

KERNFORSCHUNGSZENTRUM KARLSRUHE

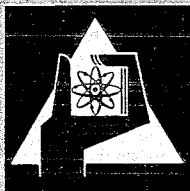
August 1969

KFK 1096

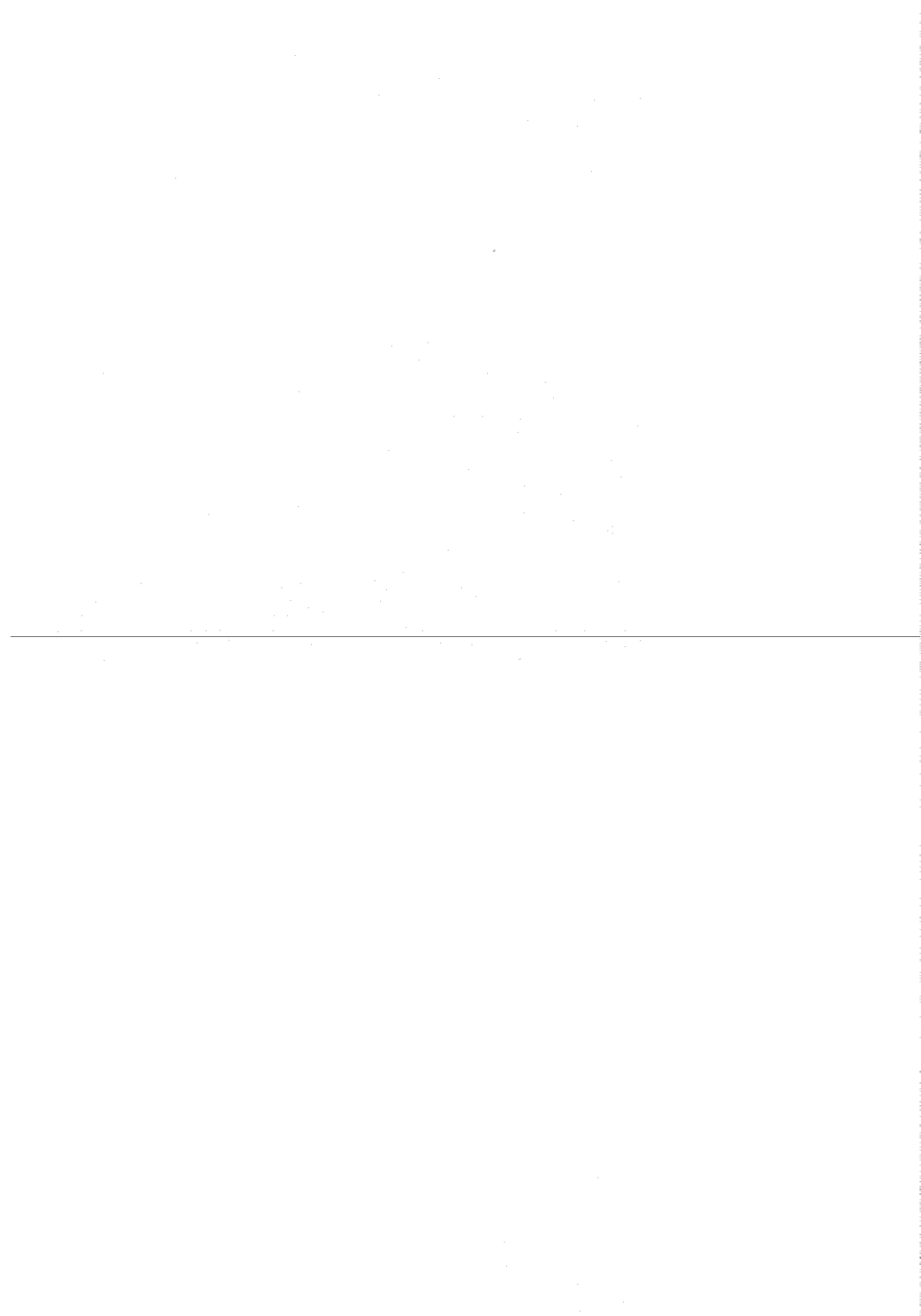
Institut für Angewandte Kernphysik

Band Mixing in Deformed Odd-Mass Nuclei

W. Michaelis, F. Weller, H. Ottmar, U. Fanger, R. Gaeta, H. Schmidt



GESELLSCHAFT FÜR KERNFORSCHUNG M
KARLSRUHE



Reprint from
"NEUTRON CAPTURE
GAMMA-RAY SPECTROSCOPY"

