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ON THE PROBLEM OF OVERSHOOTS IN THE CONTROL OF NUCLEAR REACTORS

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The time dependent behaviour of two reactors, one uncontrolled and the other controlled by means of a specified automatic control system, are analysed and compared. Conditions are established to rule out overshoots over the final stationary values of power and temperature, and upper limits of the overshoots are calculated (in the Appendix) when these conditions are not satisfied. Finally some attention is given to the influence of the delayed neutrons.

List of symbols

- k = multiplication factor
- δk = actual reactivity = k-1
- δk^* = actual reactivity increased by the amount which is suppressed by the negative temperature effect due to the heating up of the U rods ($\delta k^* = \delta k + \Gamma T$)
- ℓ = mean life of the neutrons in the reactor
- m = heat capacity of the whole reactor fuel (uranium specific heat x fuel mass)
- Q = reactor power
- T = mean fuel temperature referred to the room temperature
- α = heat transmission coefficient
- β_i = fraction of delayed neutrons of the ith group ($\beta = \sum_i \beta_i$)
- Γ = negative temperature coefficient due to the heating up of the fuel
- λ_i = decay constant corresponding to the ith groups of delayed neutrons.

One of the main problems in operation of nuclear reactors is to increase the power level from a steady value to a higher one without any overshoot over the final stationary values of power and temperature (Fig.1).

+) Comision Nacional de Energia Atomica - Buenos Aires

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In this paper the conditions for this will be estabslished in two different cases. The first is the case of a reactor without any automatic control system, in which the reactivity is suddenly increased to a positive value by pulling out a control rod and which becomes stabilized only by the negative

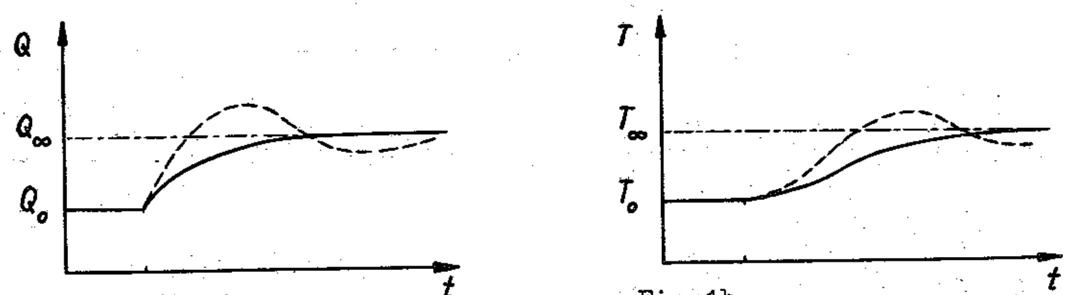

Fig.1a Fig.1b

reactivity arising from the increasing temperature. And the second is the case of a reactor controlled by means of a specified control system in which the power demand is suddenly increased to a higher level. Under certain assumptions the equations which rule both cases are the same.

1. Reactor without automatic control system.

We start from the well known equations:[1)]

$$(1) \qquad \frac{1}{Q}\frac{dQ}{dt} = \frac{\delta k}{\ell}$$

$$(2) \qquad \delta k = \delta k(o) - \Gamma(T - T(o)) = \delta k^* - \Gamma T$$

$$(3) \qquad Q = \alpha T + m\frac{dT}{dt}$$ (newtonian cooling)

supposing at the moment, a unique mean life time ℓ of the neutrons in the reactor, and further assuming that the reactor has been operated up to t=0 in a steady state characterized by

1) Only the temperature effect due to the fuel has been considered here. This generally is admissible, as the heating up of the moderator occurs with a much larger time constant than that which corresponds to the fuel. Hence the temperature effect due to the moderator, like the poisoning effect, must be compensated later by pulling out gradually a control or shim rod.

