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Projekt Schneller Brüter

Fast Neutron Scattering Models, Slowing Down and Formation of Reactor Spectra

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Abstract

In this paper a review is presented covering the models of neutron slowing down calculations in both thermal and fast reactors. To understand the physical nature of the processes which form the reactor spectrum, slowing down is investigated in an infinite medium. The performance of space dependent calculations in homogeneous as well as in heterogeneous media is outlined.

Zusammenfassung

In diesem Bericht wird ein Überblick über die Modelle zur Berechnung der Abbremsung von Neutronen sowohl in thermischen als auch in schnellen Reaktoren gegeben. Zum besseren Verständnis der physikalischen Prozesse, die ein Reaktorspektrum erzeugen, wird auch die Abbremsung in einem unendlich ausgedehnten Medium untersucht. Die Berechnung ortsabhängiger Neutronenspektren in homogenen und heterogenen Medien wird kurz beschrieben.

Fast Neutron Scattering Models, Slowing Down and Formation of Reactor Spectra

by

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1. Introduction

In a fission event two or three neutrons are born with a mean energy of about 2 MeV. In order to obtain a self sustaining chain reaction, it is necessary that the absorption of one neutron in a fissile nucleus produces more than one (fission) neutron. This condition can be expressed by

$$\eta(E) = \bar{\nu}(E)\,\frac{\sigma_f(E)}{\sigma_a(E)} > 1$$

$\bar{\nu}(E)$ is the mean number of neutrons per fission, $\sigma_f(E)$ and $\sigma_a(E)$ are the microscopic fission and absorption cross sections respectively. In a reactor medium the neutron excess $\eta-1$ is needed to account for inevitable neutron losses by absorption in coolant and structural materials as well as for neutron leakage out of the reactor system. The form of the energy dependence of the fission and absorption cross sections of fissile materials in the energy range between 0 and 10 MeV has a very important consequence. In fig. 1 the neutron gain per absorption for U^{233}, U^{235} and Pu^{239} is shown. From the point of view of neutron economy those energy ranges are of considerable interest for the reactor designer, where the η-value is definitely larger than one. This is the case for neutrons with slow (thermal) and fast (larger than 50 keV) energies. Thus two reactor concepts are attractive: the thermal reactor in which the fission is induced by thermal neutrons, and the fast reactor in which the fission is mainly induced by neutrons of energies about 100 keV. There can be developed another interesting feature from

fig. 1. For energies above 10 keV $\eta(Pu^{239})$ is larger than two. Thus another neutron is available and can, in a Pu^{239}-U^{238}-mixture, be captured by U^{238} which leads by β decay to Pu^{239}:

$$U^{238} + n \xrightarrow{(n,\gamma)} U^{239} \xrightarrow[23.5']{\beta^-} Np^{239} \xrightarrow[2.35d]{\beta^-} Pu^{239}$$

If there is more plutonium produced than consumed, such a reactor is named a breeder reactor.

Another characteristic can be seen also directly from fig. 1. If for some reason in a large Pu fueled fast reactor the spectrum hardens, this is followed by an enhanced neutron multiplication.

These general remarks make it obvious that in a thermal reactor with the appropriate neutron moderator content, the discussion of formation of the energy distribution of neutrons, that means the neutron spectrum, is influenced by the three main effects:

a) the slowing down of neutrons with energies of about 10 MeV to energies of some eV,

b) the thermalization of neutrons below some eV which includes the possible energy gain of a neutron in a collision with a moderator atom,

c) the effect of heterogeneity in a reactor lattice on the neutron spectrum.

In a fast power reactor the neutron spectrum is strongly dependent on

a) fission and absorption cross sections of heavy isotopes in the resonance region,

b) the inelastic scattering mainly of U^{238},

c) the elastic down scattering by structural and coolant materials.

The importance of the investigation of neutron spectra in reactors can easily be demonstrated. First of all the calculation of the neutron spectrum, as accurately as possible, is one of the necessities in thermal and fast reactor design work in order to fix properly the physical design parameters

as for instance enrichment, burn-up, and reactivity coefficients. Especially in fast reactors the neutron spectrum strongly influences the Doppler and coolant density coefficients as well as the breeding gain.

In this paper a review is presented covering the models of slowing down calculations in both thermal and fast reactors. To understand the physical nature of the processes which form the reactor spectrum, slowing down is investigated in an infinite medium. The performance of space dependent calculations in homogeneous as well as in heterogeneous media is outlined.

2. Neutron Slowing Down Processes and Calculational Models

In this chapter we describe the main features of the neutron slowing down processes in reactor media and their mathematical representation.

2.1 Inelastic Scattering

If a neutron collides with a nucleus, the ratio σ_{in}/σ_t is a measure for the probability that this neutron will be scattered inelastically (σ_{in} and σ_t are the inelastic and total microscopic cross sections respectively). The inelastic scattering is a threshold reaction, and the threshold energy decreases from some MeV for light materials to about 50 keV or even less for heavy nuclei (for Pu^{239} the threshold is about 8 keV, for U^{238} 45 keV). The minimum energy of the neutron necessary for inelastic scattering is equal to the energy amount required to reach the first excited state of the target nucleus. Having formed a compound nucleus, the neutron will be emitted with a lower energy, and the residual nucleus is left in an excited state which decays subsequently by γ emission. Thus inelastic scattering is very important in a reactor by reducing the energy of fast neutrons above 100 keV. The degradation of fission neutrons below the fission threshold of U^{238} is responsible for the fact, that no self sustaining chain reaction can be formed in a block of natural uranium, although the mean η-value above 1.5 MeV is larger than 2 for U^{238}. This is so, because neutrons with energies about 2 MeV are scattered to about 60% elastically with a negligible energy loss, about 30% suffer an inelastic collision, and only about 7% induce fission and the rest is being captured.

The inelastic scattering process is characterized by the scattering cross section $\sigma_{in}(E'\to E,\mu)$ (E' is the neutron energy before and E the energy after the collision, μ is the cosine of the scattering angle). For heavy and medium nuclei the angular distribution of the inelastically scattered neutrons is normally assumed to be isotropic. We may therefore decompose the cross section in an amplitude factor, given by $\sigma_{in}^{tot}(E')$, and an energy transfer probability $P_{in}(E'\to E)$, thus

$$\sigma_{in}(E'\to E) = \sigma_{in}^{tot}(E') \cdot P_{in}(E'\to E)$$

with

$$\int_0^\infty P_{in}(E'\to E)dE = 1$$

For an inelastic scattering process on a single level with excitation energy E^k, the transfer probability is simply a δ-function ($\sigma_{in}^{k}(E')$ is the excitation cross section for level k).

$$P_{in}^{k}(E'\to E) = \delta(E'-E^k-E) \cdot \sigma_{in}^{E_k}(E')/\sigma_{in}^{tot}(E')$$

The energy of the neutron after an inelastic collision is given by the difference of the incident energy and the excitation of the level considered:

$$E'-E_k = E$$

Then it follows that

$$P_{in}(E'\to E) = \sum_k \frac{\sigma_{in}^{k}(E')}{\sigma_{in}^{tot}(E')} \delta(E'-E_k-E)$$

The inelastic scattering probability into a group interval ΔE is therefore given by

$$P_{in}(E'\to \Delta E) = \int_E^{E+\Delta} \frac{\sum_k \sigma_{in}^{k}(E')}{\sigma_{in}^{tot}(E')} \cdot \delta(E'-E_k-E'')dE''$$

$$P_{in}(E' \to \Delta E) = \begin{cases} \dfrac{\sum_k \sigma_{in}^{k}(E')}{\sigma_{in}^{tot}(E')} & \text{for } \begin{cases} E'' = E' - E_k \\ E \leq E'' \leq E + \Delta E \end{cases} \\ 0 & \text{otherwise} \end{cases}$$

At high neutron energies (for U^{238} E>2 MeV) many levels contribute to the inelastic cross section. Experimentally they are not resolved. Thus a model based on single level excitation is meaningless. In this energy region it is assumed that the levels are distributed statistically. According to this picture the compound nucleus behaves like a solid or liquid and the excitation energy can be characterized by a temperature. The inelastic scattering can then be described as an evaporation process of the compound nucleus and the probability that the energy of the emitted particle reaches the unit energy interval at E is proportional to

$$P_{in}^{V}(E' \to E) \sim E\, e^{-E/kT} = E \cdot e^{-E/\Theta(E')}$$

(k is Boltzman's constant). $\Theta(E') = kT$ represents the temperature of the residual nucleus. This is based on the assumption that in an inelastic collision the neutron loses most of its kinetic energy which then heats up the residual nucleus. According to Weißkopf the nuclear temperature is then determined by the level density W(E) of the residual nucleus.

$$\frac{1}{\Theta(E')} = \frac{d}{dE} \ln W(E)\Big/_{E=E'}$$

This formula is analogous to the thermodynamic relation between entropy and temperature, implying that lnW(E) can be considered as the entropy of the residual nucleus in an energy interval near E.

There are some formulas according to which the nuclear temperature can be fitted to experiment. It can be shown [1] that a suitable choice of the free parameters represent the experimental nuclear temperature equally well. Often the Weißkopf formula is taken:

$$\Theta(E') = \sqrt{E'/\gamma A}$$

where A is the mass number of the nucleus and γ a parameter, evaluated in [1] to 0.16, valid for $A \geq 27$.

The normalized transfer probability is given by

$$P_{in}^{V}(E' \to E) = \frac{E}{\theta^2(E')} e^{-E/\theta(E')}$$

The integration is performed to infinity because in the cases, where the evaporation model is used, $E' >> \theta$ holds.
Consequently

$$\sigma_{in}^{V}(E' \to E) = \sigma_{in}(E') \cdot \frac{E}{\theta^2(E')} e^{-E/\theta(E')}$$

The simple Maxwellian form of the evaporated neutron spectrum usually underestimates the high energy contribution, but in reactor calculations this is not serious anyhow because of low neutron flux intensities in these regions.

2.2 Elastic down scattering

Elastic scattering is one of the simplest nuclear reactions. In the energy range down to some eV the collisions between a neutron and the atoms of light materials, by which the neutron energy is mainly degraded, can be treated according to the laws of classical physics. Let us first recall the kinetics of the process of an elastic collision.
The relationship between incident and final neutron energy in the laboratory system for a collision process with a nucleus of mass number A is given by

$$E = E' \frac{A^2+1+2A\cos\theta}{(A+1)^2}$$

E' is the neutron energy before the collision process, E the energy after the collision, θ is the scattering angle in the CMS-system, A the mass number. With

$$\alpha = \left(\frac{A-1}{A+1}\right)^2$$

it follows

$$E = \frac{E'}{2} \{(1+\alpha) + (1-\alpha)\cos\theta\}$$

Furtheron the cosine of the scattering angle in the laboratory system, μ_L, can be expressed as

$$\mu_L = \frac{1}{2}\{(A+1)\sqrt{\frac{E}{E'}} - (A-1)\sqrt{\frac{E'}{E}}\}$$

The maximum energy loss is obtained for $\Theta = \pi$:

$$E_{min} = \alpha E'$$

Thus the energy interval, which can be reached by a neutron of energy E' in an elastic collision is

$$\Delta = E'(1-\alpha)$$

In a central collision with a hydrogen atom the neutron, therefore, can lose all of its energy, if the atom is assumed to be at rest. The probability that a neutron of energy E' will be scattered down to energy E is given by

$$P(E'\to E) = \begin{cases} \dfrac{4\pi\sigma_s(\Theta)}{E'(1-\alpha)\sigma_s} & \text{for } \alpha E' < E < E' \\ 0 & \text{otherwise} \end{cases}$$

$\sigma_s(\Theta)$ and σ_s are the differential and the total cross sections, respectively. Very important in the calculation of neutron slowing down by moderator atoms in thermal reactors is the case of isotopic scattering in the centre of mass system.
Then

$$\sigma_s(\Theta) = \frac{\sigma_s}{4\pi} \qquad \text{and therefore}$$

$$P(E'\to E) = \begin{cases} \dfrac{1}{(1-\alpha)E'} & \text{for } \alpha E' < E < E' \\ 0 & \text{otherwise} \end{cases}$$

Thus the energy distribution $f_1(E)$ of neutrons with energy E after an elastic scattering process is a constant, independent of E:

$$f_1(E)dE = \frac{dE}{(1-\alpha)E'}$$

The energy distribution at E of those neutrons which have suffered two collisions, is given by

$$f_2(E) = \int_E^{E'} \frac{f_1(E'')dE''}{(1-\alpha)E''} = \frac{1}{(1-\alpha)^2 E'} \ln\frac{E'}{E} \qquad \text{for } \alpha E' \leq E \leq E'$$

$$f_2(E) = \int_{\alpha E'}^{E/\alpha} \frac{f_1(E'')}{(1-\alpha)E''} dE'' = \frac{1}{(1-\alpha)^2 E'} \ln\frac{E}{\alpha^2 E'} \qquad \text{for } \alpha^2 E' < E < \alpha E'$$

$$f_2(E) = 0 \qquad \text{otherwise}$$

To find the distribution of neutrons, which have collided n times, it is convenient to introduce the lethargy variable

$$u = \ln\frac{E'}{E} \quad \text{and} \quad g = 2\ln\frac{A+1}{A-1}, \quad \alpha = e^{-g}$$

(g is the maximum lethargy gain).
We obtain [2]

$$f_n(u) = \begin{cases} \dfrac{e^{-u}}{(n-1)!(1-\alpha)^n} \sum_{k=0}^{\bar{k}} (-1)^k \binom{n}{k} (u-kg)^{n-1} & \text{for } 0 \leq u \leq ng \\ 0 & \text{otherwise} \end{cases}$$

The k-summation has to be carried out to that value $\bar{k}$ that each term $(u-kg)>0$.