1950

THE UNIVERSITY OF CHICAGO
LIBRARY

BAND MIXING IN DEFORMED ODD-MASS NUCLEI

W. MICHAELIS, F. WELLER, H. OTTMAR,
U. FANGER, R. GAETA*, H. SCHMIDT
Institut für angewandte Kernphysik,
Kernforschungszentrum Karlsruhe,
Karlsruhe, Federal Republic of Germany

Abstract

BAND MIXING IN DEFORMED ODD-MASS NUCLEI. The level structure of several deformed odd-mass nuclei has been studied at the Karlsruhe research reactor FR-2 using radiative capture of thermal neutrons. High-resolution measurements of the gamma-ray spectrum have been performed with a Ge(Li) anti-Compton spectrometer and a Ge(Li) pair spectrometer. The high accuracy of the data allows the application of Ritz' combination principle up to energies above 1,5 MeV. Coincidence relationships have been obtained by means of a Ge(Li)-NaI(Tl) coincidence system. Detailed data are given for the transition diagram of ^{167}Er . The experiments reveal the occurrence of considerable band mixing. A theoretical treatment is presented within the framework of the unified model. The calculations take into account pair correlation, quasiparticle-phonon interaction, Coriolis coupling and rotation-vibration interaction. Various properties of the deexcitation mechanism are determined to a large extent by band mixing. Examples are given for branching ratios and partial gamma-ray half-lives both in ^{167}Er and ^{169}Yb .

1. Introduction

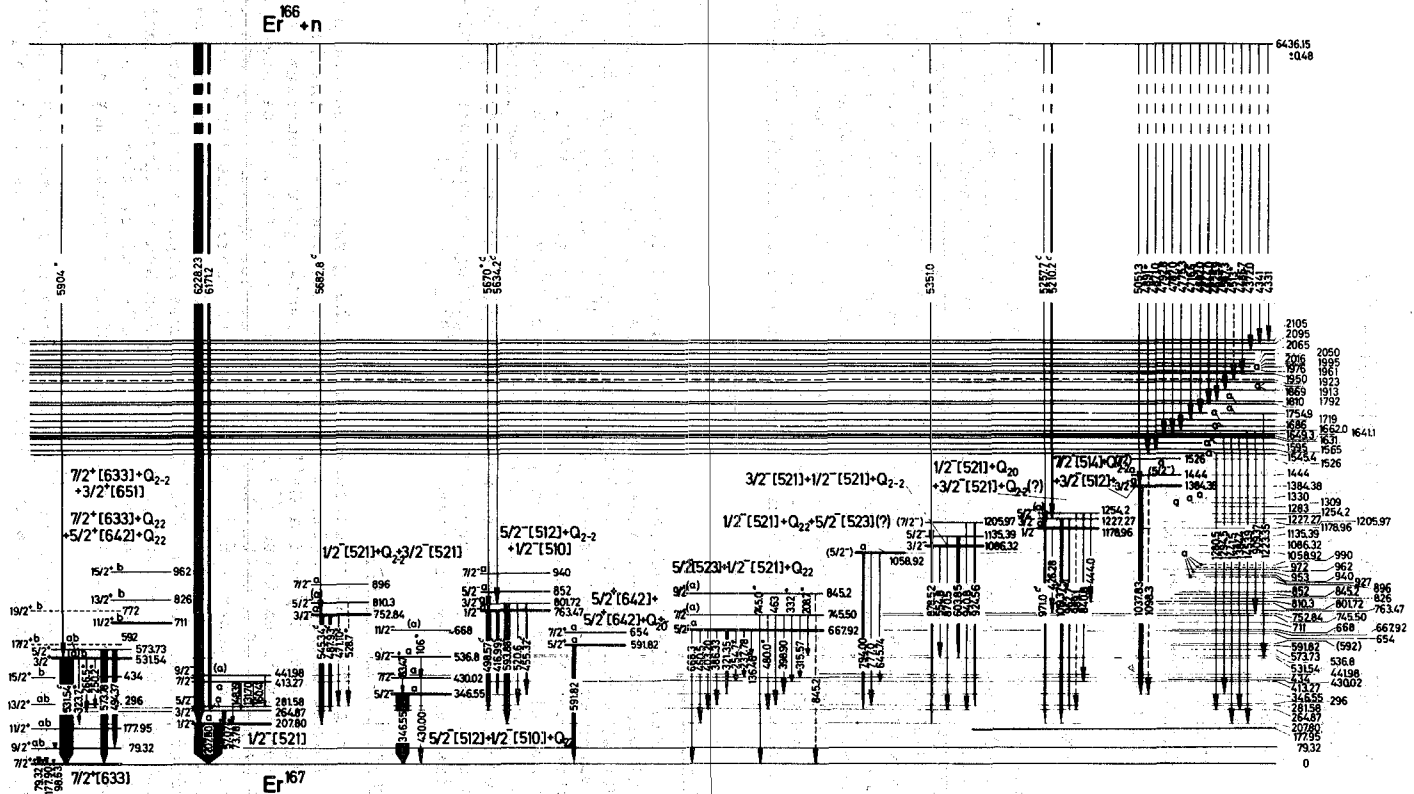
The rapid development of experimental techniques in gamma-ray spectroscopy has made feasible detailed investigations of nuclear structure by means of the radiative neutron capture process. In a recent paper on thermal neutron capture in ^{168}Yb we have pointed out the occurrence of considerable band mixing effects in ^{169}Yb [1]. Similar results have also been published on ^{165}Dy [2]. The purpose of this presentation is to provide a brief summary of experimental and theoretical data obtained for the isotonic nucleus ^{167}Er and to discuss some obvious consequences of band mixing on branching ratios and partial gamma-ray halflives both in ^{167}Er and ^{169}Yb .

