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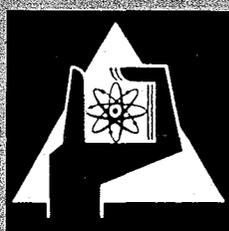
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Neutron Scattering from Solid Hydrogen

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⁺ This is a slightly modified version of a paper presented at
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Gesellschaft für Kernforschung mbH Karlsruhe

Abstract:

By inelastic neutron scattering the quadrupole interaction constant ratio Γ/Γ_0 in solid hydrogen has been determined to be 0.74, 0.77, and 0.81 for ortho concentrations $c = 0.68, 0.27$ and 0.04 , respectively.

Moreover the phonon spectrum of solid hydrogen in the hcp-phase ($c = 0.68$) was obtained from the scattering distribution.

Zusammenfassung:

Mit Hilfe von inelastischer Neutronenstreuung wurde die Quadrupol-Quadrupol Wechselwirkung in festem Wasserstoff bestimmt. Für Ortho-Konzentrationen von 68, 27 und 4 % ergab sich Γ/Γ_0 zu 0.74, 0.77 und 0.81. Außerdem wurde die Frequenzverteilung der Phononen für festen Wasserstoff in der hcp-Phase und der Ortho-Konzentration von 68 % aus der Neutronenstreuverteilung erhalten.

Neutron Scattering from Solid Hydrogen

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Kernforschungszentrum Karlsruhe

I. Introduction

There are two groups in Western Germany working experimentally on neutron scattering from solid hydrogen. These two groups are located at the two research centers of Karlsruhe and Jülich. In Karlsruhe the width of the rotational state $J = 1$ was measured in solid hydrogen by studying the $0 \rightarrow 1$ rotational transition. Moreover, we were able to assess the frequency distribution of the phonons from the scattering distribution by means of a method of extrapolation. Recently we made an experiment which yields the quadrupole coupling constant Γ at very low ortho concentration. Thereby the splitting of the ortho molecule pairs was measured with an energy resolution of $6^{\circ}/_{00}$ of the $0 \rightarrow 1$ energy transfer. All the experiments were conducted at a temperature of 4.3 K, that means above the λ -point of solid hydrogen. In Jülich, an experiment is planned at a temperature below the λ -point of solid hydrogen. The aim of this experiment is to measure the spectrum of the angular momentum waves.

It is known that the incoherent scattering length of the H atom is large in relation to its coherent scattering length. The ratio of incoherent to coherent total cross section is about 44. Moreover, the absolute value of the total bound cross section is relatively large, namely 81.5 barn. Condensed hydrogen consists of ortho and para molecules with the nuclear spins of 1 and 0, respectively. The hydrogen molecules are known to rotate almost freely also in the condensed state. The ortho molecules are in a rotational state with $J = 1$, the para molecules with $J = 0$. Because of the symmetry of the molecule there is a strong correlation between the nuclear spin of the molecules and their rotational state. Scattering from the ortho molecules has an incoherent and a coherent fraction. Scattering between the ortho- and para states is

purely incoherent, since it is coupled with a spin flip process ^{1,2)}. The incoherent fractions accordingly result in a high intensity for these scattering processes. Incoherent scattering can be employed to measure frequency distributions and individual molecular excitations. The rotational transition from $J = 0$ to $J = 1$ is an example of a molecular excitation. This is a para-ortho transition and, hence, purely incoherent.

II. Determination of the Quadrupole Coupling Constant

II.1 Width of the rotational line

Figure 1 shows the scattering process schematically.

Neutrons with an energy of incidence E_0 and a wave number vector \vec{k}_0 are scattered from the sample of solid hydrogen under an angle θ . In inelastic scattering, in which they excite a para molecule with the rotational state of $J = 0$ into an ortho molecule with $J = 1$, the neutrons lose the energy of the first rotational state of about 14.7 meV. Correspondingly, they leave the sample with an energy E reduced by the rotational energy and a wave vector \vec{k} . The energy transfer of 14.7 meV connected with the $0 \rightarrow 1$ transition is directly visible as a relatively sharp line in the inelastic neutron scattering distribution.

For the intensity of the $0 \rightarrow 1$ line ³⁾ the approximate result, if we assume a gaussian curve with a variation of ΔE for the shape of the line, treat the molecules as rigid rotators, consider the spin correlation for the scattering of neutrons from the nuclei of a molecule, and describe the movement of the centers of mass of a molecule by a Debye Waller factor in the Debye approximation is given by the expression:

$$\left(\frac{d^2\sigma}{d\Omega dE}\right)_{0 \rightarrow 1} = (1 - c) \cdot b_{inc}^2 \frac{k}{k_0} \cdot 6 \cdot j_1^2 \left(\frac{\chi_{0 \rightarrow 1} a}{2}\right) \cdot e^{-\frac{3\pi^2 \kappa^2}{4M\theta_D}} \cdot \frac{e^{-\frac{(E - E_{J=1})^2}{2\Delta E^2}}}{\sqrt{2\pi\Delta E^2}}$$

In this formula, c is the ortho concentration, b_{inc} the bound incoherent scattering length, k_0 and k are the wave numbers of the incident and scattered neutrons, respectively, j_1 is a spherical Bessel function, $\hbar \kappa_{0 \rightarrow 1}$ is the momentum transfer of the scattered neutrons, M the mass of the molecule, Θ_D the Debye temperature.

The width of this line is given essentially by the resolution of the equipment, which is known⁴⁾, and the quadrupole quadrupole interaction between the ortho molecules. Hence, an exact knowledge of the physical line width allows conclusions with respect to the quadrupole coupling parameter Γ .

Figure 2 shows the rotating crystal time-of-flight spectrometer we used to measure the inelastic scattered neutrons.

Neutrons of a thermal beam are reflected from the 111 planes of a rotating Al crystal. On the basis of Bragg's equation $n\lambda = 2d \sin \theta$, specific wavelengths and energies, respectively of the n order correspond to each scattering angle 2θ . These monochromatic neutrons are impinging upon the sample of solid hydrogen contained in a helium cryostat set up at the place of the sample (the cryostat is not shown in the picture). The cryostat cools the hydrogen sample with cold helium gas which is maintained at a temperature of 4.3 K. The neutrons scattered from the sample can be registered by a maximum of nine detectors. Since the crystal rotates, Bragg's condition is fulfilled only during a short interval of time. Therefore, two short neutron bursts are produced at each rotation. The energies of the scattered neutrons are determined from the measurement of the time-of-flight they require to cover the distance between the sample and the detector. The starting pulse is generated by a magnetic trigger attached to the rotating crystal.

Figure 3 shows two time-of-flight spectra measured with this system. The spectra were measured at 4.3 K and two different ortho concentrations of 68 and 27 %. The $0 \rightarrow 1$ and the $1 \rightarrow 0$ transitions are visible, and so is the elastic line and a broad spectrum which is due to the excitation of single and multi phonon processes. An enlargement of the line width of the $0 \rightarrow 1$ peak at the higher ortho concentration is clearly recognizable. The phonon spectrum will be discussed later. The peaks were evaluated by a least squares

fit of a gaussian distribution. For $c = 0.68$ the background due to multi phonon processes was described by an exponential function. For $c = 0.27$ the background can be represented approximately by a straight line, due to the lower intensity of multi phonon scattering which is caused by the scattering from ortho molecules.

Figure 4 shows the $0 \rightarrow 1$ line at 27 % ortho concentration. The fit with a gaussian distribution can be carried out very well. The two curves correspond to two different scattering angles. The dashed line corresponds to the unfolded curve where the resolution of the equipment has been folded out.

Figure 5 shows the $0 \rightarrow 1$ line at 68 % ortho concentration. One example of a spectrum with background and three examples of different scattering angles with the background subtracted are pictured. Obviously, at this high ortho concentration the line can not be described in a satisfying way by a gaussian curve. Instead, there is a structure, which may have however no major influence upon the line width. One possible explanation is that even above the λ -point collective modes of the angular momentum waves still exist. A spectrum which is similar to the angular momentum wave spectrum would explain the structure of the rotational line. Furthermore the steep descent of the line to higher energies is then due to the fact that angular momentum waves which have an energy of about 12 K are not excited at 4.3 K.

Elliott and Hartmann ⁵⁾ carried out a theoretical treatment of neutron scattering from condensed hydrogen. They based their argument on the assumption that the ortho molecules are unoriented. Thus, the theory holds for temperatures above the λ -point. Moreover, they considered only the interaction between two ortho molecules and they used in the perturbation theory the nine possible wave functions of state of a pair of free ortho molecules. For the mean square of the energy transfer of the $0 \rightarrow 1$ rotational transition their result is then as follows:

$$\overline{\Delta \epsilon^2} = c \cdot \frac{70}{9} \cdot \Gamma^2 \cdot \sum_{j(\neq i)} \left(\frac{R_0}{R} \right)^{10}$$

Here, $\Gamma_0 = \frac{6}{25} \frac{Q^2}{R_0^5}$ is the quadrupole coupling parameter. Q is the quadrupole moment of the H_2 molecule, and R_0 is the distance of the nearest neighbors of the molecules in the hcp lattice. The root of the mean square energy transfer $\sqrt{\Delta E^2}$ is connected with the full line width at half height of a gaussian curve $\Delta E_{0 \rightarrow 1}$ by

$$\Delta E_{0 \rightarrow 1} = 2.358 \cdot \sqrt{\Delta E^2_{0 \rightarrow 1}}$$

In this model only the electric quadrupole quadrupole (EQQ) interaction was considered. Van der Waals and valence forces, which also have a contribution that is effective between the ortho molecules, have an effect of only about 2 % relative to the quadrupole forces. We used the formula by Elliott and Hartmann to calculate Γ from our measured line width.