For $n \gg 1$ and $\left|\frac{u}{ng} - \frac{1}{2}\right| \ll 1$ one obtains [2]

$$f_n(u) \approx \frac{e^{-u}}{(1-\alpha)^n} g^{n-1} \sqrt{\frac{6}{n\pi}} e^{-6n(\frac{u}{ng} - \frac{1}{2})^2}$$

This asymptotic expression deviates from the exact formula for A=4 and n=10 only by 4%, for n =20 already the exact results are represented. It can easily be shown that

$\int_0^\infty f_n(u)du = 1$, and therefore

$f_n(u)$ is the probability that the neutron reaches the lethargy u in n collisions. The maximum of this nearly symmetric probability distribution is

$$n \approx \frac{2u}{\xi} \approx u/\zeta$$

where ζ is the logharithmic energy decrement:

$$\zeta = 1 + \frac{\alpha}{1-\alpha} \ln\alpha = \frac{1}{2}\xi - \frac{1}{12}\xi^2 + \ldots$$

In the absence of absorption the summation $f(u)du = \sum_{n=1}^{\infty} f_n(u)\cdot du$ is also equal to the total number of neutrons, which arrive at u in du and will suffer their next collision in this interval [2], so f(u) must be identical with the collision density. It follows

$$f(u) = 1/\zeta$$

When the scattering is <u>anisotropic</u> in the centre of mass system, the transfer probability $P(E'\to E)$ is no longer a constant. In a forward scattering process $P(E'\to E)$ is larger near $E=E'$ than near $E=\alpha E'$, since neutrons lose comparatively little energy in small angle scattering.

In order to treat this problem more generally, we try to find an expression in the laboratory system for the elastic scattering transfer function $\sigma^L(E'\to E, \vec{\Omega}'\to\vec{\Omega})$. This is the cross section for changing the energy E' and the direction $\vec{\Omega}'$ of a neutron into an unit energy interval at E and an unit solid angle around $\vec{\Omega}$. In an isotropic medium the angular dependence is determined by $\mu_L = \vec{\Omega}'\cdot\vec{\Omega}$. The final energy E is uniquely determined by the deflection $\mu_L(E)$ and thus E is a spurious variable in the expression for the transfer function. Therefore it is allowed to write [compare also 3]

$$\sigma^L(E'\to E, \mu_L) = \sigma^L(E', \mu_L)\cdot\delta(\mu_L(E)-\mu_L)\cdot\frac{d\mu_L}{dE}$$

The Dirac-δ-function $\delta(\mu_L(E)-\mu_L)$ expresses the fact that $\sigma^L(E'\to E,\mu_L)dE=0$ if $\mu_L \neq \frac{1}{2}(A+1)\sqrt{\frac{E}{E'}} - \frac{1}{2}(A-1)\sqrt{\frac{E'}{E}}$

It is a usual technique to expand the scattering cross section into Legendre Polynomials

$$\sigma^L(E'\to E,\mu_L) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} K_\ell(E',E)\cdot P_\ell(\mu_L)$$

The kernel $K_\ell(E',E)$ is

$$K_\ell(E',E) = \int_{-1}^{+1} d\mu_L \cdot \sigma^L(E',\mu_L)\cdot\delta(\mu_L(E)-\mu_L)\cdot \frac{d\mu_L}{dE} \cdot P_\ell(\mu_L)$$

$$= \sigma^L(E',\mu_L(E))\cdot \frac{d\mu_L(E)}{dE} \cdot P_\ell(\mu_L(E))$$

The differential cross section $\sigma(E',\mu)$ is usually evaluated from experiment in the CMS-system in terms of Legendre Polynomials. Since

$$\sigma^L(E',\mu_L)d\mu_L = \sigma^c(E',\mu_c)d\mu_c,$$

we expand

$$\sigma^c(E',\mu_c) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{2} B_\ell^c(E')\, P_\ell(\mu_c)$$

Therefore we have

$$K_\ell(E',E) = \sum_{\ell'=0}^{\infty} \frac{2\ell'+1}{2} \cdot P_{\ell'}(\mu_c(E))\cdot P_\ell(\mu_L(E)) \frac{d\mu_c(E)}{dE} \cdot B_{\ell'}^c(E')$$

with

$$\frac{d\mu_c}{dE} = \frac{2}{(1-\alpha)E'}$$

For <u>isotropic scattering</u> in the CMS-system it is

$$B_\ell^c(E') = \sigma_s(E')\cdot\delta_{\ell'o}$$

Then

$$K_o(E',E) = \frac{\sigma_s(E')}{(1-\alpha)E'} \equiv \sigma_s(E') \cdot P(E' \to E)$$

$$P(E' \to E) = \frac{1}{(1-\alpha)E'} \quad ,$$

as has been defined earlier.
The first anisotropic moment yields:

$$K_1(E',E) = \frac{\sigma_s(E')}{2(1-\alpha)E'} \cdot \{(A+1)\sqrt{\frac{E}{E'}} - (A-1)\sqrt{\frac{E'}{E}}\}$$

The treatment of anisotropic scattering in reactor physics is important especially in the presence of hydrogen. For small fast reactors with a high neutron leakage the anisotropy in the CMS-system of all materials is essential.

2.3 Scattering processes of neutrons below some eV

The energy range below 1 eV is most important for an accurate prediction of the neutron properties of thermal reactors. Although the scattering of slow neutrons is not a direct subject of this paper, we treat the problem of neutron thermalization in a more qualitative way in order to understand the formation of neutron spectra in thermal reactors.

When the neutrons have been slowed down below 1 eV due to elastic collisions with the moderator atoms, then the neutron energy becomes comparable with the energy of thermal motion of the moderator atoms, and, although the average energy of thermal motion is 0.025 eV, due to the Maxwellian distribution of velocities there is a certain fraction of atoms with energies in the range of 1 eV. These will effect the neutrons in such a way that now a neutron may gain energy in a collision process as well as lose energy: up-scattering therefore has to be considered in the theory of neutron scattering.

If there is no absorption and leakage, the neutrons finally will reach thermal equilibrium with the moderator nuclei and assume a Maxwellian distribution:

$$N_{as}(E) \tilde{\propto} \frac{E}{kT} e^{-E/kT}$$

(T is the moderator temperature, k Boltzmann's constant). This distribution will be assumed irrespective of the type of moderator.

Two main problems arise in the theory of thermalization of neutrons:

a) the moderator temperature,

b) the state of chemical binding.

Here we will not discuss in detail the mathematical description of the approximation for the scattering kernels of various moderators, but rather give a qualitative representation to understand the physical nature of the processes involved.

Wick [4], van Hove [5] and Zemach and Glauber [6] first gave the general quantum mechanical formulation of the scattering of slow neutrons with gases and molecules. This is done in first Born approximation, using the Fermi pseudo potential as a contact potential to describe the interaction of the neutron with the scattering partner.

The simplest model used in thermalization theory is assuming that the moderator atoms behave like an ideal gas [7]. The effect of chemical binding then is treated by introducing the effective mass concept of Sachs and Teller (for D in D_2O $A_{eff} \tilde{\propto} 3.6$).

The scattering kernels $K_1(E'E)$ can be calculated straight forward. The most important kernel K_o is given by

$$K_o(\varepsilon';\varepsilon) = \frac{b^2}{2\varepsilon'} (F_1+F_2)$$

where

$$b^2 = \frac{1}{4}\left(\sqrt{A}+\frac{1}{\sqrt{A}}\right)^2; \quad \varepsilon' = \frac{E'}{kT}; \quad a^2 = \frac{1}{4}\left(\sqrt{A}-\frac{1}{\sqrt{A}}\right)^2$$

$$F_1 = \phi(b\sqrt{\varepsilon}-a\sqrt{\varepsilon''}) \mp \phi(b\sqrt{\varepsilon}+a\sqrt{\varepsilon''})$$

$$F_2 = e^{-(\varepsilon-\varepsilon')}\left[\phi(b\sqrt{\varepsilon''}-a\sqrt{\varepsilon}) \pm \phi(b\sqrt{\varepsilon''}+a\sqrt{\varepsilon})\right]$$

$$\phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt;$$

upper sign: $\varepsilon'<\varepsilon$ up-scattering

lower sign: $\varepsilon'>\varepsilon$ down-scattering

The total scattering cross section is then

$$\sigma_s(\varepsilon) = \int K_o(\varepsilon,\varepsilon')d\varepsilon' = (1+\frac{1}{2\varepsilon A})\phi(\sqrt{\varepsilon A}) + \frac{e^{-\varepsilon A}}{\sqrt{\varepsilon \pi A}}$$

To take into account the effects of chemical binding more properly, one has to look for the possible translation, vibration and rotational states of the molecules [see also ref. 15]:

a) Water

The frequency distribution of the H atom in H_2O was approximated by Nelkin [8] by four δ-functions. The translation of the molecule in water is taken as the movement of a free gaseous atom of weight A=18, according to a δ-function for ω=o. The hindered rotation is described by an oscillator with $\hbar\omega$=0.06 eV. From the vibration states at 0.2 eV and 0.474 and 0.488 the latter are combined to one δ-function with a double weight.

The model mostly used nowadays is that of Haywood [9] with a continuous frequency distribution in the range of hindered rotation and translation.

b) Zirconiumhydride

Because of the large mass difference of the H and Zr atoms the H-atoms vibrate nearly as independent oscillators (Einstein-model). The optical vibrations of H- and Zr-atoms are centred around 0.14 eV (acustic vibrations are below 0.02 eV). Best model now is that of Slaggie [10], who determined the frequency distribution from lattice dynamics, assuming central forces between the neighbours.

c) Graphite

The C atoms of graphite cristallize in layers. It is assumed that the vibrations of the atoms vertical to the layer are not coupled to the vibrations within the layer, where the chemical binding is very strong. According to Young, Koppel and Parks [11] the theoretical frequency distribution is therefore split up into two parts, parallel and vertical to the cristal layer.

3. Slowing down in infinite media

To study the fundamental procedure of neutron slowing down in a medium we first want to avoid the difficulties arising from neutron leakage. Therefore, we investigate the slowing down in an infinite homogeneous medium with an homogeneous source.
This treatment of an infinite medium is not so academic, as it might appear at first sight. The characteristics of many real systems are determined to a large extend by the properties of the corresponding infinite media. For instance the large natural uranium graphite reactors have to be rather large to go critical, neutron leakage is rather small. This becomes true more and more also for light water reactors and in near future for fast reactors, because economy and power requirement tend to very large systems (1000 MWe and even more).
The balance equation, which determines the neutron spectrum, is given by

$$\Sigma_t(E)\ \phi(E) = \int\{P_e(E'\to E)\ \Sigma_e(E') + P_{in}(E'\to E)\cdot\Sigma_{in}(E')\}\ \phi(E')dE' + S(E)$$

P_e, P_{in} are the elastic and inelastic transition probabilities, respectively, $S(E)$ may be the production term due to fission:

$$S(E) = S_f(E) = \chi(E) \int_0^\infty \nu(E')\ \Sigma_f(E')\ \phi(E')dE'$$

with $\chi(E)$ the fission spectrum, $\nu(E)$ the neutron gain per fission, $S(E)$ may include an external source.

