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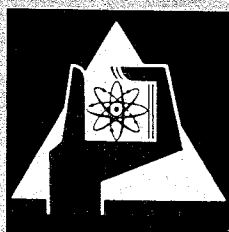
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A Method of Evaluating Progressive Substitution Experiments
for the Determination of Bucklings and Critical Radii

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A Method of Evaluating Progressive Substitution Experiments
for the Determination of Bucklings and Critical Radii⁺⁾

by

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^{+) Work performed within the association in the field of fast reactors between the European Atomic Energy Community and Gesellschaft für Kernforschung mbH., Karlsruhe}

Abstract

It is shown that the variation in critical radius upon substitution of a new composition in a central zone of a reactor can be approximated by a three-parameter equation, one of the parameters being $\Delta B_r/B_r$, the relative difference of the radial bucklings in the two reactor zones. A comparison with multigroup calculations shows the validity of the equations derived. Modifications are introduced which make them applicable for a considerable range of difference in the nuclear parameters of the two compositions. It is also shown that the presence of a reflector does not greatly interfere with the method. Finally an evaluation is given of three substitution experiments which were performed on the critical facilities SNEAK and MASURCA.

INTRODUCTION

The method of progressive substitution serves to determine physics parameters of reactor compositions built into the central zone of a zoned critical experiment. In the past it has been used extensively to measure material bucklings in thermal lattices (see for example ref. 1 to 3). It is the aim of the present work to provide a simple but physically meaningful model for the interpretation of progressive substitution experiments in fast criticals. The parameters asked for are the material buckling and the critical radius of a reactor with a core consisting entirely of the composition under investigation.

The substitution experiment starts with a core consisting entirely of a reference composition (composition 2) which must be available in sufficient quantity to build the whole core, and should have nuclear characteristics not too different from the composition under investigation (composition 1). Starting from the core center composition 2 is replaced stepwise by composition 1 until the central zone has reached its maximum size. After each step the change in reactivity or in critical radius is measured.

From these data one seeks to derive sufficient information to determine the parameters of interest.

In the practical cases which will be discussed the experiment was performed in cylindrical geometry: the central zone extended axially throughout a cylindrical reference core and its size was increased stepwise in radial direction.

In deriving a method of interpretation special attention was given to the effect of spectral mismatch between the two regions, the presence of a radial reflector and larger differences in the material bucklings.

CALCULATIONAL METHODS

One-Group Perturbation Theory

This approach makes use of the conventional formalism of perturbation theory. The differences in the nuclear parameters of the two compositions as well as the perturbation of the flux upon introduction of the central zone are assumed to be small, so that second order terms are negligible. Also the spectral interaction at the boundary of the two zones is neglected. Under these conditions the change in reactivity when the central zone is substituted can be written as

$$\frac{\Delta k}{k} = \frac{1}{\int_{V_0} SS^+ dV} \left\{ \int_{V_1} \phi^+ \Delta A \phi dV - \int_{V_1} \frac{d\phi^+}{dr} \Delta D \frac{d\phi}{dr} dV \right\} \quad (1)$$

where $A = (v-1)\Sigma_f - \Sigma_c - DB_r^2_{\text{transv.}}$

S and S^+ are the real and adjoint sources respectively

V_0 is the total volume of the reactor and

V_1 is the volume of the central zone.

In many instances it is more expedient to work with the change in critical radius (Δr_c) rather than with the reactivity change $\Delta k/k$. The change in radius compensating for a reactivity change $\Delta k/k$ is approximately given by

$$\frac{D\Delta B_r^2}{\int_{V_0} SS^+ dV} = \frac{\Delta k}{k} \quad (2)$$

or
$$\frac{\Delta r_c}{r_0} = - \frac{\Delta k}{k} \frac{1}{2DB_r^2} \frac{\int_{V_0} SS^+ dV}{\int_{V_0} \phi\phi^+ dV} \quad \text{where } r_0 = \frac{2.405}{B_r} \text{ is the core radius plus reflector savings.} \quad (3)$$

Using Eq. 1, 3 and the general neutron balance equation

$$A = DB_r^2 \quad (4)$$

one finally gets

$$\Delta r_c = \frac{r_0}{2 \int_{V_0} \phi\phi^+ dV} \left\{ - \frac{\Delta A}{A} \int_{V_1} \phi\phi^+ dV + \frac{1}{B_r^2} \frac{\Delta D}{D} \int_{V_1} \frac{d\phi}{dr} \frac{d\phi^+}{dr} dV \right\} \quad (5)$$

From Eq. 4 follows

$$\frac{\Delta B_r}{B_r} = \frac{1}{2} \left(\frac{\Delta A}{A} - \frac{\Delta D}{D} \right) \quad (6)$$

and Eq. 5 can be rewritten

$$\Delta r_c = -r_o \left\{ \frac{\frac{\Delta B_r}{B_r} \int_{V_1} \phi \phi^+ dV}{\int_{V_o} \phi \phi^+ dV} + \frac{\frac{\Delta D}{2D} \int_{V_1} \left(\phi \phi^+ - \frac{1}{B_r^2} \frac{d\phi}{dr} \frac{d\phi^+}{dr} \right) dV}{\int_{V_o} \phi \phi^+ dV} \right\} \quad (7)$$

Under the assumption that the fundamental distribution of the fluxes is known with sufficient accuracy Eq. 5 could be used to determine $\Delta B_r/B_r$ and $\Delta D/D$ from the changes in critical radius found after the steps in a substitution experiment, and to extrapolate the critical radius to the case of complete substitution, where the core consists entirely of the composition under investigation.

However, in most substitution experiments with fast cores the difference in the neutron spectra of the two zones causes significant reactivity effects at the zone boundary, so that the change in reactivity or critical radius is not described well by the above formalism even if the difference in material bucklings in the two zones is small. It is therefore necessary to use a more detailed analysis which includes the neutron spectrum and its spatial dependence.

The Overlapping-Group Method

In order to make the analysis as representative as possible of the true spectra while still working with a small number of groups Storrer and Chaumont (ref. 4) introduced the concept of overlapping groups. In their description the neutron flux at each point of the reactor is synthesized of two components having the equilibrium spectra of the two compositions. To give a basis for further discussions the basic equations of Storrer and Chaumont will be briefly recapitulated:

The space- and energy-dependent flux is assumed to be given by

$$\Psi(E, r) = g_1(r) \varphi_1(E) + g_2(r) \varphi_2(E) \quad (8)$$

where $\varphi_1(E)$ and $\varphi_2(E)$ represent the equilibrium spectra for the inner and outer zone and $g_1(r)$ and $g_2(r)$ are space-dependent functions yet to be determined.

Inserting the flux as given by Eq. 8 into the general diffusion equation and integrating over all energy while weighting with the equilibrium adjoint of either of the two compositions yields the two overlapping-group equation for the flux in zone i:

$$D_{(11)i} \Delta g_1(r) + D_{(12)i} \Delta g_2(r) + A_{(11)i} g_1(r) + A_{(12)i} g_2(r) = 0 \quad (9)$$

$$D_{(21)i} \Delta g_1(r) + D_{(22)i} \Delta g_2(r) + A_{(21)i} g_1(r) + A_{(22)i} g_2(r) = 0 \quad (10)$$

with

$$D_{(k,l)i} = \int \varphi_k^+(E) D_i(E) \varphi_l(E) dE \quad (11)$$

and

$$A_{(k,l)i} = \int dE \left\{ \varphi_k^+(E) \left[-\Sigma_{t,i}(E) \varphi_l(E) + \int \Sigma_i(E' \rightarrow E) \varphi_l(E') dE' \right] \right\} \quad (12)$$

where the first term in the square bracket of Eq. 12 represents the loss by collision and lateral leakage and the second both the slowing-down source and the fission source.

For each zone i equations 9 and 10 yield two sets of eigenfunction and eigenvalue.

The first is the eigenfunction

$$\phi_{Bi} = f_i(r) \varphi_i(E) \quad (13)$$

and eigenvalue

$$E_{Bi} = B_i^2 = \frac{A_{(ii)i}}{D_{(ii)i}} = \frac{A_{(12)i}}{D_{(12)i}} = \frac{A_{(21)i}}{D_{(21)i}} \quad (14)$$

where $f_i(r)$ is the fundamental mode distribution for zone i (in the cylindrical core zero order Bessel function) and B_i is the corresponding radial buckling.

The second solution is given by

$$\phi_{\mu i} = \begin{cases} h_1(r) \left(\varphi_2(E) - \frac{A_{(12)1}}{A_{(11)1}} \varphi_1(E) \right) \text{ in zone 1} \\ h_2(r) \left(\varphi_1(E) - \frac{A_{(21)2}}{A_{(22)2}} \varphi_2(E) \right) \text{ in zone 2} \end{cases} \quad (15)$$

with the eigenvalue

$$\mu_i^2 = - \frac{1}{B_i^2} \frac{\text{Det}_i(A)}{\text{Det}_i(D)} \quad (16)$$

where the $h_i(r)$ are the solutions of the equation

$$\Delta h_i(r) = \mu_i^2 h_i(r) \quad (17)$$

(in the cylindrical case: zero order hyperbolic Bessel functions)

and

$$\text{Det}_i(A) = A_{(11)}i^A_{(22)}i^{-A}_{(12)}i^A_{(21)}i \quad (18)$$

$$\text{Det}_i(D) = D_{(11)}i^D_{(22)}i^{-D}_{(12)}i^D_{(21)}i \quad (19)$$

The physical meaning of these results is that the flux in each zone consists of one component having the equilibrium spectrum for the respective composition and spatially the fundamental mode distribution and of a second component determined by the difference of the equilibrium spectra and a spatial distribution decreasing away from the zone boundary with a relaxation constant μ_i .

The total flux in each zone may be written as

$$\phi_i = X_i \phi_{Bi} + \bar{X}_i \phi_{\mu i} \quad (20)$$

where the X_i and \bar{X}_i are constants yet to be determined.

The above formalism which was derived up to this point by Storrer and Chaumont (ref. 4) gives the possibility to find the change in critical radius or in reactivity upon introduction of a central zone by different approaches.

A direct continuation of the theory is the evaluation of the change in critical radius by solving the criticality determinant. For this purpose the reactor is approximated by a bare two zone core. The fundamental and transient eigenfunctions f and h are then definitely determined by the boundary conditions at the core edge and center (zero flux and derivative, respectively).

In order to simplify calculations these eigenfunctions will now be rewritten in the form

$$F_i(\rho_i) = F_i(B_i r) = f(r) \quad (21)$$

$$\text{and } H_i(\bar{\rho}_i) = H_i(\mu_i r) = h(r) \quad (22)$$

The radial derivatives then become

$$\frac{df_i(r)}{dr} = B_i \frac{dF(\rho_i)}{d(\rho_i)} = B_i F'(\rho_i) \quad (23)$$

$$\text{and } \frac{dh_i(r)}{dr} = \mu_i \frac{dH(\bar{\rho}_i)}{d\bar{\rho}_i} = \mu_i H'_i(\bar{\rho}_i) \quad (24)$$

the derivative with respect to ρ being denoted by a prime. The mathematical form of the functions F and H are shown for different geometry in Table I.

At the zone boundary the overlapping group theory requires continuity in the flux and in the energy integrated current, weighted by either of the two equilibrium adjoint spectra. This requirement leads to the following set of equations in the coefficients X

$$\begin{aligned}
 X_1 F_1(\rho_{z1}) - \frac{A_{(12)1}}{A_{(11)1}} \bar{X}_1 H_1(\bar{\rho}_{z1}) &= \bar{X}_2 H_2(\bar{\rho}_{z2}) ; \\
 \bar{X}_1 H_1(\bar{\rho}_{z1}) &= X_2 F_2(\rho_{z2}) - \frac{A_{(21)2}}{A_{(22)2}} \bar{X}_2 H_2(\bar{\rho}_{z2}) ; \\
 D_{(11)1} X_1 B_1 F_1'(\rho_{z1}) + \left(D_{(12)1} - \frac{A_{(12)1}}{A_{(11)1}} D_{(11)1} \right) \bar{X}_1 \mu_1 H_1'(\bar{\rho}_{z1}) &= \\
 = D_{(12)2} X_2 B_2 F_2'(\rho_{z2}) + \left(D_{(11)2} - \frac{A_{(21)2}}{A_{(22)2}} D_{(12)2} \right) \bar{X}_2 \mu_2 H_2'(\bar{\rho}_{z2}) ; & \quad (25)^+ \\
 D_{(21)1} X_1 B_1 F_1'(\rho_{z1}) + \left(D_{(22)1} - \frac{A_{(12)1}}{A_{(11)1}} D_{(21)1} \right) \bar{X}_1 \mu_1 H_1'(\bar{\rho}_{z1}) &= \\
 = D_{(22)2} X_2 B_2 F_2'(\rho_{z2}) + \left(D_{(21)2} - \frac{A_{(21)2}}{A_{(22)2}} D_{(22)2} \right) \bar{X}_2 \mu_2 H_2'(\bar{\rho}_{z2}) . &
 \end{aligned}$$

When the reactor is critical the determinant (DET) of this set of equations disappears. When the reactor is near critical the change Δr_c which is necessary to bring about criticality is given by

$$\Delta r_c = - \frac{DET}{dDET/dr_o} \quad (26)$$

(the core radius r_o entering into DET through the eigenfunctions F and H).

