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Projekt Schneller Brüter

Flow and Temperature Distribution Including Coolant Mixing in Sodium Cooled Fuel Elements with Eccentric Geometry

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Equation (8) should read:

$$r_o \varrho u^{*2} = \frac{\partial}{\partial \varphi} \left[\varrho \, \varepsilon_{M\varphi} \, \overline{u}^{+} \cdot \frac{\partial u^{*}}{\partial \varphi} \cdot \frac{2 \, \hat{y}}{d+\hat{y}} \right] - \frac{dp}{dx} \, \frac{1}{2} \, \hat{y} \, (d + \hat{y}) \qquad (8)$$

The relation by Bender should read:

Bender: $$\varepsilon_M = 0.01 \cdot \nu \cdot Re \qquad (9b)$$

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FLOW AND TEMPERATURE DISTRIBUTION INCLUDING COOLANT MIXING IN SODIUM COOLED FUEL ELEMENTS WITH ECCENTRIC GEOMETRY

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Abstract

A theoretical analysis of the temperature and velocity distribution in symmetrical and asymmetrical triangular rod clusters is presented. The analysis is applied to sodium coolant with fully developed turbulent flow. Coolant mixing is taken into account by correlations for the eddy diffusivities of momentum and energy. The thermal heat flux at the outer boundary of the fuel pin is variable in circumferential and axial direction.

Kurzfassung

Es wird eine theoretische Analyse der Temperatur- und Geschwindigkeitsverteilung in symmetrischen und asymmetrischen Brennstabbündeln mit Dreiecksanordnung dargestellt. Die Analyse wird auf Natrium als Kühlmittel bei voll turbulenter Strömung angewandt. Die Kühlmittelvermischung wird durch Beziehungen für die turbulenten Impuls- und Energieaustauschgrößen berücksichtigt. Der Wärmefluß an der Oberfläche eines Brennstabes ist variabel in Umfangs- und Längsrichtung.

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1. Introduction

One of the important design characteristics of sodium cooled fast reactors are tightly packed fuel rod bundles with high rod power and small fuel rod diameters. As a consequence of these close spacings already small deviations from the design specifications, as they can occur for instance during manufacturing of the rod bundle, will cause considerable variations in the peripheral velocity and temperature distributions around the fuel rod. The asymmetrical temperature distributions will increase the original rod bowing. As a result the thermal and mechanical stresses can lead to fuel rod failure.

A number of theoretical analyses have been published dealing with the aspects of circumferential temperature and velocity variations for the symmetrical case [1, 2, 3, 4]. The present paper is an extension of an earlier reported work by Fischer/Shimamune [5]. It deals with a theoretical analysis of the temperature and velocity distribution in symmetrical and asymmetrical triangular rod clusters. The analysis is applied to liquid metal coolants with fully developed turbulent flow. Coolant mixing is taken into account by correlations for the eddy diffusivities of momentum and energy. The thermal heat flux at the outer boundary of the fuel pin is variable in circumferential direction. In addition the variation of the geometrical asymmetry along the axial direction of the rod cluster is considered. Heat flux distribution in axial direction can be an arbitrary function. The analysis is restricted to a rod cluster of bare rods without spacers.

2. The Mathematical Model

The analysis is applied to an asymmetrical rod cluster shown by fig. 1. One rod deviates from its nominal position by an angle Θ and a distance ξ. The cross section of the coolant flow area around the central rod is divided into twelve elements. These elements are again divided into an arbitrary number of smaller segments. Each segment is bounded by the rod wall, two velocity gradient lines and the maximum velocity line. The analysis implies the following assumptions for the axial cross section shown in fig.1:

(i) There is no net current of energy and momentum across the line of maximum velocity.

(ii) The influence of the fluctuations of the axial coolant velocity shall be negligibly small in comparison with the mean value.

(iii) The pressure is constant.

(iv) The coolant flow is fully turbulent and steady.

(v) The power density in the fuel is constant.

(vi) The heat flux at the outer boundary of the cladding is a function of the circumferential direction.

(vii) The coolant properties in each segment are treated as functions of the segment-averaged temperatures, and, therefore, are functions of the angle φ.

(viii) The coolant is incompressible.

(ix) The mean values of the velocity components in radial and circumferential direction are zero.

2.1 The Velocity Distribution

The coolant velocity distribution is determined by the Navier-Stokes equation.

$$\frac{\partial \mathfrak{r}}{\partial t} + (\mathfrak{r}\nabla)\mathfrak{r} = -\frac{1}{\rho}\,\mathrm{grad}\;p + \frac{\mu}{\rho}\Delta\mathfrak{r} \tag{1}$$

and the continuity equation

$$\mathrm{div}\;\mathfrak{r} = 0 \tag{2}$$

With the assumption $u_r = 0$, $u_\varphi = 0$ and $\frac{\partial \mathfrak{r}}{\partial t} = 0$ we obtain the following equation:

$$\frac{1}{\rho r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right) + \frac{1}{\rho}\frac{\partial}{r\,\partial\varphi}\left(\mu\frac{\partial u}{r\,\partial\varphi}\right) = \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{3}$$

Introducing the relations

$$\tau_r = \mu_r \frac{\partial u}{\partial r} \qquad (4a)$$

$$\tau_\varphi = \mu_\varphi \frac{\partial u}{r\partial \varphi} \qquad (4b)$$

for the local shear stresses in radial and circumferential direction, it then follows from equation (3):

$$\frac{\partial(r\tau_r)}{\partial r} + \frac{\partial \tau_\varphi}{\partial \varphi} = \frac{r\partial p}{\partial x} \qquad (5)$$

In the above relations (4a,b) the shear stresses are composed of the laminar and the turbulent momentum transfer terms according to the laminar viscosity and turbulent eddy diffusivities.

$$\tau_r = (\mu_d + \varrho \varepsilon_{M_r}) \frac{\partial u}{\partial r} \qquad (5a)$$

$$\tau_\varphi = (\mu_d + \varrho \varepsilon_{M_\varphi}) \frac{\partial u}{r\partial \varphi} \qquad (5b)$$

The numerical solution of the above equation (5) in three dimensions using reasonable relations for the eddy diffusivities seems to be too expensive in term of calculation effort. Therefore, we have to introduce further simplifying assumptions to find a solution:

(i) To reduce the three-dimensional to a two-dimensional problem we use the following equation for the dimensionless radial velocity distribution:

$$u^+ = A + B \ln y^+ \qquad (7)$$

where u^+ and y^+ are defined as

$$u^+ = u / \left(\frac{\tau_w}{\varrho}\right)^{1/2} \qquad (7a)$$

$$y^+ = \frac{(r-r_o)\,(\tau_w/\varrho)^{1/2}}{\nu} \qquad (7b)$$

(ii) The eddy diffusivity in circumferential direction is a function of the radius only in the vicinity of the wall [6]. Therefore it is here assumed to be independent of the radial direction.