2. Experimental Procedure

High-resolution measurements of the gamma-ray spectrum have been performed using a Ge(Li) anti-Compton spectrometer in the low-energy region [3, 4] and a Ge(Li) five-crystal pair spectrometer for the high-energy transitions [2]. Coincidence relationships have been studied by means of a Ge(Li)-NaI(Tl) coincidence system [5] coupled to an on-line computer [6]. The energy calibration is based on the decay lines of ^{192}Ir , ^{137}Cs , ^{60}Co and ^{88}Y and on capture lines from the product nuclei ^2H and ^{15}N . The procedures applied in spectrum stabilization, spectrum analysis, calibration and nonlinearity correction are described in detail in Ref. [4]. A brief discussion of the technique used for analysing complex coincidence spectra may be found in Ref. [7].

The measurements on ^{167}Er have been performed with a sample of Er_2O_3 enriched to 95.6 % in ^{166}Er . In spite of this relatively

* Visiting scientist from Junta de Energía Nuclear, Ciudad Universitaria, Madrid.

FIG. 1. Transitions of ^{167}Er .

high enrichment care had to be taken in the isotope assignment of the gamma-ray lines. Due to the unfavourable cross section, the capture contribution of ^{166}Er was only 68.8 %. The main interference resulted from the target nucleus ^{167}Er which contributed with 31.1 % to the total capture cross section. Neutron capture in other erbium isotopes was negligible. In order to obtain a reliable isotope assignment each run was repeated with a target of natural erbium. More than 350 gamma-ray lines have been observed from the enriched sample.

The data were also examined for the possibility of contributions from likely chemical contaminants and, in fact, several lines were identified as arising from Sm and Gd.

3. Transition Diagram of ^{167}Er

As yet no exhaustive investigation of the ^{167}Er neutron capture transition diagram has been performed and thus the information about the deexcitation mechanism was still limited. Most of the available data have come from studies with the Risø bent crystal spectrometer [8] and with a Ge(Li) pulse-height spectrometer [9]. In the first study 47 lines of the low-energy spectrum were assigned to ^{167}Er and 24 transitions were fitted into a level scheme. The second work examined the high-energy spectrum using a sample of very high enrichment (99.97 %). The most recent investigations of ^{167}Er levels by means of charged-particle reactions are those in Refs. [10] and [11]. The decay of ^{167}Ho (3 h) and ^{167}Tm (9.6 d) to ^{167}Er has recently been studied with Ge(Li) and scintillation counters both in single and coincidence mode [12].