Using the original core radius for calculating the right side of Eq. 26, one obtains for Δr_c the change in critical radius brought about by substituting the central zone.

Evaluating the determinant one finds that it can be written in the form

+) The ρ_{zi} and $\bar{\rho}_{zi}$ are the values of $B_i r$ and $\mu_i r$, respectively, at the zone boundary. The underlined expressions are zero (Eq. 14)

$$\begin{aligned}
 \text{DET} &= a \left(F_1'(\rho_{z1})F_2(\rho_{z2}) - F_1(\rho_{z1})F_2'(\rho_{z2}) \right) \\
 &+ b \left(F_1'(\rho_{z1})F_2(\rho_{z2}) + F_1(\rho_{z1})F_2'(\rho_{z2}) \right) \\
 &+ c F_1(\rho_{z1})F_2(\rho_{z2}) \\
 &+ d F_1'(\rho_{z1})F_2'(\rho_{z2})
 \end{aligned} \tag{27}$$

with

$$\begin{aligned}
 a &= \frac{1}{2} \left\{ \text{Det}_1(D) \left(-B_1\mu_1 - B_2\mu_1 \frac{D(12)2^A(21)2}{D(11)1^A(22)2} \right) H_1'(\bar{\rho}_{z1})H_2(\bar{\rho}_{z2}) + \right. \\
 &\quad \left. + \text{Det}_2(D) \left(B_1\mu_2 \frac{D(21)1^A(12)1}{D(22)2^A(11)1} + B_2\mu_2 \right) H_1(\bar{\rho}_{z1})H_2'(\bar{\rho}_{z2}) \right\}
 \end{aligned}$$

$$\begin{aligned}
 b &= \frac{1}{2} \left\{ \text{Det}_1(D) \left(-B_1\mu_1 + B_2\mu_1 \frac{D(12)2^A(21)2}{D(11)1^A(22)2} \right) H_1'(\bar{\rho}_{z1})H_2(\bar{\rho}_{z2}) + \right. \\
 &\quad \left. + \text{Det}_2(D) \left(B_1\mu_2 \frac{D(21)1^A(12)1}{D(22)2^A(11)1} - B_2\mu_2 \right) H_1(\bar{\rho}_{z1})H_2'(\bar{\rho}_{z2}) \right\}
 \end{aligned}$$

$$c = \mu_1\mu_2 \frac{\text{Det}_1(D)\text{Det}_2(D)}{D(11)1 D(22)2} H_1'(\bar{\rho}_{z1})H_2'(\bar{\rho}_{z2})$$

$$\text{and } d = B_1B_2 \frac{(D(11)1^D(22)2^{-D}(21)1^D(12)2)^D (D(11)1^D(22)2^{-D}(12)1^D(21)2)^D}{D(11)1 D(22)2} \times$$

$$\times H_1(\bar{\rho}_{z1})H_2(\bar{\rho}_{z2})$$

Forming of this expression the derivative with respect to r_o one notes that the expressions $(F'_1(\rho_{z1})F_2(\rho_{z2}) - F_1(\rho_{z1})F'_2(\rho_{z2}))$, b, c and d are sufficiently small so that the derivative is essentially given by

$$\frac{dDET}{dr_o} = a \frac{d}{dr_o} (F'_1(\rho_{z1})F_2(\rho_{z2}) - F_1(\rho_{z1})F'_2(\rho_{z2})) \quad (28)$$

For equal B_1 and B_2 the expression $(F'_1(\rho_{z1})F_2(\rho_{z2}) - F_1(\rho_{z1})F'_2(\rho_{z2}))$ disappears. Therefore we find by differentiation with respect to B_1

$$(F'_1(\rho_{z1})F_2(\rho_{z2}) - F_1(\rho_{z1})F'_2(\rho_{z2})) = \Delta B r_z (F''_1(\rho_{z1})F_2(\rho_{z2}) - F'_1(\rho_{z1})F'_2(\rho_{z2})) \quad (29)$$

Using the general differential equation

$$F''(\rho) + \frac{n-1}{\rho} F'(\rho) + F(\rho) = 0 \quad (30)$$

(n=1 plane geometry ; n=2 cylinder ; n=3 sphere)

and the boundary condition

$$F_2(\rho_o) = 0 \quad \rho_o = B_2 r_o \quad (31)$$

one can show that with good approximation

$$\frac{d}{dr_o} (F'_1(\rho_{z1})F_2(\rho_{z2}) - F_1(\rho_{z1})F'_2(\rho_{z2})) = -F'_1(\rho_o)F'_2(\rho_o) \left(\frac{\rho_o}{\rho_{z2}}\right)^{n-1} B_2 \quad (32)$$

The results for Δr_c then becomes

$$\begin{aligned} \Delta r_c = & \frac{1}{B_2 F'_1(\rho_o) F'_2(\rho_o)} \left(\frac{\rho_{z2}}{\rho_o}\right)^{n-1} \left\{ \frac{\Delta B}{B} \rho_{z2} (F''_1(\rho_{z1})F_2(\rho_{z2}) - F'_1(\rho_{z1})F'_2(\rho_{z2})) \right. \\ & + \frac{1}{a} \left(b(F'_1(\rho_{z1})F_2(\rho_{z2}) + F_1(\rho_{z1})F'_2(\rho_{z2})) + cF_1(\rho_{z1})F_2(\rho_{z2}) + \right. \\ & \left. \left. + dF'_1(\rho_{z1})F'_2(\rho_{z2}) \right) \right\} \quad (33) \end{aligned}$$

where a, b, c, and d are the factors appearing in the four terms of Eq. 27.

Eq. 33 gives the change in critical radius in four terms in different combinations of the fundamental eigenfunctions. It can be easily shown that the coefficients b/a , c/a , and d/a which still contain the transient functions are essentially constant except when the central zone radius is very small or very large. Thus, in principle Eq. 33 can be used directly for the evaluation of substitution experiments. However, the physical meaning of the expression becomes more evident if it is rewritten, making some further simplifications:

We will allow the indices 1 and 2 to be dropped and assume the same parameter (eigenfunction, eigenvalue, diffusion constant) to apply for both compositions where this does not introduce serious error (as in the case when differences are involved). We define

$$\frac{\Delta D}{D} = \frac{D_{(11)1}^{-D} (11)2}{D_{(11)1}} = \frac{D_{(22)1}^{-D} (22)2}{D_{(22)2}} \quad (34)$$

One then gets for a and b

$$a = B \left(-\mu_1 \text{Det}_1(D) H_1'(\bar{\rho}_{z1}) H_2(\bar{\rho}_{z2}) + \mu_2 \text{Det}_2(D) H_1(\bar{\rho}_{z1}) H_2'(\bar{\rho}_{z2}) \right) \quad (35)$$

and with

$$\frac{A_{(21)2}}{A_{(22)2}} = \frac{D_{(21)2}}{D_{(22)2}} \quad \text{and} \quad \frac{A_{(12)1}}{A_{(11)1}} = \frac{D_{(12)1}}{D_{(11)1}} \quad (\text{from Eq. 14})$$

$$\begin{aligned} b &= \frac{\mu_1 \text{Det}_1(D)}{2D_{(11)1}} \left(-B_1 D_{(11)1} + B_2 D_{(11)2} - \frac{B_2 \text{Det}_2 D}{D_{(22)2}} \right) H_1'(\bar{\rho}_{z1}) H_2(\bar{\rho}_{z2}) + \\ &+ \frac{\mu_2 \text{Det}_2(D)}{2D_{(22)2}} \left(-\frac{B_1 \text{Det}_1(D)}{D_{(11)1}} + B_1 D_{(22)1} - B_2 D_{(22)2} \right) H_1(\bar{\rho}_{z1}) H_2'(\bar{\rho}_{z2}) \quad (36) \\ &= \frac{1}{2} \left(-\mu_1 \text{Det}_1(D) H_1'(\bar{\rho}_{z1}) H_2(\bar{\rho}_{z2}) + \mu_2 \text{Det}_2(D) H_1(\bar{\rho}_{z1}) H_2'(\bar{\rho}_{z2}) \right) \left(\Delta B + B \frac{\Delta D}{D} \right) \\ &- \frac{1}{2} \frac{\text{Det}_1(D) \text{Det}_2(D)}{D_{(11)1} D_{(22)2}} \left(\mu_1 B_2 H_1'(\bar{\rho}_{z1}) H_2(\bar{\rho}_{z2}) + \mu_2 B_1 H_1(\bar{\rho}_{z1}) H_2'(\bar{\rho}_{z2}) \right) \end{aligned}$$

We may now rewrite Eq. 33 in the form

$$\begin{aligned} \Delta r_c &= \frac{1}{B_2 F_1'^2(\rho_o) \rho_o^{n-1}} \left\{ \frac{\Delta B}{B} (\rho_z^n F''(\rho_z) F(\rho_z) - \rho_z^{n-1} F'^2(\rho_z) + \rho_z^{n-1} F(\rho_z) F'(\rho_z)) \right. \\ &\left. + \rho_z^{n-1} \frac{\Delta D}{D} F(\rho_z) F'(\rho_z) \right\} + Q \quad (37) \end{aligned}$$

where the differences in the F_1 and F_2 functions have been neglected and all terms not containing $\Delta B/B$ or $\Delta D/D$ explicitly were included in the rest

term Q.

Using Eq. 3o it is easy to show that

$$\rho_z^n F''(\rho_z) F(\rho_z) - \rho_z^n F'^2(\rho_z) + \rho_z^{n-1} F(\rho_z) F'(\rho_z) = -2 \int_0^{\rho_z} \rho^{n-1} F^2(\rho) d\rho \quad (38)$$

$$\rho_z^{n-1} F(\rho_z) F'(\rho_z) = - \int_0^{\rho_z} \rho^{n-1} (F^2(\rho) - F'^2(\rho)) d\rho \quad (39)$$

and $\rho_0^{n-1} F'^2(\rho_0) = \frac{2}{\rho_0} \int_0^{\rho_0} \rho^{n-1} F^2(\rho) d\rho \quad (4o)$

Our final equation for the change in critical radius therefore becomes

$$\begin{aligned} \Delta r_c = & \frac{r_o}{\int_0^{\rho_o} \rho^{n-1} F^2(\rho) d\rho} \left\{ - \frac{\Delta B}{B} \int_0^{\rho_z} \rho^{n-1} F^2(\rho) d\rho - \frac{\Delta D}{2D} \int_0^{\rho_z} \rho^{n-1} (F^2(\rho) - F'^2(\rho)) d\rho \right. \\ & + \frac{\rho_z^{n-1}}{2BD^2 (\text{Det}_1(D) \mu_1 R_1 + \text{Det}_2(D) \mu_2 R_2)} \left(\mu_1 \mu_2 \text{Det}_1(D) \text{Det}_2(D) R_1 R_2 F^2(\rho_z) \right. \\ & + (\mu_1 B_2 R_1 - \mu_2 B_1 R_2) \text{Det}_1(D) \text{Det}_2(D) F(\rho_z) F'(\rho_z) \\ & \left. \left. - B_1 B_2 ({}^D(11)1^D(22)2^{-D}(21)1^D(12)2) ({}^D(11)1^D(22)2^{-D}(12)1^D(21)2) F'^2(\rho_z) \right) \right\} \quad (41) \end{aligned}$$

with $R_i = \left| \frac{H'_i}{H_i} \right|$

The first two terms of Eq. 41 correspond exactly to the perturbation formula (Eq. 7). The other terms are different from 0 only if the spectra in the two medii are not the same and they are proportional to the surface of the zone interface. They may therefore be interpreted as interface terms caused by the interaction of different spectra at the zone boundary. In practical applications it is generally sufficient to consider the contribution proportional to $F^2(\rho_z)$, omitting the last two terms because

$$\mu_1 \mu_2 \begin{cases} \gg |\mu_1 B_2 R_1 - \mu_2 B_1 R_2| \\ \gg B_1 B_2 \end{cases} \quad (42)$$

This leaves us with an equation in three terms which is physically interpretable and well suited for the evaluation of substitution experiments.