With these assumptions for one segment eq.(5) is reduced to:

$$r_o \varrho u^{*2} = \frac{\partial}{\partial \varphi} \left[\varrho \varepsilon_{M\varphi} \bar{u}^{+} + \frac{\partial u^*}{\partial \varphi} \cdot \frac{2 \hat{y}}{d+\hat{y}} \right] - \frac{dp}{dx} \frac{1}{2} \hat{y} (d + \hat{y}) \tag{8}$$

Introducing different relations for the peripheral eddy diffusivity it is possible to solve eq.(8) numerically now. Then the velocity distribution in the rod cluster is obtained for constant dp/dx.

2.1.1 Eddy Diffusivity in Circumferential Direction

To study the influence of different formulas for the eddy diffusivity in circumferential direction equation (8) has been solved using relations given by Nijsing, Bender and Rapier [1, 2, 3]:

Nijsing: $$\varepsilon_M = 0.0115 \, \nu \cdot Re^{7/8} \, \frac{u}{\bar{u}} \, \frac{d_e}{De} \tag{9a}$$

Bender: $$\varepsilon_M = 0.02 \cdot \nu \cdot Re \tag{9b}$$

Rapier: $$\varepsilon_M = \frac{u^* \, \hat{y}}{10} \tag{9c}$$

The obtained peripheral dimensionless velocity distributions have been compared with experimental results for water reported by Nijsing [1]. As can be seen by fig.2 the semi-empirical formula by Nijsing shows the best agreement with the experimental data, especially for small P/D ratios. Consequently the relation by Nijsing will be used further on. In addition it has been shown that the eddy diffusivities in the radial direction are one order of magnitude smaller than those in the circumferential direction [4].

3. The Temperature Field in the Coolant

For a differential fluid element the following heat balance equation in cylindrical coordinates can be derived as follows:

$$- \frac{\partial}{\partial r} \left\{ (\lambda + \varrho c_p \, \varepsilon_{Hr}) \frac{\partial T}{\partial r} r \right\} - \frac{\partial}{\partial \varphi} \left\{ (\lambda + \varrho c_p \, \varepsilon_{H\varphi}) \frac{\partial T}{r \partial \varphi} \right\} + \frac{\partial}{\partial x} (u \, r \, \varrho c_p \, T) = 0 \tag{10}$$

If we assume constant fluid properties λ, ϱ, c_p, ε_{Hr} and $\varepsilon_{H\varphi}$ for each channel segment and then integrate equation (10) in radial direction, it holds:

$$q = \frac{1}{d} \frac{\partial}{\partial x} \left\{ u \, \varrho \, c_p \, T \, (d + \hat{y}) \, \hat{y} \right\} - \frac{1}{r_o} \frac{\partial}{\partial \varphi} \left\{ (\lambda + \varrho \, c_p \, \varepsilon_{H\varphi}) \frac{\partial T}{\partial \varphi} \cdot \ln \left(1 + \frac{\hat{y}}{r_o}\right) \right\} \tag{11}$$

where the temperature T and u are mean values in each channel segment.

Equation (11) describes the circumferential temperature distribution in a cross section at fixed axial coordinate of the triangular rod cluster. To take into account the axial dependency of the values of ϱ, T and u, equation (10) has to be integrated over discrete elements in axial direction, too. By this procedure the following system of difference equations for a two-dimensional x, φ -mesh, such as shown in fig.3, is obtained.

$$\begin{aligned} q(\varphi)_{n+1/2} = & \left\{ \frac{c_p(\varphi)_{n+1} - c_p(\varphi)_n}{\Delta x} \left(u(\varphi) \cdot \varrho(\varphi) \cdot T(\varphi)\right)_{n+1/2} + \right. \\ & \left. + \left(u(\varphi) \cdot \varrho(\varphi) \cdot c_p(\varphi)\right)_{n+1/2} \frac{T(\varphi)_{n+1} - T(\varphi)_n}{\Delta x} \right\} \frac{(d + \hat{y}) \, \hat{y}}{d} + \\ & - \frac{1}{r_o} \frac{\partial}{\partial \varphi} \left\{ \left(\lambda(\varphi)_{n+1/2} + \varepsilon_{H\varphi}(\varphi)_{n+1/2} \cdot \varrho(\varphi)_{n+1/2} \cdot c_p(\varphi)_{n+1/2}\right) \frac{\partial T(\varphi)_{n+1/2}}{\partial \varphi} \cdot \right. \\ & \left. \cdot \ln \left(1 + \frac{\hat{y}}{r_o}\right) \right\} \end{aligned} \tag{12}$$

Equation (11) can be solved numerically applying an iterative calculation scheme which accounts for the velocity distribution described by equation (8). This is possible because eq.(8) and eq.(12) are coupled only by the variation of the material properties of the fluid.

The thermal eddy diffusivity in peripheral direction can be calculated for sodium as coolant by the following relation [7]:

$$\frac{\varepsilon_{H\varphi}}{\varepsilon_{M\varphi}} = 1 - \frac{(0.2/Pr) - 2}{(\varepsilon_{M\varphi}/\nu)^{0.9}} \tag{13}$$

The heat flux distribution at the outer boundary of the fuel rod can be calculated by applying an iterative method which incorporates difference schemes for the asymmetrical two-dimensional temperature field in the fuel, gap and clad. An attempt to avoid the exact calculation of the temperature field in the fuel rod has been reported earlier by Fischer/Shimamune [5]. This method uses an expression composed of cosine-functions and a trial parameter E.

$$q(\varphi) = q_{av} \frac{\left[1 - E \left(\cos\varphi + \frac{1}{2} \cos 2\varphi - \frac{1}{2} \right) \right]}{1 + E/2} \qquad (14)$$

where $0 \lesssim E < 1$

The trial parameter E has to be chosen always in such a way that both the heat flux distribution at the surface of the fuel rod as well as the temperature distribution of the coolant are consistent. The later method is used for the present investigation. This approximation will be only valid for deviations along the line $0' - 0_1$ (fig.1).