For the sum $\sum_{j(\neq i)} \left(\frac{R_0}{R}\right)^{10}$ the value of 12.312 results for the hcp lattice according

to Wallace and Patrick ⁶⁾. The table (Fig. 6) shows the line widths of the rotational lines $\Delta E_{0 \rightarrow 1}$ measured at the ortho concentration c and momentum transfer $\hbar \kappa$. The width ΔE_{App} caused by the finite resolution of the equipment was calculated. The line widths do not depend on κ . From $\overline{\Delta E_{0 \rightarrow 1}}$, an average over the measurements with different κ , and the same ortho concentration, we obtained Γ . The ratio Γ/Γ_0 is below unity, where Γ_0 is calculated simply by using the above formula with $R_0 = 3.755 \text{ \AA}$. For Γ_0 we used a value of 0.0866 meV given by A.B. Harris ⁷⁾. For Γ/Γ_0 at an ortho concentration $c = 0.27$ a value of 0.77 results, for $c = 0.68$ a value of 0.74 .

II.2 Measurement of the Ortho Pair Spectrum Levels

Figure 7 shows the principle arrangement of our neutron scattering experiment aiming for a high energy resolution to measure the splitting of the ortho molecule pairs directly if the contribution of the higher ortho molecule clusters can be neglected. Therefore we chose a sample containing only 4 % ortho hydrogen. The probability that an ortho molecule is surrounded only by para molecules is in this case 61 % and the probability for just one ortho

molecule neighbour is 23 %. The temperature of the sample was 4.4 K. We used a pseudostatistical chopper ⁸⁾ which enables one to measure the scattered distributions like in a conventional time-of-flight experiment if one cross correlates the scattered neutron distribution with the special pattern of the chopper. The chopper consists of a disk with an arrangement of slits. With this pattern the disk is half of the time transparent and half of the time black for neutrons. Therefore a factor of roughly 50 in intensity can be gained in comparison with a rotating crystal. In our case we gained only a factor of 6.8 because the peaks to be measured were about of the same height as the mean value of the time-of-flight spectrum. The cross correlating will be done by a computer. We used a copper single crystal as a monochromator. After the sample there is a flight path of 9 m. For the $0 \rightarrow 1$ transition we achieved in this way an energy resolution of about 6^o/oo. The energy of the incident neutrons is $E_0 = 19.26$ meV.

Figure 8 shows a photograph of this arrangement. The direct beam, the cryostat and the flight path can be seen.

If we take only the EQQ interaction between the ortho molecule pairs into account we expect a 4 level scheme ⁹⁾ in which the splitting is given by -4Γ (2), 0 (4), Γ (2), 6Γ (1) (the numbers in parentheses denote the degeneracy). This splitting should be seen in the scattered distribution.

Figure 9 shows the folded spectrum which corresponds to a conventional time-of-flight spectrum. The main peak originates from an excitation of a para molecule into an ortho molecule which is surrounded only by para molecules. This peak is broadened because of three reasons:

- (a) The width of the resolution function of the spectrometer for the rotational line was calculated to be about 100 μ eV corresponding to an energy resolution of 6^o/oo for the $0 \rightarrow 1$ energy transfer.
- (b) The EQQ interaction between ortho molecules with mutual distances larger than the nearest neighbour distance. This effect causes a broadening of 51 μ eV [5].
- (c) The line $E_0 - 14.79 - \Gamma = (4.47 - \Gamma)$ meV which is broadened by a) and b) is included in the peak.

Furthermore the main peak contains the unsplit level of the ortho

molecule pair spectrum at $E_0 - 14.79 = 4.47$ meV.

The other two lines of the pair spectrum corresponding to a splitting of $+6\Gamma$ and -4Γ can be seen as weak lines on both sides of the main peak. Especially the last one is certainly out of statistics. The ordinate shows the counts which are to be expected with a conventional rotating crystal chopper. Because of the pseudostatistical technique the absolute error is about $1/2 \sqrt{\text{counts}}$. The expected spectrum was fitted to the measured points. This spectrum has been calculated by superposing gaussian lines corresponding to the contribution of soloists and pairs. The contribution of higher ortho molecule clusters was approximated by a gaussian distribution by means of the theory of Elliott et al. [5]. The width of this distribution turned out to be 7 times as large as the width of the soloist- and pair lines. Thus, the higher ortho clusters produce only a flat background which hardly effects the spectrum. Γ and the width of the soloist- and pair lines W were used as parameters for the least squares fit and determined to be $\Gamma = (70.0 \pm 0.03) \mu\text{eV}$ and $W = 112 \mu\text{eV}$. This yields $\Gamma/\Gamma_0 = 0.81 \pm 0.03$ which is in very good agreement with NMR experiments [11].

Our experimental results show that Γ/Γ_0 is larger for low concentration or dilute ortho hydrogen than for concentrated ortho hydrogen. In both cases the value is smaller than one. This result is supported by calculations of A.B. Harris [12]. Γ is changed in the crystal by the interaction of phonons with orientational states and by dielectric screening. The interaction with the phonons can be divided into a static and a dynamic part. Whereas the static part is the same for concentrated and dilute ortho hydrogen and leads to a reduction of Γ , the dynamic part is different for both cases and causes Γ to decrease and increase for concentrated and dilute ortho hydrogen, respectively. The dielectric screening leads to an effective decrease of the EQQ interaction by about the same amount in both cases. The dielectric screening which is caused by three body interactions also changes the separations of the EQQ pair levels relatively to each other. In fact it is rather difficult to observe these relative shifts of the EQQ pair levels. Taking three body interactions into account, the 6Γ peak, for example, is expected to be shifted by about $0.02\Gamma_0$ to lower energies if the remaining part of the spectrum is fitted by a superposition of gaussian functions accor-

ding to the contributions of soloists and ortho molecule pairs (EQQ spectrum) as we have done.

III. Phonon frequency distribution of concentrated ortho hydrogen in the hcp phase

Besides the $0 \rightarrow 1$ transition, the incoherent neutron scattering distribution yields the phonon spectrum as another item of information. The differential cross section for neutron scattering from a system of identical nuclei with a statistical orientation of the nuclear spin can be written as

$$\frac{d^2\sigma}{d\Omega dE} = \frac{\sigma_b}{4\pi k_B \cdot T} \cdot \frac{k}{k_0} \cdot e^{-\frac{\beta}{2}} \cdot s(\alpha, \beta)$$

where σ_b means the bound total cross section.

$$\beta = \frac{E - E_0}{k_B \cdot T} = - \frac{\mathcal{E}}{k_B \cdot T}$$

is proportional to the energy transfer and

$$\alpha = \frac{\hbar^2 |\vec{k}_0 - \vec{k}|^2}{2Mk_B \cdot T} = \frac{\hbar^2 \chi^2}{2Mk_B \cdot T}$$

is proportional to the square of the momentum transfer.

$S(\alpha, \beta)$ is the scattering law. This formula applies also to the scattering from phonons in solid hydrogen in the incoherent approximation where the scattering law S reduces to the self term S_s , i.e., where coherent scattering may be neglected. Only the ortho molecules contribute to the scattering law S_s for the excitation of phonons, because the scattering in the para state is coherent and negligible. It has been shown by Egelstaff et al.¹³⁾ that the phonon spectrum $\mathcal{Q}(|\mathcal{E}|)$ can be obtained from S_s by means of a method of extrapolation. The procedure may be described by the following relation:

$$\left(\frac{S_s}{\alpha}\right)_{\alpha \rightarrow 0} = \frac{\mathcal{P}(\beta)}{2\beta \sinh \beta/2} = \frac{k_B \cdot T \cdot \rho(\mathcal{E})}{2\beta \cdot \sinh \beta/2}$$

The $\rho(\mathcal{E})$ obtained in each case has the character of a frequency spectrum. In the case of a harmonic solid this relation follows from the phonon expansion.