For easier comparison with results below Eq. 41 may be rewritten, using Eq. 14 and 16, as:

$$\Delta r_c = \frac{r_o}{\int_0^{\rho_o} \rho^{n-1} F^2(\rho) d\rho} \left\{ -\frac{\Delta B}{B} \int_0^{\rho_z} \rho^{n-1} F^2(\rho) d\rho - \frac{\Delta D}{2D} \int_0^{\rho_z} \rho^{n-1} (F^2(\rho) - F'^2(\rho)) d\rho \right.$$

$$- \frac{\rho_z^{n-1}}{\text{Det}_1(D)} \left[\frac{B}{\mu} \frac{\text{Det}_1 A}{A^2} \left(R_1 R_2 F^2(\rho_z) + \frac{B_2}{\mu_2} R_1 F(\rho_z) F'(\rho_z) \right. \right.$$

$$- \left. \left. \frac{B_1}{\mu_1} R_2 F(\rho_z) F'(\rho_z) \right) \right] - \frac{\mu}{B} \frac{(A_{(11)} I^A_{(22)} 2^{-A} (21) I^A_{(22)} 2^A (A_{(11)} I^A_{(22)} 2^{-A} (12) I^A_{(21)} 2^A)}{A^2 \text{Det}_2 A}$$

$$\left. \times F'^2(\rho_z) \right\} \quad (41a)$$

As the derivation of Eq. 41 and 41a is rather complicated and the influence of the various approximations is not easy to estimate it will be helpful to look at another mathematical approach before describing practical applications.

Calculation of Δr_c Using an Overlapping-Group Perturbation Formalism

We know from perturbation theory that a reactivity change is given exactly by the equation

$$\frac{\Delta k}{k} = \frac{\int_V \phi_o^+ R \phi_p dV}{\int_V S_o^+ S_p dV} \quad (43)$$

where the real fluxes and source are given for the perturbed case while the adjoint quantities are for the original reactor. In first order perturbation calculations the unperturbed fluxes are being used throughout. Avoiding this approximation we will now use the information provided by the overlapping group theory to include the perturbation of the real fluxes into the consideration.

The perturbed flux in the central zone is given by

$$\phi_p = F_1(\rho_1)\varphi_1(E) + \alpha \left(\varphi_2(E) - \frac{A_{(12)1}}{A_{(11)1}} \varphi_1(E) \right) H_1(\bar{\rho}_1) \quad (44)$$

α being an unknown coefficient. (This expression is equivalent to Eq. 2o, the coefficient of the fundamental component can be set equal to one without loss of generality.

The unperturbed adjoint flux is

$$\phi_o^+ = F_2(\rho_2)\varphi_2^+(E) \quad (45)$$

Before using these fluxes to calculate $\Delta k/k$ it is convenient to rewrite Eq. 43 in the form

$$\frac{\Delta k}{k} = \frac{\int_{V_1} \phi_o^+ (M_p - M_o) (\phi_o + \delta\phi) dV}{\int_{V_o} S_o^+ S_p dV} = \frac{\int_{V_1} \phi_o^+ P \phi_o dV + \int_{V_1} \phi_o^+ M_p \delta\phi dV}{\int_{V_o} S_o^+ S_p dV} \quad (46)$$

where M_p and M_o are the perturbed and the unperturbed diffusion operator. The term containing $M_o \delta\phi$ disappeared because of the relationship

$$\int_{V_1} \phi_o^+ M_o \delta\phi dV = 0 \quad (47)$$

The first term $\int_{V_1} \phi_o^+ P \phi_o dV$ corresponds to normal perturbation theory and does not need further evaluation. For calculating the second term we set $\delta\phi$ equal to the transient term and find

$$\begin{aligned} \int_{V_1} \phi_o^+ M_p \delta\phi dV &= \alpha \int_{V_1} F_2(\rho_2)\varphi_2^+(E) M_p \left(\varphi_2(E) - \frac{A_{(12)1}}{A_{(11)1}} \varphi_1(E) \right) H_1(\bar{\rho}_1) dV \\ &= \alpha \int_{V_1} \left\{ \left[A_{(22)1}^{-A_{(21)1}} \frac{A_{(12)1}}{A_{(11)1}} \right] F_2(\rho_2) H_1(\bar{\rho}_1)^{-\mu_1} B_2 \left[D_{(22)1}^{-D_{(21)1}} \frac{A_{(12)1}}{A_{(11)1}} \right] \right. \\ &\quad \left. F_2'(\rho_2) H_1'(\bar{\rho}_1) \right\} dV \end{aligned} \quad (48)$$

Using Eq. 3o and the corresponding equation for the transient functions

$$H''(\bar{\rho}) + \frac{n-1}{\bar{\rho}} H'(\bar{\rho}) - \mu^2 H(\bar{\rho}) = 0 \quad (49)$$

one can calculate the volume integrals

$$\int_0^z r^{n-1} F_2(\rho_2) H_1(\bar{\rho}_1) dr = \frac{\mu_1 H_1'(\bar{\rho}_{z1}) F_2(\rho_{z2}) - B_2 H_1(\bar{\rho}_{z1}) F_2'(\rho_{z2})}{\mu_1^2 + B_2^2} r_z^{n-1} \quad (50)$$

$$\text{and } \int_0^z r^{n-1} F_2'(\rho_2) H_1'(\bar{\rho}_1) dr = \frac{\mu_1 H_1(\bar{\rho}_{z1}) F_2'(\rho_{z2}) + B_2 H_1'(\bar{\rho}_{z1}) F_2(\rho_{z2})}{\mu_1^2 + B_2^2} r_z^{n-1} \quad (51)$$

Introducing these expressions in Eq. 48 and eliminating the diffusion coefficients by using Eq. 14 and 16 we get

$$\frac{1}{2\pi} \int_{V_1} \phi^+ M_p \delta\phi dV = \alpha \frac{\text{Det}_1(A)}{A_{(11)1}} \frac{r_z^{n-1}}{\mu_1} H_1'(\bar{\rho}_{z1}) F_2(\rho_{z2}) \quad (52)$$

The remaining task now is to calculate α , the coefficient of the transient. A direct calculation from the system of linear equations (Eq. 25) representing the boundary conditions in the overlapping group formalism leads to complicated results which so far could not be evaluated meaningfully.

An alternative approach which certainly is a rather crude approximation is to disregard the difference of the eigenvalue and the fundamental distributions for the two compositions and to postulate continuity of values and derivatives of the overlapping group fluxes at the zone boundary in the following form

$$F(\rho_z) = \alpha H_1(\rho_z) + \bar{\alpha} H_2(\rho_z) \quad (53)$$

$$BF'(\rho_z) = \alpha \mu H_1'(\rho_z) + \bar{\alpha} \mu H_2'(\rho_z)$$

$\bar{\alpha}$ being the transient coefficient in the outer zone. This yields the result

$$\alpha = \frac{F(\rho_z) H_2'(\rho_z) - \frac{B}{\mu} F'(\rho_z) H_2(\rho_z)}{H_1(\rho_z) H_2'(\rho_z) - H_1'(\rho_z) H_2(\rho_z)} \quad (54)$$

and therefore

$$\frac{1}{2\pi} \int_{V_1} \phi^+ M_p \delta\phi dV = \frac{1}{\mu} \frac{\text{Det}_1(A)}{A_{(11)1}} r_z^{n-1} \left(\frac{R_1 R_2 F^2(\rho_z)}{R_2 + R_1} + \frac{B}{\mu} \frac{R_1 F(\rho_z) F'(\rho_z)}{R_2 + R_1} \right) \quad (55)$$

Collecting all contributions and rewriting the result in terms of change in critical radius one finds

$$\Delta r_c = \frac{r_o}{\int_0^{\rho_o} \rho^{n-1} F^2(\rho) d\rho} \left\{ - \frac{\Delta B}{B} \int_0^{\rho_z} \rho^{n-1} F^2(\rho) d\rho - \frac{\Delta D}{2D} \int_0^{\rho_z} \rho^{n-1} (F^2(\rho) - F'^2(\rho)) d\rho \right. \\ \left. - \frac{\rho_z^{n-1}}{2(R_1+R_2)} \frac{\text{Det}_1(A)}{A^2} \frac{B}{\mu} \left(R_1 R_2 F^2(\rho_z) + \frac{B}{\mu} R_1 F(\rho_z) F'(\rho_z) \right) \right\} \quad (56)$$

(the influence of the perturbation on the normalization integral was neglected)

We see that this expression is almost identical with the first four terms in Eq. 41a (except for the factor $\frac{\text{Det}_1(D)}{\text{Det}_2(D)}$ in the denominator of the interface terms). The fifth term (which approximately would compensate the fourth one) and the last term do not appear. As only the first three terms give significant contributions this discrepancy is of little practical importance although the investigation of its origin might be quite interesting.

It should further be noted that the difference in the fundamental flux distribution for the two zones as well as the changes in the normalization integral were neglected in the evaluation. Similar simplifications were made in deriving Eq. 41a. The terms going beyond first order perturbation theory in both expressions therefore represent only the effect of spectral overlap, while the overall flux distribution is still considered to be essentially unchanged.

Summary of the Results from Overlapping Group Calculations

The two approaches used to find the change in critical radius upon introduction of a central zone yield expressions of the form

$$\Delta r_c = \frac{r_o}{\int_0^{\rho_o} \rho^{n-1} F^2(\rho) d\rho} \left\{ - \frac{\Delta B}{B} \frac{r}{r_o} \int_0^{\rho_z} \rho^{n-1} F^2(\rho) d\rho - \frac{\Delta D}{2D} \int_0^{\rho_z} \rho^{n-1} (F^2(\rho) - F'^2(\rho)) d\rho \right. \\ \left. - \frac{R_1 R_2 \rho_z^{n-1}}{2(R_1+R_2)} \frac{B}{\mu} \frac{\text{Det}_1 A}{A^2} F^2(\rho_z) \right\} \quad (57)$$

where contributions which are considerably smaller than the three terms given have been omitted. Eq. 57 is the basis for all practical applications to be discussed in this report.

In evaluating a substitution experiment one fits the expression to measured data with $\Delta B_r/B_r$, $\Delta D/D$ and $B/\mu \text{Det}_r A/A^2$ being the parameters to be adjusted. (In general it leads to better results to calculate $\Delta D/D$ from multigroup data and to use it as a fixed parameter.) The change in the critical radius for complete substitution may be found directly from ΔB_r or by extrapolating the measured data using Eq. 57 with the fitted parameters. A more detailed discussion of practical applications will be found further below.

The Influence of the Reflector

So far expressions for the change in critical radius upon introduction of the central zone have been derived for bare reactors only. A reflector which is present in most practical cases introduces a third zone into the problem.

In the first order perturbation formalism only the normalization integral is affected by the reflector. The same conclusion may be drawn for the expression derived from the overlapping group formalism, as long as the central zone is small enough so that the spatial dependence of the spectra near the zone boundary is the same as in a bare core.

As the central zone approaches the core blanket boundary the spectral interaction of all three reactor zones makes an analysis very difficult. This does not affect the buckling determination which usually can be done at smaller zone radii but it introduces an uncertainty of about 0.5 cm when in a reflected reactor the critical radius is extrapolated to the case of complete substitution. One may try to avoid this difficulty in the following way: For a number of zone radii r_z one makes multigroup calculations of the change in critical radius Δr_c for both the reflected and unreflected case. Assuming that the differences between the Δr_c for the reflected and bare reactor are calculated correctly (even though their absolute values may be wrong) one now subtracts these differences from the measured Δr_c which are thereby supposedly corrected to correspond to the bare core and can be extrapolated to the case of complete substitution without difficulty. After the

extrapolation the calculated differences between the reflected and bare case are added again to the substitution curve, yielding the final result. This method was used quite successfully for predicting the final critical radius in the French substitution experiment MASURCA 1B \rightarrow 1A'. It presupposes, however, that the spectral interaction between the reflector and both core zones is correctly given by the multigroup calculations, which is certainly not always the case.