3.1 Slowing down in nonabsorbing media and in special mixtures

The first statement is that the relatively simple equation in its general form cannot be solved analytically. Only for the simple case of moderation in hydrogen without absorption an exact solution is possible and yields the well known 1/E dependence for the neutron flux:
With

$$\Sigma_{in} = 0,\ S(E) = S\cdot\delta(E-E_o);\ \Sigma_a = 0,\ \Sigma_e = \text{constant}$$

$$P_e(E'\to E) = 1/E';\ E_o = \text{source energy},\ S = \text{source strength}$$

we obtain

$$F(E) = \Sigma_s \phi(E) = \int_E^{E_o} \frac{\Sigma_s \phi(E')}{E'} dE' + S \cdot \delta(E-E_o)$$

F(E) is the collision density.
The solution is

$$F(E) = \frac{S}{E} + S \cdot (E-E_o), \text{ thus}$$

$$\phi(E) = \frac{S}{E\Sigma_s} \qquad \text{for } E<E_o$$

As a more realistic case we consider a homogeneous mixture of hydrogen (H) with a heavy absorber (A), which does not moderate neutrons. Then it follows for a monoenergetic source at energy E_o

$$(\Sigma_t^H(E) + \Sigma_a^A(E))\phi(E) = \int_E^{E_o} \frac{\Sigma_s^H(E')}{E} \phi(E')dE' + S(E_o)$$

The solution is

$$F(E) = \frac{S}{E} e^{-\int_E^{E_o} \frac{\Sigma_a(E')}{\Sigma_t(E')} \frac{dE'}{E'}}$$

Here

$$\Sigma_a(E') = \Sigma_a^H + \Sigma_a^A \text{ and } \Sigma_t \equiv \Sigma_t^H + \Sigma_a^A; \; F(E) = \Sigma_t(E)\,\phi(E)$$

For $\Sigma_a = o$ we have the previously obtained result.

For $\Sigma_a/\Sigma_t = \text{const.}$, we obtain

$$F(E) = \frac{S}{E} \left(\frac{E}{E_o}\right)^{\Sigma_a/\Sigma_t}$$

In this case the collision density F(E) with decreasing energy does not raise as steep as in the case of no absorption, because neutrons vanish per absorption.

For the elastic slowing down of neutrons by nuclei with A>1 we have already discussed the form of the spectrum after n collisions. Again in the case of no absorption we obtain asymptotically ($E<E_o$, more precisely: $E<\alpha^3 E_o$):

$$\phi(E) = \frac{S}{\Sigma_s} \cdot \frac{1}{E}$$

The results are true also for a non-monoenergetic source, e.g. the fission spectrum $\chi(E)$, if we are interested in neutron energies below the fission spectrum.
Thus in a thermal reactor the fission spectrum of neutrons will be affected mainly by elastic down scattering with the moderator atoms and approaches the form 1/E, at least in finite energy intervals of suitable length, if weak absorption is assumed.

3.2 Slowing down in media with resonance absorption

For simplicity we restrict our discussion on the resolved resonance region and assume that the resonance width is small compared to the level distance. Then one resonance can be described by the single level Breit-Wigner formula. For nuclei at rest (zero temperature T=o) we have for a capture resonance:

$$\sigma_c(E) = \frac{\sigma_c^o}{1+x^2}; \qquad \sigma_c^o = \text{peak value of the resonance}$$

$$x = \frac{(E-E_o)}{\Gamma/2}$$

E_o = resonance energy; Γ = total width of the resonance.

The total cross section is given by

$$\sigma_t(E) = \frac{\sigma_t^o}{1+x^2} + (\sigma_t^o \sigma_P G)^{1/2} \cdot \frac{2x}{1+x^2} + \sigma_P$$

Here is :

$$\sigma_t^o = \sigma_c^o + \sigma_s^o; \qquad \sigma_P = \text{potential cross section}$$

$$G = g_I \cdot \frac{\Gamma_n}{\Gamma}; \qquad g_I = \begin{cases} 1 & \text{for } I=o \\ 1/2\ \frac{2J+1}{2I+1} & \text{for } I \neq o \end{cases}$$

I = spin of resonance nucleus
J = spin of compound nucleus.

If we assume that the resonance width is small compared to the degradation interval of neutrons in an elastic collision, then almost all neutrons, which collide with the nucleus at resonance energies have had there last collision above the resonance of interest. Since the absorption outside of a resonance is normally very small, we therefore are allowed to identify the collision density for a unit energy interval of the resonance with the collision density outside the resonance.

The number of neutrons, which are absorbed in a resonance per unit time, is given by (the collision density is taken proportional to 1/E):

$$A = \int_{Res} F_{Res}(E) \cdot \frac{\Sigma_a(E)}{\Sigma_t(E)}\, dE \approx \int_{Res} F_{as}(E) \cdot \frac{\Sigma_a(E)}{\Sigma_t(E)}\, dE = \int_{Res} \frac{\Sigma_a(E)}{\bar{\xi}\Sigma_t(E)}\, \frac{dE}{E}$$

$$\approx \int_{-\infty}^{+\infty} \frac{\Sigma_a(E)}{\bar{\xi}\Sigma_t(E)}\, \frac{dE}{E}$$

($\bar{\xi}$ is the mean logarithmic energy loss in a mixture, $\bar{\xi} = \frac{\sum_j \xi_j \Sigma_j^s}{\Sigma_s}$, F_{as} is the asymptotic collision density outside the resonance).

This procedure is well known as Narrow-Resonance (NR) Approximation and is widely used in reactor physics, especially for fast reactors, where the collision density naturally is not assumed to vary as 1/E. As a consequence the resonance absorption has to be evaluated numerically, while for the simple case given above, the absorption in a series of single resonances can be determined analytically. We will come back to the procedure used in fast reactors, in section 4.1.

In a mixture of a heavy resonance absorber (e.g. U^{238}) with a moderator (e.g. hydrogen) one may assume as in section 3.1 that the heavy nucleus does not change the neutron energy in an elastic collision. Then the collision density within the resonance to be used in the calculation of resonance absorption is given by

$$F(E) = \frac{1}{\xi_M(\Sigma_t(E)-\Sigma_s^A(E))} \cdot \frac{1}{E} \qquad \text{and consequently}$$

$$A = \int_{-\infty}^{-\infty} \frac{\Sigma_a(E)}{\xi_M(\Sigma_a(E)+\Sigma_s^M(E))}\, \frac{dE}{E}$$

(M stands for moderator).

This procedure is known as NRIM-Approximation (narrow resonance infinite mass) and is often applied in thermal reactor physics for the determination of the absorption in the low energy, broad resonances.

Both methods differ only with respect to the treatment of neutron scattering on heavy isotopes.

As a conclusion from this section we keep in mind that in a thermal reactor the global 1/E dependence of the neutron spectrum shows a fine structure in the resonance region. This fine structure in NR-approximation is proportional to $1/\Sigma_t(E)$ at each resonance for thermal reactors as well as for fast reactors. The collision density is a slowly varying function with energy in this region.

3.3 Scattering resonances

In fast reactors the elastic slowing down of neutrons is predominantly due to collisions with nuclei of the coolant (Na, gas, steam), the structure (Fe, Ni, Cr) and in oxide fueled reactors with oxygen. The resonances of these materials are broad resonances and cannot be treated in NR or in NRIM approximation sufficiently accurate. Because an analytical solution of the slowing down equation is not possible, this equation has to be solved numerically, using as many energy points as are necessary to cover the broad scattering resonances in question. This was first done by H. Hummel and A. Rago [12], representing the scattering cross sections by about 700 small energy intervals (groups) from 9 keV up to 4 MeV in their code ELMOE (elastic moderation). Based on this principle now in many fast reactor laboratories the calculation of resonance scattering is treated in many groups in order to account for the variation of the collision density within the resonance. A typical ELMOE spectrum is shown in fig. 9. A semi-analytical approach was given by M. Segev [13], assuming that the resonance structure of each isotope is superposed to a constant background, which approximates the effect of the other isotopes in the mixture. In the range of structural resonances the mixture is described by two quantities: the background cross section and the effective lethargy gain. Every pair of these quantities thus yields another fine structure of the neutron spectrum. This procedure can be used in an effective way to prepare average

cross sections in broader energy intervals.
Another way to deal with the scattering resonances is given in section 4.1.

3.4 Thermal spectra

The main procedures of calculating the infinite medium thermal neutron spectrum will briefly be outlined. Using the dimensionless variable $\varepsilon = E/kT$, we have to solve for isotropic scattering the equation:

$$\Sigma_t(\varepsilon)\phi(\varepsilon) - \int_0^\infty K_o(\varepsilon',\varepsilon)\,\phi(\varepsilon')d\varepsilon' = 0$$

This sourcefree homogeneous integral equation has to be solved with the inhomogeneous boundary condition

$$\phi(\varepsilon) \sim 1/\varepsilon \qquad \text{for } \varepsilon\to\infty$$

The solution of the thermalization equation is usually obtained numerically.

For a free gas the kernel $K_o(\varepsilon',\varepsilon)$ is given in 2.3. If the mass of the scatterer A>>1, the basic equation can be transformed into a differential equation (heavy gas model). In order to take into account approximately various types of scattering models, Cadilhac [14] arrived at

$$\frac{d}{d\varepsilon}\cdot\frac{\Sigma_o}{G(\varepsilon)}\varepsilon M(\varepsilon)\frac{d}{d\varepsilon}\left[1+H(\varepsilon)\,\Sigma_s(\varepsilon)\right]\frac{\phi(\varepsilon)}{M(\varepsilon)} = \Sigma_a(\varepsilon)\,\phi(\varepsilon)$$

$M(\varepsilon) = \varepsilon e^{-\varepsilon}$; Σ_o = scattering cross section of the free atom.
$G(\varepsilon)$ and $H(\varepsilon)$ have been determined by Cadilhac for the most important moderators. For $H(\varepsilon) \equiv 0$ and $G(\varepsilon) \equiv 1$ the differential equation for a free gas evolves.

The approximate solutions give satisfactory results for higher energies $\varepsilon \gtrsim 5$ if no additional resonance absorber (for instance Plutonium) has to be included. For energies $\varepsilon < 1$ the gas approximations for a moderator yield higher neutron flux densitites than the more sophisticated models as well as experimental data (see for instance ref. [15]).

4. Calculation of space dependent neutron spectra

The calculation of neutron spectra in a real reactor design is based on a balance equation, which in general depends for the stationary case on the space variable $\vec{r}$, the energy variable E, and the direction of neutron flight $\vec{\Omega}$. In a homogeneous medium this neutron transport equation is

$$\vec{\Omega}\nabla\psi(\vec{r},E,\vec{\Omega}) + \Sigma_t(E)\,\psi(\vec{r},E,\vec{\Omega}) = \int d\Omega' \int dE'\, \Sigma(E'\to E,\vec{\Omega}'\to\vec{\Omega})\,\psi(\vec{r},E',\vec{\Omega}')$$

$$+ \frac{1}{k_{eff}} \cdot \frac{\chi(E)}{4\pi} \int\int \nu(E')\,\Sigma_f(E')\,\psi(r,E',\Omega')dE'd\Omega'$$

The transfer cross section $\Sigma(E'\to E,\vec{\Omega}'\to\vec{\Omega})$ includes elastic, inelastic and (n,2n) processes.

The transport equation has to be solved with the appropriate boundary conditions. $\frac{1}{k_{eff}}$ is the eigenvalue of the homogeneous equation. The procedure which is used quite commonly, is not to require the neutron balance for each unit energy interval but rather for finite energy groups.