Figure 10 shows a plot of S_s/α as a function of α on a logarithmic scale. The measured points then should be roughly on a straight line because of the Debye Waller factor, which has the form of $e^{-\lambda\alpha}$. From the slope an average of the Debye Waller coefficient λ can be calculated. The points up to $\alpha \approx 60$ are very well arranged on a straight line. At small β -values, the resulting S_s/α -values are too high because of the finite width of the elastic line, mainly due to the resolution of the equipment. Correspondingly, the extrapolation and the resulting frequency distribution are relatively inaccurate at small β -values. The solid lines represent an attempt to reconstruct the scattering law S_s by means of the phonon spectrum obtained via a phonon expansion. Moreover, we averaged over the polarization vectors of the phonons, which is correct only for cubic crystals¹⁴⁾. Nevertheless, the values of S are quite reasonable. For a hcp lattice the Debye Waller factor is anisotropic. Correspondingly, we obtained a Debye Waller factor averaged over the polarization vectors of the phonons. The root of the mean square shift of the molecules of 0.588 \AA calculated in this way is in good agreement with the value of 0.6 \AA obtained by Mertens and Biem¹⁵⁾ for fcc para-hydrogen. Furthermore the value $\theta = 109.5 \text{ K}$ (at $T = 4.3 \text{ K}$) obtained for the Debye temperature is reasonable. From sonic velocities¹⁶⁾ a value of $\theta(T = 0) = 115 \text{ K}$ was deduced.

Figure 11 shows the extrapolated phonon spectrum for solid hydrogen with $c = 0.68$ and $T = 4.3 \text{ K}$. For comparison, we have included the phonon spectrum by Mertens and Biem for hcp para hydrogen. This calculation was carried up to neighbours of the second order.

Both spectra agree in the number and the relative position of the peaks. The

width of the calculated spectrum is smaller than the measured one. This discrepancy may vanish by inclusion of neighbours of higher order in the calculation. The width of the theoretical spectrum was found to be 8.1 meV with one shell and 9.4 meV with two shells of neighbours. Another discrepancy is the last peak.

There may be no difference between ortho- and para hydrogen in the spectrum of the lattice vibrations above the λ -point. The quadrupole quadrupole forces may cancel each other, because the molecules are not very much orientated.

Figure 12 shows the dispersion curves for hcp para hydrogen by Mertens and Biem. The calculation has carried out in the Random Phase Approximation.

Figure 13 shows the specific heat calculated from our extrapolated phonon spectrum. For comparison, we have included the experimental $C_v(T)$ values by Ahlers¹⁷⁾. These measurements were carried out on para hydrogen. The values by Ahlers were converted to a molar volume corresponding to our measurement of $22.68 \text{ cm}^3/\text{mol}$. The values calculated from the frequency distribution are higher than the experimental ones over the entire range of temperature. This deviation may be due to insufficient knowledge of the frequency distribution at low energy transfer. However, the experimental values by Ahlers result in a very high Debye temperature of $\Theta(T=0) = 127 \text{ K}$.

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Figure captions

Fig. 1 Scattering process, schematically

Fig. 2 Rotating crystal time-of-flight spectrometer at the FR2

Fig. 3 Time-of-flight spectra $E_0 = 21.75$ meV, $c = 0.68$, $\mathcal{J} = 55.2$

Fig. 4 $0 \rightarrow 1$ Rotational lines $c = 0.27$

Fig. 5 $0 \rightarrow 1$ Rotational lines $c = 0.68$

Fig. 6 Table

Fig. 7 Experimental arrangement with high energy resolution

Fig. 8 Photograph of the high resolution arrangement

Fig. 9 Ortho molecule pair spectrum

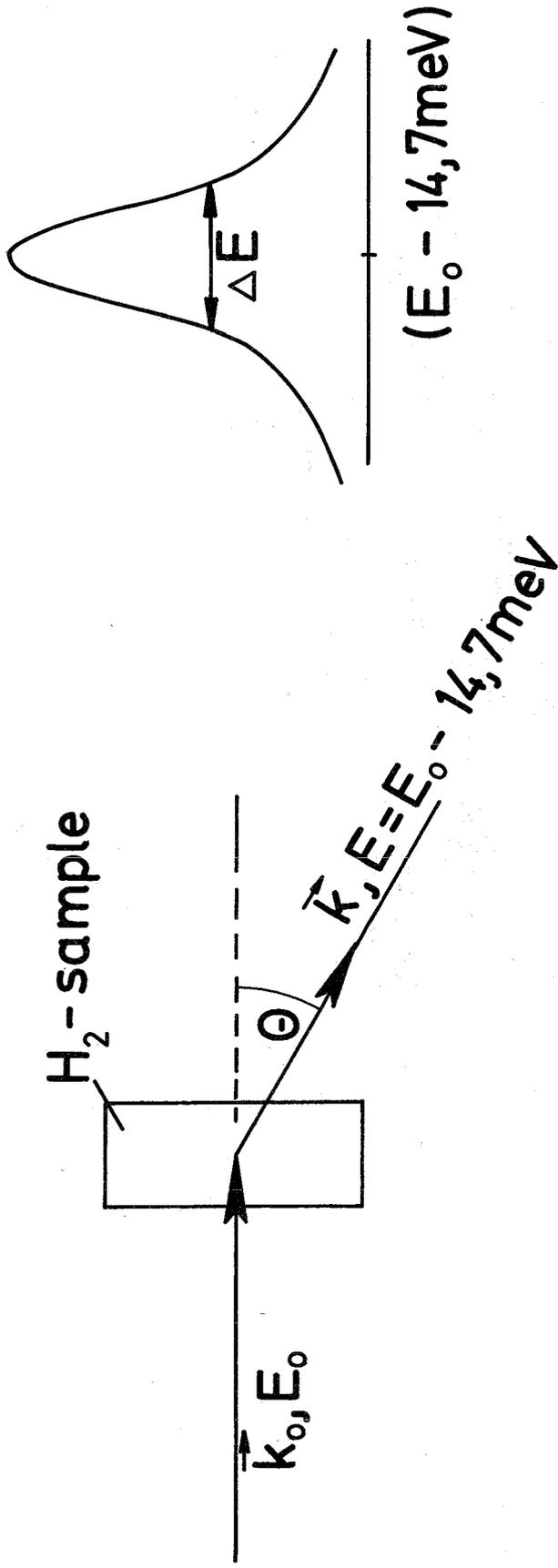
Fig. 10 $S(\alpha, \beta)/\alpha$ of solid hydrogen $c = 0.68$

Fig. 11 Phonon spectrum of solid hydrogen $c = 0.68$

Fig. 12 Phonon dispersion curves of para hydrogen in the hcp phase

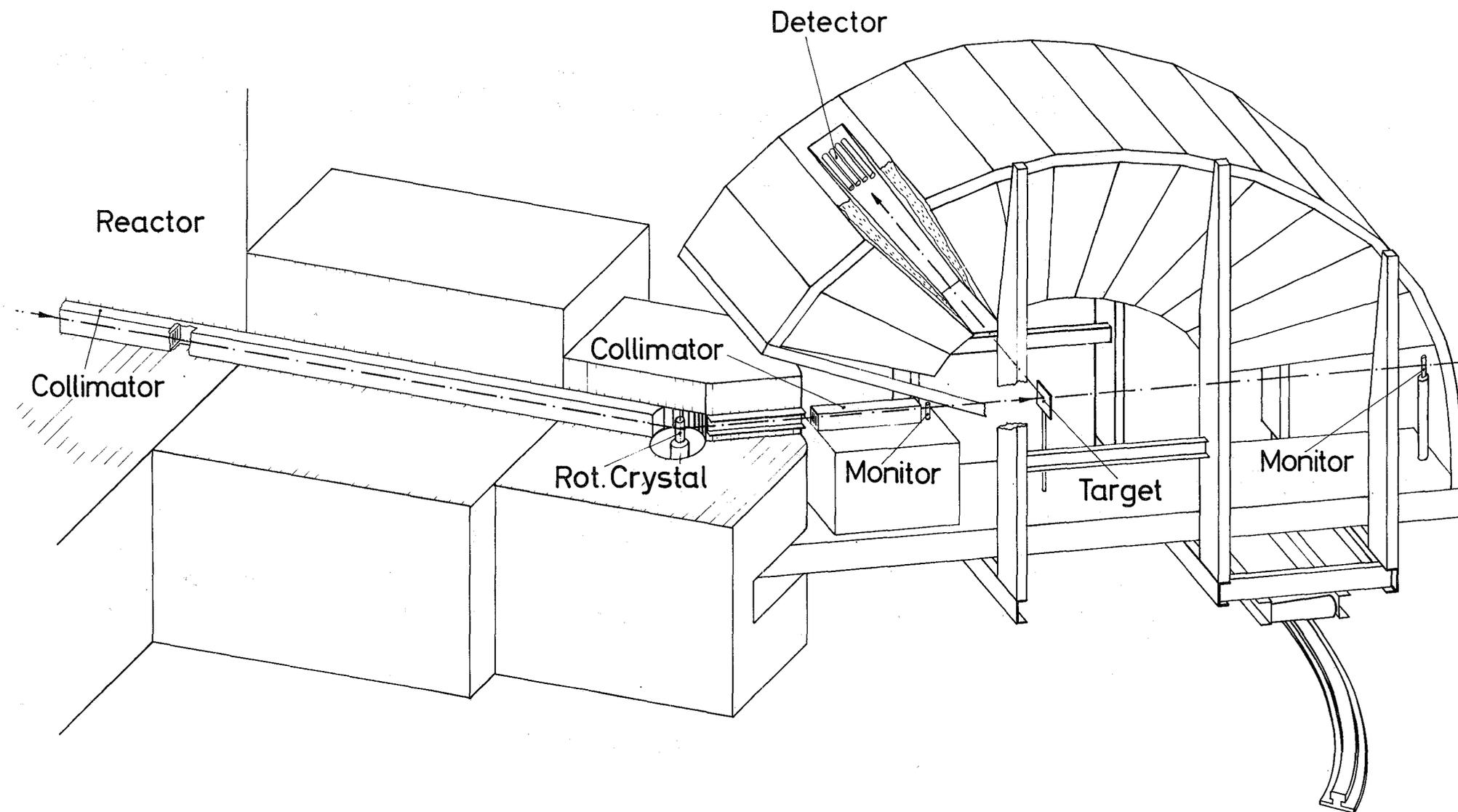
Fig. 13 Specific heat of solid hydrogen $c = 0.68$