In many instances it is convenient to calculate the change in critical radius for complete substitution ($\Delta_o r_c$) from the ΔB_r found by the fit. For the bare reactor one has approximately

$$\Delta_o r_c = -r_c \frac{\Delta B_r}{B_r} \quad (58)$$

In order to study the reflected case one has to look at the individual terms of Eq. 57. For complete substitution the transient term must be dropped as there remains no boundary between the two core zones. The terms with $\int_0^{\rho_o} \rho^{n-1} F^2(\rho) d\rho$ will be essentially the same for complete substitution in the bare and reflected case. A significant difference in $\Delta_o r_c$ is caused only by the diffusion term. It is

$$\Delta_o r_c(\text{bare}) - \Delta_o r_c(\text{refl.}) = \left[\frac{\Delta D}{2D} \frac{\int_0^{\rho_o} \rho^{n-1} F'^2 d\rho}{(\rho_o - B_r RS)} \bigg/ \int_0^{\rho_o} \rho^{n-1} F^2 d\rho \right] r_o(\text{bare}) \quad (59)$$

where RS stands for reflector savings.

To a good approximation we can then write

$$\Delta_o r_c(\text{bare}) - \Delta_o r_c(\text{refl.}) = \frac{\Delta D}{2D} \frac{B_r RS \rho_o^{n-1} F'^2(\rho_o)}{(1/2) \rho_o^n F^2(\rho_o)} r_o(\text{bare}) = \frac{\Delta D}{D} RS \quad (60)$$

$$\text{or } \Delta_o r_c(\text{refl.}) = -r_o(\text{bare}) \frac{\Delta B_r}{B_r} - RS \frac{\Delta D}{D} \quad (61)$$

Eq. 60 is equivalent with the well known result of one-group diffusion theory that the reflector savings is proportional to the diffusion coefficient of the core material. The difference of spectral interaction between the blanket and either core composition is still not taken in account and the uncertainty of about 0.5 cm quoted above for the evaluation of a direct fit still remains. Similar as in the case of direct extrapolation a correction

may be found by comparing multigroup calculations for the reflected and the unreflected case.

NUMERICAL EXPERIMENTS

In order to test the validity of the results derived by overlapping group theory a large number of multigroup calculations were performed on a test case of substitution of plutonium for uranium fuel. The composition for the core and blanket zones for this test were chosen according to preliminary data then available for SNEAK 3A-2 and 3B-2 (ref. 8 and 9). The atomic densities are shown in Table 2. The substitution is performed in cylindrical geometry, the unperturbed critical radius is 57.85 cm (bare) or 44.05 cm (reflected).

Direct Calculations

The first numerical experiment was the calculation of the change in critical radius as a function of the central zone radius for the unreflected model reactor 1) directly by multigroup diffusion theory and 2) by the overlapping group formalism using Eq. 33 (with $F_1(\rho) = F_2(\rho) = J_0((B_1 + B_2)r/2)$) and Eq. 41. The nuclear parameters in these equations (D, μ, B_p) were derived from multigroup constants and the fluxes found by the multigroup calculations. The ratios $R_i = |H_i^1/H_i|$ were set equal to unity in order to see if such a simplification was possible.

The results are shown in Fig. 1 and 2. We see that the overlapping group results give a good approximation to the multigroup data and that the simplifications made in going from Eq. 33 to Eq. 41 had no significant effect. Further we see that all terms beyond the third, that is, the interface terms proportional to $F(\rho_z)F'(\rho_z)$ and to $F'(\rho_z)^2$ only yield very small corrections. This confirms that, as stated in Eq. 57, the change in critical radius can be described essentially by three terms: A buckling and a diffusion contribution corresponding to the result of first order perturbation theory and in addition an interface term proportional to the square of the flux at the zone boundary.

Fitting of Core Parameters

The results of multigroup calculations were used not only to test the validity of the expressions derived by overlapping group theory but also to test the possibility of deriving differences in buckling and critical radii from the shape of the substitution curve.

The multigroup calculations give the critical radius as a function of the central zone radius. To this function, Eq. 57 was fitted by the least squares method in the same way as this is done in the evaluation of an actual experiment. The resulting values for $\Delta B_r/B_r$ and Δr_c (extrapolated) are compared to the corresponding values found by multigroup calculations for the case of complete substitution.

In the cylindrical geometry of our test case we have

$$\int_0^{\rho_z} \rho^{n-1} F^2(\rho) d\rho = \int_0^{\rho_z} \rho J_0^2(\rho) d\rho = \frac{\rho_z^2}{2} (J_0^2(\rho_z) + J_1^2(\rho_z)) \quad (62)$$

$$\frac{1}{2} \int_0^{\rho_z} \rho^{n-1} (F^2(\rho) - J_0^2(\rho)) d\rho = \frac{1}{2} \int_0^{\rho_z} \rho^{n-1} (J_0^2(\rho) - J_1^2(\rho)) d\rho = \frac{1}{2} \rho_z J_0 J_1 \quad (63)$$

$$\frac{\int_0^{\rho_z} \rho^{n-1} F^2(\rho) d\rho}{\int_0^{\rho_z} \rho^{n-1} J_0^2(\rho) d\rho} = \frac{\rho_z}{2.405} = \frac{3.086}{B_r} \quad (64)$$

For the direct calculations described above the ratios R_1 and R_2 were set equal to unity. However, during the work on fitting procedures it was found that deviations between multigroup data and fit curve can be decreased by about a factor of 10 if at least R_1 is calculated explicitly. (R_2 was left equal to unity, this should not have any significant effect, except possibly for very large zone radii.)

Thus we get

$$\frac{R_1 R_2}{R_1 + R_2} \approx \frac{1}{1 + \frac{H_1(\bar{\rho}_z)}{H_1'(\bar{\rho}_z)}} = \frac{1}{1 + (I_0(\mu_1 r_z) / I_1(\mu_1 r_z))} \quad (65)$$

Using relations 62-65 the fitting equation becomes

$$\Delta r_c = x_1 \rho_z^2 (J_0^2(\rho_z) + J_1^2(\rho_z)) + x_2 \rho_z J_0(\rho_z) J_1(\rho_z) + x_3 \frac{\rho_z J_0^2(\rho_z)}{1 + (I_0(\bar{\rho}_z)/I_1(\bar{\rho}_z))} \quad (66)$$

with $\frac{\Delta B_r}{B_r} = -0.648 B_r x_1$

and $\frac{\Delta D}{D} = -0.648 B_r x_2$

The quantity $x_3 = -\frac{\text{Det}_1 A/A^2}{0.648 \mu_1}$ serves here as a fitting parameter only and is of no further practical interest.

In Tables 3 and 4 results of the fitting procedure are shown for the test case described above. The multigroup calculations giving the change in critical radius as a function of the central zone radius were performed for the reflected and the unreflected reactor. In the reflected case the resulting $\Delta B_r/B_r$ was decreased by 2.5% in order to account approximately for the smaller normalization integral. All calculations were performed with $\Delta D/D$ (or x_2) as a variable and as a fixed quantity.

Fits were made including an increasing number of steps, starting with the first four. The results are always given in the line corresponding to the last step included.

The results with $\Delta D/D$ as a free parameter show rather large scatter while when $\Delta D/D$ is fixed we get very good data for the buckling and the critical radius in the unreflected case as well as for the buckling in the reflected case, as long the central zone boundary does not approach the blanket.

$\Delta_0 r_c$ for complete substitution in the reflected case was calculated by extrapolation to the core-blanket boundary and also using Eq. 60 with very similar results. It is generally found 0.5 cm smaller than the multigroup value. As stated above, this is caused by different spectral interaction between the blanket and the two core zones which is not taken in account by the method.

Extension of the Method to Large Buckling Differences

The equations used above were derived under the assumption that the two core media have about the same material buckling. It is, naturally, of considerable interest, whether an expression similar to Eq. 57 can be used for the evaluation of a substitution experiment when the difference in the bucklings is larger. An attempt was made to modify Eq. 57 so as to describe such a case more realistically and the result was tested on model cases with a $\Delta B_r/B_r$ of -10% and -30%.

A mathematical treatment of the overlapping group theory which is valid for large buckling differences would certainly be very involved. Eq. 57 therefore was modified using some physical considerations.

We have seen, when deriving Eq. 57 from the perturbation formalism that it represents essentially a reactivity balance.

Considering the integrals in the first two terms as weighting integrals it becomes clear that the situation is described more correctly if ρ_z is redefined by basing its value on the actual critical radius after each substitution step:

$$\rho_{zm} = \rho_{z2} \frac{r_o}{r_o + \Delta r_c} \quad (67)$$

As above, r_o is the critical radius for a bare core of the original composition.

When the sum on the right hand side of Eq. 57 actually describes a reactivity difference, then it must be proportional to the difference in $(1/r_c^2)$ rather than to Δr_c . An expression which fulfills this demand but becomes equal to Δr_c for small ΔB is

$$\Delta_m(r_c) = \frac{r_o^3}{2} \left[\frac{1}{r_o^2} - \frac{1}{(r_o + \Delta r_c)^2} \right] \quad (68)$$

This will be used on the left side of Eq. 57 instead of Δr_c .

Finally, we know that the equation must also be satisfied for the case of complete substitution of a bare core where only the first term remains, while all others disappear.

In order to yield an identity in this case $\Delta B_r/B_r$ must be replaced by

$$\frac{\Delta_{mo}(r_c)}{r_o} = \frac{r_o^2}{2} \left[\frac{1}{r_o^2} - \frac{1}{r_1^2} \right] \quad (69)$$

r_1 being the bare critical radius after complete substitution.

$(\Delta_{mo}(r_c))/r_o$ takes the place of $-\Delta B_r/B_r$ as a fitting parameter. The two quantities are related to each other by

$$\frac{\Delta B_r}{B_r} = \frac{r_o^{-1} - r_1^{-1}}{r_o^{-1}} = -1 + \sqrt{1 - 2(\Delta_{mo}(r_c)/r_o)} \quad (70)$$

The modified form of Eq. 57 therefore becomes

$$\Delta_m(r_c) = \frac{r_o}{\int_0^{\rho_{zm}} \rho^{n-1} F^2(\rho) d\rho} \left\{ \frac{\Delta_{mo}(r_c)}{r_o} \int_0^{\rho_{zm}} \rho^{n-1} F^2(\rho) d\rho - \frac{\Delta D}{2D} \int_0^{\rho_{zm}} \rho^{n-1} (F^2(\rho) - F'^2(\rho)) d\rho - \frac{R_1 R_2 \rho_{zm}^{n-1}}{2(R_1 + R_2)} \frac{B \text{Det} A}{\mu A^2} F^2(\rho_z) \right\} \quad (71)$$

In order to test Eq. 71 the original test case was modified by decreasing the Pu-concentration in the central zone first to 13.5×10^{20} (Modification 1), then to 12.0×10^{20} atoms/cm³ (Modification 2). Changes in critical radii as found by multigroup calculations were fitted to Eq. 71, and the values for $\Delta B_r/B_r$ and the extrapolated critical radius were deduced from fits for different numbers of steps. The calculations were done for the reflected case only and $\Delta D/D$ was used as a fixed parameter.

The results are given in Tables 5 and 6. Again we note that the dependence on the number of steps included is only very slight as long as the central zone radius does not become too large. The agreement with the values found by direct multigroup calculations is quite good, the buckling of the central zone as deduced from the fit having accuracy of about 2% even in the most unfavourable case. Of a similar quality are the values for the extrapolated critical radius. Of course, the result of two numerical experiments do not provide a definite proof for the general validity of the method. However, they give a strong indication that substitution may be used successfully even if the differences in the material bucklings of the two zones are large.

EXPERIMENTS ON CRITICAL FACILITIES

The Substitution Experiment MASURCA 1B → 1A'

This experiment was particularly suitable for a test of the progressive substitution method since it was continued until the central zone boundary reached the blanket, that is, until the original composition was completely replaced by the new one.

MASURCA 1B was a uranium graphite core with a depleted uranium reflector (see Table 2). It was replaced by the plutonium graphite composition of MASURCA 1A' in successive radial steps starting at the center of the assembly. The radius of the original core was about 33.2 cm. An evaluation of the experiment was performed after the substituted zone had reached a radius of $r_z = 20.716$ cm.

In Table 7, the first three lines show the information (ref. 5) received on changes in critical radius up to this point. Evaluations were made using these data directly or correcting them for the presence of a reflector using the results of multigroup calculations, as described above: Line 4 - 7 of Table 7 show the results of multigroup calculations for a bare and a reflected system, the calculated difference $\Delta r_c(\text{refl.}) - \Delta r_c(\text{bare})$ and the measured data corrected by these values.

The data were then fitted to Eq. 66 using two types of least square procedures. In the first method (which was also used in the numerical experiments) the expression

$$\sum_n (\Delta r_{c,\text{calc.}} - \Delta r_{c,\text{meas.}})^2$$

was minimized (direct fit). The integer n refers to the individual steps.