4.1 The multigroup procedure

Integration over a finite energy interval ΔE_i yields

$$\Omega\nabla\psi^i(\vec{r},\vec{\Omega}) + \Sigma_t^i \cdot \psi^i(\vec{r},\vec{\Omega}) = \sum_j \int \Sigma^{j\to i}(\vec{\Omega}'\to\vec{\Omega})\,\psi^j(\vec{r},\vec{\Omega}')d\Omega'$$

$$+ \frac{1}{k_{eff}} \cdot \frac{1}{4\pi}\,\chi_i \sum_j (\nu\Sigma_f)^j \int \psi^j(\vec{r},\vec{\Omega}')d\Omega'$$

It is:

$$\psi^i(\vec{r},\vec{\Omega}) = \int_{\Delta E_i} \psi(E,\vec{r},\vec{\Omega})dE_i; \quad \Sigma_t^i = \frac{\int_{\Delta E_i} \Sigma_t(E)\cdot\psi(E,\vec{r},\vec{\Omega})dE}{\int_{\Delta E_i} \psi(E,\vec{r},\vec{\Omega})dE}$$

$$\Sigma^{j\to i}(\Omega'\to\Omega) = \frac{\int_{\Delta E_j} dE \int_{\Delta E_i} dE'\, \Sigma(E'\to E,\vec{\Omega}'\to\vec{\Omega})\,\psi(E',\vec{r},\vec{\Omega}')}{\int_{\Delta E_j} \psi(E,\vec{r},\vec{\Omega})dE}$$

Thus the reaction rates are preserved.
One difficulty arises by the fact that all the group constants now are dependent on space and angle. We assume, only for the purpose of constructing the group constants, that

$$\psi(E,\vec{r},\vec{\Omega}) = \phi(E)\cdot F(\vec{r},\vec{\Omega})$$

Then those space and angular dependences of the group cross sections, which stem from $\psi(\vec{r},\vec{\Omega},E)$, drop out. $\phi(E)$ is used then as a weighting function. The separational ansatz is justified in the central parts of larger reactors, where most of the contribution to the neutron multiplication arises. The separational ansatz is one simple possibility to make the group cross sections only energy-dependent. If we have across the core a strongly varying neutron flux density, then only for the purpose of constructing group constants we integrate the transport equation over finite energy interval, finite space regions of not necessarily different composition and over the solid angle. Then we obtain for each zone (ΔV) a set of group cross sections:

$$\Sigma_t^i = \frac{\int_{\Delta E_i} dE \int_{\Delta V} dV \int_{4\pi} \Sigma_t(E)\cdot\psi(E,\vec{r},\vec{\Omega})\, d\Omega}{\int_{\Delta E_i} dE \int_{\Delta V} dV \int_{4\pi} \psi(E,\vec{r},\vec{\Omega}) d\Omega}$$

$$= \frac{\int_{\Delta E_i} \Sigma_t(E)\cdot\bar{\phi}(E)dE}{\int_{\Delta E_i} \bar{\phi}(E)dE}$$

Here

$$\bar{\phi}(E) = \frac{1}{\Delta V} \int_{\Delta V} dV \int \psi(E,\vec{r},\vec{\Omega})d\Omega$$

is the average scalar flux density in zone ΔV.

If one expands the transfer cross section into Legendre polynomials according to 2.2, the same procedure holds for the expansion coefficients.

Because the group constants already include the solution of the problem, namely ψ or ϕ, the whole procedure in principle is iterative.

In the resonance region $\phi(E) \sim 1/\Sigma_t(E)$ is a strongly varying function with energy, which also depends on the composition and temperaturs of the medium. The collision density then is a slowly varying function in an energy group which is not too broad. Therefore the following definition of effective group cross sections is more practicle in the resonance range:

$$\Sigma_a^i = \frac{\int_{\Delta E_i} \bar{\phi}(E)\, \Sigma_a(E) dE}{\int_{\Delta E_i} \bar{\phi}(E) dE} = \frac{\int_{\Delta E_i} \frac{\Sigma_a(E)}{\Sigma_t(E)} \bar{F}(E) dE}{\int_{\Delta E_i} \frac{\bar{F}(E)}{\Sigma_t(E)}\, dE}$$

$$\approx \frac{\int_{\Delta E_i} \frac{\Sigma_a(E)}{\Sigma_t(E)}\, dE}{\int_{\Delta E_i} \frac{dE}{\Sigma_t(E)}} \qquad \text{if the groups are sufficiently small.}$$

Having defined the group constants, the remaining group equation is solved numerically by further discretisation in space and angle. This multigroup scheme is applied to thermal as well as to fast reactors.

A useful approximation for fast reactors is to expand the angular dependent flux density and the transfer cross sections into Legendre Polynomials P_N, truncating the series after the term P_1, yielding two equations for the scalar neutron flux density ϕ and the net neutron current density J. If we further assume that Fick's law

$$J(\vec{r},E) = -D(E)\, \nabla\, \phi(\vec{r},E)$$

holds (the diffusion coefficient D has to be evaluated by one of the P_1-equations), then we obtain the well known multigroup diffusion equation for one zone of the reactor and N groups:

$$-D^i \Delta\, \phi^i(\vec{r}) + \Sigma_t^i \phi^i(\vec{r}) = \sum_{j=1}^{i} \Sigma_o^{j\to i}\, \phi^j(\vec{r}) + \frac{1}{k_{eff}}\, \chi_i \sum_{j=1}^{N} (\nu\Sigma_f)^j \phi^j(\vec{r})$$

Starting from the energy dependent P_1-equations, the diffusion coefficient, which is defined as 1/3 of the reciprocal transport cross section has to be properly weighted to give the corresponding group constant. For a more

complete discussion of this point see ref. [16].

As mentioned earlier in section 3.3, the calculation of the group constants for elastic moderation requires special attention. The design calculations for fast reactors are usually done in a multigroup scheme with about 30 groups. Because the degradation interval is often much smaller than the group width, the inner group spectrum is very important for an accurate calculation of neutron down scattering. This is especially true if scattering resonances are located near the lower group boundary. There are two main problems to be dealt with: at first the proper self-shielding of a structural resonance itself and secondly the effect of a structural resonance on the elastic moderation by another light or medium weight isotope via background contribution in $\Sigma_t(E)$. The last effect can easily be taken into account by using the detailed microscopic cross section information of all materials in the mixture to calculate from about 1000 energy points the macroscopic effective group constant for elastic moderation in NR approximation [17]. This REMO procedure (removal) has a remarkable influence on the neutron spectrum near the large resonances of oxygen, iron and sodium.
Both effects of the structural resonances can be handled with ELMOE type calculations as mentioned in 3.3. But these calculations up to now suffer from the fact, that only space independent spectra can be obtained.

4.2 Slowing down in heterogeneous reactors

Multigroup methods based on the integro-differential transport or diffusion equation are well established. For heterogeneous lattice calculations in thermal reactors as well as in zero power fast critical assemblies powerful collision probability methods have been established, which are based on the integral form of the transport equation. For the calculation of heterogeneous thermal reactor spectra the multigroup code THERMOS, developed by H.C. Honeck [18], is widely used. The code ZERA by D. Wintzer [19] uses a similar technique and is especially developed to take into account the spatial dependent resonance self-shielding effects in fast critical assemblies so that flux and reaction rate fine structure measurements across a cell properly can be interpreted.

The calculation of neutron spectra in thermal lattices is sometimes performed in such a way that the fuel rods or fuel plates are mathematically treated as δ-functions to which effective source strengths and effective cross sections are attributed (heterogeneous method [20]). W. Häfele has used this method to calculate the slowing down spectrum in a finite thermal reactor lattice [21]. H. Kunze has used also this procedure to determine the space dependent thermal spectrum [22] within the frame of the heavy gas model.

5. Results of spectrum calculations and spectral sensitivity of some nuclear reactor parameters

In this section we discuss briefly some reactor spectra in thermal and fast reactors.

5.1 Thermal reactors

In fig. 2 the flux distribution in a D_2O reactor of plane geometry for different lethargies are shown. They have been calculated by W. Häfele [21] with the heterogeneous method. The lines represent the fuel plates. For $u \approx 5.5$ ($E \approx 10$ keV) nearly a cosine distribution results, while for $u \approx 2$ ($E \approx 250$ keV) the originally δ-peaked fluxes in the fuel plates have already been broadened. For $u = 19$ ($E \approx 1/40$ eV) the neutrons are thermalized and the flux densities have their maxima in the moderator.
In fig. 3 the spectra in a 2 media D_2O cell (uranium metal rod radius ρ=1.6 cm, cell radius 9.5cm) the results of H. Kunze [22] are compared with a consistent P_1 thermalization calculation of H. Küsters [23] for a free gas approximation with an effective mass.
Fig. 4 shows the neutron spectrum in graphite [24]. Here we have an excellent agreement of the cristal model with experiment, while the gas model results in a much too soft spectrum.

5.2 Fast Reactors

Fast power reactors can be treated in nuclear calculations as homogeneous mixtures of the different materials, because the mean free paths of the neutrons are larger than the pin diameter and the pitch. Fig. 5 shows the core arrangement of the 300 MWe sodium cooled prototype Na2 [25]. Each subassembly contains 169 pins which are 6 mm in diameter. In fig. 6 smoothed neutron flux spectra of fast reactors with different fuel and coolant are compared [26]. The maximum of the neutron distribution is centered around 200 keV, the spectrum of the steam cooled reactor has more "low" energy neutrons then the spectra of the sodium cooled reactors. These spectra have been calculated with 26 groups in diffusion approximation. The energy distribution of the reaction rates in a steam cooled fast reactor is shown in fig. 7 [27]. Here it can be seen that about 50% of the production and absorption rates stem from the resonance region. In fig. 8 experimental and theoretical results for the local variation of reaction rates within a fuel plate of the fast critical assembly SNEAK in Karlsruhe are compared [28]. The assembly 3A2 is a simulation of a steam cooled fast reactor, 3A3 has twice hydrogen content of 3A2. The calculations were performed with the code ZEBA [19]. Fig. 9 compares two typical ELMOE spectra, calculated with 700 groups, with experiment [29].

5.3 Spectral sensitivity of some nuclear parameters

The knowledge of the spectral sensitivity of nuclear parameters such as Doppler-coefficient and coolant density coefficients are extremely important for a judgement on the accuracy of corresponding calculations. Fig. 10 represents the energy distribution of the Doppler coefficient in a 1000 MWe steam cooled fast reactor (D1) [30]. The contributions are centered around 1 keV, from which follows that the slowing down of neutrons from higher energies, has to be treated very carefully. The spectral distribution of the reduced steam density coefficient is shown in fig. 11 [30]. Here positive and negative contributions tend to compensate each other and thus this coefficient, which is important for the stability of the design, is extremely sensitive to calculational methods and input data.
The situation is less severe for the spectral distribution of the coolant loss coefficient which is centered for Na-cooled reactors around 100 keV.

In the last fig. 12 the spectral distribution of the influence of fission products on reactivity is shown [31]. The "difference line" contains the influence of fission products on the void reactivity change and has again positive and negative contributions.

These examples show clearly that a proper calculation of fast reactors requires a large effort in calculating the neutron slowing down processes from 10 MeV to some hundred eV.

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Figure Captions

Fig. 1 The energy distribution of $\eta(E)$ for various fissile nuclei

Fig. 2 The formation of neutron spectra in a plane D_2O reactor (heterogeneous method [21])

Fig. 3 Comparison of heterogeneous thermal neutron spectra obtained by H. Kunze [22] and H. Küsters [23]. The Cell: uranium metal rod (ρ=1.6 cm), D_2O moderator (cell radius R=9.5 cm)

Fig. 4 Neutron spectrum in graphite [24]

Fig. 5 Core arrangement of 300 MWe prototype Na-2 [25]

Fig. 6 Neutron spectra in fast reactors [26]

Fig. 7 Energy distribution of the production and absorption rates in a steam-cooled fast reactor [27]

Fig. 8 Comparison of experimental and calculated local variation of reaction rates within a fuel zone of SNEAK [28]

Fig. 9 Flux fine structure in the centre of ZPR3-48 and assembly 4Z [29]

Fig. 10 Energy distribution of the Doppler coefficient in a steam-cooled fast breeder [30]

Fig. 11 Energy distribution of the reduced steam density coefficient for a steam-cooled fast reactor [30]

Fig. 12 Energy distribution of the fission product influence on reactivity [31]