The present research suggests a considerably extended transition diagram as represented in Fig. 1. The intensities of the gamma rays are expressed by the arrow width. Most of the lines were fitted into the diagram using the Ritz combination principle. Transitions marked with the letter "c" have been clearly observed in coincidence measurements and their position in the decay scheme is well established. Dashed arrows mean that the available data suggest the existence of these gamma rays and the position shown, but the assignment is considered to be somewhat tentative. Transitions labelled with an asterisk have been adopted from previous investigations provided that the assignment is consistent with the present study. For the sake of completeness the results obtained from charged-particle reactions have also been included in the decay scheme. Levels labelled with the letter "a" have also been observed in (d,p) reactions [10]. Letter "b" indicates excited states detected in inelastic scattering of deuterons [11].

Analysis of the experimental data and results of theoretical calculations described in section 4 suggest the following spectroscopic interpretation (bandhead energies and dominant structure):

0 keV	$7/2^+$ [633]	90 %	$7/2^+$ [633] + Q_{20}	6 %
208 keV	$1/2^-$ [521]	92 %		
347 keV	$5/2^-$ [512]	86 %	$1/2^-$ [510] + Q_{22}	11 %
532 keV	$7/2^+$ [633] + Q_{2-2}	81 %	$3/2^+$ [651]	15 %
592 keV	$5/2^+$ [642]	79 %	$5/2^+$ [642] + Q_{20}	12 %
	$3/2^+$ [651]	7.5 %		

668 keV	$5/2^- [523]$	81 %	$1/2^- [521] + Q_{22}$	16 %
711 keV	$7/2^+ [633] + Q_{22}$	94 %	$5/2^+ [642] + Q_{22}$	5 %
753 keV	$1/2^- [521] + Q_{2-2}$	61 %	$3/2^- [521]$	37 %
763 keV	$5/2^- [512] + Q_{2-2}$	56 %	$1/2^- [510]$	38 %
1059 keV	$1/2^- [521] + Q_{22}$	80 %	$5/2^- [523]$	15 % (?)
1086 keV	$3/2^- [521]$	60 %	$1/2^- [521] + Q_{2-2}$	37 %
1179 keV	$1/2^- [521] + Q_{20}$	93 %	$3/2^- [521] + Q_{2-2}$	4 % (?)
1384 keV	$3/2^- [512]$	41 %	$7/2^- [514] + Q_{2-2}$	38 %
	$1/2^- [510] + Q_{2-2}$	16 %		

A detailed analysis and discussion of the level structure will be given elsewhere [13].

4. Theoretical Considerations

In order to arrive at a better understanding of the level structure, theoretical calculations have been performed which take into account pair correlation, quasiparticle-phonon interaction, rotation-vibration interaction and Coriolis coupling. Since a solution with the exact Hamiltonian is beyond the possibilities of present theoretical nuclear physics, it is convenient to use a phenomenological approach. Such a procedure is justified, if only few additional parameters are introduced and an extensive set of data is predicted which can be examined experimentally. The calculations that will be outlined here very briefly reveal the energy and structure of individual levels, absolute transition rates and partial gamma-ray half-lives, multipolarity admixtures and branching ratios. They use a generally valid set of single-particle energies. The BCS treatment is performed with the constant $G_n = 0.021 \hbar \omega_0 = 26/A \text{ MeV}$ ($\hbar \omega_0 = 41 A^{-1/3} \text{ MeV} = 7.445 \text{ MeV}$). The parameter $\xi = \hbar/J_0$ is fitted where J_0 is the undisturbed moment of inertia. Excitation energies for the β and γ quadrupole vibrations are taken from the neighbouring even nuclei.

The Hamiltonian is written as [14, 15]

$$H = H_0 + H'$$

with

$$H_0 = H_N + H_{OC}$$

Here H_N is the Nilsson Hamiltonian [16] and H_{OC} describes the undisturbed collective motion, i.e. the rotation of the nucleus and the β and γ vibrations. The interactions between the various modes of motion are taken into account by H' . This term includes the Coriolis coupling, the particle-phonon interaction and the rotation-vibration interaction.