The second method takes in account that the quantity which is measured actually is the difference in critical radius between one step and the next. Therefore one minimizes the expression

$$\sum_n [(\Delta r_{c,n} - \Delta r_{c,n-1})_{\text{calc}} - (\Delta r_{c,n} - \Delta r_{c,n-1})_{\text{meas}}]^2$$

(difference method.)

The calculations were performed both with $\Delta D/D$ as a free parameter and with a fixed value of $\Delta D/D = 0.0193$ which was derived from multigroup calculations. Also found by multigroup calculations was the value used for the decay constant of the spectral perturbation $\mu_1 = 0.167$.

When reflector-corrected measured data were used for the fit the extrapolated radius for complete substitution was found from the curve after reintroducing the influence of the reflector by adding to the curve the multigroup data for $\Delta r_c(\text{reflected}) - \Delta r_c(\text{bare})$.

The results for the direct and the difference method of fitting were practically the same so that it will suffice to present the data as found by the difference method. These are given in Table 8 and in Fig. 3 and 4.

Also shown is the final result which was found as the experiment was continued to complete substitution. When the reflector correction were applied and $\Delta D/D$ was treated as a fixed parameter the extrapolated critical radius was consistently very close to the correct value. When $\Delta D/D$ was treated as a free parameter the data found from the first few steps are grossly erroneous, only after the sixth step is included the correct result is approximated.

The data found without reflector correction give a very similar result for $\Delta B_r/B_r$, however, the extrapolated critical radius is in error by about 0.5 cm.

The evaluation of the MASURCA experiment was performed with data which were provided as the substitution proceeded. A reevaluation of the data after the experiment was completed gave small changes in the critical radii (see Fig. 3 and 4). Using these new data in a final analysis one will probably find somewhat reduced errors when $\Delta D/D$ is used as a free parameter while otherwise it appears that the results will remain essentially unchanged. In their own evaluation the MASURCA group reached results very similar to those quoted above (ref. 6 and 7).

Experiments on SNEAK

Two substitution experiments were performed in the course of the work on the assemblies SNEAK-3A and 3B which were simulations of steam-cooled fast reactors (the hydrogen being introduced in the form of polyethylene foils) with uranium fuel only and a central plutonium fuelled zone, respectively. (For compositions, see Table 2.)

The first substitution consisted of the stepwise introduction of the plutonium zone in the uranium core of SNEAK-3A-2. As in the MASURCA-experiment, the plutonium zone extended axially through the whole core. The radius was increased in 10 substitution steps. The final zone radius was 29.91 cm, which is close to 2/3 of the total core radius of 44.9 cm. Only very small changes in critical radius were observed during the experiments.

The evaluation was performed with $\Delta D/D$ as a fixed and as a free parameter. The only reflector correction applied was to decrease the resulting differences in bucklings and diffusion constants by 2.5% in order to take in account the influence of the reflector on the normalization integral. The method of reducing the measured data to the bare case, using a series of multigroup calculations was not applied here, because the amount of calculations necessary seemed to be quite large for a rather uncertain improvement. (A correction of this type included in a previous report of the experiment (ref. 10) was of preliminary nature and based on the compositions of the test core.) In the light of the good agreement in the MASURCA case between the evaluation which was reflector corrected by multigroup calculations and the measured critical radius after complete substitution one might reconsider the above standpoint. However, the total effect of the correction (~ 0.5 cm in Δr_c corresponding to 0.3% in Δk) is not large enough to be of great practical importance.

A small correction to the measured points was taking in account the influence of the irregular core boundary on the interface term. This effect was calculated from the value of the interface term taken from a preliminary evaluation and the ratio between an estimated effective interface area and the interface area for a circular zone periphery.

The results of the evaluation are given in Table 9 and in Fig. 5 and 6. Consistent values are found only when $\Delta D/D$ is taken in a fixed parameter. For variable $\Delta D/D$ a comparable result is found only when all substitution steps are included in the evaluation.

The second substitution experiment performed in SNEAK consisted of introducing a central zone with modified structural materials (simulating Inconel) into the core of SNEAK-3B-2. The substitution was performed in 5 steps with a final radius of 20.59 cm. Since SNEAK-3B-2 already has two

core zones (with plutonium and uranium fuel) the actual core configuration becomes quite complicated and is not covered by the theory as presented above. However, it was assumed, that as long as the substituted "Inconel" zone does not approach the outer boundaries of the Pu-fuelled core zone (at $r=29.9$ cm), a two-zone treatment will describe the problem sufficiently well. The original core was assumed to consist entirely of the Pu-composition and to have the corresponding radial buckling. The values for Δr_c (extrapolated) and ΔB_r resulting from the evaluation therefore are to be interpreted with respect to the pure plutonium core. The results as given in the second part of Table 9 and in Fig. 7 and 8 appear to be of the same quality as those found for the other experiments discussed above. Not apparent from the figures and tables is the fact that there was practically no interface effect found in the evaluation of the Inconel substitution. This indicates that the interface effect found for the other experiments was due mostly to the interaction of the different fuel materials.

CONCLUSIONS

The results found in this report show that the effect of progressive substitution in a central zone is quite well described by three-term expressions such as Eq. 57 or for large differences in the bucklings of the two zones Eq. 71. Eq. 57 contains the first order perturbation terms plus a contribution due to the spectral overlap at the zone-interface. In addition to this Eq. 71 takes in account the effect of the change in the fundamental flux distribution and the normalization integral, as estimated by physical considerations. The evaluation of numerical and actual experiments with cores of 30 to 50 cm radius showed that it is possible to measure changes in radial bucklings with an accuracy in the order of $10^{-3} B_r$ for very similar bucklings and of $2 \times 10^{-2} B_r$ for large buckling differences. For the extrapolation of the critical radius there remained an uncertainty of about $10^{-2} r_0$ due to the unknown spectral effect of the reflector. A reflector correction using multigroup calculations yielded very good results in the one case of the MASURCA 1B \rightarrow 1A' substitution but the general validity of this method has yet to be proved.

In any case, it appears that the accuracies reached are sufficient to make the method an interesting tool for the evaluation of zoned critical experiments.

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Table 1: FLUX DISTRIBUTION FUNCTIONS FOR DIFFERENT GEOMETRIES^{a)}

	Plane Geometry		Cylinder		Sphere	
	Zone 1	Zone 2	Zone 1	Zone 2	Zone 1	Zone 2
$F(\rho)$	$\cos \rho_1$	$\sin (\rho_o - \rho_2)$	$J_o(\rho_1)$	$J_o(\rho_2) - Y_o(\rho_2) \frac{J_o(\rho_o)}{Y_o(\rho_o)}$	$\frac{\sin \rho_1}{\rho_1}$	$\frac{\sin (\rho_o - \rho_2)}{\rho_2}$
$F'(\rho)$	$-\sin \rho_1$	$-\cos (\rho_o - \rho_2)$	$-J_1(\rho_1)$	$-J_1(\rho_2) + Y_1(\rho_2) \frac{J_o(\rho_o)}{Y_o(\rho_o)}$	$\frac{\cos \rho_1}{\rho_1} - \frac{\sin \rho_1}{\rho_1^2}$	$-\frac{\cos (\rho_o - \rho_2)}{\rho_2} - \frac{\sin (\rho_o - \rho_2)}{\rho_2^2}$
$H(\bar{\rho})$	$\cosh \bar{\rho}_1$	$\sinh(\bar{\rho}_o - \bar{\rho}_2)$	$I_o(\bar{\rho}_1)$	$-I_o(\bar{\rho}_2) + K_o(\bar{\rho}_2) \frac{I_o(\bar{\rho}_o)}{K_o(\bar{\rho}_o)}$	$\frac{\sinh \bar{\rho}_1}{\bar{\rho}_1}$	$\frac{\sinh(\bar{\rho}_o - \bar{\rho}_2)}{\bar{\rho}_2}$
$H'(\bar{\rho})$	$\sinh \bar{\rho}_1$	$-\cosh(\bar{\rho}_o - \bar{\rho}_2)$	$I_1(\bar{\rho}_1)$	$-I_1(\bar{\rho}_2) - K_1(\bar{\rho}_2) \frac{I_o(\bar{\rho}_o)}{K_o(\bar{\rho}_o)}$	$\frac{\cosh \bar{\rho}_1}{\bar{\rho}_1} - \frac{\sinh \bar{\rho}_1}{\bar{\rho}_1^2}$	$-\frac{\cosh(\bar{\rho}_o - \bar{\rho}_2)}{\bar{\rho}_2} - \frac{\sinh(\bar{\rho}_o - \bar{\rho}_2)}{\bar{\rho}_2^2}$

a) ρ_o is defined as $B_2 r_o$, where r_o is the outer radius of the bare reactor

Table 2: CORE COMPOSITIONS (10^{20} Atoms/cm³)

Isotope	Test Core			MASURCA			SNEAK			
	U-Zone	Pu-Zone	Blanket	Assy. 1B	Assy. 1A'	Blanket	Assy. 3A2	Assy. 3B2 Pu-Zone	Inconel Zone	Blanket
Pu239		14.5 ^a			16.97			14.76	14.76	
Pu240		1.3			1.56			1.33	1.33	
Pu241		0.13			0.15			0.11	0.11	
Pu242		0.006			0.02			0.06	0.06	
U235	20.22	0.35	1.54	25.20	0.125	1.76	20.31	0.56	0.56	1.625
U238	81.06	86.4	399.3	58.06	55.48	418.98	81.04	81.86	81.86	399.414
Al	128.9	129.0					129.10	125.6	127.4	
C	8.6	8.6	0.14	564.7	564.7		9.32	9.73	9.56	0.14
Co							0.19	0.14	-	
Cr	36.7	34.1	11.9	10.45	12.28	10.45	34.53	33.72	17.41	11.8
Fe	123.3	124.0	40.1	38.92	48.94	38.92	121.85	119.7	61.99	39.55
H	16.4	16.4					17.92	18.49	18.49	
Mg	0.37						0.64	1.31	1.33	
Mn							1.94	2.23	1.47	0.87
Ni	19.0	18.8	10.2	8.77	6.04	17.10	18.54	17.55	95.00	9.84
O	144.8	145.0					145.29	122.2	122.2	
Si	1.88		0.46				1.88	2.54	1.79	0.46
Ti	0.55		0.20				0.40	0.38		
Mo							0.39	0.29	8.85	0.19
Nb									0.05	

a) The Pu239 concentration in the test core was reduced to 13.5×10^{20} in Modification 1 and 12.0×10^{20} in Modification 2

Table 3: RESULTS OF PARAMETER FIT TO TEST CASE (BARE REACTOR)

Step	r_z	Δr_c (Multi-group)	Results of Fit after this Step			
			$\Delta D/D$ free parameter		$\Delta D/D$ fixed parameter (0.0026)	
			$\frac{\Delta B_r}{B_r}(\%)$	$\Delta_o r_c$ (extrap.)	$\frac{\Delta B_r}{B_r}(\%)$	$\Delta_o r_c$ (extrap.)
1	3.07	-0.015				
2	9.21	-0.022				
3	14.06	0.051				
4	20.59	0.289	-6.05	3.50	-2.80	1.62
5	25.49	0.513	-2.34	1.35	-2.74	1.59
6	31.3	0.809	-2.55	1.47	-2.72	1.57
7	35.0	1.036	-3.28	1.90	-2.80	1.62
8	40.0	1.257	-2.99	1.73	-2.82	1.63
Multigroup results			-2.74	1.628	-2.74	1.628

Table 4: RESULTS OF PARAMETER FIT TO TEST CASE (REFLECTED REACTOR)

Step	r_z	Δr_c	Results of Fit after this Step					
			$\Delta D/D$ free parameter			$\Delta D/D$ fixed parameter (0.0026)		
			$\frac{\Delta B_r}{B_r}(\%)$	$\Delta_o r_c$ (extrap.)	$\Delta_o r_c$ Eq. 60	$\frac{\Delta B_r}{B_r}(\%)$	$\Delta_o r_c$ (extrap.)	$\Delta_o r_c$ Eq. 60
1	3.07	-0.016						
2	9.21	-0.024						
3	14.06	0.062						
4	20.59	0.303	-3.07	1.64	1.65	-2.82	1.58	1.59
5	25.49	0.555	-3.29	1.70	1.71	-2.85	1.60	1.61
6	31.3	0.911	-3.78	1.83	1.81	-2.96	1.65	1.67
7	35.0	1.168	-4.10	1.92	1.87	-3.09	1.71	1.75
Multigroup results			-2.74	2.18		-2.74	2.18	