THE ENERGY DISTRIBUTION OF $\eta(E)$ FOR VARIOUS FISSILE NUCLEI

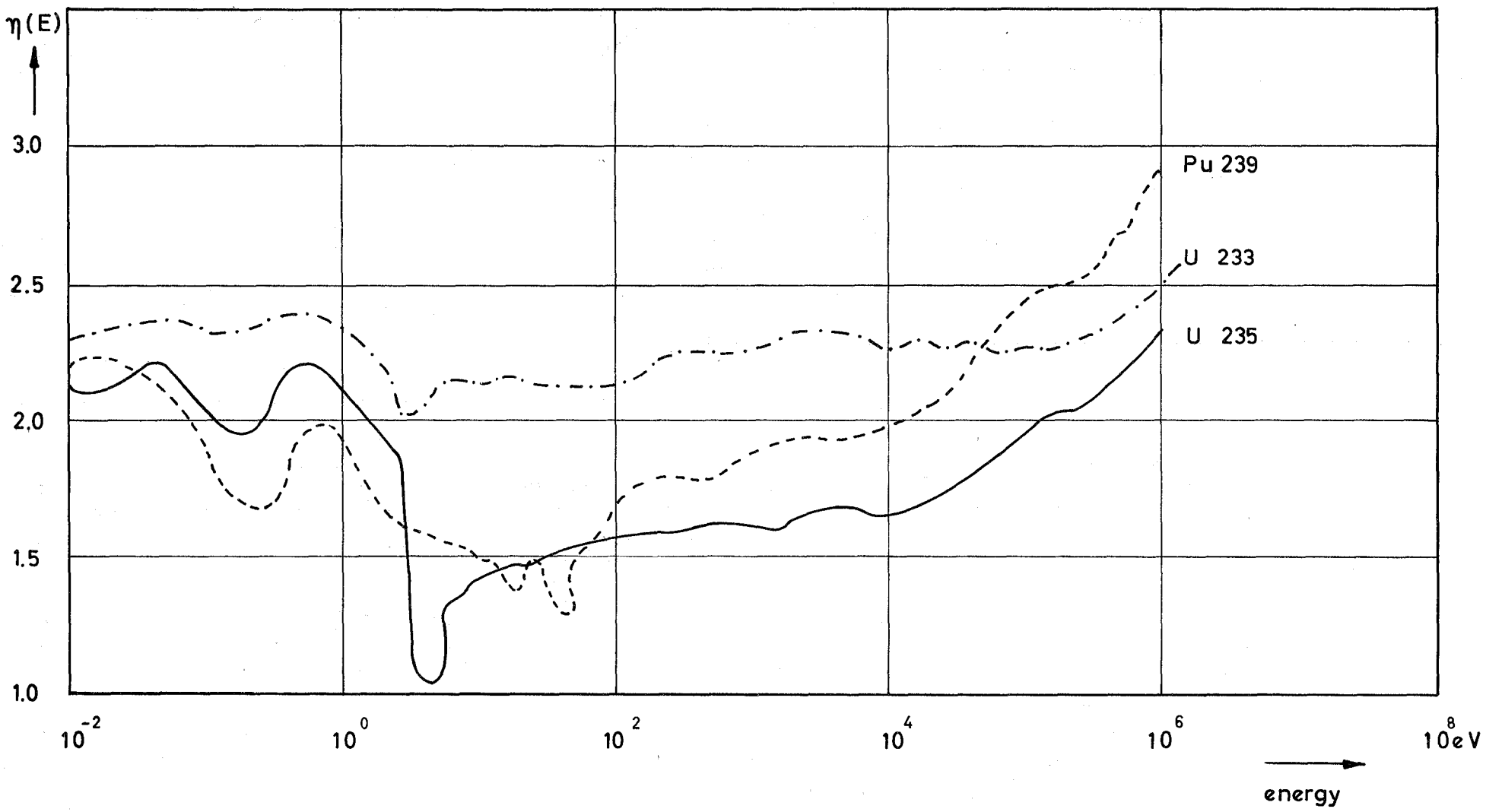
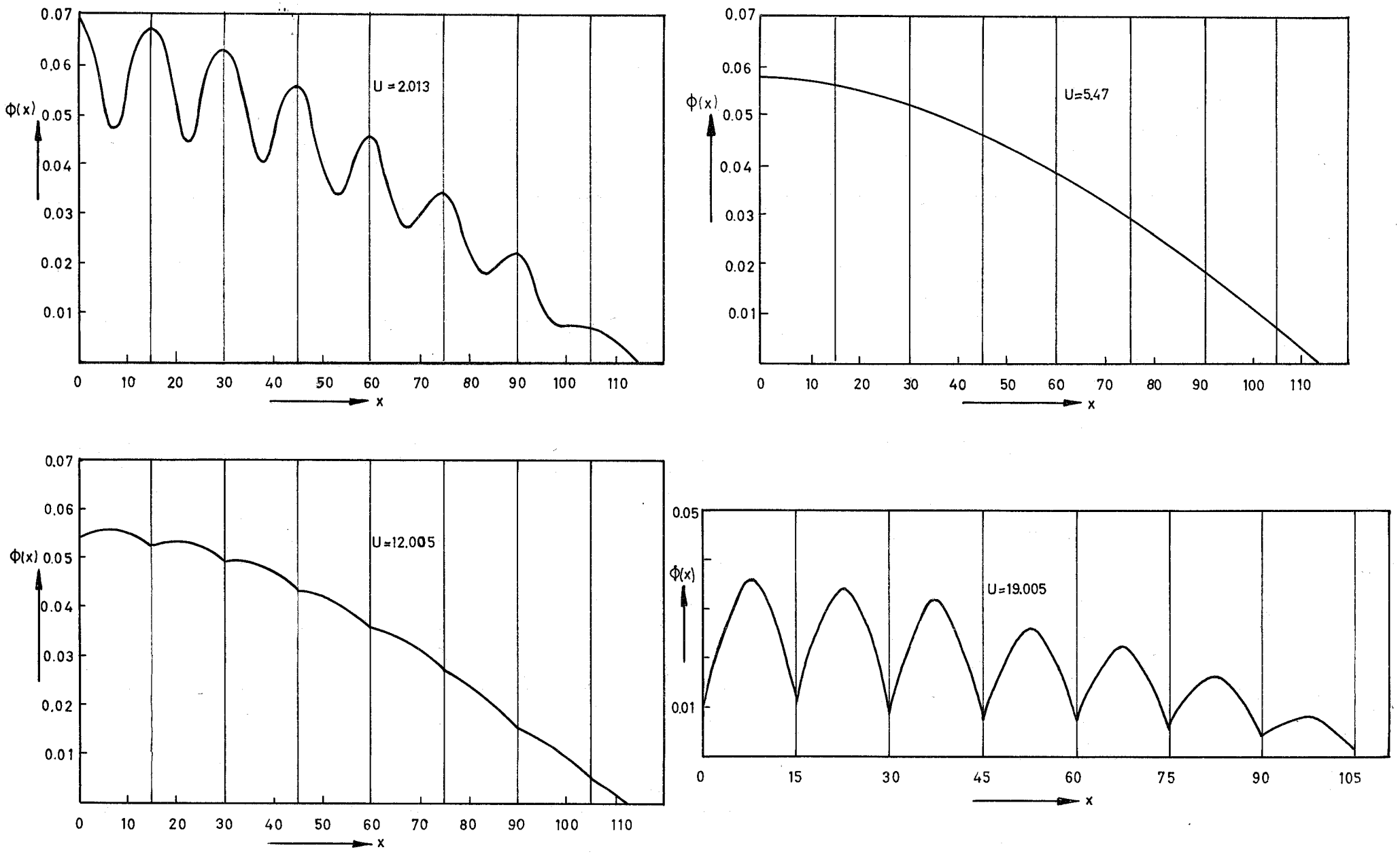
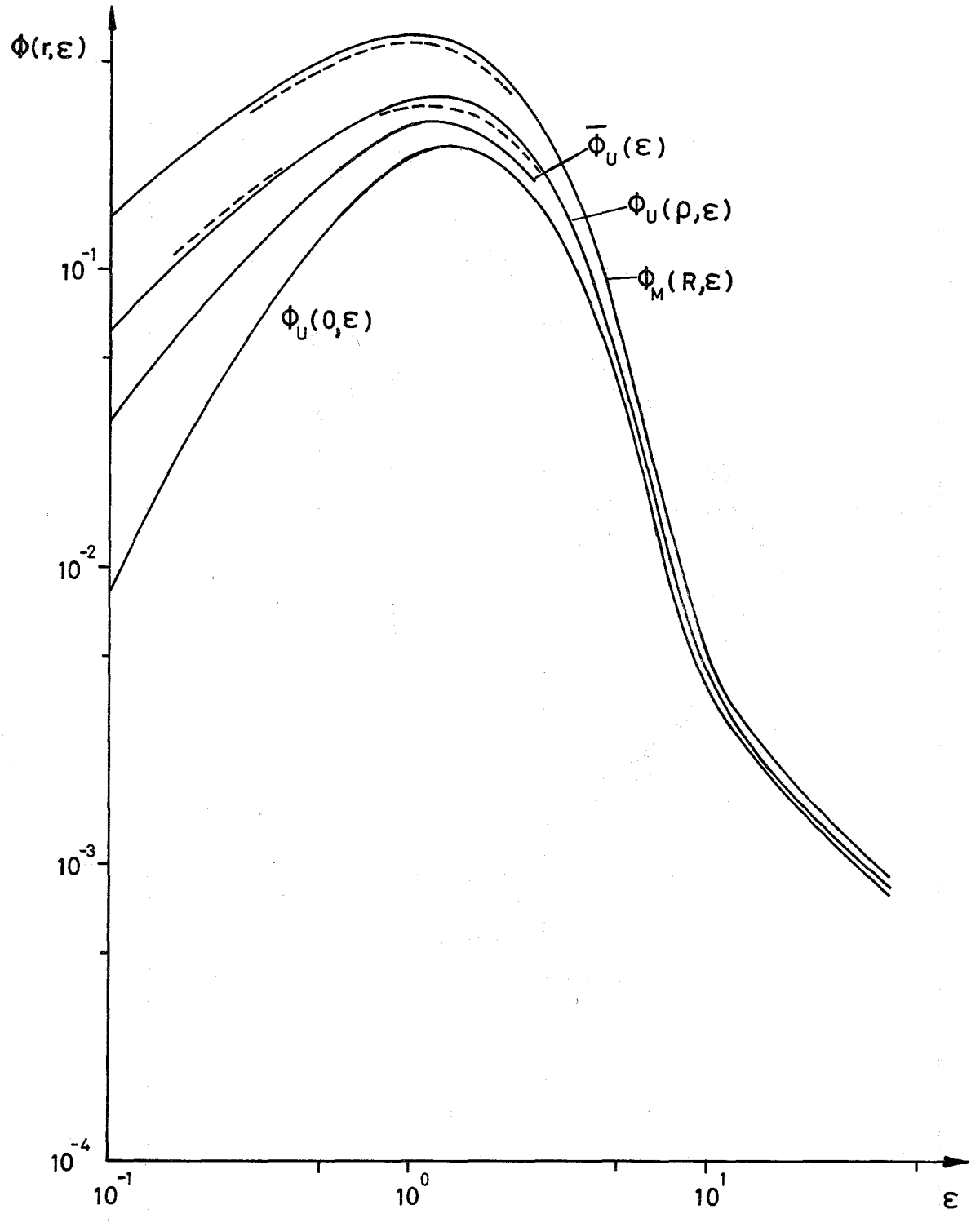


FIG. 1



THE FORMATION OF NEUTRON SPECTRA IN A PLANE D_2O REACTOR (HETEROGENEOUS METHOD [21])

FIG. 2



COMPARISON OF HETEROGENEOUS THERMAL NEUTRON SPECTRA OBTAINED BY H.KUNZE [22] AND H.KÜSTERS [23] . THE CELL: URANIUM METALL ROD(ρ = 1.6cm), D_2O MODERATOR (CELL RADIUS R=9.5cm) (H.KUNZE, dashed lines)