It has to be mentioned, however, that the method described in this section can only be applied to problems where the radial and peripheral net mass flow will be negligibly small. This will only be realistic besides the symmetrical case, for eccentric rod clusters with axially constant deviations of fuel rods along the total height of the cluster and for sufficiently small bowing of the rod.

3.1 Simplified Method of Solution

A first approximation for the solution of the temperature and velocity field in an asymmetrical rod cluster can be obtained by introducing the following simplifying assumptions:

In a first iteration step for one axial cross section of the cluster the axial gradient of the temperature distribution is considered to be constant in circumferential direction.

With this assumption then equations (11) and (8) are solved to obtain the solution of the temperature and velocity profiles as a function of the angle φ.
This procedure is repeated for each axial row of the two-dimensional net given by fig.3. The results of these calculations can then be used to introduce the angle dependent axial gradients for the temperature and velocity. Then a new iteration step has to be started. This procedure has to be followed until the difference between two subsequent steps will be sufficiently small.

4. Discussion of Numerical Results

First numerical results were obtained for a rod cluster with the following specifications:

rod diameter	D = 0.47 cm
(P/D)	1.12
axial length of the cluster	H_c = 80 cm
max. heat flux	q_{max} = 526 W/cm
shape factor for the axial heat flux distribution	φ_{ax} = 0.82
inlet temperature	T_{in} = 200°C
averaged outlet temperature	T_{out} = 535°C

The simplified method of solution, described above, was applied using an approximation for the peripheral dependency of (dT_f/dx). The symmetrical rod cluster configuration (ξ = 0) as well as two asymmetrical cases were investigated. For both asymmetrical cases the fuel rod was deviated along the line $0' - 0_1$ as shown in fig.1. The deviation was constant along the axial length of the rod cluster and was varied from ξ = 0.1 mm to ξ = 0.2 mm. For ξ = 0.2 mm this means a reduction of 20 °/o for the width of the minimum coolant channel segment in comparison to the symmetrical case.

The peripheral distributions for the temperature, the coolant velocity, the friction velocity and the axial temperature gradients were calculated for different axial heights. In all the following figures, however, results are only shown for axial segments at position x = 8 cm and x = 40 cm and angle variation from $\varphi = 0$ to $\varphi = \pi$. The angle distributions from $\varphi = \pi$ to $\varphi = 2\pi$ are symmetric to the distribution from $\varphi = \pi$ to $\varphi = 0$.

Fig.4a shows the coolant velocity distribution normalised to the mean velocity for the axial zone at x = 8 cm. Due to the hexagonal geometry of the coolant channel the normalised velocity distribution for the symmetrical case is sinusoidal with a 9 °/o difference between the maximum and minimum values. For both the symmetrical and the asymmetrical case (ξ = 0.2 mm) the period of the oscillatory velocity distribution is $\pi/3$. However, for the asymmetrical case an additional sinusoidal distribution having a period of 2π is superposed which results in a 24 °/o difference between the maximum and minimum value. From fig.4b it can be seen that for the symmetrical case variations of the relative peripheral velocity distribution are negligibly small in axial direction, whereas for the asymmetrical case (ξ = 0.2 mm) the difference between maximum and minimum values now increases up to 30 °/o at axial position x = 40 cm.

Fig.5a and 5b show the peripheral distributions of the normalised friction velocity at axial positions x = 8 and x = 40 cm for ξ = 0 and ξ = 0.2 mm. The results give a behaviour similar to that already described for the velocity distributions. At position $x = 8$ cm the difference between maximum and minimum values for $\xi = 0$ mm is 5 %, for $\xi = 0.2$ mm 16 %. For the symmetrical case no variation is observed in axial direction. For $\xi = 0.2$ mm the maximum difference of the relative friction velocity increases to 19 %.

Fig. 6a and 6b represent the peripheral variation of the coolant temperature. For the symmetrical case and $x = 8$ cm the peripheral temperature distribution is again sinusoidal with a maximum coolant temperature difference of 3,5 °C. At $x = 40$ cm this difference increases to 6 °C. In addition also the coolant temperature distributions for $\xi = 0.1$ mm and $\xi = 0.2$ mm are shown. For $\xi = 0.1$ mm the maximum coolant temperature difference in peripheral direction is now 38 °C at $x = 8$ cm and it increases to 63 °C at $x = 40$ cm. If ξ is increased to 0.2 mm the maximum coolant temperature difference changes to 73 °C at $x = 8$ cm and 120 °C at $x = 40$ cm. This temperature difference in circumferential direction seems to be very high, but it has to be taken into account that for $\xi = 0.2$ mm a reduction of the smallest coolant channel width of 20 % occurs.

With the results of this first iteration step now improved values for the peripheral variation of the axial gradient of the coolant temperature are available and shown in fig.7. As can be seen there are no significant variations around the mean value for $x = 8$ cm and $\xi = 0$ mm. For $x = 40$ cm there exists nearly no variation of the axial coolant temperature gradient in peripheral direction. For the asymmetrical case ($\xi = 0.2$ mm) the difference between maximum and minimum value of $dT/dx(\varphi)$ is 2.1 °C/cm at $x = 8$ cm and only 1.5 °C/cm at $x = 40$ cm. This shows that the influence of the circumferential variation of (dT/dx) will be of negligible significance for the coolant temperature around the rod. Further iteration steps, therefore, will hardly improve these results.

5. Conclusions

It has been shown that already small deviations of the fuel rod in a tightly packed rod bundle can result in considerable coolant temperature variations around the rod in sodium cooled fuel elements,too. The circumferential temperature variations increase along the axial height of the bundle. Although coolant mixing in the form of turbulent eddy diffusivities was accounted for, these temperature variations can be in the order of 80 - 100 °C for the cases considered here. The results obtained are restricted to the case where the rod has a constant deviation over the total height of the bundle. This assumption is an extremely conservative approach in simulating realistic axial dependent deviations of the rod as they can be caused by thermal bowing. In addition the turbulences caused by spacers would also decrease the circumferential temperature variations. Further developments of the calculational procedure used in this paper will include the numerical treatment of the differential equations for the temperature field in the fuel, the gap and the canning.