Table 5: RESULTS OF PARAMETER FIT TO TEST CASE, MODIFICATION 1

($C_{Pu9}=1.35 \times 10^{20}$) Reflected Reactor, $\Delta D/D$ fixed parameter (0.0062)

Step	r_z	Δr_c	Results of Fit after this Step		
			$\frac{\Delta_{mo}(r_c)}{r_o}$	$\frac{\Delta B_r}{B_r}$	$\Delta_o r_c$ extrapolated
1	3.17	0.027			
2	9.21	0.373			
3	14.06	0.962			
4	20.59	2.088	0.1114	0.1184	7.68
5	25.49	3.084	0.1120	0.1191	7.73
6	31.3	4.327	0.1133	0.1206	7.84
7	35.0	5.114	0.1151	0.1226	7.99
8	40.0	6.132	0.1160	0.1236	8.07
9	45.0	7.081	0.1175	0.1254	8.20
Multigroup results				0.1210	8.42

Table 6: RESULTS OF PARAMETER FIT TO TEST CASE, MODIFICATION 2

($C_{Pu9}=1.2 \times 10^{20}$) Reflected Reactor, $\Delta D/D$ fixed parameter (0.0117)

Step	r_z	Δr_c	Results of Fit after this Step		
			$\frac{\Delta_{mo}(r_c)}{r_o}$	$\frac{\Delta B_r}{B_r}$	$\Delta_o r_c$ extrapolated
1	3.17	0.090			
2	9.21	0.953			
3	14.06	2.258			
4	20.59	4.639	0.2319	0.2678	21.0
5	25.49	6.723	0.2321	0.2680	21.02
6	31.3	9.368	0.2350	0.2720	21.45
7	35.0	11.112	0.2372	0.2750	21.78
8	40.0	13.449	0.2401	0.2790	22.55
9	45.0	15.691	0.2422	0.2819	22.55
10	55.0	19.804	0.2457	0.2868	23.10
Multigroup results				0.2893	23.92

Table 7: DATA USED FOR THE EVALUATION OF THE MASURCA SUBSTITUTION EXPERIMENT

Substitution Step Nr.		1	2	3	4	5	6				
Radius of central Pu-Zone	0	5.98	10.78	11.96	15.82	17.941	20.716	25.0	30.0	37.4 ^a	48.92 ^b
Critical Radius (measured)	33.18	33.20	33.41	33.46	33.66	33.796	34.027				
Δr_c (measured)	-	0.02	0.23	0.28	0.48	0.62	0.85				
Calculated Δr_c (refl.)	-	0.10	0.42	0.53	0.96	1.22	1.60	2.22	2.97	4.18	-
Calculated Δr_c (bare)	-	0.10	0.41	0.53	0.93	1.18	1.53	2.08	2.66	3.33	3.80
Difference (refl.-bare)	-	0	0.01	0.01	0.03	0.05	0.07	0.14	0.31	0.85	
Δr_c (measured) (with reflector correction)	-	0.02	0.22	0.27	0.45	0.57	0.78				

- a) complete Pu-core for reflected case (calculated)
- b) complete Pu-Core for unreflected case (calculated)

Table 8: RESULTS OF MASURCA SUBSTITUTION EXPERIMENT

Steps Used	with reflector correction			without reflector correction		
	$\Delta B_r / B_r$	$\Delta D / D$	Δr_c (extrap.)	$\Delta B_r / B_r$	$\Delta D / D$	Δr_c (extrap.)
1 - 4	-0.04062	fixed (0.0193)	2.26	-0.04533	fixed	1.73
1 - 5	-0.04079		2.22	-0.04475		1.70
1 - 6	-0.04290		2.35	-0.04805		1.83
1 - 4	+0.2254	-0.6347	-1.65	+0.2214	-0.6360	-1.80
1 - 5	+0.07422	-0.2710	0.16	+0.06482	-0.2592	-0.03
1 - 6	-0.04417	+0.02232	2.38	-0.05211	+0.03045	1.92

Measured r_c after complete substitution = 2.323 cm

Table 9: RESULTS OF SNEAK SUBSTITUTION EXPERIMENTS

Experiment	Step	r_z	Δr_c	Result of Fit after this Step				
				$\Delta D/D$ free parameter			$\Delta D/D$ fixed parameter ⁺⁾	
				$\frac{\Delta B_r}{B_r}$	$\frac{\Delta D}{D}$	$\Delta_o r_c$	$\frac{\Delta B_r}{B_r}$	$\Delta_o r_c$
SNEAK-3A2 + Uranium SNEAK-3B2 + Plutonium	1	3.07	-0.017					
	2	6.86	-0.068					
	3	9.21	-0.138				0.0111	-1.26
	4	14.06	-0.188	-0.6877	1.515	20.0	-0.0210	0.53
	5	18.67	-0.26	0.0233	-0.0454	-0.79	-0.0174	0.34
	6	23.17	-0.315	0.0194	-0.0365	-0.70	-0.0151	0.21
	7	25.49	-0.366	0.0327	-0.0687	-1.01	-0.0129	0.08
	8	25.86	-0.366	0.0314	-0.0655	-0.98	-0.0124	0.05
	9	29.60	-0.326	-0.0097	0.0397	0.01	-0.0131	0.09
	10	29.91	-0.322	-0.0133	0.0488	0.10	-0.0133	0.095
SNEAK-3B2+ SNEAK-3B-Inconel	1	3.07	0.031					
	2	6.86	0.152					
	3	9.21	0.260	0.3873	-0.9176	-10.5	-0.0055	1.26
	4	15.35	0.680	-0.1950	0.3332	7.0	-0.0173	1.87
	5	20.59	1.099	-0.0093	-0.0905	1.63	-0.0166	1.84

⁺⁾ = 0.0502 (U→Pu)

= -0.0733 (Inconel)

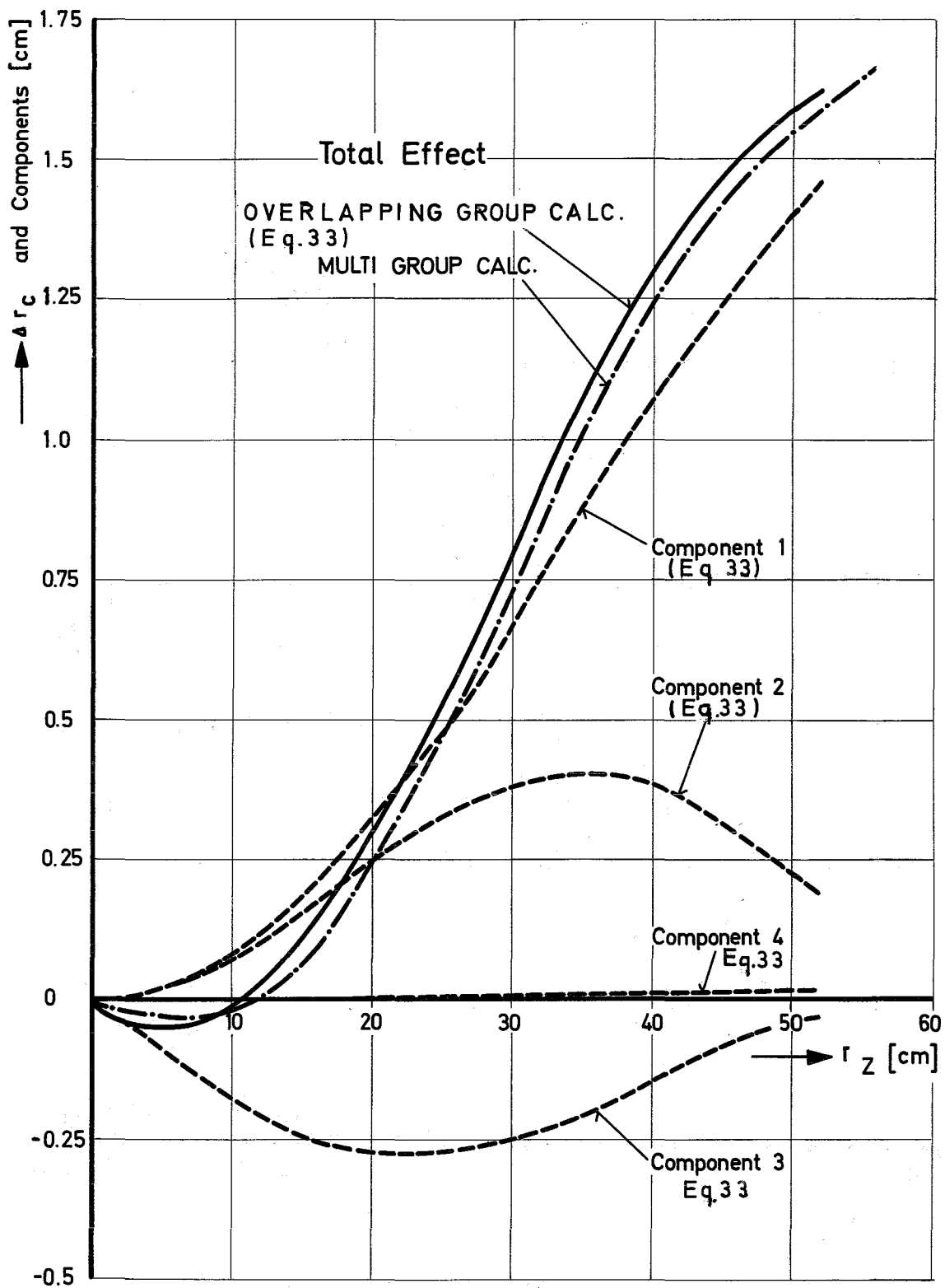


Fig.1 Comparison of Multigroup and Overlapping Group Calculations for Test Case (using Eq. 33)

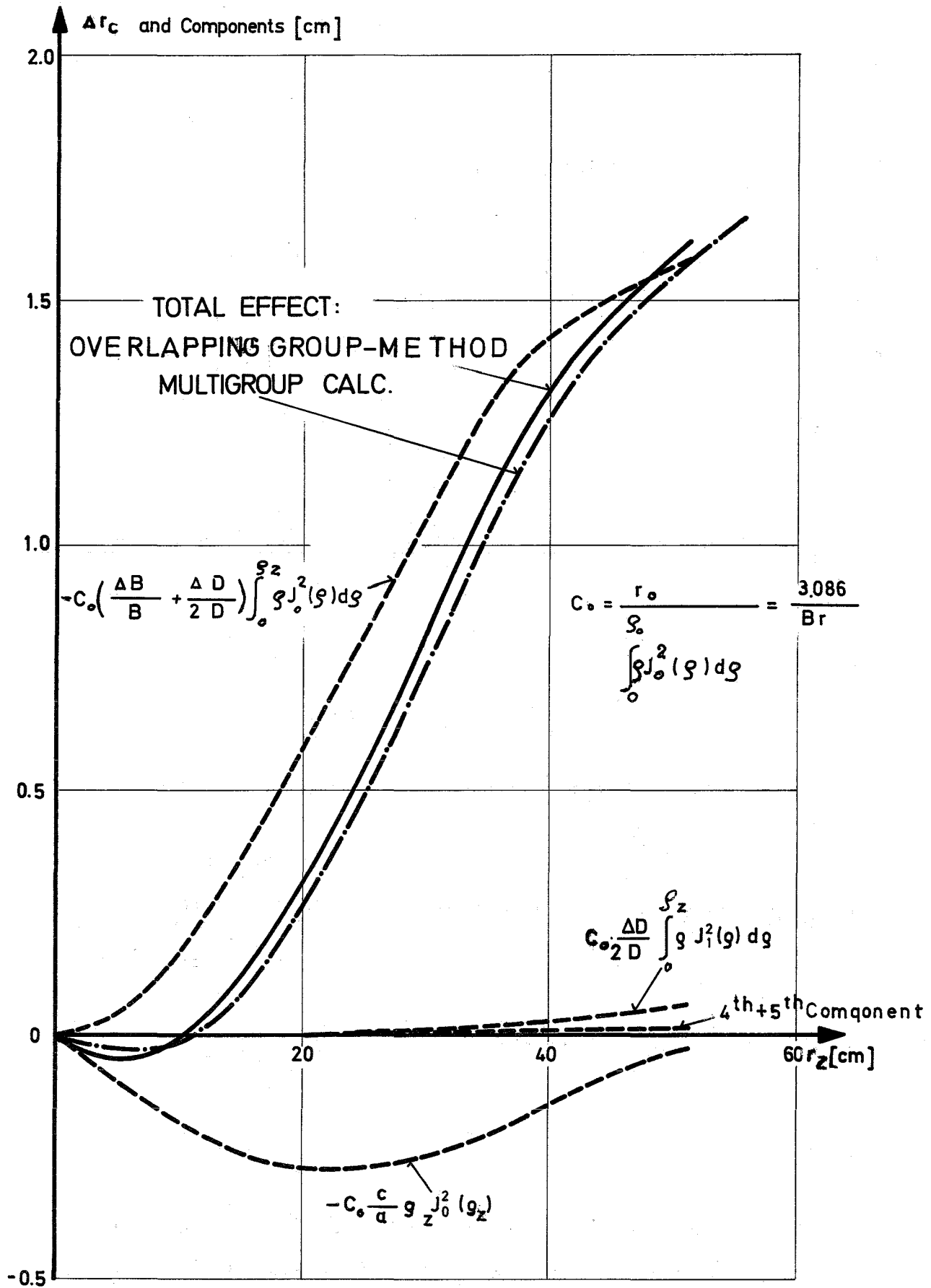


Fig. 2 Comparison of Multigroup and Overlapping Group Calculation for Test Case (using Eq. 41)