0→1 Rotational Line



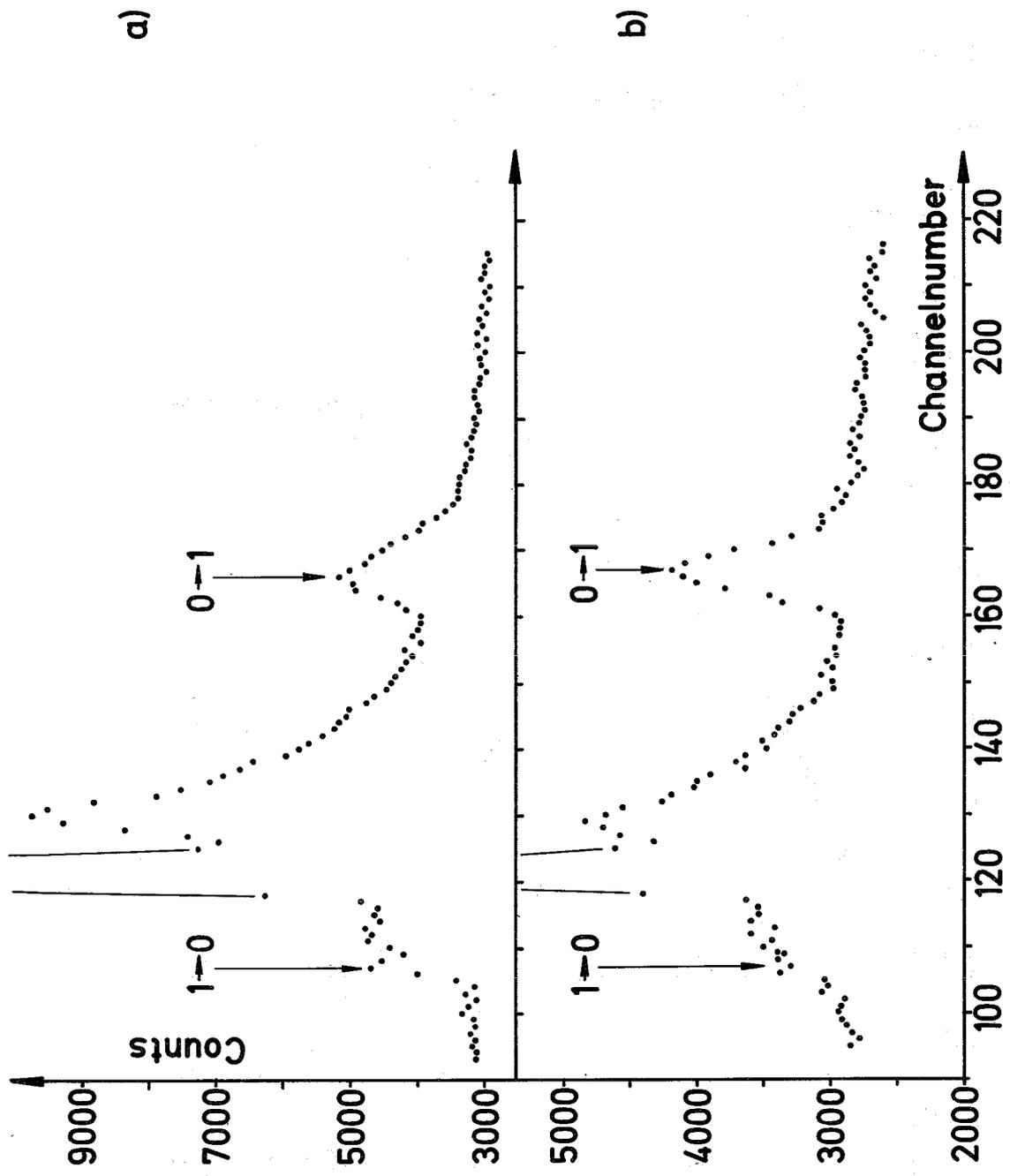
$$\left(\frac{d^2 G}{d\Omega d\varepsilon} \right)_{0 \rightarrow 1} \approx (1-c) \cdot b_{\text{inc}}^2 \cdot \frac{k}{k_0} \cdot 6j_1^2 \left(\frac{\kappa_{0 \rightarrow 1} \cdot a}{2} \right) e^{-\frac{3\hbar^2 \kappa_{0 \rightarrow 1}^2}{4M\Theta_D}} \cdot \frac{(\varepsilon - E_{J=1})^2}{\sqrt{2\pi \Delta\varepsilon^2}}$$

Fig.1



Rotating crystal time-of-flight spectrometer at the FR2

Fig. 2



Time of Flight Spectra $E_0=21.75\text{meV}$; $T=4.3^\circ\text{K}$; $c=0.68$;
 $\nu=55.2^\circ$

Fig.3

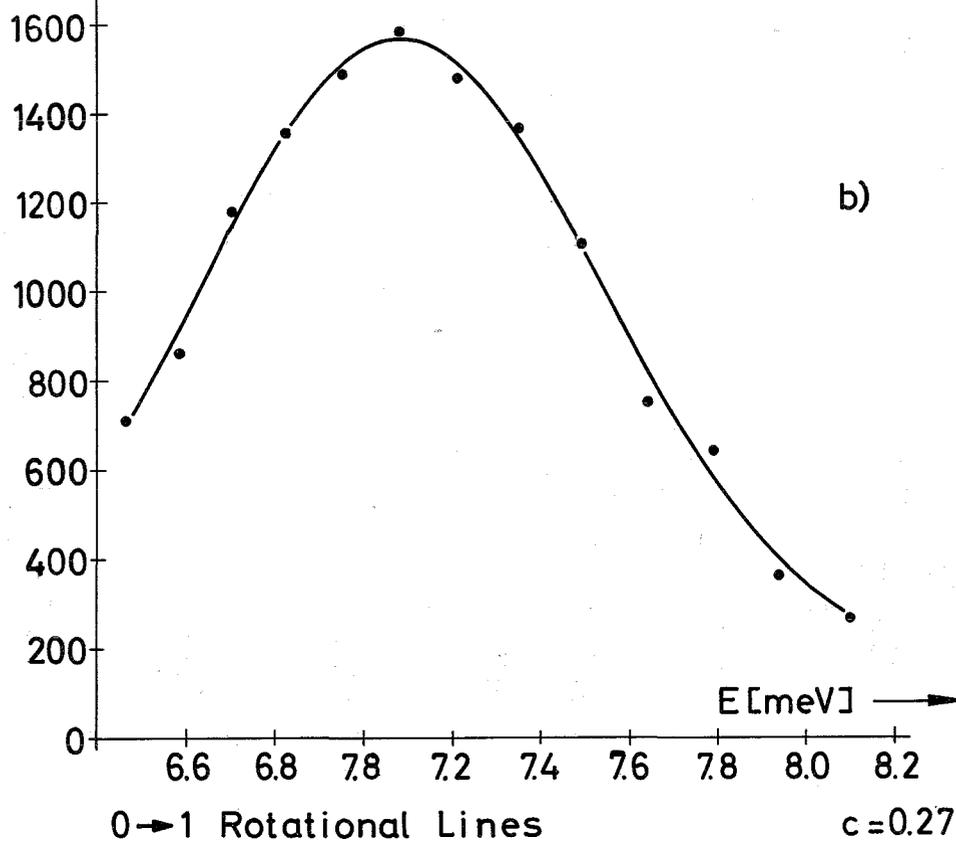
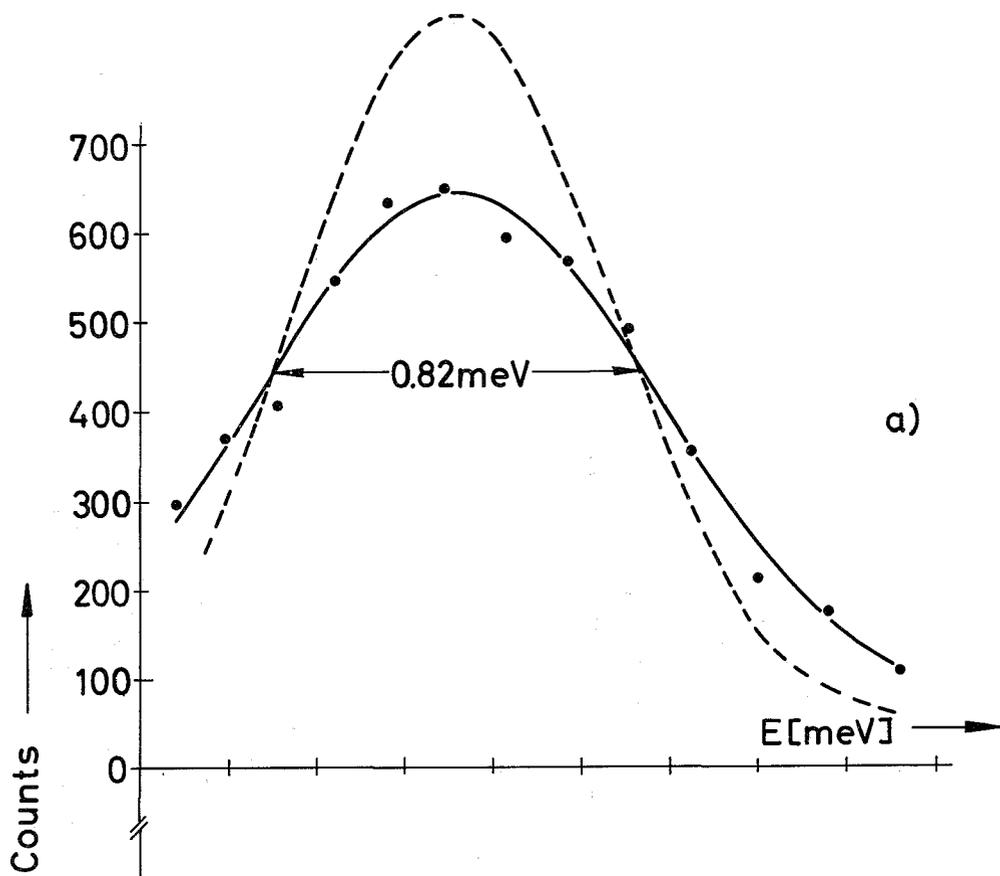