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Nomenclature

A	Constant, normally taken to be 5.5
B	Constant, normally taken to be 2.5
c_p	Specific heat of fluid at constant pressure
d, D	Outer diameter of clad
D_e	Equivalent hydraulic diameter of subchannel
E	Parameter of heat flux distribution
τ	Velocity vector
P	Pitch of fuel rod
p	Static pressure
Pr	Prandtl number = $(c_p \mu_d)/\lambda$
q	Heat flux of fuel rod
q_{av}	Average heat flux of fuel rod
Re	Reynolds number in symmetrical subchannel = $(D_w u_0 \rho)/\mu$
r	Radial distance in cylindrical coordinate
r_0	Radius of fuel rod = d/2
T	Coolant temperature
T_w	Clad temperature
u	Local flow velocity
$\bar{u}$	Average flow velocity in an axial zone
u^+	Generalized flow velocity = u/u^*
$\bar{u}^+$	Average generalized flow velocity = $\bar{u}/u^*$
u^*	Friction velocity = $\sqrt{\tau_w/\rho}$
$\bar{u}^*$	Mean value of u^* around the circumference of the wall
x	Axial coordinate
y	Perpendicular distance from rod wall
y^+	Generalized distance from rod wall
$\hat{y}$	Radial distance from wall to maximum-velocity-line
$\hat{y}^+$	Generalized radial distance from wall to maximum-velocity-line
δ_e	Equivalent hydraulic diameter of segment
ε_M	Eddy diffusivity of momentum
ε_H	Eddy diffusivity of heat
θ	Angular coordinate of deviations, degree
λ	Thermal conductivity of coolant
μ	Coefficient of shear stress
μ_d	Dynamic viscosity
ν	Kinematic viscosity

ξ	Length of deviation
ϱ	Coolant density
τ_r	Fluid shear stress on circumferential plane
τ_φ	Fluid shear stress on radial plane
τ_w	τ_r at rod wall
φ	Angular coordinate

Subscripts

n	Number of the axial zone
r	Radial direction
φ	Circumferential direction

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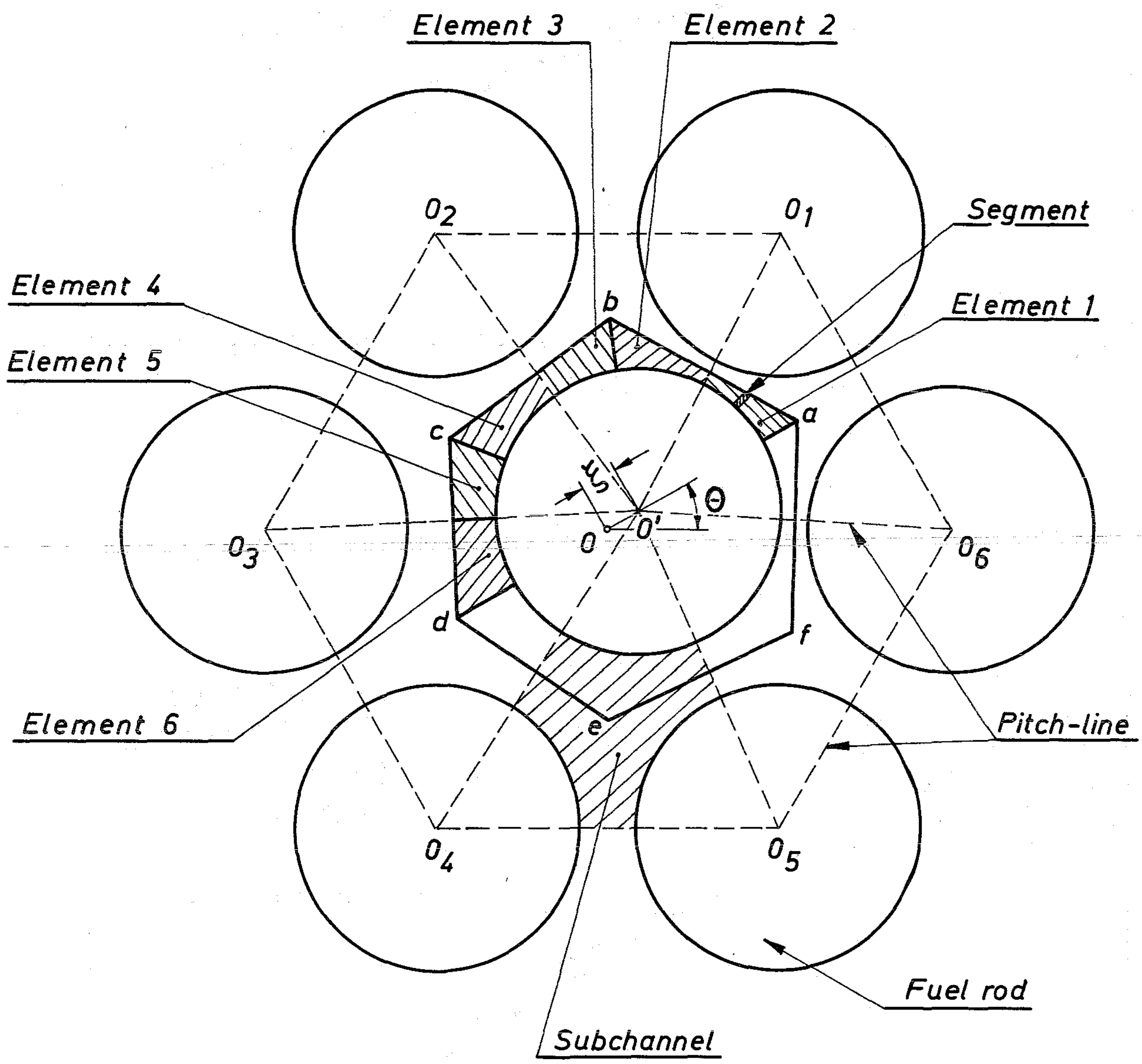
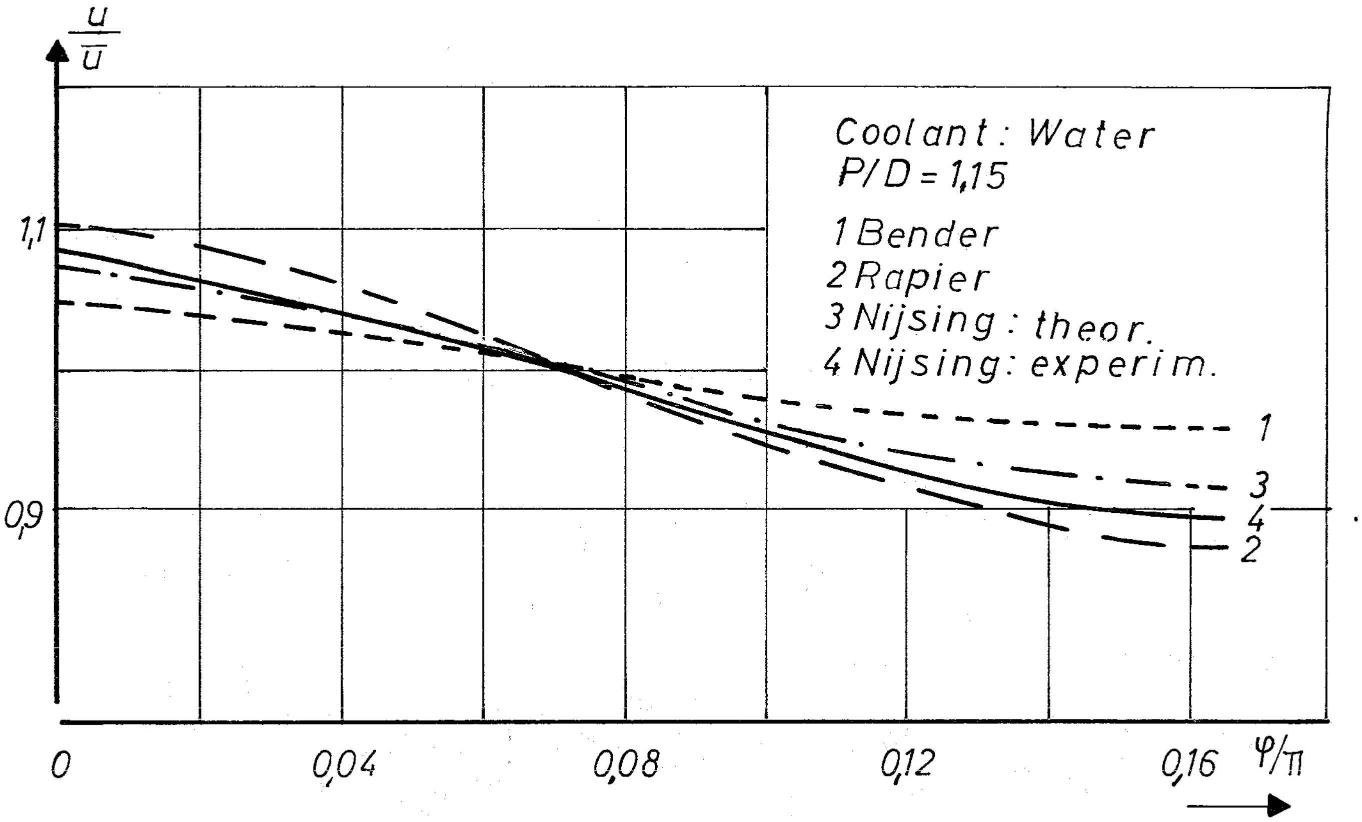