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$$Q_o > 0$$

(4)
$$T_o = \frac{Q_o}{\alpha}$$

$$\delta k_o = \delta k_o^* - \Gamma T_o = 0$$

At the time t=0 the reactivity is increased suddenly to the value $\delta k(o)$ and the variation of Q and T will now be analysed. As initial conditions we have now:

(5)
$$Q(o) = Q_o$$

$$T(o) = T_o$$

$$\delta k(o) = \delta k^* - \Gamma T_o > 0 \quad .$$

Combining (1) and (2) we obtain

(6)
$$T = \frac{1}{\Gamma}\left[\delta k^* - \ell \frac{d(\ln Q)}{dt}\right]$$

which on differentiation gives

(7)
$$\frac{dT}{dt} = -\frac{\ell}{\Gamma}\frac{d^2(\ln Q)}{dt^2} \quad .$$

Combining (6), (7) and (3) and introducing the coefficients

(8)
$$\frac{m\ell}{\Gamma} = C$$

$$\frac{\alpha\ell}{\Gamma} = B$$

$$\frac{\alpha\,\delta k^*}{\Gamma} = Q_\infty$$

and the function

(9)
$$\ln\frac{Q}{Q_\infty} = s$$

we obtain the equation

(10)
$$C\ddot{s} + B\dot{s} = Q_\infty(1 - e^s) \quad .$$

This is the equation which describes the motion of a unit mass under the influence of a force

(11) $$f = \frac{Q_\infty}{C}(1-e^s) = -\frac{dU}{ds}$$

against a frictional resistance proportional to its velocity [5] :

(12) $$r = \frac{B}{C}\dot{s}$$

The force f is the gradient of a potential U, and therefore when B=0 the unit mass would oscillate with a constant amplitude depending only on the initial conditions. As $B > 0$ the motion is damped and approaches gradually the equilibrium point s=0, $\dot{s}$=0 for $t \to \infty$. According to the way in which this equilibrium point is reached we may distinguish three different cases:

1. Damped oscillations around s=0 (Fig.2a)
2. Aperiodic motion with overshoot over s=0 (Fig.2b)
3. Aperiodic motion without any overshoot over s=0 (Fig.2c).

The stable equilibrium point s=0, $\dot{s}$=0 corresponds to the final steady state $\frac{Q}{Q_\infty} = 1$; $\dot{Q} = 1$

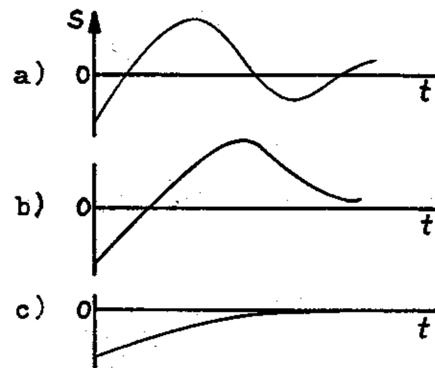

Fig.2

A condition which rules out the cases 1. and 2. will now be established. For this purpose we shall introduce a function y(t) which is a solution of

(13) $$C\ddot{y} + B\dot{y} = -Q_\infty y$$

and satisfies the initial conditions

(14) $$y(0) = s(0)$$
$$\dot{y}(0) = \dot{s}(0) \; .$$

This function, like s(t), also approaches the final stationary value 0 when t becomes infinite. Now considering that

(15) $$e^s - 1 \geq s$$
$$\text{for } -\infty < s < +\infty$$

and owing to the continuity of the functions y(t) and s(t), it can be shown that

(16) $$\boxed{s_{max} \leq y_{max}}\,.$$

Therefore the well known conditions:

(17) (Aperiodicity condition) $$\frac{B^2}{4CQ_\infty} \geq 1$$

(18) $$-\frac{\dot{y}(0)}{y(0)} \leq \frac{B}{2C}$$

which rule out any overshoot of y over its endvalue 0 are also sufficient for s(t) to have no overshoot over 0.

Replacing the coefficients in (17) by their values given in (8) we get:

(17a) $$\boxed{\frac{\alpha \ell}{4m\delta k^*} \geq 1}$$

and after some calculations the condition (18) becomes:

(19) $$-\frac{1-\delta k_o^*/\delta k^*}{\ln\left(\frac{\delta k_o^*}{\delta k^*}\right)} \leq \frac{\alpha \ell}{2m\delta k^*}\,.$$

Now it is easy to show that:

(20) for $$0 \leq \frac{-(1-x)}{\ln x} \leq 1 \quad \text{for} \quad 0 \leq x \leq 1$$

and therefore equation (19) is always fulfilled when $\frac{\alpha \ell}{2m\delta k^*} \geq 1$. It follows from this that the condition (17a) alone is sufficient to insure the function $s(t) = \ln Q/Q_\infty$ to have no overshoot over 0 and therefore also sufficient to insure the power Q to rise from Q_o to Q_∞ without any overshoot. That (17a) is also a necessary condition can be realized from the fact that for very small values of y and s, equations (10) and (13) are equivalent (considering only the first term of the Taylor-serie of $1-e^s$).