Fig. 4

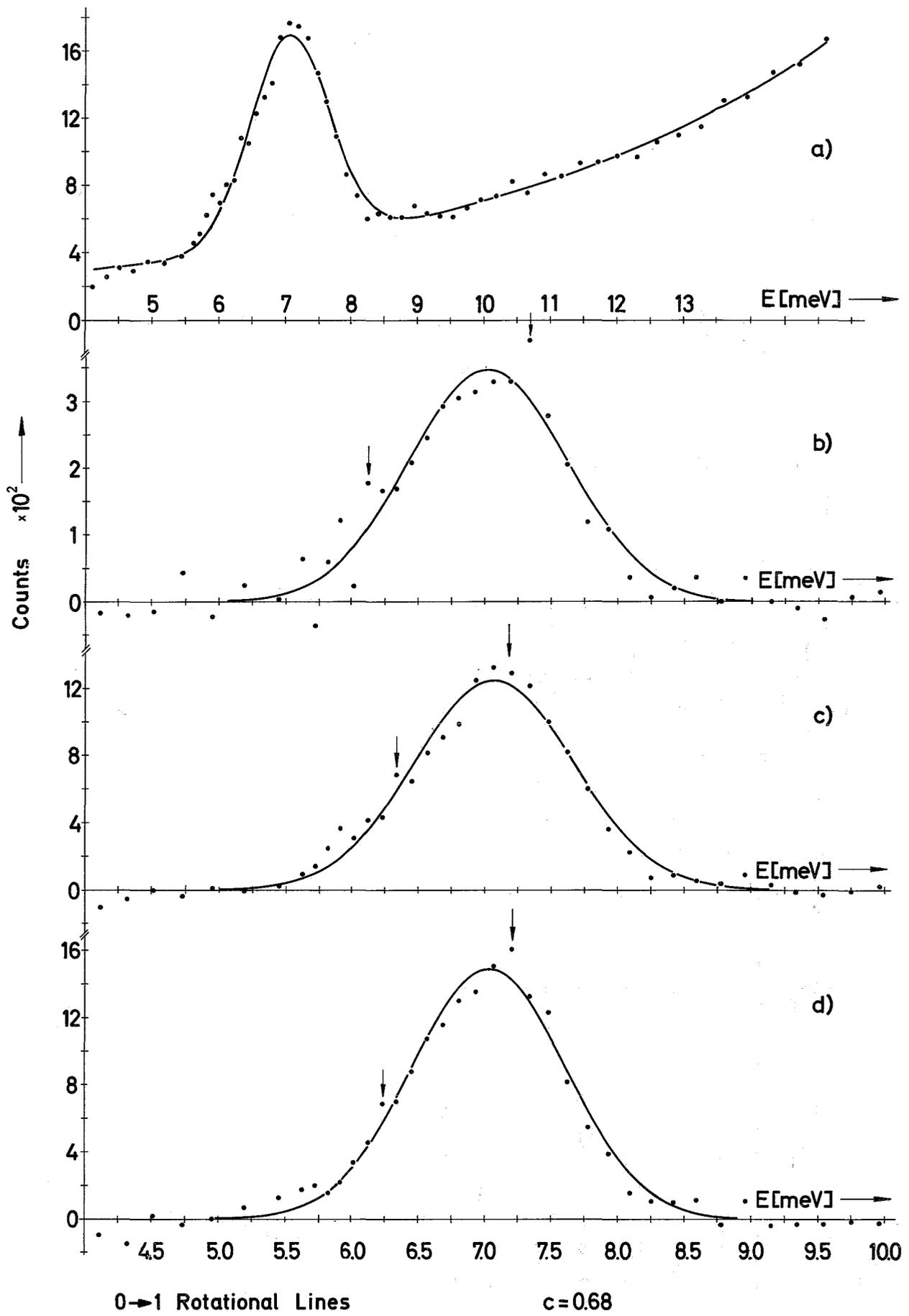


Fig. 5

Table

c	$\kappa^2 [A^{-2}]$	$\Delta E_{\text{App}} [\text{meV}]$	$\theta [^\circ]$	$\Delta E_{0 \rightarrow 1} [\text{meV}]$	$\overline{\Delta E_{0 \rightarrow 1}} [\text{meV}]$	Γ	$\frac{\Gamma}{\Gamma_0}$
0.27	5.28	0.71	43.9	0.82 ± 0.07			
0.27	7.07	0.71	55.2	0.79 ± 0.07	0.80	0.0667	0.77
0.68	3.82	0.66	32.5	1.24 ± 0.07			
0.68	5.28	0.66	43.9	1.23 ± 0.07	1.22	0.0641	0.74
0.68	7.07	0.67	55.2	1.18 ± 0.07			