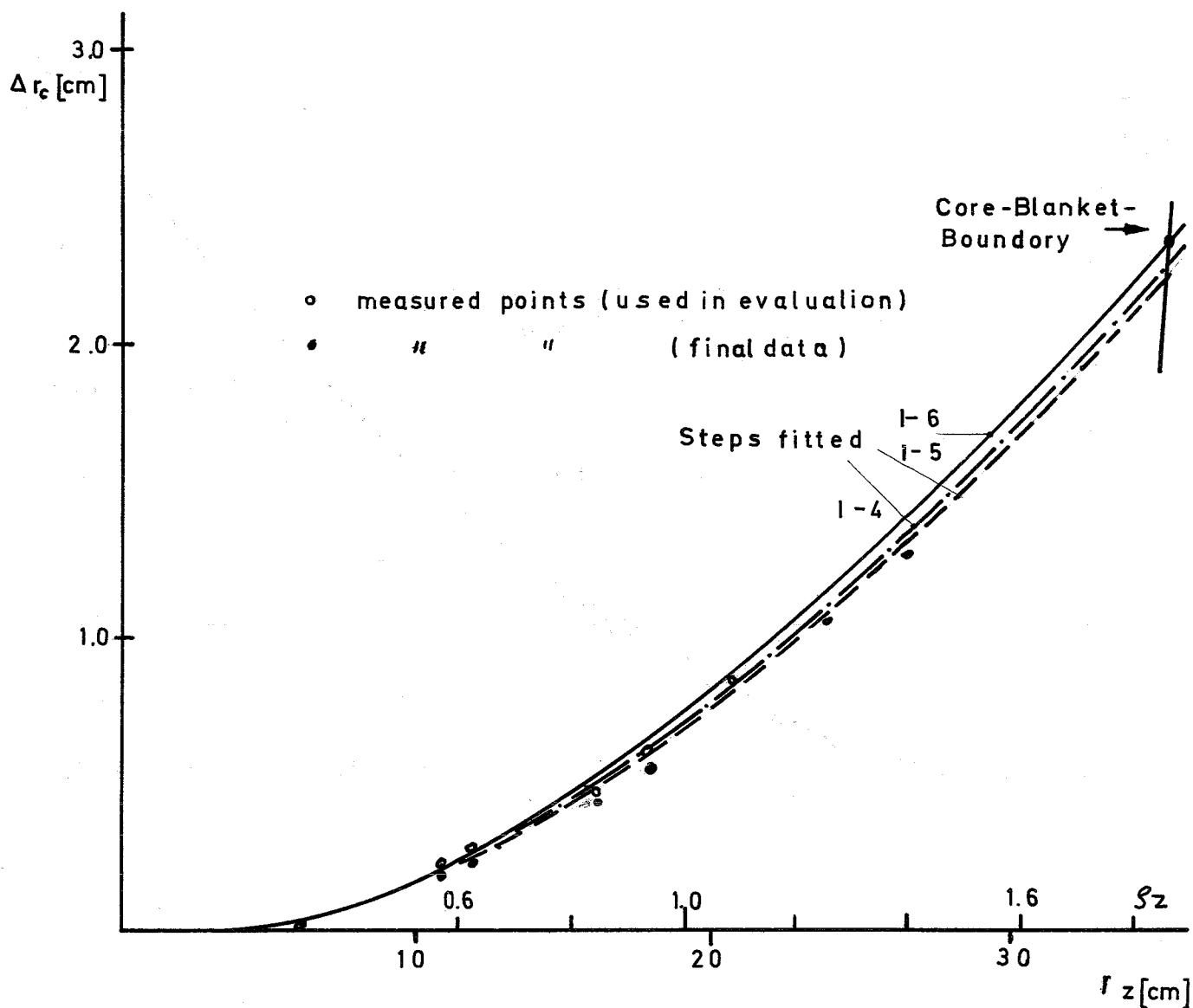


Fig.3 SUBSTITUTION MASURCA IB-IA'

Least Squares Fits to measured data (with reflector correction)

Difference method $\frac{\Delta D}{D} = 0.0193$, fixed

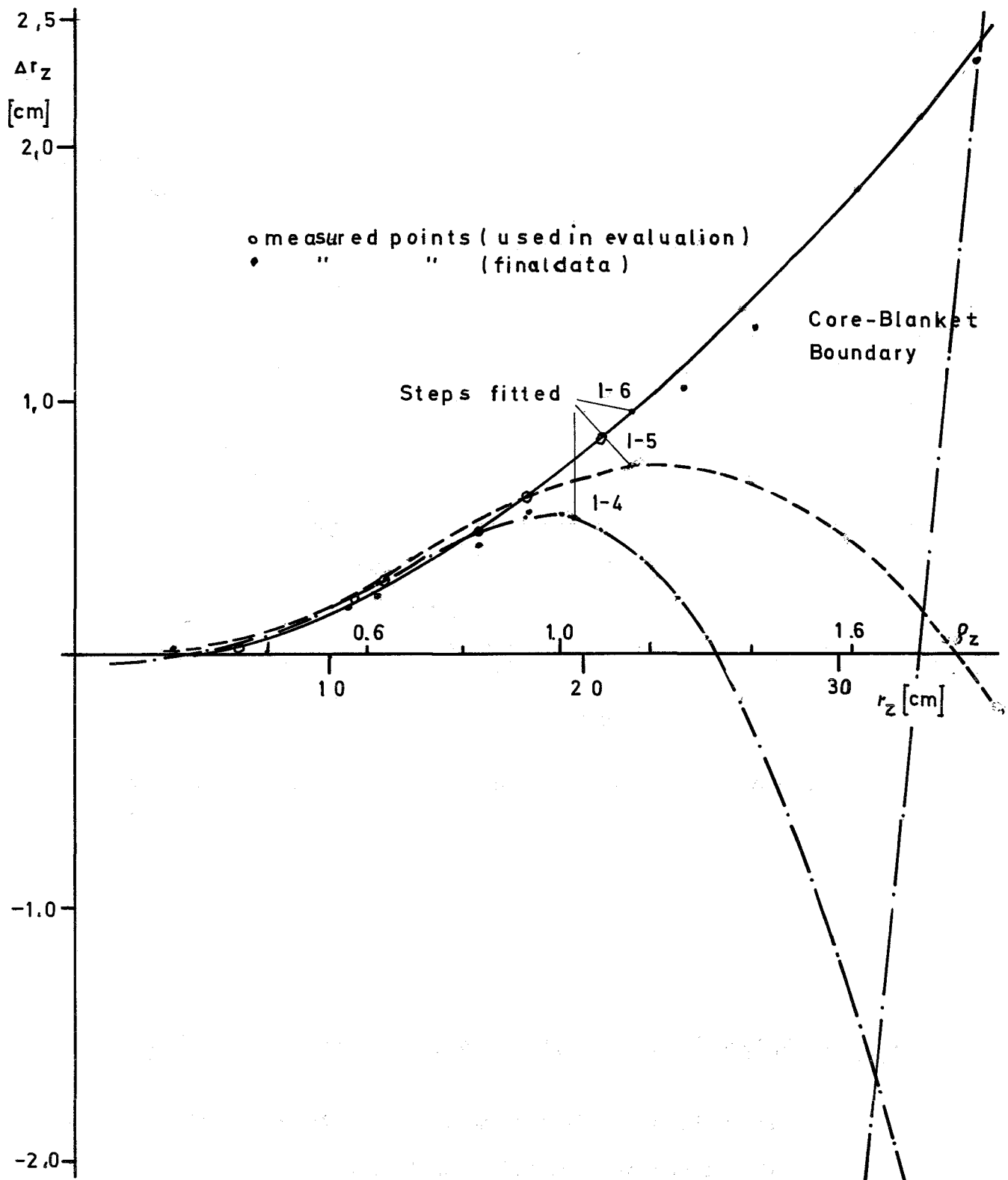


Fig.4 SUBSTITUTION MASURCA IB-IA'

Least Squares Fits to measured data (with reflector correction)
 Difference method, $\frac{\Delta D}{D}$ is free parameter