FIG. 3

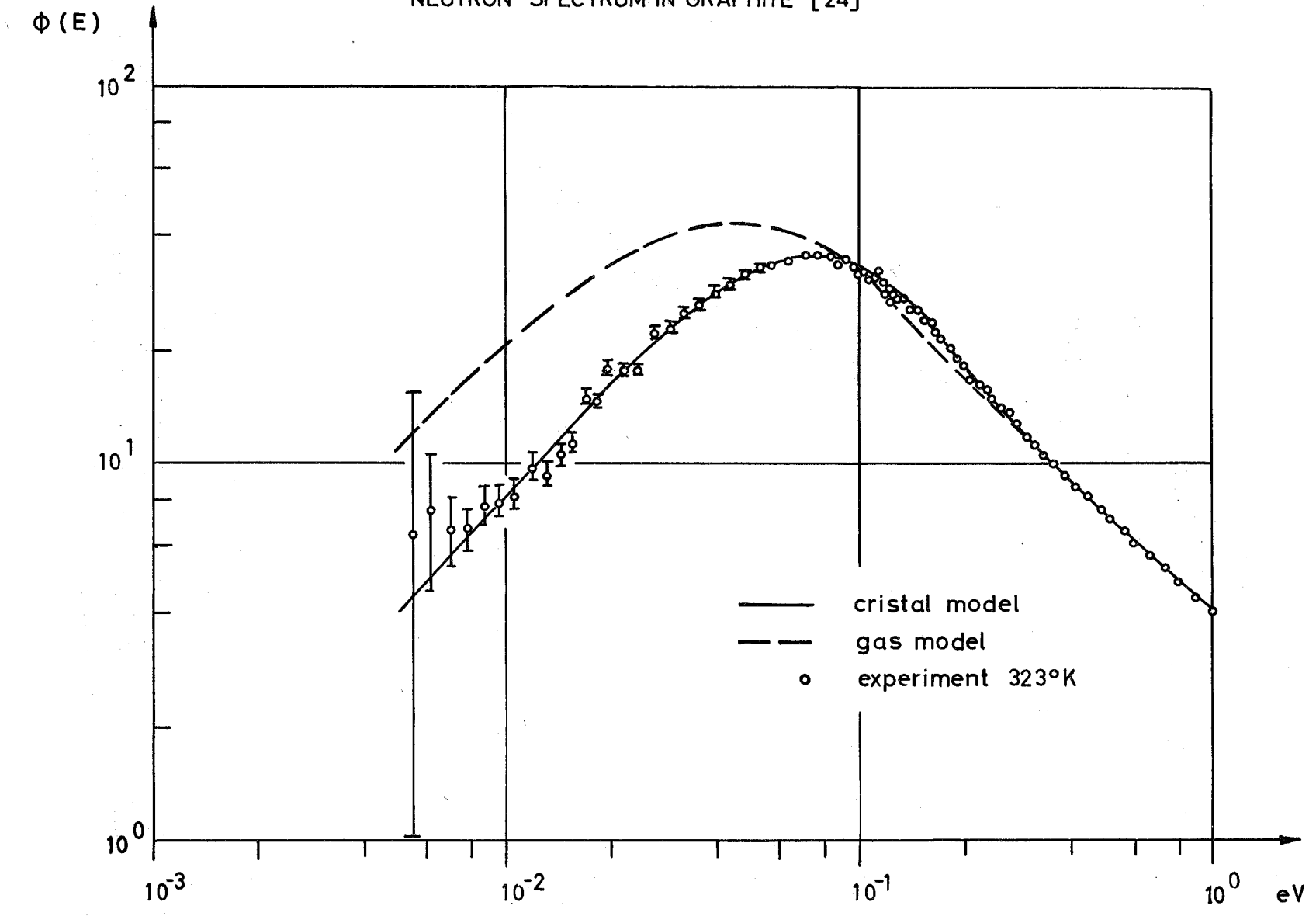
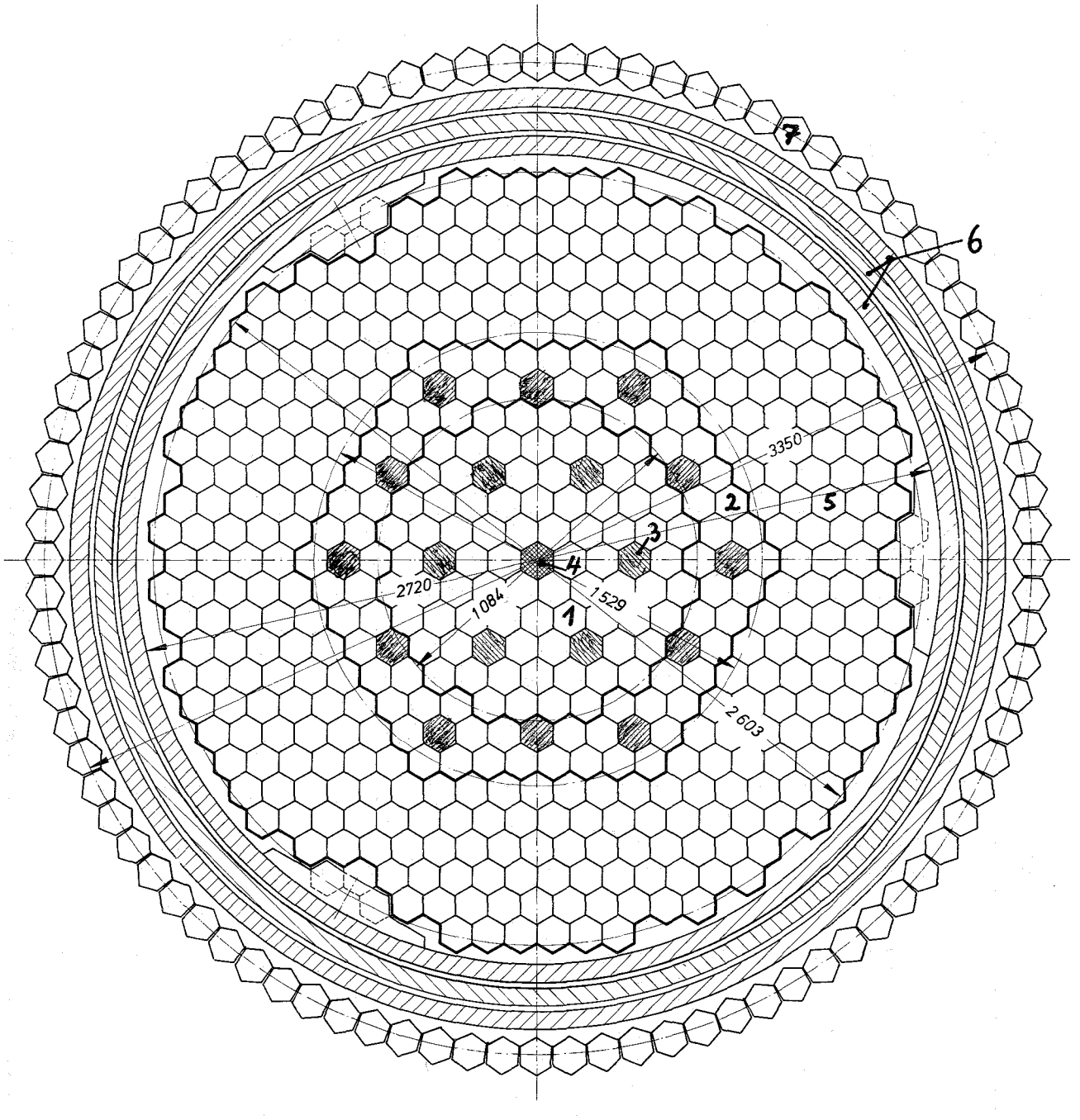

NEUTRON SPECTRUM IN GRAPHITE [24]
Φ (E)
10^2
10^1
10^0
10^-3
10^-2
10^-1
10^0
eV
cristal model
gas model
experiment 323°K

FIG. 4



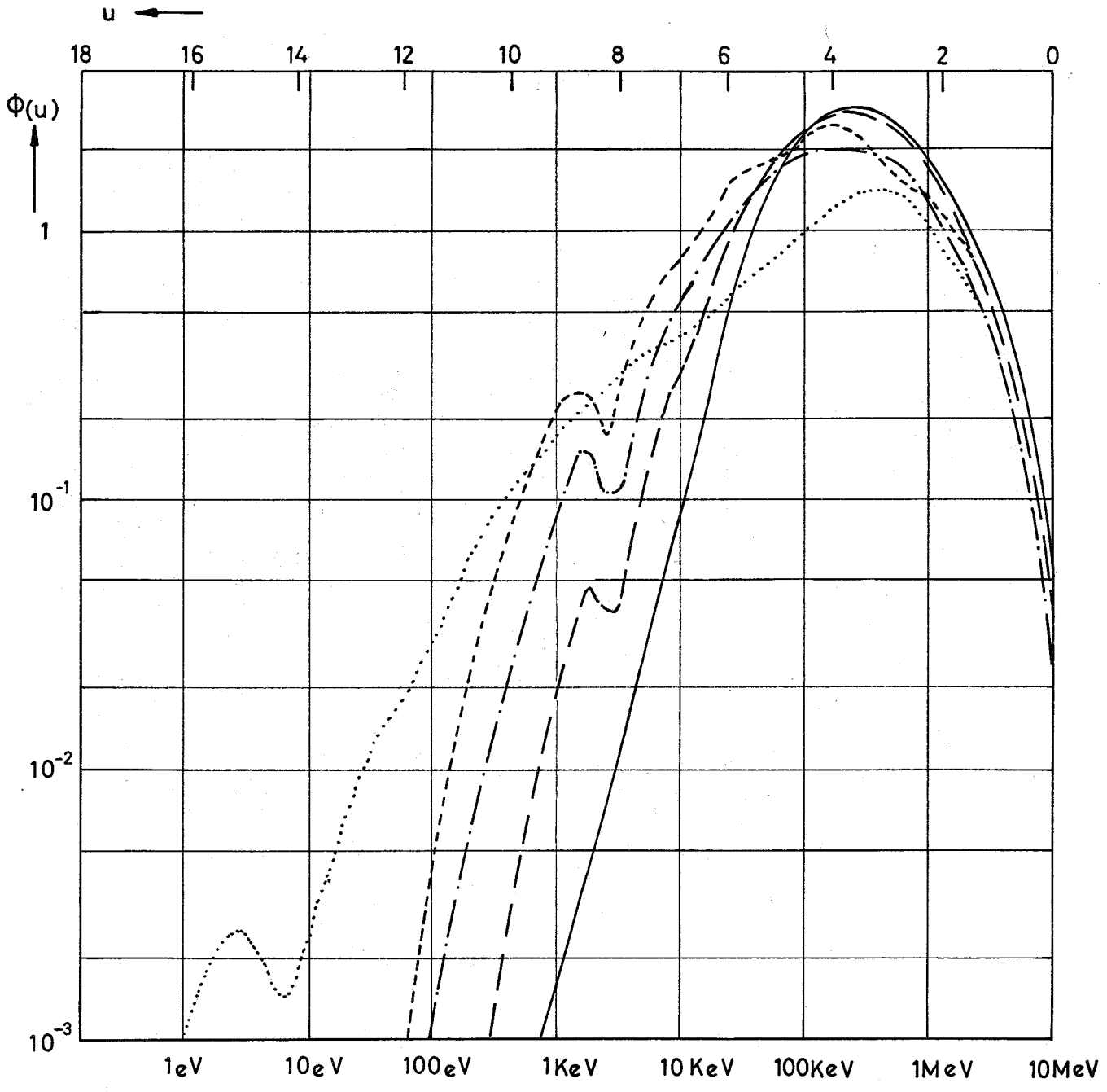
1 SUBASSEMBLY INNER ZONE 78
2 SUBASSEMBLY OUTER ZONE 72
3 CONTROL ROD
4 INSTRUMENTATION POSITION
5 BREEDER ELEMENT 321
6 NEUTRON SHIELD
7 SPENT FUEL STORAGE