$\Gamma_0 = 0.0866 \text{ meV}$

Fig.6

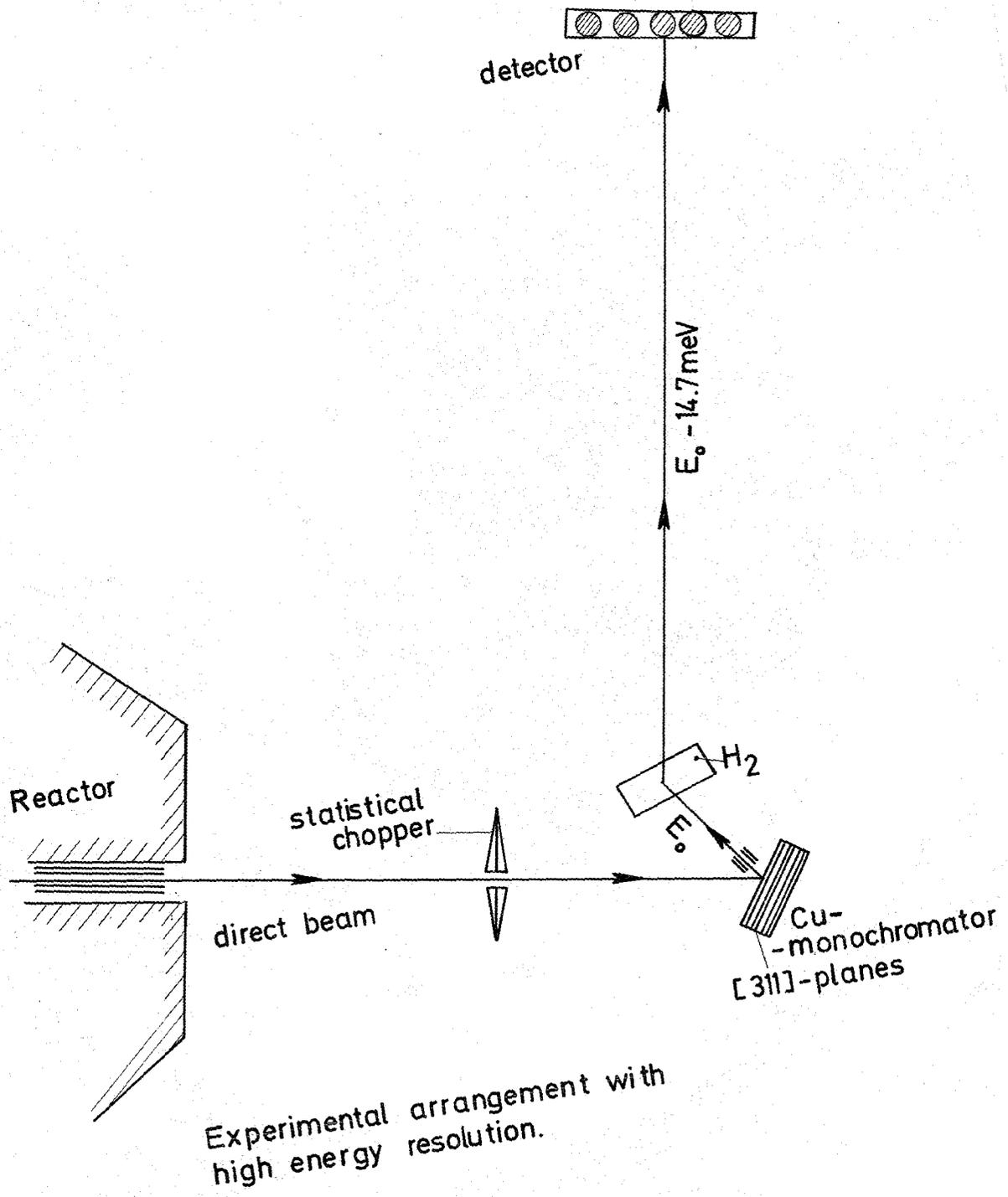


Fig.7

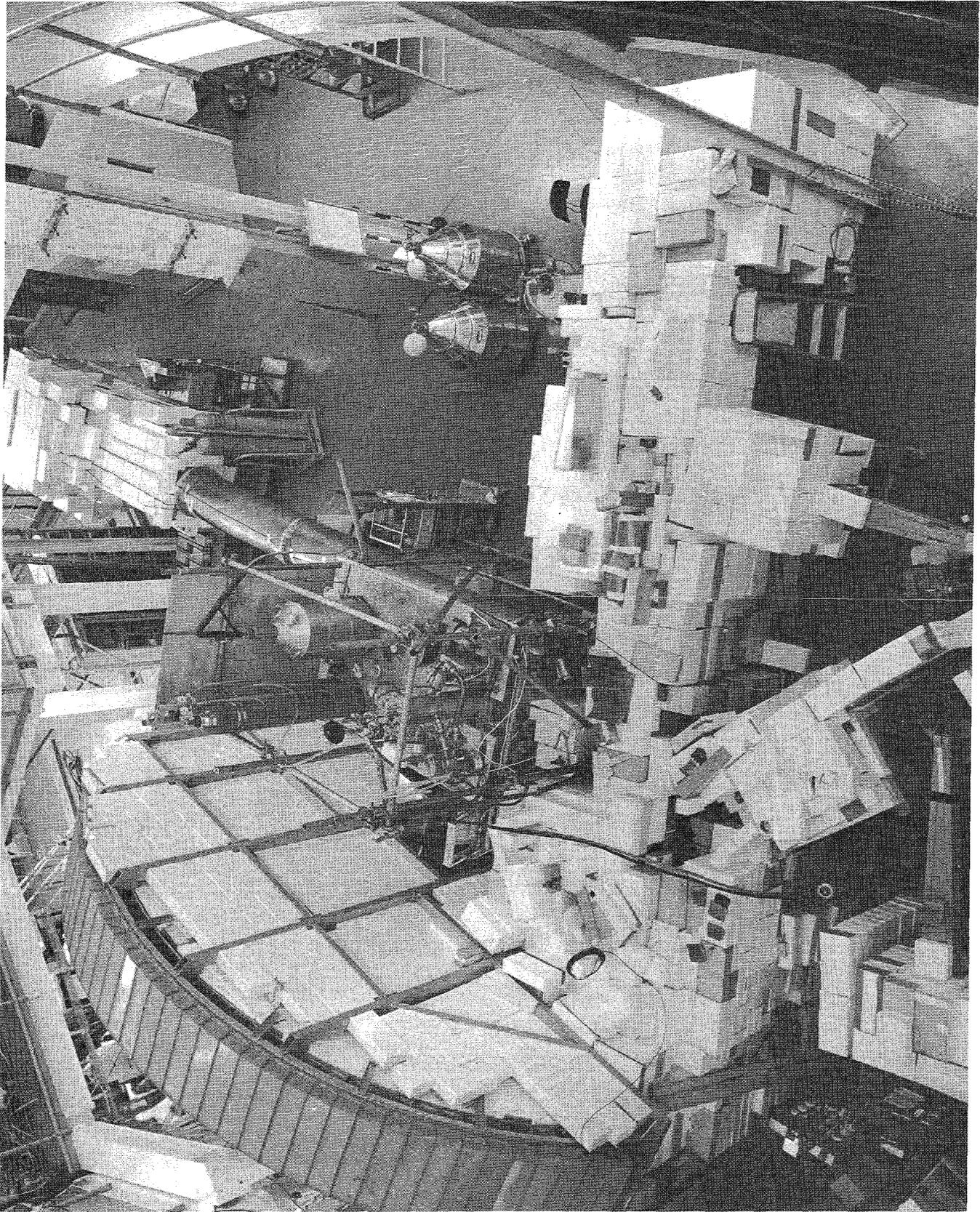


Fig. 8