Fig 1: Cross-section of asymmetrical triangular rod cluster.



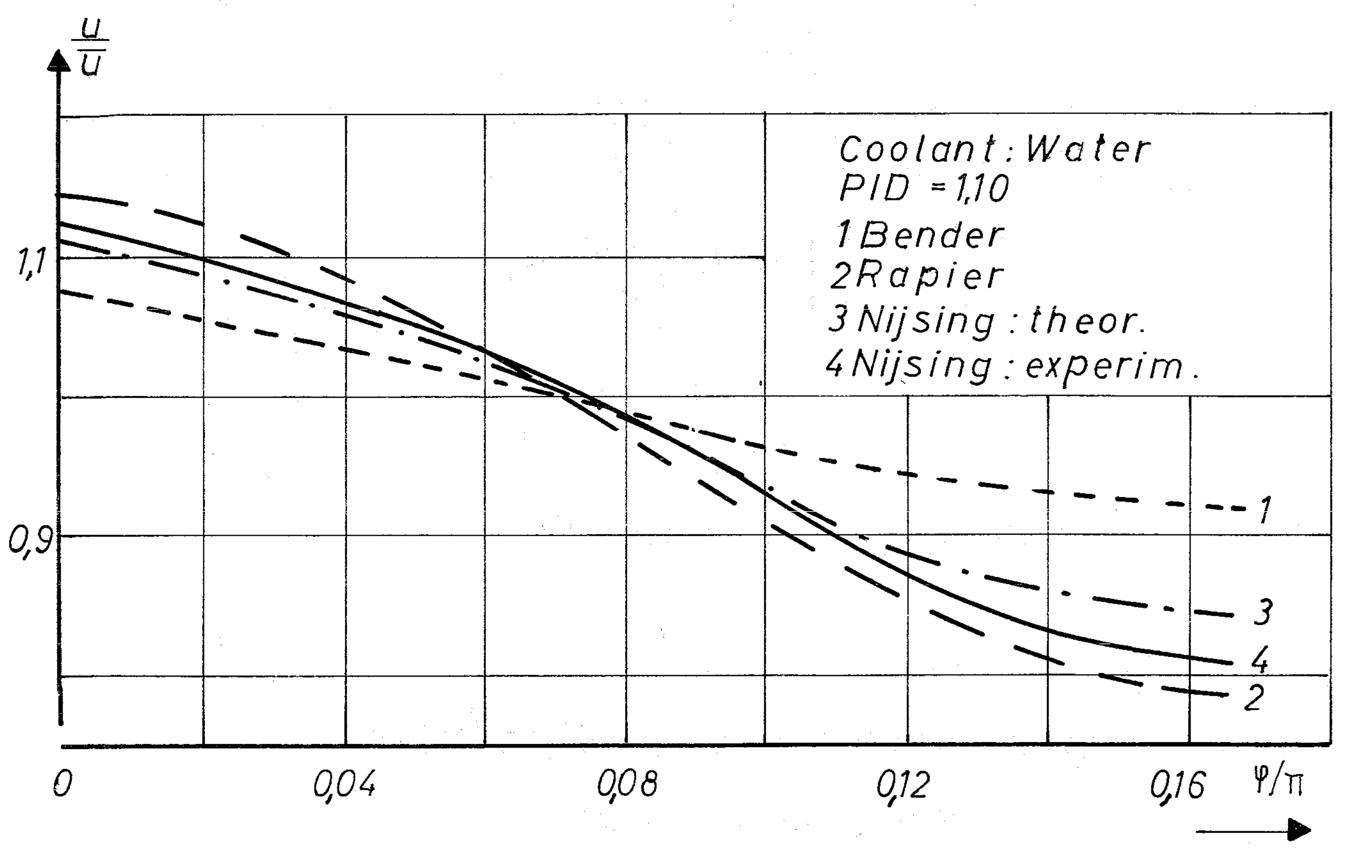


Fig 2: Comparison of different relations for the eddy diffusivity ε_{φ_M}.