Neglect of pair correlations leads to unreasonable matrix elements and serious disagreement with experimental results. Therefore

$$H_N = \sum_{\nu} \epsilon_{\nu} a_{\nu}^{\dagger} a_{\nu}$$

has to be replaced by

$$H_{QP} = \sum_{\nu} E_{\nu} \alpha_{\nu}^{\dagger} \alpha_{\nu}$$

with the usual notation

$$E_{\nu} = [(\epsilon'_{\nu} - \lambda)^2 + \Delta_{\nu}^2]^{1/2}$$

and

$$\epsilon'_{\nu} = \epsilon_{\nu} - G V_{\nu}^2$$

ϵ_{ν} and E_{ν} denote the particle and quasiparticle energies, respectively. The operators a and α are correlated via the Bogolyubov-Valatin transformation. The coefficients V_{ν} and U_{ν} ($V_{\nu}^2 + U_{\nu}^2 = 1$) are determined separately for each state from the well-known variational problem. Thus the blocking effect is fully taken into account. All quasiparticle matrix elements can be reduced to particle matrix elements. In many cases the pairing factors reduce the matrix elements considerably.

For calculating the structure of ^{167}Er and ^{169}Yb eleven Nilsson orbits near the Fermi level together with their γ^+ , γ^- and β vibrational bands have been taken as a basis. The contribution of each of these configurations to the structure of the individual levels results from the diagonalization. More details of the theoretical treatment may be found in Refs. [1] and [13].

In Fig. 2 the calculated excitation energies for ^{167}Er are compared with the experimental level scheme. In general, the agreement is surprisingly good. Some obvious consequences of band mixing on other nuclear properties are discussed in the following sections.

5. Branching Ratios

The occurrence of strong band mixing is responsible for various phenomena which cannot be explained within the framework of simple models. It is beyond the scope of this presentation to give a detailed discussion here. As an example let us select the deexcitation of the $1/2^-$ rotational band observed at 763 keV in ^{167}Er (cf. Fig. 1). The only Nilsson state with spin and parity $1/2^-$ near the Fermi level is the orbit $1/2^- [521]$. This state, however, is well established to occur at 208 keV excitation energy. Thus it is reasonable to assume that the band at 763 keV corresponds to the γ^- vibrational band based upon the configuration $5/2^- [512]$ and, in fact, a collective E2 transition leaving the bandhead is observed. In other respects, however, the deexcitation shows clear anomalies. From both the first and second member of the band transitions proceed to the $1/2^- [521]$ Nilsson band which in intensity considerably exceed the E2 transitions to the "own" intrinsic configuration. In addition, the branching ratio to the $3/2^-$ and $1/2^-$ levels is exceptional. A reasonable explanation for these anomalies is provided by assuming a strong admixture of the $1/2^- [510]$ Nilsson state in the $5/2^- [512] + Q_{2-2}$ excitation. It is true that the $1/2^- [510]$ orbit is expected at much higher energies, but it is connected with the $5/2^- [512]$ orbit by a strong E2 matrix element. As mentioned above an admixture of 38 % is predicted by the calculations described in section 4. On the other hand, the rotational band at 208 keV consists only to 92 % of the Nilsson state $1/2^- [521]$.