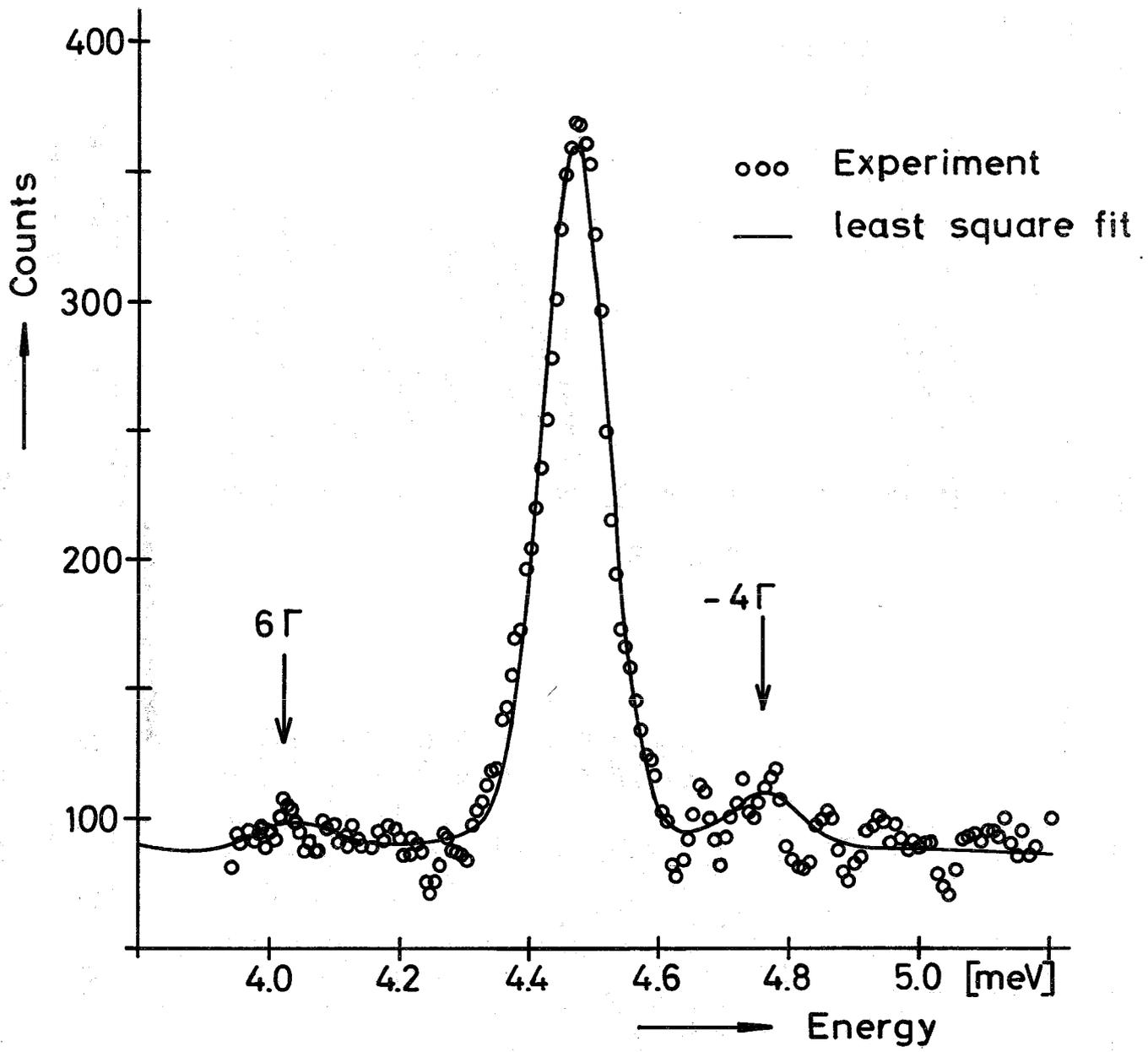
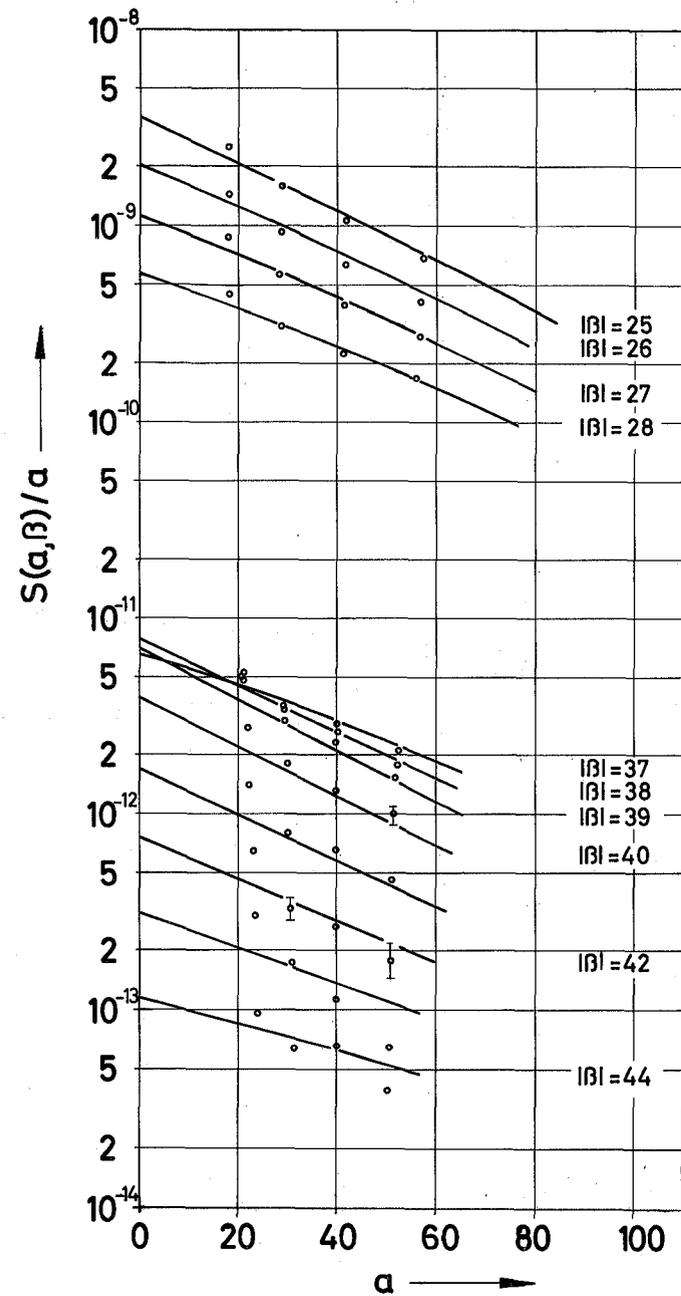
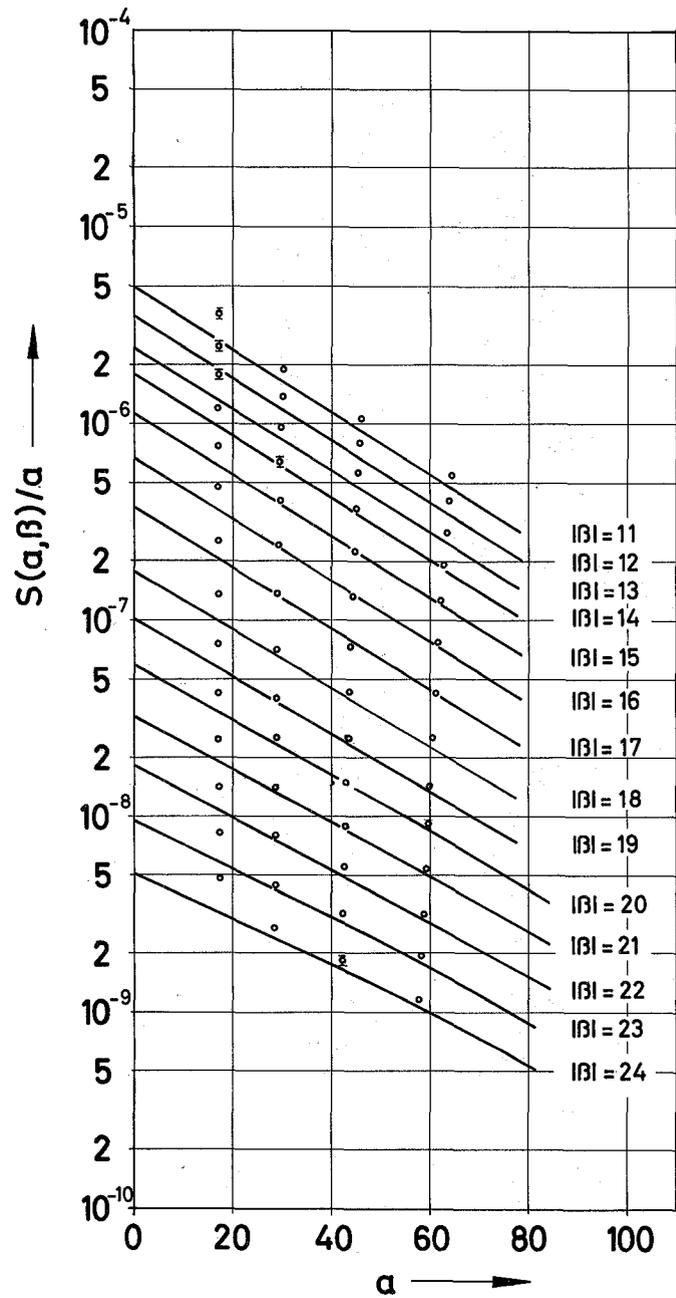
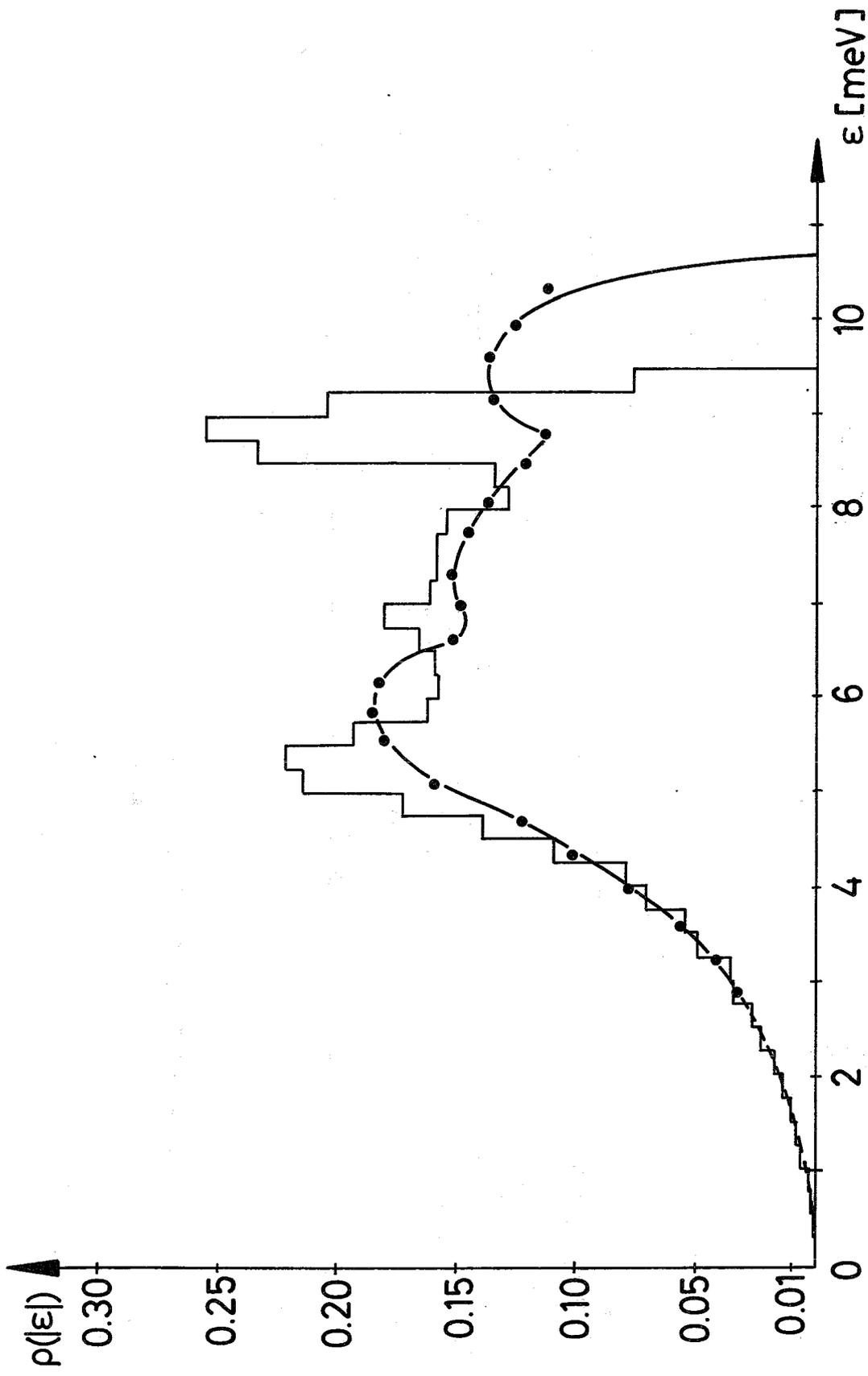