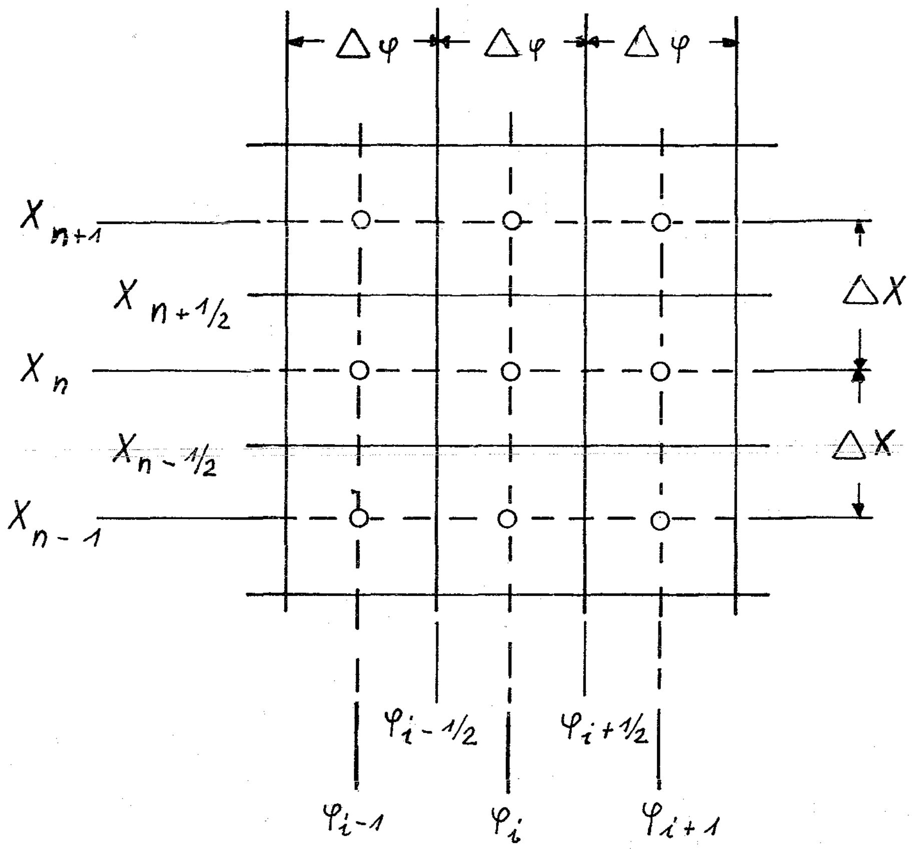


Fig 3: Network for the stepwise numerical calculation.