CORE ARRANGEMENT OF 300 MWe

PROTOTYPE NA-2

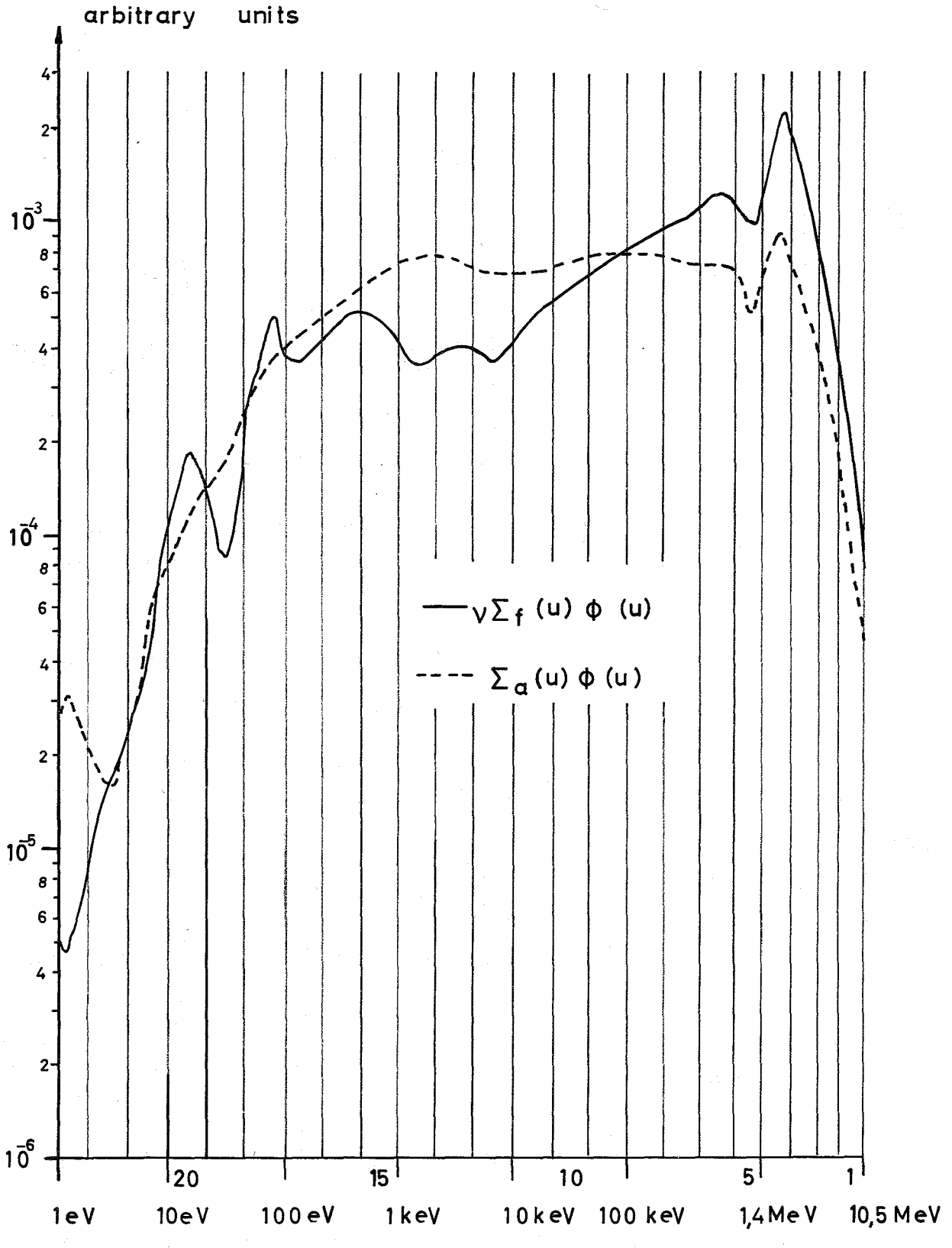
FIG. 5

NEUTRON FLUX SPECTRA OF FAST REACTORS



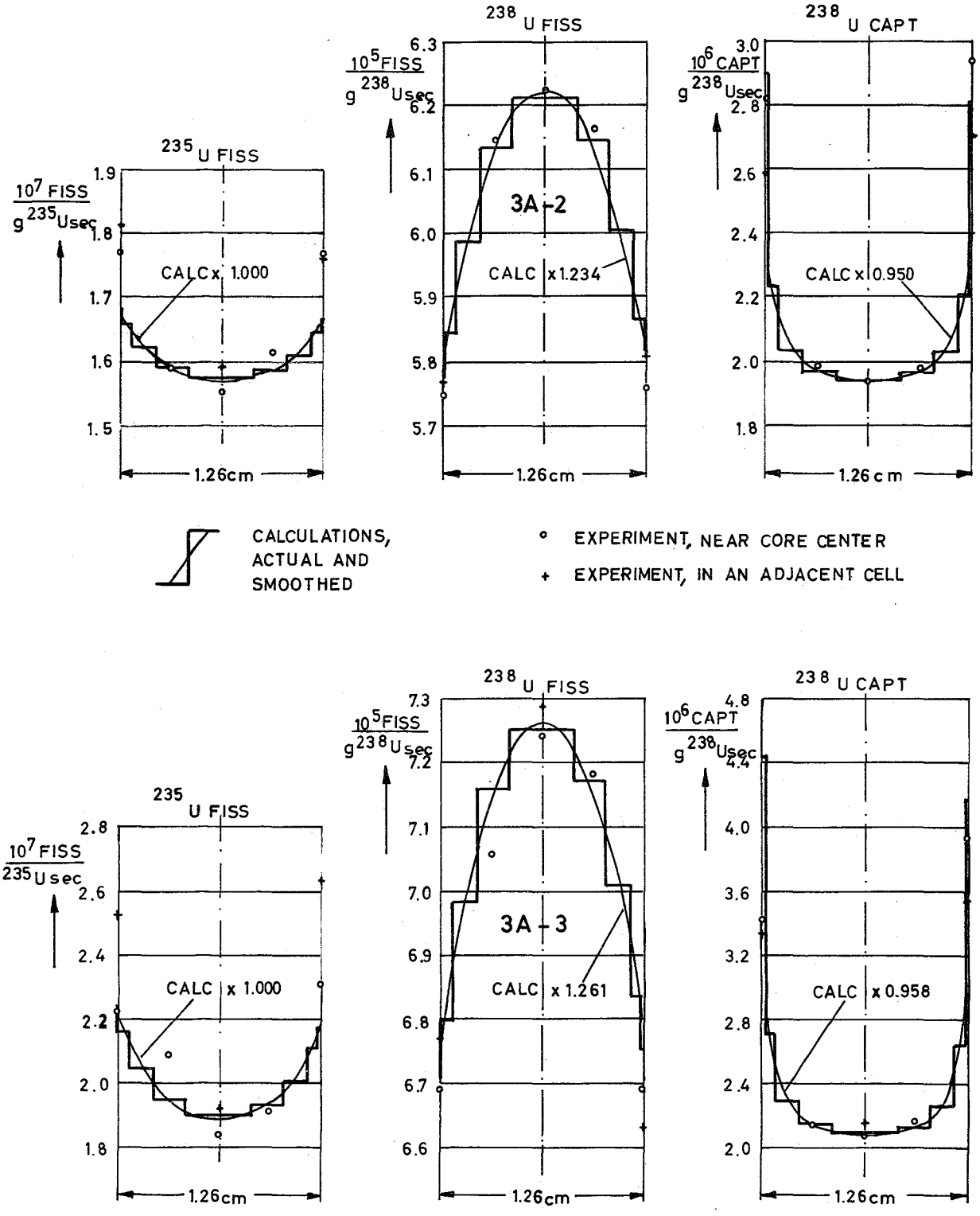
		σ_f of Pu-239.b
———	EFFBR	1.61
— —	1000 MWe Breeder Metal Fuel Na cooled	1.61
—·—	1000 MWe Breeder Carbide Fuel Na cooled	1.71
- - - -	1000 MWe Breeder Oxide Fuel Na cooled	1.78
·····	1000 MWe Breeder Oxide Fuel Steam cooled	2.01

FIG. 6



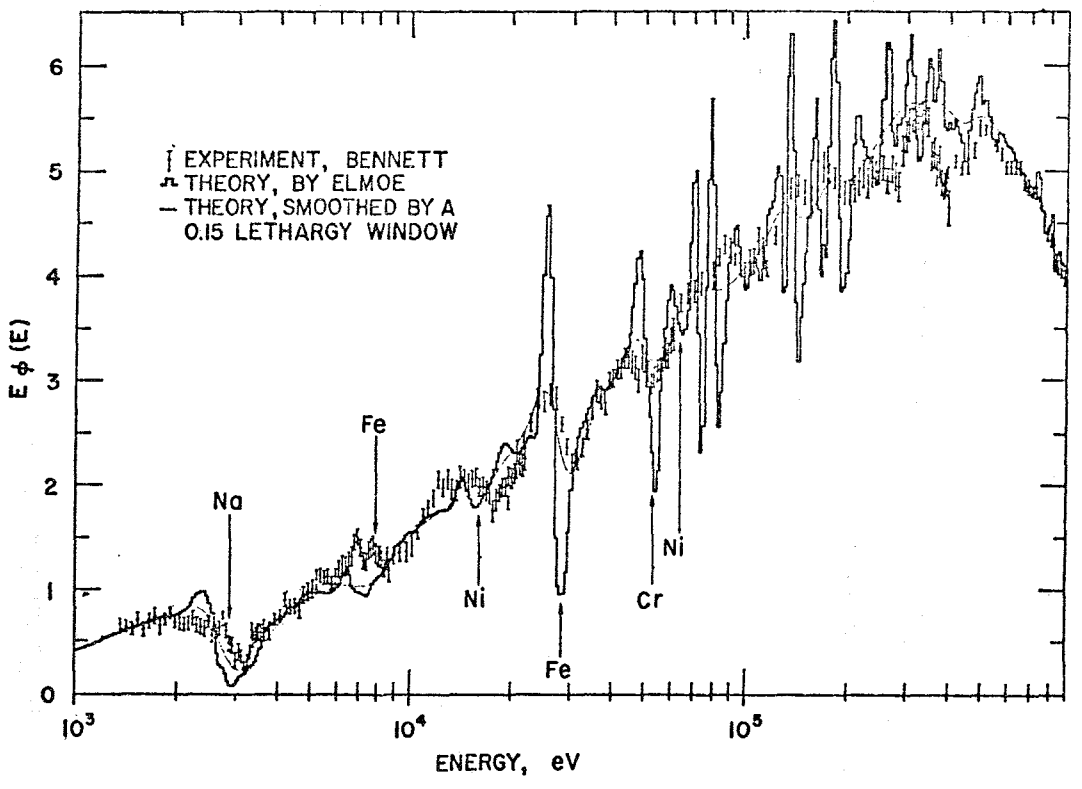
ENERGY DISTRIBUTION OF THE PRODUCTION AND ABSORPTION RATES IN A STEAM-COOLED FAST REACTOR [27]

FIG. 7

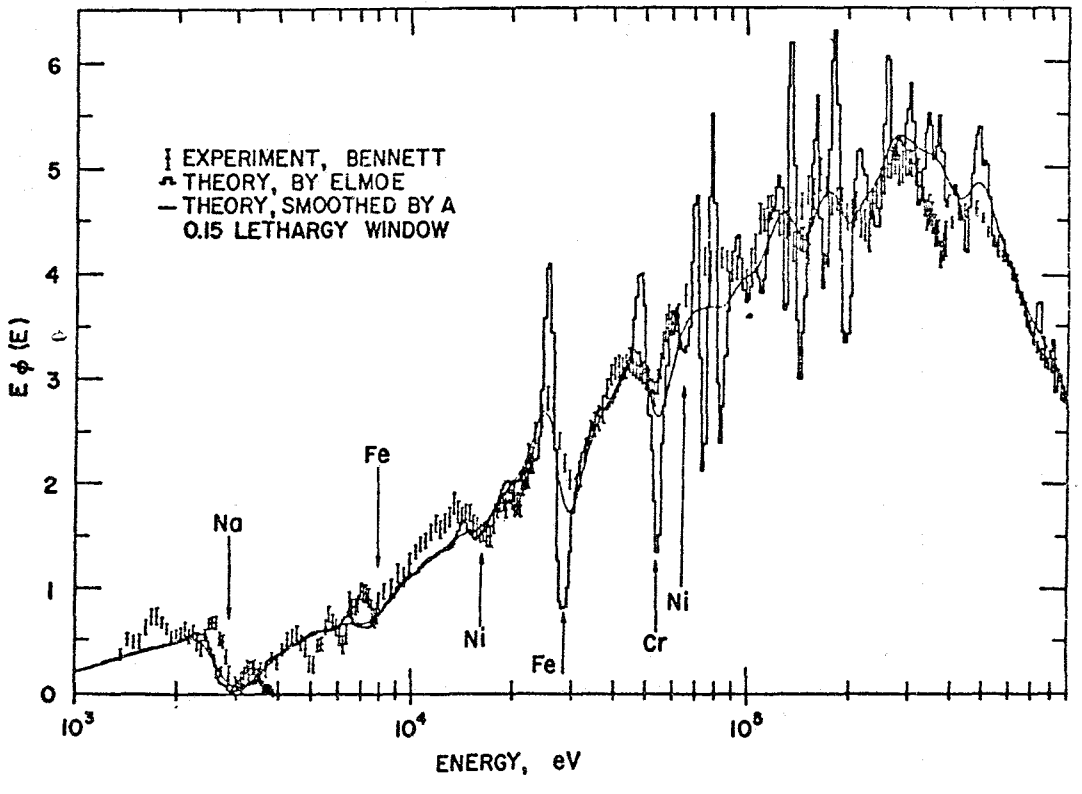


COMPARISON OF EXPERIMENTAL AND CALCULATED LOCAL VARIATION OF REACTION RATES WITHIN THE FUEL ZONE (20% ENRICHED URANIUM) OF SNEAK-ASSEMBLIES 3A-2 (UPPER GRAPHS) AND 3A-3 (LOWER GRAPHS). THE CALCULATIONS ARE BASED ON THE SNEAK SET.