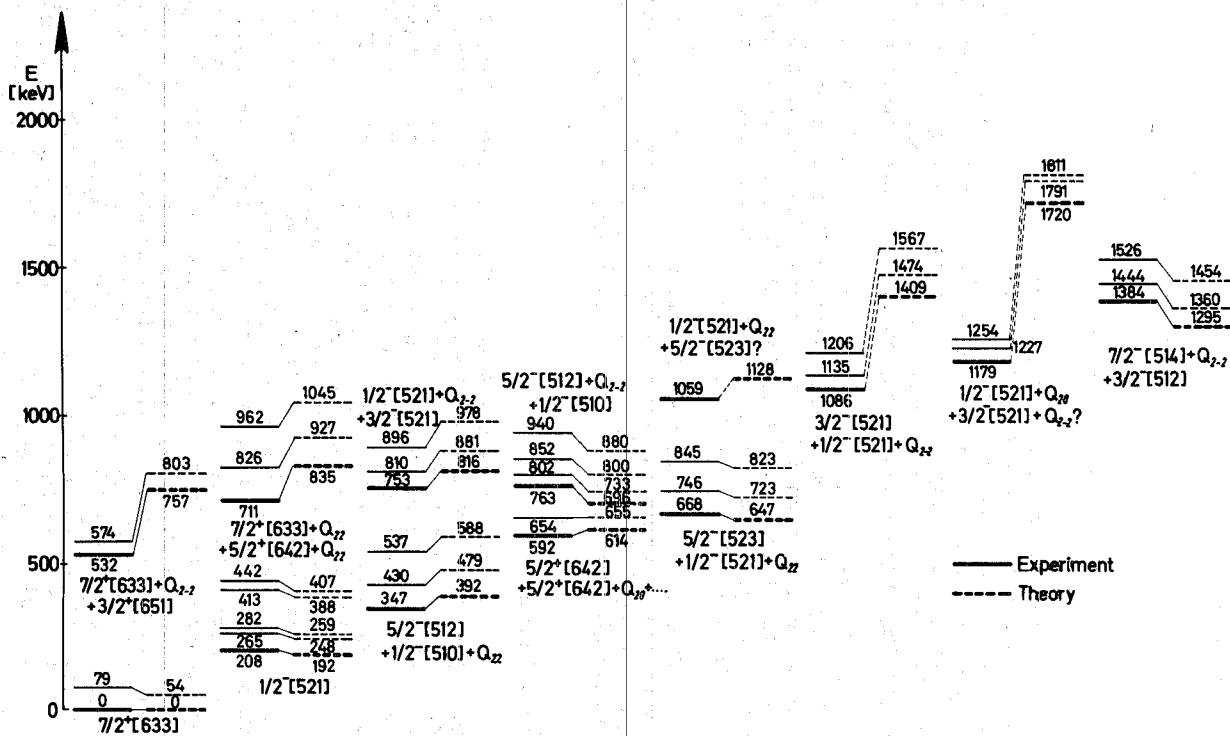


FIG. 2. Comparisons of experimental and theoretical excitation energies in ^{167}Er .

Table I

Transitions from the band
 $5/2^- [512] + 0_{2-2} + 1/2^- [510]$
 ^{167}Er

Intensity ratio	Assuming a pure Nilsson band	Assuming a collective nature		Exp.
		$H' = 0$	$H' \neq 0$	
$\frac{I(1/2 \ 1/2 \rightarrow 3/2 \ 1/2)}{I(1/2 \ 1/2 \rightarrow 1/2 \ 1/2)}$	7.0	-	1570	> 15
$\frac{I(1/2 \ 1/2 \rightarrow 3/2 \ 1/2)}{I(1/2 \ 1/2 \rightarrow 5/2 \ 5/2)}$	-	0	6.1	3.6 ± 0.4

Table II

Partial gamma-ray halflives $T_{1/2 \gamma}$

Nucleus	Initial con-figuration	Final con-figuration	Multi-polar-ity	$\frac{2}{\sigma_{\text{theor.}}}$	$T_{1/2 \gamma} [\text{nsec}]$		
					$H' = 0$	$H' \neq 0$	Exp.
^{167}Er	$1/2^- 1/2 [521]$	$7/2^+ 7/2 [633]$	E3	-	3.6 sec	4.3 sec	5.5 sec^b
	$3/2^- 1/2 [521]$	$1/2^- 1/2 [521]$	M1+E2	0.74	13.5	23.5	39.4^a
	$5/2^- 5/2 [512]$	$7/2^+ 7/2 [633]$	E1	-	0.07	0.1	1.4^c
^{169}Yb	$5/2^- 5/2 [512]$	$7/2^+ 7/2 [633]$	E1	-	0.1	0.3	1.3^a 4.2^b
		$1/2^- 1/2 [521]$	E2	∞	21 000	630	102^a 320^b
		$3/2^- 1/2 [521]$	M1+E2	0.055	190 000 (M1: K-forb)	135	103^a 260^b
		$5/2^- 1/2 [521]$	M1+E2	0.009	630 000 (M1: K-forb)	110	84^a 220^b

^a Ref. [17] ^b Ref. [18] ^c Ref. [12]

The influence of the interaction term H' in the Hamiltonian is demonstrated in Table I. Both the assumption of a pure Nilsson state and a pure vibrational wave function reveal disagreement with the experimental data while the band mixing resulting from H' provides a reasonable explanation for the observed anomalies. Obviously, the transition to the $1/2^-$ state is strongly retarded compared to that reaching the $3/2^-$ level and, in fact, in none of the hitherto existing investigations of the nuclei ^{165}Dy , ^{167}Er and ^{169}Yb such a transition has been detected. When judging the theoretical branching ratio in the second line of Table I, one should notice that we are not dealing with a simple procedure such as that applied in the Alaga rule, but that the nuclear wave functions of three states directly enter into the result.