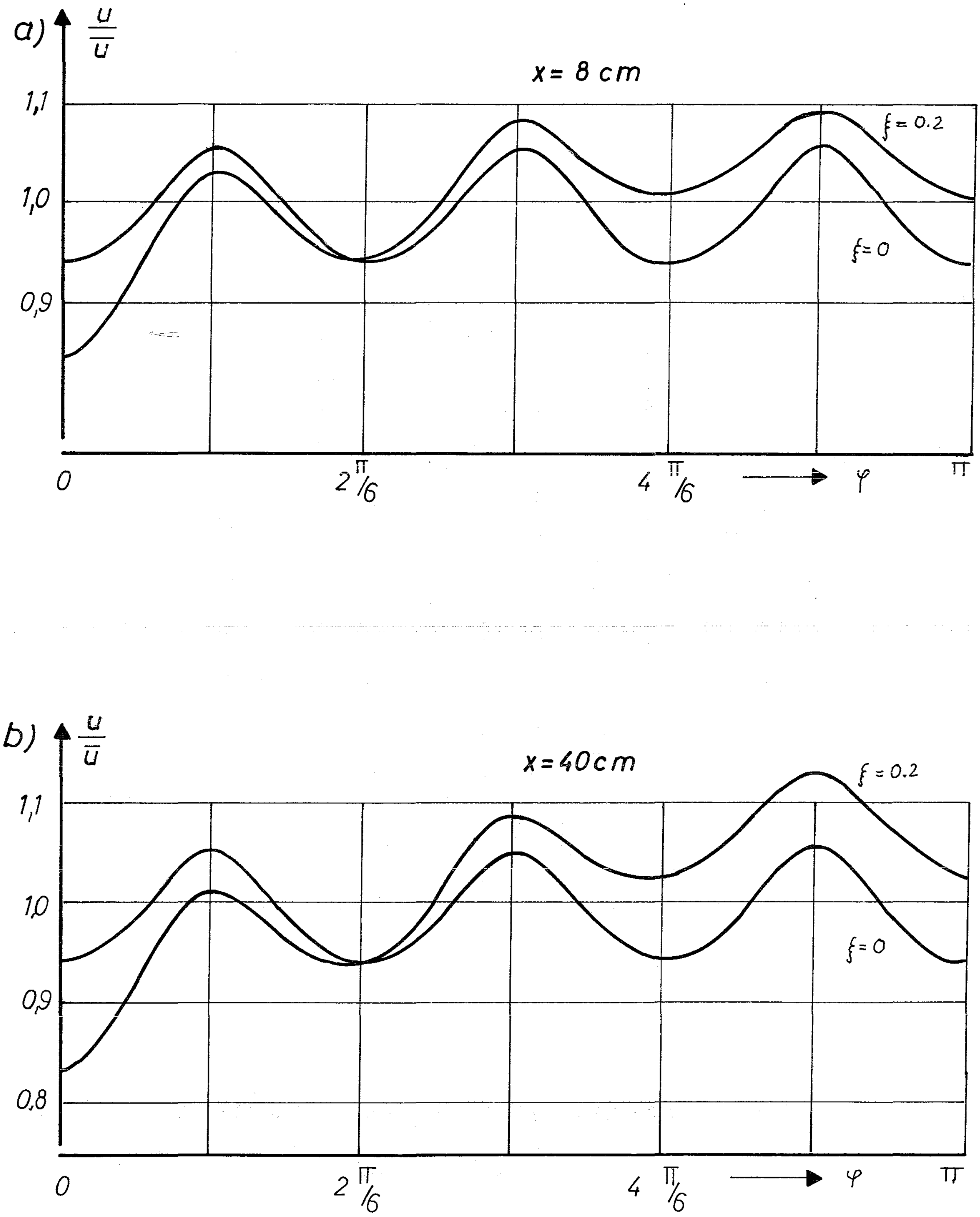


Fig 4: Flow velocity $\frac{u}{\bar{u}}$ vs angle φ

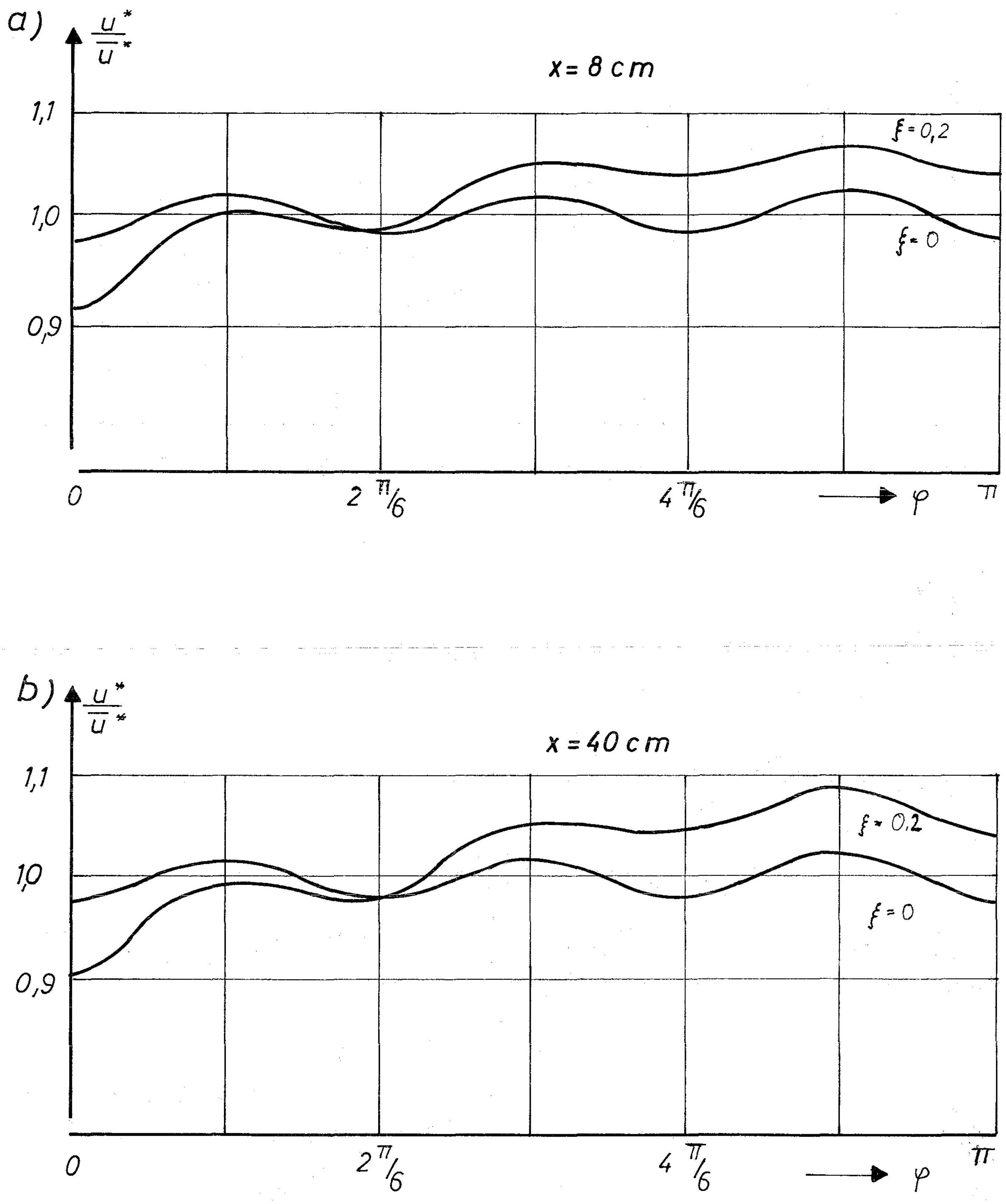


Fig 5: Friction velocity $\frac{u^x}{\overline{u}^x}$ vs angle φ