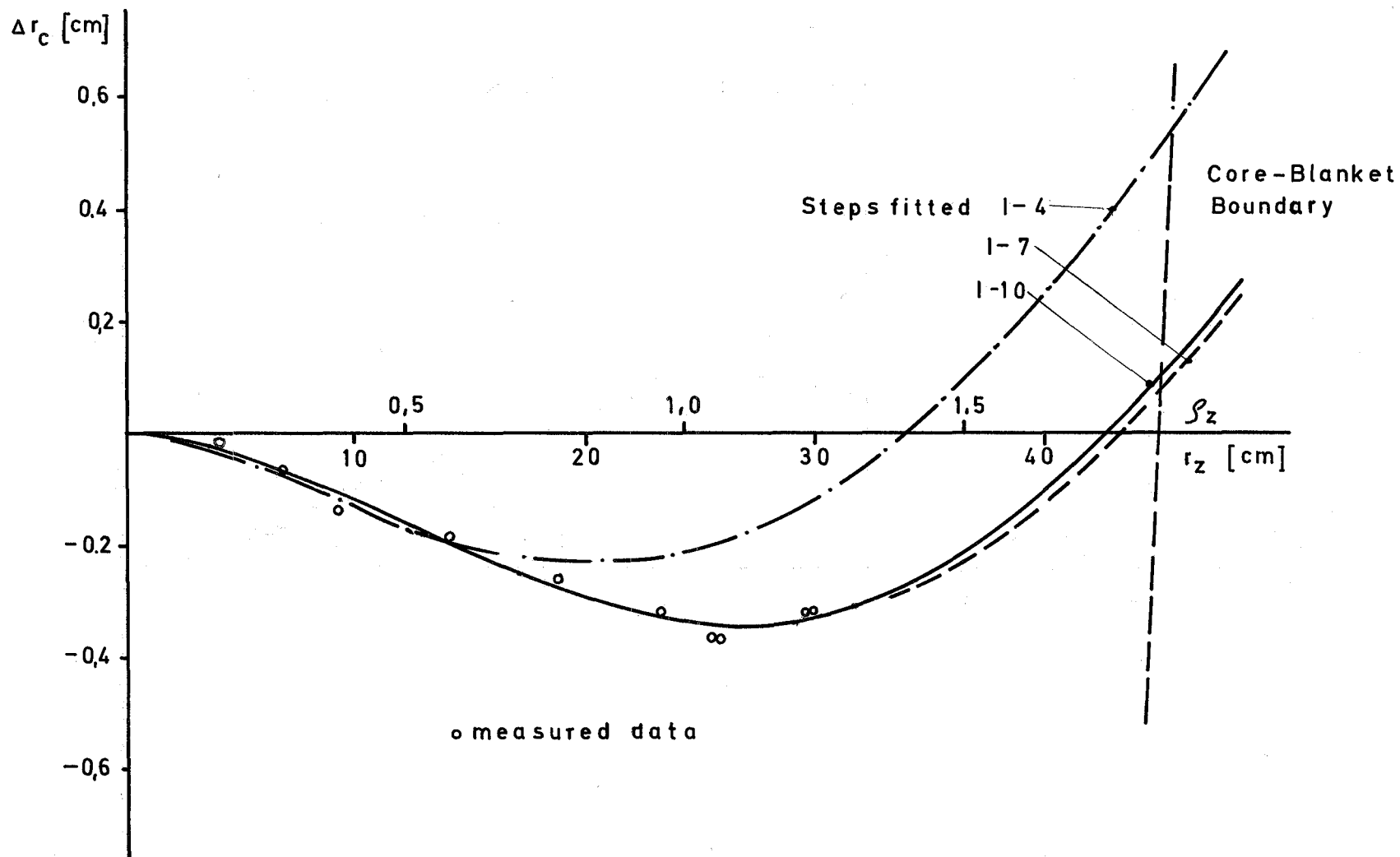


Fig. 5 SUBSTITUTION SNEAK 3A2 → 3B2

Least Square Fits to measured data, direct method

$$\frac{\Delta D}{D} = 0,0502, \text{fixed}$$

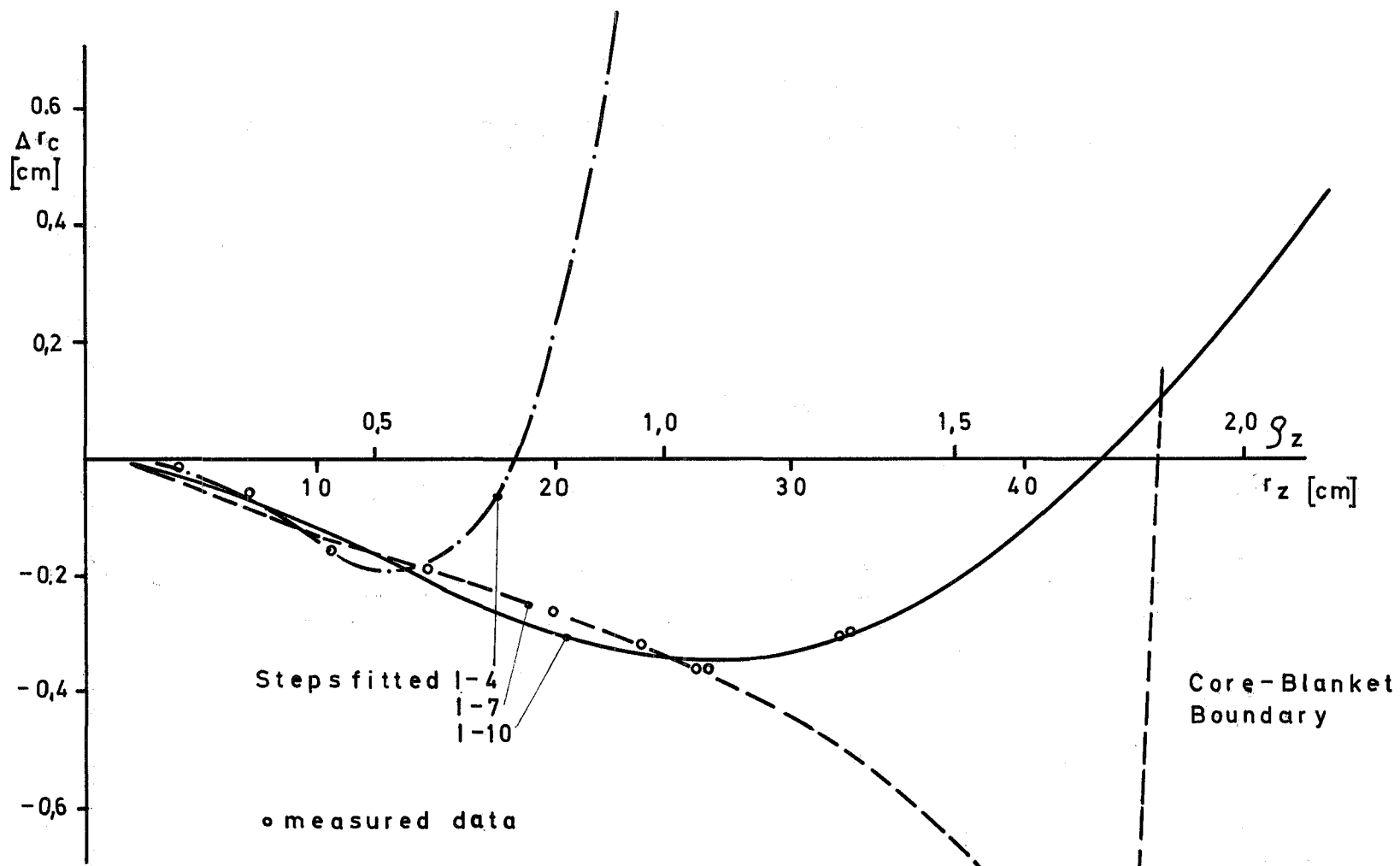


Fig.6 SUBSTITUTION SNEAK 3A2-3B2

Least Squares Fits to measured data, direct method

$\frac{\Delta D}{D}$ is free parameter

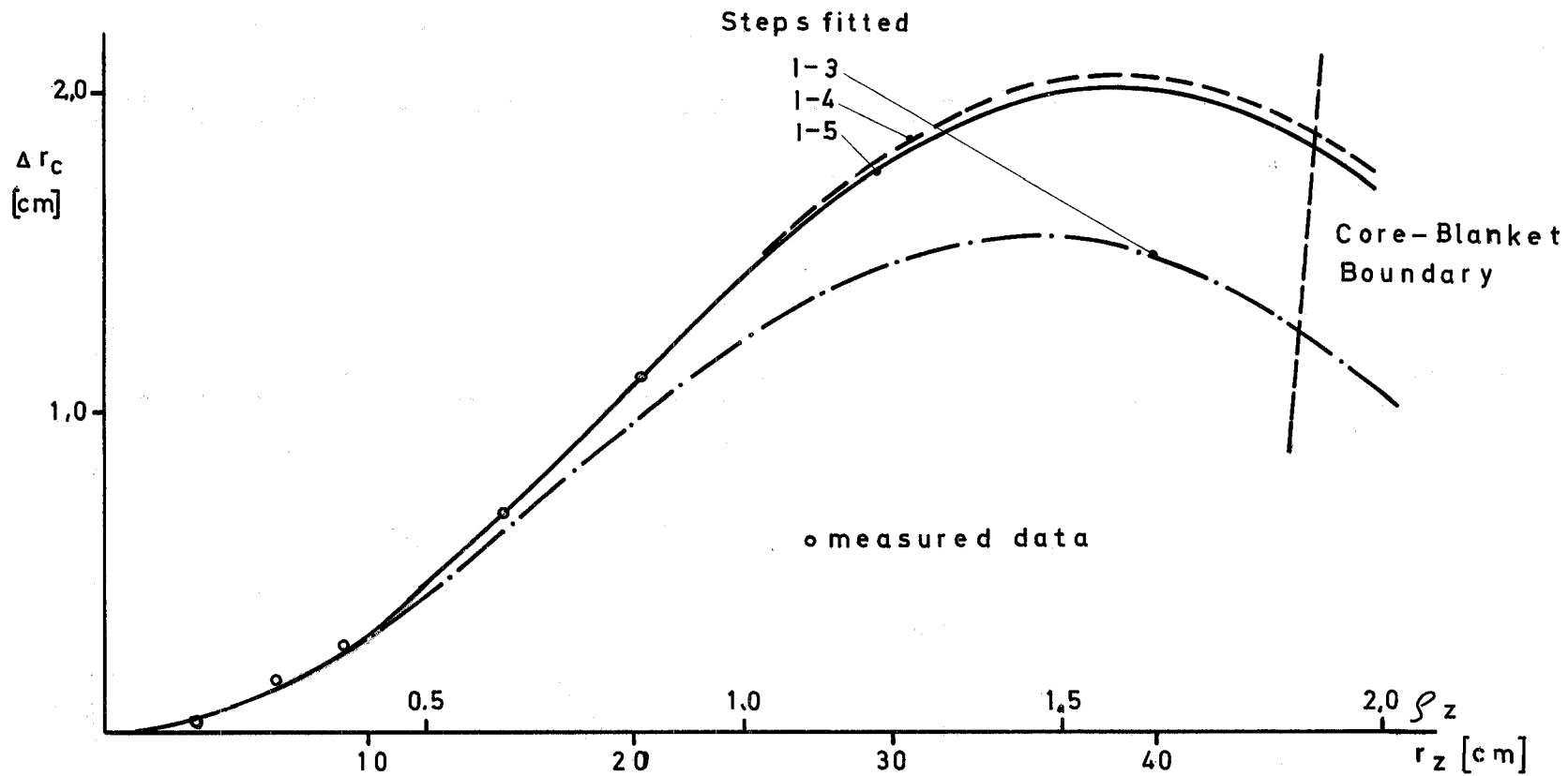


Fig.7 SNEAK 3B2 Inconel Substitution

Least Squares Fits to measured data, direct method

$$\frac{\Delta D}{D} = -0,0733 \text{ fixed}$$

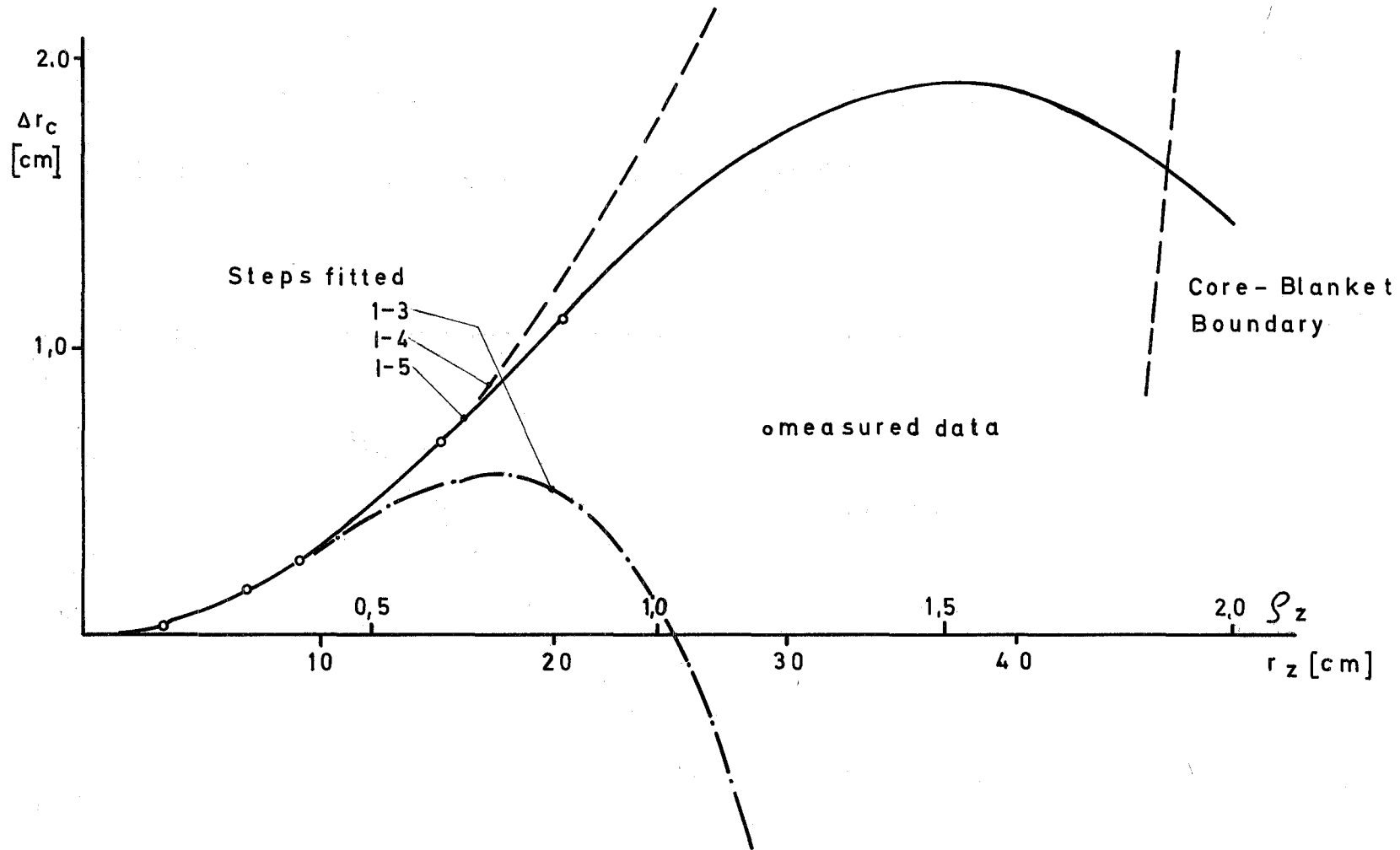


Fig. 8 SNEAK 3B2 Inconel Substitution

Least Squares Fits to measured data

$\frac{\Delta D}{n}$ is free parameter