So far we have dealt only with the reactor power level. When the condition (17a) is satisfied this level will rise continously approaching the final stationary value Q_∞ asymptotically. Then, owing to (1) and (2) the temperature can also not surpass its new stationary value $T_\infty = \delta k^*/\Gamma$. But when (17a) is not fulfilled Q will reach a maximum Q_{max} larger than Q_∞ and therefore also T will surpass T_∞ . Formulae for calculating upper limits of Q_{max} and T_{max} are given in the Appendix.

We shall evaluate now the condition (17a) numerically with coefficients corresponding to the English graphite moderated natural uranium reactor BEPO (3).

$$\alpha \cong 0{,}02\ MW/^\circ C \qquad \frac{m}{\alpha} \cong 200\ sec$$
$$m \cong 4\ MW\ sec/^\circ C$$

Taking into account the influence of the delayed neutrons, the value

$$\bar{\ell} = \ell + \sum_i \frac{\beta_i}{\lambda_i} \cong 0{,}1\ sec$$

can be adopted as neutron mean life.

The negative temperature coefficient corresponding to the heating up of the uranium rods can be quoted as:

$$\Gamma \cong 1{,}5 \times 10^{-5}/^\circ C$$

Introducing these values into (17a) we obtain

$$T_\infty = \frac{\delta k^*}{\Gamma} \leqslant \frac{1}{4} \cdot \frac{0{,}1}{200} \cdot \frac{1}{1{,}5 \cdot 10^{-5}} = 8{,}3\ ^\circ C$$

This is the maximum temperature which could be reached in this case without overshoots. Of course, this result would be improved by lower values of $\frac{m}{\alpha}$ and Γ .

2. Automatically controlled reactor.

Now we come to the second case. The control system we shall consider here is of the type utilized in **Saclay** and is shown schematically in fig.3. Let us suppose that the response of the system to an increase in the arbitrarily value

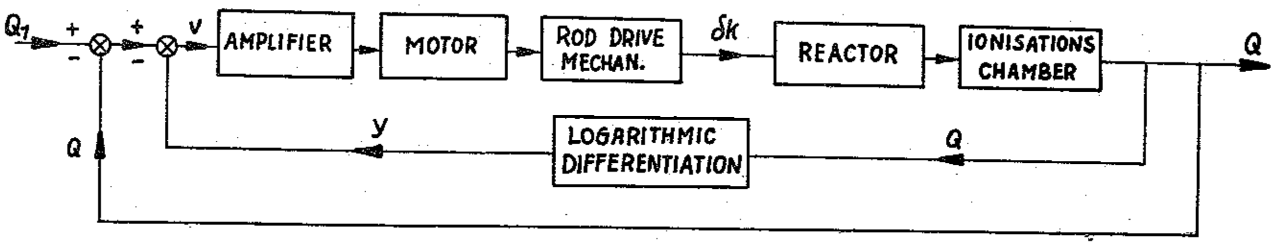

Fig. 3

of the power demand is fast in comparison to the thermal time constant of the reactor. Reactivity changes due to temperature rise can then be neglected during the period of transition to the new stationary power level and it will not be necessary to introduce the temperature in the analysis of the behaviour of the system during this period. When finally the slowly rising temperature effect becomes appreciable, the system will be able to compensate for it automatically. In the figure Q is the output of the ionization chamber which is proportional to the reactor power and Q_1 is the increased power demand. From the error signal Q_1-Q another signal y is substracted, which is proportional to the logarithmic derivative of Q, and this difference, v, is feed into the amplifier. This is described by the following equation:

$$v = (Q_1 - Q) - B\,\frac{d(\ln Q)}{dt} = Q_1\left(1 - \frac{Q}{Q_1}\right) - B\,\frac{d\left(\ln \frac{Q}{Q_1}\right)}{dt} \qquad (21)$$

The speed of the motor and therefore also the velocity of the control rod driven by it are proportional to the output of the amplifier. Thus, supposing that the reactivity released by a control rod is a linear function of its displacement, and remembering equation (1) we have

(22) A being a factor of proportionality $$Av = \frac{d(\delta k)}{dt} = \ell \frac{d^2(\ln Q/Q_1)}{dt^2} .$$

Combining (21) and (22) and calling $\frac{\ell}{A} = C$ we obtain:

(23) $$C \frac{d^2(\ln Q/Q_1)}{dt^2} + B \frac{d(\ln Q/Q_1)}{dt} = Q_1 (1 - \frac{Q}{Q_1}) .$$

This equation is identical with (10) and therefore again the conditions:

(17b) $$\frac{B^2}{4CQ_1} \geqslant 1$$

(18b) $$- \frac{\dot{Q}(0)}{Q(0) \ln[Q(0)/Q_1]} \leqslant \frac{B}{2C}$$

must be fulfilled in order to have no overshoots during the transition to the new stationary power level Q_1.