6. Partial Gamma-Ray Halflives

In general, the low-lying excited states are expected to be essentially characterized by pure wave functions and, in fact, only small admixtures are predicted for these states by the calculations described in section 4. Nevertheless such small admixtures may have a decisive influence on the partial gamma-ray halflives. This is demonstrated in Table II where the theoretical values both for $H' = 0$ and $H' \neq 0$ are compared with experimental data. In all cases the inclusion of band mixing yields an improvement of the theoretical predictions. This is particularly conspicuous for K-forbidden $M1$ transitions.

7. Conclusions

Considerable band mixing effects occur in deformed odd-mass nuclei. Experimentally these phenomena can thoroughly be studied by means of the radiative neutron capture process. A better understanding can be obtained within the framework of the unified model, if the interaction between all possible modes of nuclear motion is taken into account. The inclusion of pair correlations is essential for achieving agreement with experimental data. Various properties of the deexcitation mechanism are determined to a large extent by band mixing, even for low-lying states where the admixtures are small.

References

- [1] W. Michaelis, F. Weller, H. Schmidt, G. Markus, and U. Fanger, Nuclear Physics A119 (1968) 609
- [2] G. Markus, W. Michaelis, H. Schmidt, and C. Weitkamp, Z. Phys. 206 (1967) 84
- [3] W. Michaelis and H. Küpfer, Nuclear Instr. and Meth. 56 (1967) 181
- [4] W. Michaelis and F. Horsch, contribution to this conference
- [5] U. Fanger, KFK 887 (1969)
- [6] G. Krüger and G. Zipf, KFK 371 (1965)
- [7] U. Fanger, R. Gaeta, W. Michaelis, H. Ottmar, and H. Schmidt, contribution to this conference
- [8] H.R. Koch, Zeitschrift für Physik 187 (1965) 450

- [9] W.V. Prestwich and R.E. Coté,
Phys. Rev. 162 (1967) 1112
- [10] R.A. Harlan and R.K. Sheline, Phys. Rev. 168 (1968) 1373
- [11] M. Rozkos, F. Sterba, J. Sterbova, B. Elbek, and P.O. Tjøm;
Int. Symp. on Nuclear Structure, Dubna, USSR, July 1968
- [12] L. Funke, W. Andrejtscheff, H. Graber, U. Hagemann,
K.-H. Kaun, P. Kemnitz, W. Meiling, H. Sodan, F. Stary,
and G. Winter, Nuclear Physics A 118 (1968) 97
- [13] W. Michaelis, F. Weller, H. Ottmar, U. Fanger, R. Gaeta,
G. Markus, and H. Schmidt, Nuclear Physics, to be published
- [14] A. Faessler, Nuclear Physics 59 (1964) 177
- [15] A. Faessler, Nuclear Physics 85 (1966) 679
- [16] S.G. Nilsson, Mat. Fys. Medd. Dan. Vid. Selsk. 29, No. 16 (1955)
- [17] H. Nabilek, SGAE PH-78/1968
- [18] K.E.G. Löbner and S.G. Malmkog, Nuclear Physics 80 (1966) 505

The first part of the paper is devoted to a general discussion of the problem of the structure of the group of automorphisms of a free group. The main result of this part is the following theorem:

Theorem 1. Let G be a free group of rank n and let Γ be a subgroup of G . Then the following conditions are equivalent:

- (1) Γ is a free group of rank n .
- (2) Γ is a free group of rank n and the natural map $\pi: G \rightarrow G/\Gamma$ is an isomorphism.

The second part of the paper is devoted to a study of the structure of the group of automorphisms of a free group. The main result of this part is the following theorem:

Theorem 2. Let G be a free group of rank n and let Γ be a subgroup of G . Then the following conditions are equivalent:

- (1) Γ is a free group of rank n .
- (2) Γ is a free group of rank n and the natural map $\pi: G \rightarrow G/\Gamma$ is an isomorphism.

The third part of the paper is devoted to a study of the structure of the group of automorphisms of a free group. The main result of this part is the following theorem:

Theorem 3. Let G be a free group of rank n and let Γ be a subgroup of G . Then the following conditions are equivalent:

- (1) Γ is a free group of rank n .
- (2) Γ is a free group of rank n and the natural map $\pi: G \rightarrow G/\Gamma$ is an isomorphism.