FIG. 8

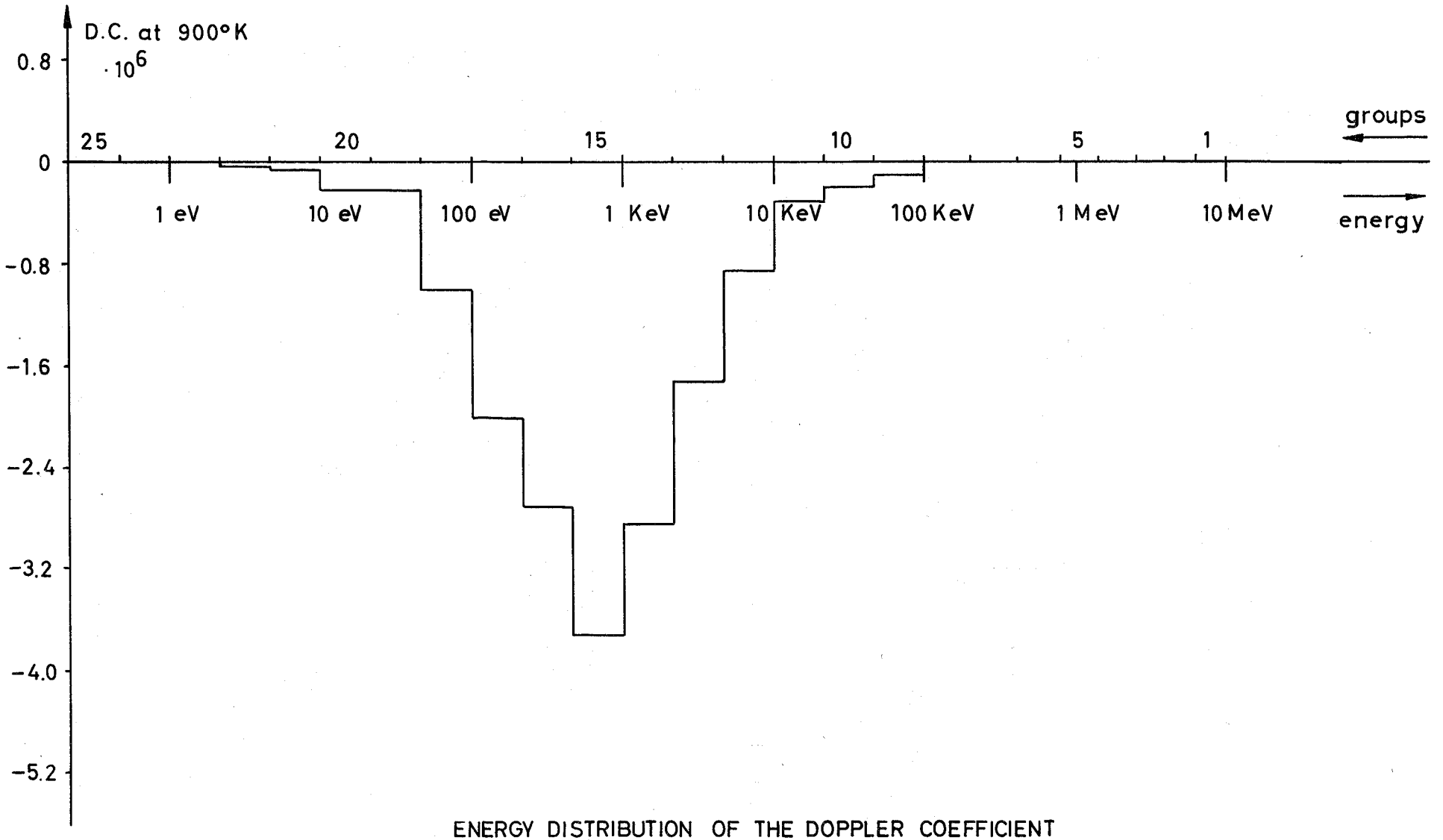


Comparison of Theoretical with Experimental Neutron Spectra in Assembly 4S. The histogram gives the result of the ELMOE calculation; the continuous curve shows the same results through a Gaussian window.



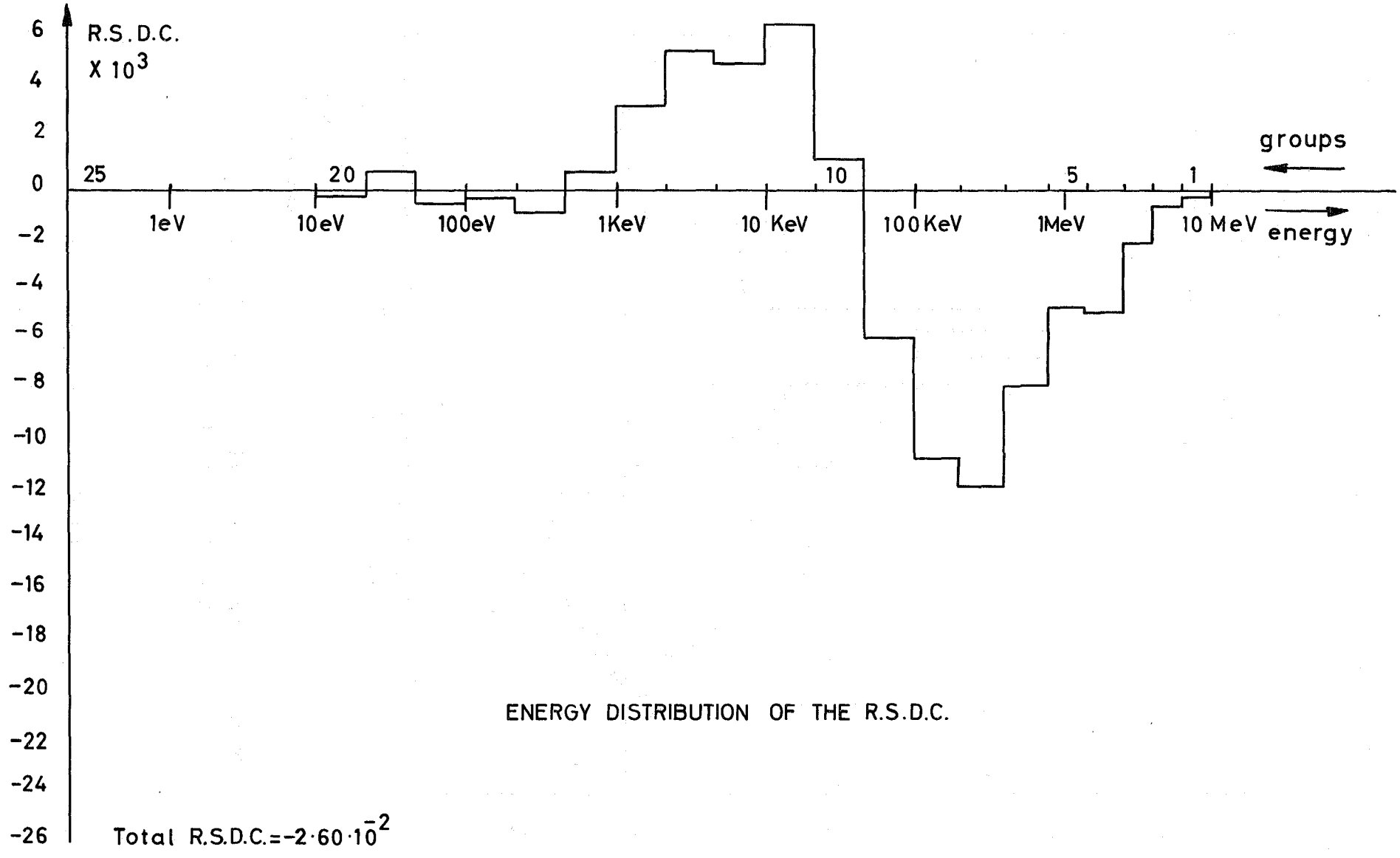
Comparison of Theoretical with Experimental Neutron Spectra in Assembly 4Z. The histogram gives the result of the ELMOE calculation; the continuous curve shows the same results through a Gaussian window.

Fig. 9



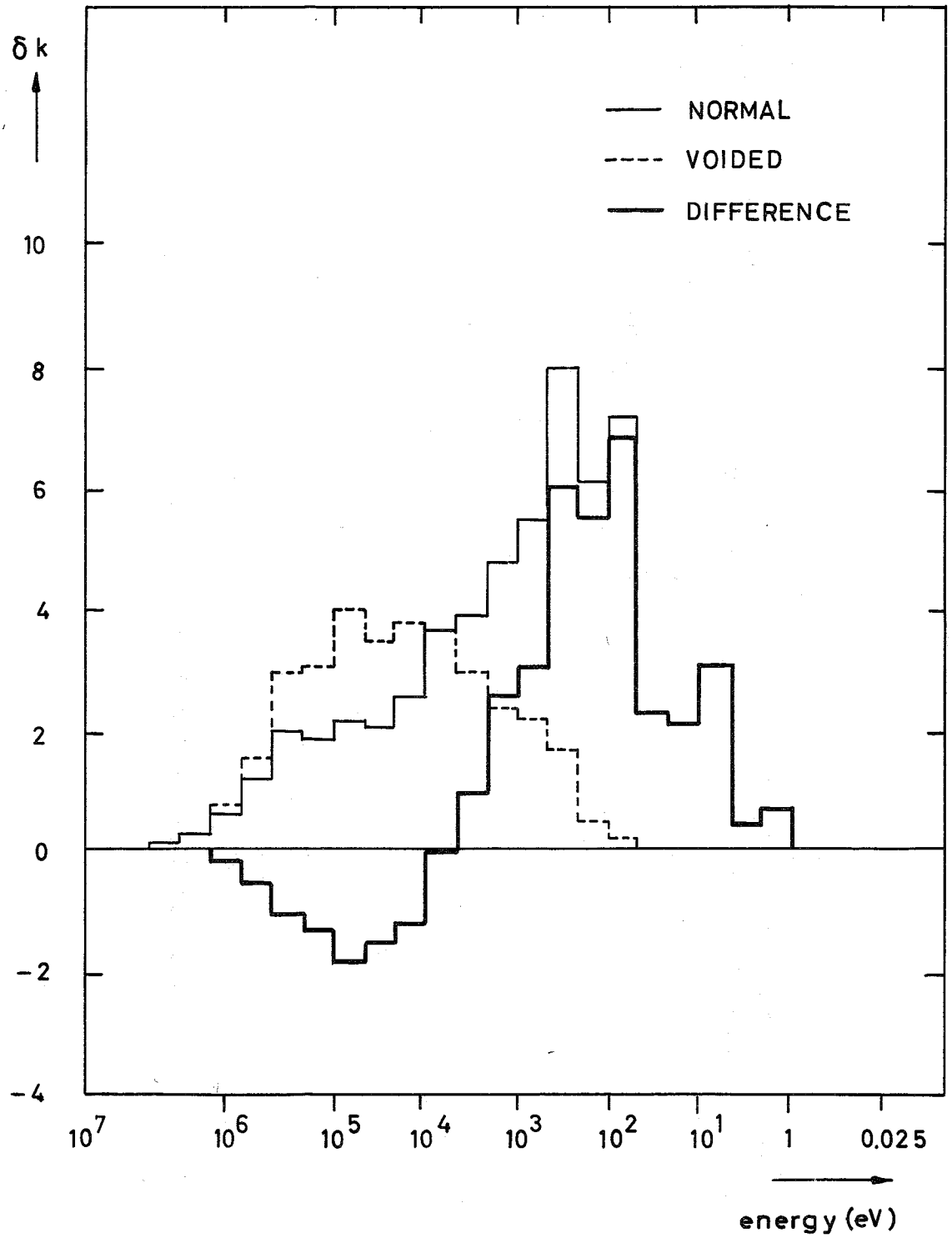
ENERGY DISTRIBUTION OF THE DOPPLER COEFFICIENT

FIG. 10



ENERGY DISTRIBUTION OF THE R.S.D.C.

FIG. 11



ENERGY DISTRIBUTION OF THE FISSION PRODUCT INFLUENCE ON REACTIVITY [31]

FIG. 12