Now let us examine the behavour of the system just after the power demand has been increased from Q_o to Q_1. In the first time the imput v to the amplifier is so large that the amplifier becomes saturated and therefore the control rods are pulled out with a constant limiting velocity w_s. During this period equations (22) and (23) do not apply. Owing to the continous increasing of the reactivity δk (and therefore of y) which follows the progressive withdrawal of the control rods, the imput signal to the amplifier is gradually reduced. Finally the amplifier saturation desappears and after a short time v becomes 0. At this instant Q has some value intermediate between Q_o and Q_1. It is convenient to consider this moment as starting point t=0:

(24) $$V(0) = C\left[\frac{d^2(\ln Q/Q_1)}{dt^2}\right]_{t=0} = 0 \quad .$$

According to (23) we have then the following initial conditions:

(25) $$\frac{\dot{Q}(0)}{Q(0)} = \frac{Q_1}{B}\left[1 - \frac{Q(0)}{Q_1}\right] \quad .$$
$$Q_0 < Q(0) < Q_1$$

Again because of equation (20) it is easy to show that with these initial conditions the fulfilment of the condition (17b) insures that of (18b) too.

But in the present case there is another condition which must be fulfilled in order to rule out any overshoot over Q_1 1) . It refers to the limiting velocity w_s of the control rods, which of course must be large enough to allow for the maximum reactivity change rate required after t=0. Now we shall calculate an upper limit for this maximum reactivity change rate.

When $\left|\frac{d(\delta k)}{dt}\right| = \ell \left|\frac{d^2(\ln Q/Q_1)}{dt^2}\right|$

reaches a maximum, then its first time derivative becomes 0. Therefore, by differentiating the equation (23) we obtain:

(26) $$\left[\frac{d^3(\ln Q/Q_1)}{dt^3}\right]_{\dot{\delta k} = \dot{\delta k}_{max}} = -\frac{1}{C}\left[\dot{Q}\right]_{\dot{\delta k} = \dot{\delta k}_{max}} - \frac{B}{C}\left[\frac{d^2(\ln Q/Q_1}{dt^2}\right]_{max} = 0$$

It follows that:

(27) $$\left[\frac{d^2(\ln Q/Q_1}{dt^2}\right]_{max} = \frac{1}{B}\left[\dot{Q}\right]_{\dot{\delta k} = \dot{\delta k}_{max}} \leq \frac{1}{B}\left(\frac{\dot{Q}}{Q}\right)_{max} \cdot Q_1$$

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1) This condition has already been established by Bonnaure et al. [1] in another way.

where $\left(\frac{\dot{Q}}{Q}\right)_{max}$ means the maximum value of the logarithmic derivative $\frac{\dot{Q}}{Q}$. Owing to (24) and (25) this maximum value is found to be:

$$(28) \qquad \left(\frac{\dot{Q}}{Q}\right)_{max} = \frac{\dot{Q}(0)}{Q(0)} = \frac{Q_1}{B}\left(1 - \frac{Q(0)}{Q_1}\right) \leqslant \frac{Q_1}{B} \ .$$

Replacing this value in (27) we get finally

$$(29) \qquad \ell\left[\frac{d^2(\ln Q/Q_1)}{dt^2}\right]_{max} = \left[\frac{d(\delta k)}{dt}\right]_{max} \leqslant \frac{Q_1^2}{B^2}\ell \ .$$

This is the maximum reactivity change rate which must be achieved by the control rods after t=0. Therefore calling $\dot{R}_s$ the reactivity change rate corresponding to the limiting rod velocity w_s we obtain the following supplementary condition to (17b)

$$(30) \qquad \boxed{\frac{Q_1^2}{B^2}\cdot\ell = \dot{R}_s} \ .$$

By means of more thorough going developments it is also possible to show that the less restrictive condition

$$\dot{R}_s \geqslant \frac{Q^2}{2\ B^2}\cdot\ell$$

is sufficient.

