most computer facilities the user has access to the computer clock by way of a subprogram. If this is the case it may be used with the present program to monitor computation time as well as for problem termination. To this purpose the input parameter CLM specifies the maximum computation time allotted to a transient problem. The units of CLM must be the same as the ones used by the clock. In order to avoid excessive time consumption the frequency of clock interrogation may be restricted to every NCL'th step of the integration.

By definition the function ZEIT (CLO) must yield the computer clock reading $C L$ minus the current value of the argument CLO

$$
\mathrm{ZEIT}=\mathrm{CL}-\mathrm{CLO}
$$

If the computation time for any one transient problem in a job exceeds its allotted time CLM, this problem is terminated, a comment is printed out (CØMPUTING TIME CØNSUMED) and the next problem is started.

If no clock-subprogram is available a dummy function should be supplied, which returns a constant value to the calling program. In this case the parameter CLM is irrelevant.

The function ZEIT is called by the main program REX only and therefore it is not required if subroutine REXION alone is used.

## APPENDIX 0 : Annotated Sample Problem

The sample problem presented in this appendix is taken from a study 144 of reactivity parameters in conjunction with a steam cooled fast breeder reactor [8_7.

The reactor system is represented by two flow sections. The first flow section consists of two (radial) zones, one of them representing an annular bypass around the perimeter of the core, the other representing the radial blanket. The second flow section represents the core with an axial blanket at each end. The core is subdivided into two radial zones and five axial sections. The first radial zone represents the average core channel whereas the second radial zone comprises a single channel only which is identified with the central channel of the core (nominal hot channel). Geometry and properties of this channel are the same as in the average channel, however, the power density is higher (310 vs. 266).

Power release is assumed to be distributed as follows: radial blanket $10 \%$, axial blankets $2,5 \%$ each, core $85 \%$. Power is released in the fuel only (no heat sources in cladding, coolant, structure).

No orifices are assumed for the first flow section ( $F T=0$ ). Average channel and central channel of the second section (core) are orificed so as to achieve nearly equal exit temperatures for the coolant (i.e. 538.99 vs 548.32 at steady state).

Feedback coefficients and weighting factors were computed as indicated in APPENDIX G. External reactivity was calculated from mode IV $=4$ of the function RCBE described in APPENDIX $L$ ( $\cos ^{2}$-distribution of reactivity weight, constant rod acceleration). Total reactivity insertion is $\$ 8-25.00$, all rods start from the edge of the core and are accelerated by 2 g . Scram is initiated when power exceeds $25 \%$ of the starting value ( 2320 MW , nominal reactor power) and rod motion follows after a 100 msec delay.

The reactor transient arises from a rupture of a steam pipe carrying superheated steam from the reactor. Mass flow and pressure transients for this accident were calculated with a different model representing the entire reactor circuit [ 15 _7.

Output is printed every 100 steps and indicates the main features of the transient: Scram occurs at $X=0.050 \mathrm{sec}$, rod motion starts at $X=0.150 \mathrm{sec}$,
full insertion is completed at about 0.38 sec. Total excess reactivity $\Delta k=K+T$ reaches a maximum of about $\$ 0.98$, peak power is about $P_{\text {Max }}=100000 \mathrm{MW}$ at $X_{M}=0.18 \mathrm{sec}$, total energy release (up to $X=1.9$ sec) is about 10500 MWsec .

Maximum fuel temperature rise (nominal hot channel) is about $2678{ }^{\circ} \mathrm{C}$ (just below melting point) at $X=0.293 \mathrm{sec}$. Maximum cladding temperature (nominal hot channel) is about $1317^{\circ} \mathrm{C}$ at $\mathrm{X}=1.15 \mathrm{sec}$.



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[15_7 Private communication, Dipl. Ing. Malang, IRB, Gesellschaft für Kernforschung, Karlsruhe, 1968.


[^0]:    +)
    A digital model describing the dynamic behavior of all circuit components external to the reactor is being developed by the IRB (Institute for Reactor Components) at the GfK. Both programs are designed to be integrated into a single code giving a complete description of the entire reactor system.

[^1]:    ${ }^{+}$Here, as well as in all subsequent instances arbitrary numbers of components and zones are restricted only by DIMENSIONS specified in the corresponding programs. In the present version these are chosen to accomodate most cases of practical interest and may be extended whenever necessary.

[^2]:    +)
    see APPENDIX A for nomenclature

[^3]:    + For the case indicated above the coefficient of heat transfer can be obtained from:

    $$
    \left.\left.1 . / \mathrm{HFS}=\mathrm{R} 1 / \mathrm{R} 2 \cdot(\mathrm{R} 1 / \mathrm{C} 4 \cdot \mathrm{C} \varnothing 1)+1 \cdot / \mathrm{HGP}+(\mathrm{R} 2-\mathrm{RI}) / \mathrm{C} \not \mathrm{C}_{2}+\mathrm{R} 1 / \mathrm{R} 2 \cdot \mathrm{H} 23\right)\right)
    $$

    if the blanket pin is of a design similar to the fuel pin.

[^4]:    Card 37 has to be repeated as to accommodate the coordinates of all NY points.

[^5]:    +) These absolute values of the individual contributions may be used in the same sense as mentioned with the total steady state feedback for stability investigations.