Fig.9

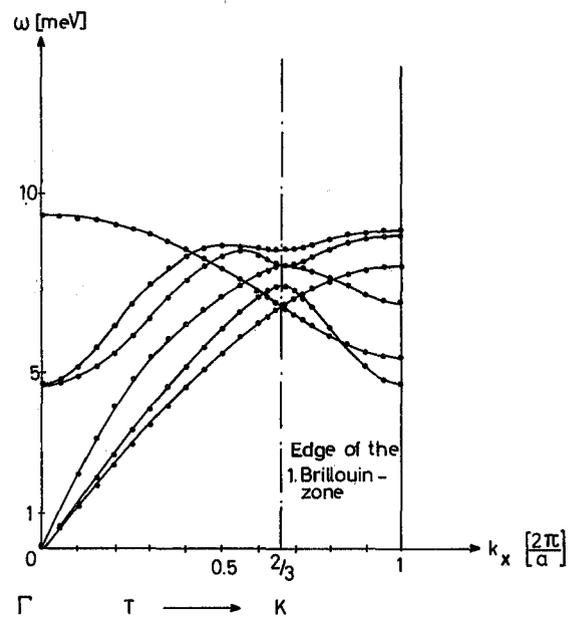
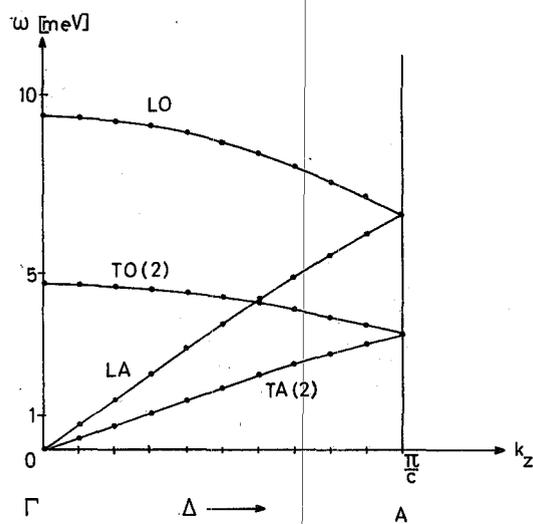
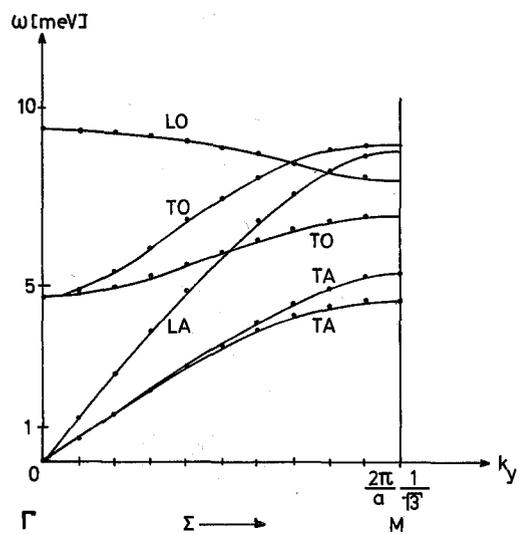
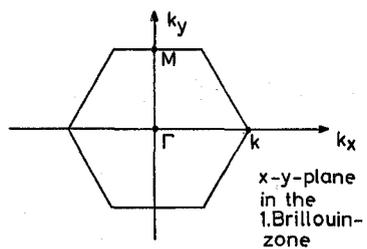


$S(a,\beta) / a$ of Solid Hydrogen, $c = 0.68$; $T = 4.3^\circ\text{K}$
 Fig.10



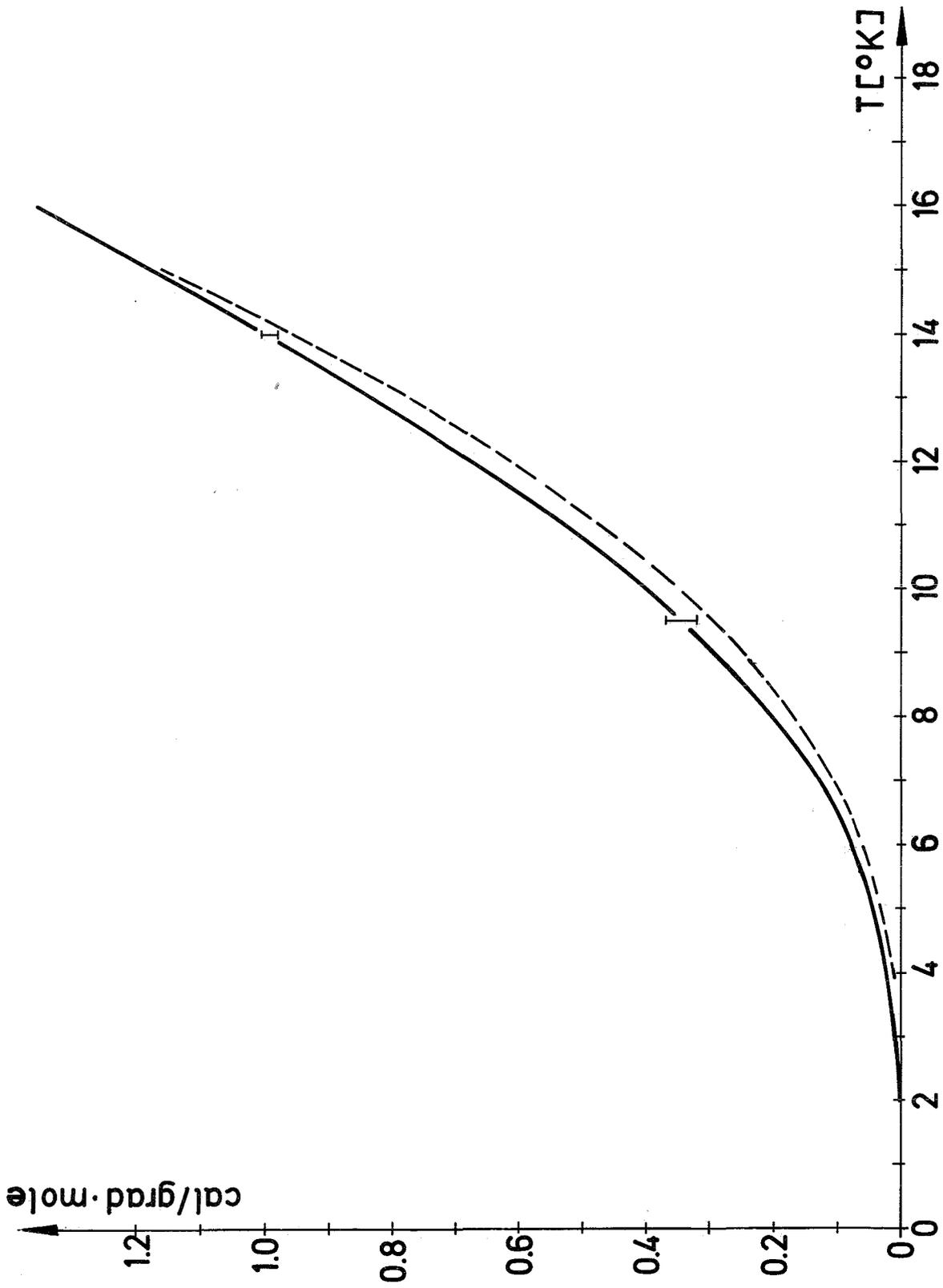
Phononspectrum of Solid Hydrogen. $c=0.68$ $T = 4.3 \text{ }^\circ\text{K}$

Fig.11



PHONON-DISPERSION CURVES OF PARA - H₂ (HCP)

Fig.12



Specific Heat $C_v(T)$ of Solid Hydrogen $c=0.68$

Fig.13