<u>Finally let uns come back again to the first case</u>, that is the reactor without automatic control system, but now taking into account one group of delayed neutrons. Then, during the first time after the reactivity jump the power will be given by the following expression [2]:

$$(31) \qquad Q = Q_o\left[\frac{\beta}{\beta-\rho}\cdot e^{\frac{\lambda\rho t}{\beta-\rho}} - \frac{\rho}{\beta-\rho}\cdot e^{-\frac{\beta-\rho}{\ell}t}\right]$$

where $\rho = \frac{\delta k}{K} \cong \delta k \leqslant 0{,}005$. After nearly 0,5 sec the second term in (31) can be neglected and therefore we can reduce the analysis of the present problem to the first case only introducing as initial conditions:

$$Q(0) = Q_0 \frac{\beta}{\beta-\rho} \cong Q_0 \frac{\beta}{\beta - \delta k(0)}$$

(32) $$\delta k(0) = \delta k^* - \delta k_0^* = \delta k^* - \Gamma T_0$$

$$T(0) = T_0$$

and as mean neutron life the value

(33) $$\ell_1 = \frac{\beta - \delta k}{\lambda} = \frac{\beta - \delta k}{\beta} \cdot \frac{\beta}{\lambda} \cong \frac{\beta - \delta k}{\beta} \cdot \bar{\ell}$$

where $\bar{\ell} = \ell + \sum_i \beta_i/\lambda_i \cong \sum_i \beta_i/\lambda_i = \frac{\beta}{\lambda}$.

It must be observed that δk is not constant but decreases with increasing temperatures. However we are always on the safe side adopting the smallest value of ℓ_1 given by:

(34) $$\ell_1 = \frac{\beta - \delta k(0)}{\beta} \cdot \bar{\ell} = \frac{\beta - \delta k^* + \delta k_0^*}{\beta} \cdot \bar{\ell} \quad .$$

Introducing now (34) into (17a) and (32) into (18) it is seen that the power and the temperature will reach their final stationary values without overshoots when the following two conditions are fulfilled:

(35) $$\boxed{\frac{\alpha \cdot \bar{\ell}}{4m\,\delta k^*} \geqslant \frac{\beta}{\beta - (\delta k^* - \delta k_0^*)}}$$

with $\delta k^* - \delta k_0^* \leqslant 0{,}005$

(36) $$\boxed{\frac{\alpha \cdot \bar{\ell}}{4m\,\delta k^*} \geqslant \frac{\beta}{2(\beta - \delta k^*)}}$$

with $\delta k^* < \beta$

Appendix. - Upper limits for the overshoots of power and temperature when the condition (17a) is not fulfilled:

$$\frac{Q_{max}}{Q_\infty} \leqslant 1 + \frac{1}{8D}(1 - \delta^2)$$

$$\frac{T_{max}}{T_\infty} \leqslant 1 + \sqrt{8D\,(e^{s_{max}} - 1 - s_{max})}$$

where

$$\delta = \frac{\delta k_o^*}{\delta k^*}$$

$$D = \frac{\alpha \, \ell}{4m \, \delta k^*}$$

$$s_{max} = \ln\left(\frac{Q_{max}}{Q_\infty}\right) \; .$$

For values of D or δ very close to 1 the following formulae gives lower values

$$\frac{Q_{max}}{Q_\infty} \leq \exp\left[\frac{(1-\delta)}{2\sqrt{D}} \cdot e^{\frac{1}{x}(\operatorname{artg} x - \pi)}\right]$$

$$\frac{T_{max}}{T_\infty} \leq \frac{Q_{max}}{Q_\infty}$$

where

$$x = \sqrt{\frac{1}{D} - 1}$$

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REFERENCES

[1] Bonnaure, P. et al. (1954): Pilotage automatique d'un réacteur nucléaire, Journal of nuclear energy I p.24

[2] Glasstone, S. and M.C.Edlund (1952): The elements of nuclear reactor theory.

[3] Moore, R.V.(1953): The control of a thermal neutron reactor. Proc.Inst.Electr.Engineers, 100 part I 9.90.

[4] Schulten, R. und H.Gaus (1954): Temperaturanstieg in thermischen Reaktoren bei Ausfall der Kühlung. Zeitschrift f.Naturforschung 9a, p.1039.

[5] Weinberg, A.W. and W.K.Ergen (1953): Some aspects of non linear reactor kinetics. Proc.of the Kjeller Conference on heavy water reactors.

[6] Chernick, J. (1951): The dependence of reactor kinetics in temperature, BNL - 173.

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