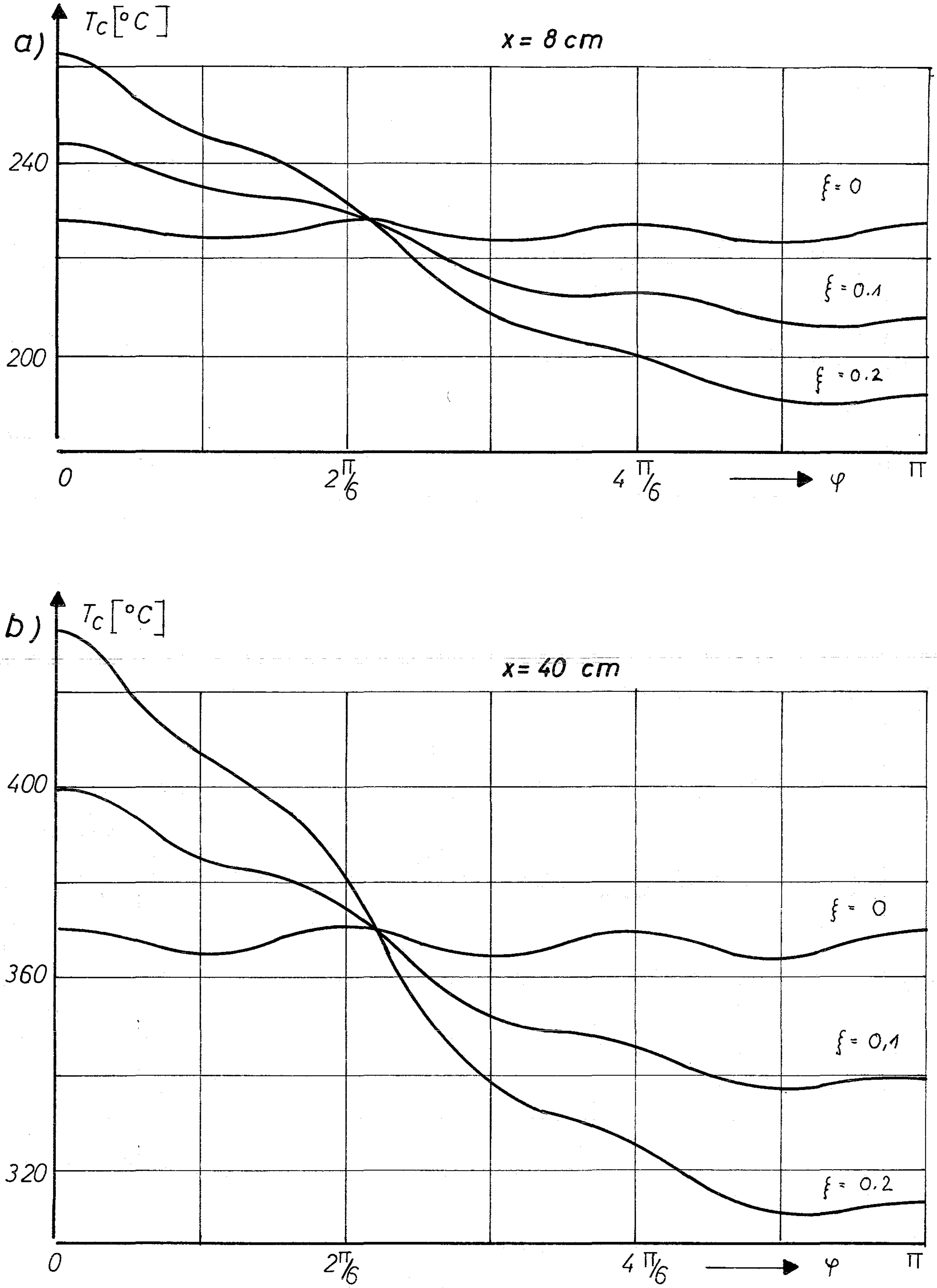


Fig 6: Coolant temperature T_c vs angle φ

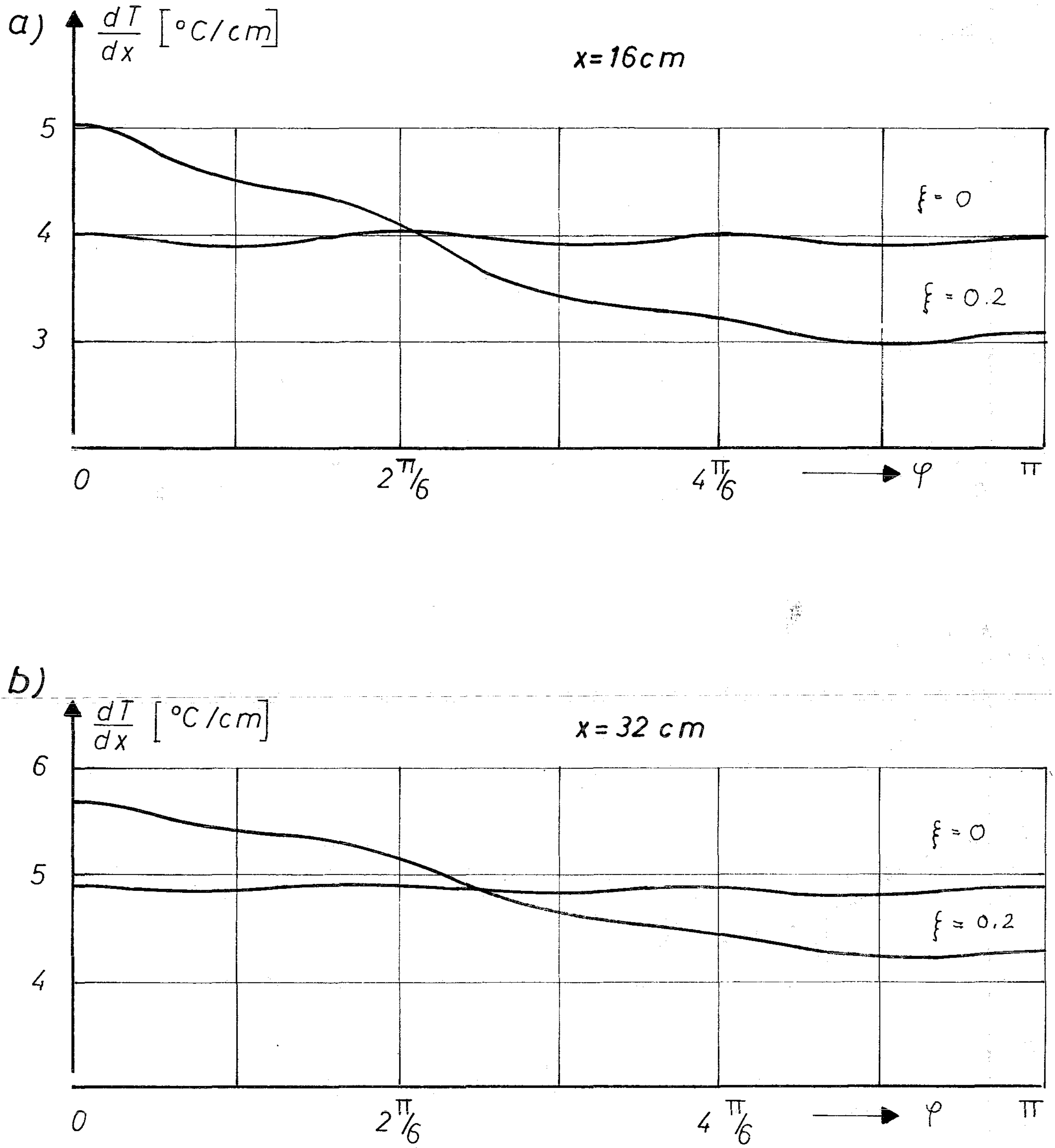


Fig 7: Axial temperature gradient $\frac{dT_c}{dx}$ vs angle φ