

# KERNFORSCHUNGSZENTRUM

# KARLSRUHE

März 1970

KFK 1134 EUR 3687 e

Institut für Reaktorentwicklung Projekt Schneller Brüter

Advanced Statistical Hot Spot Analysis

A. Amendola



GESELLSCHAFT FUR KERNFORSCHUNG M. B. H.

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#### Abstract

This report presents a general statistical method of hot spot and hot channel analysis. A definition of "hot spot" is proposed, which allows to correlate the probability of exceeding certain critical temperatures to the size of the zone in which they occur.

The assumptions of previous methods are critically reviewed and, in contrast to results of previous methods, the hot channel factors are demonstrated to be independent of the assumed spot size.

The different effects of local and global uncertainties, and the radial and axial power profile in the core are taken into account.

Together with the probability of <u>at least one</u> hot spot in the core, the probability that hot spots occur in exactly one, <u>or exactly two</u>, or <u>in exactly n</u> subassemblies is evaluated.

The report includes a description of the FORTRAN-IV code SHOSPA, which is based on the proposed method and is applicable to any type of reactor and any power profile as long as the core is constituted of bundled fuel rods.

A numerical application of the code to the sodium cooled fast reactor Na-2 is shown, together with the analysis of the influence of the assumed spot size on the hot spot factors.

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#### Zusammenfassung

In diesem Bericht wird eine allgemeine statistische Methode der Heißstellenund Heißkanal-Analyse beschrieben. Eine Definition der "Heißstelle" wird vorgeschlagen, die es erlaubt, eine Beziehung anzugeben zwischen der Wahrscheinlichkeit für das Überschreiten bestimmter kritischer Temperaturen und der Größe der Zone, in der dies auftritt.

Die Annahmen für früher angegebene Methoden werden kritisch geprüft, und es wird gezeigt, daß, im Gegensatz zu den Ergebnissen früher angegebener Methoden, die Heißkanalfaktoren unabhängig sind von der angenommenen Größe der Heißstelle.

Die unterschiedlichen Auswirkungen lokaler und globaler Unsicherheiten sowie radiale und axiale Leistungsprofile im Reaktorkern werden berücksichtigt.

Zusammen mit der Wahrscheinlichkeit für das Auftreten<u>mindestens einer</u> Heißstelle im Reaktorkern wird auch die Wahrscheinlichkeit berechnet, daß Heißstellen in genau einem, <u>genau zwei</u> oder <u>genau n</u> Brennelementen auftreten.

Dem Bericht ist eine Beschreibung des FORTRAN-IV-Rechenprogramms SHOSPA beigefügt, das die vorgeschlagene Methode zur Grundlage hat und für jeden Reaktortyp und jedes Leistungsprofil angewendet werden kann, wenn nur der Reaktorkern aus Stabbündel-Brennelementen aufgebaut ist.

Die Ergebnisse einer Anwendung des Rechenprogramms auf den natriumgekühlten schnellen Reaktor Na-2 werden berichtet, weiterhin wird der Einfluß der angenommenen Größe der Heißstelle auf die Heißstellenfaktoren analysiert.

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List of symbols

c <sub>p</sub> =	coolant specific heat
d <sub>i</sub> ,d <sub>o</sub> =	inner and outer cladding diameter
$f_c(z), f_p(z) =$	functions describing the axial profile of coolant temperature and of power respectively
F <sub>hs</sub> =	hot spot factor
G =	coolant mass flow rate
<sup>h</sup> cl-c <sup>,h</sup> f-cl <sup>=</sup>	heat transfer coefficients cladding-coolant and fuel-cladding
$h^{m}(N), h^{\sigma}(N) =$	coefficients by which the standard deviation has to be multiplied, in order to obtain the mean and the standard deviation, respectively, of the maximum deviation from nominal in N samples drawn from a normal distribution.
H =	power gradient within a subassembly
i =	subscript indicating an uncertainty, or <u>superscript</u> indicating a zone
i <sub>M</sub> =	<u>superscript</u> of the zone in which the maximum nominal temperature occurs
j =	subscript indicating cladding or fuel
kcl, kf =	thermal conductivity of cladding and fuel respectively
l,l <sub>ex</sub> =	active and extrapolation length
l <sub>s</sub> =	"spot" length
m =	mean of a normal distribution
n <sub>s</sub> =	number of "spots" at a given axial abscissa
$N_{c}, N_{c}^{eq} =$	actual and equivalent number of channels
N <sub>s</sub> =	number of subassemblies
N <sub>sp</sub> =	equivalent number of "spots" in a pin
$N_z, N_z^{eq} =$	actual and equivalent number of zones in a core
P <sub><b>r</b></sub> =	"adequate" confidence level

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P(x) =	probability that a certain value x will not be exceeded
ହ(x) =	1 - P(x), probability of exceeding a certain value x
$Q_{nhs}(f) =$	probability that in exactly n subassemblies the temperature deviation § will be exceeded
r =	radial abscissa
t =	cladding thickness
z =	axial abscissa, varying within - $\frac{1}{2}$ and + $\frac{1}{2}$
<b>z</b> <sub>M</sub> =	abscissa of the maximum temperature in axial profile
v, J =	a certain temperature and corresponding nominal value
$\mathcal{J}_{cl}, \mathcal{J}_{f} =$	inner cladding and central fuel temperature
$\vartheta_{\text{crit}} =$	critical temperature, which must not be exceeded
θ <sub>i</sub> =	coolant inlet temperature
J <sub><b>r</b></sub> =	reference temperature in equivalent number of spots
<sup>v</sup> <sub>M</sub> , v <sub>MS</sub> , v <sub>MC</sub> =	maximum temperature of the considered parameter along the pin length, in a subassembly and in the core re- spectively.
$\Delta \vartheta, \overline{\Delta \vartheta} =$	a certain temperature difference and corresponding nominal value
∆9 <sub>all</sub> =	allowable temperature difference $(\mathcal{J}_{MC} - \mathcal{J}_{i})$ .
$\Delta \vartheta_{c}(z) =$	coolant temperature increase up to height z
$\Delta \vartheta_{\mathbf{c}} =$	total coolant temperature span
<sup>∆</sup> <sup></sup> 𝔅-cl <sup>, Δ</sup> <sup></sup> <sup></sup> <sub></sub> cl-	$t^{A} \stackrel{\wedge}{}_{cl-f}^{A} \stackrel{\vee}{}_{f-r}^{f-r} = temperature drops coolant-cladding,across cladding thickness, cladding-fuel and across fuel radius.$
Δϑ f-d, max, Δί	= axial maximum of the temperature difference fuel (cladding) and coolant at inlet.
∆J = max =	$\vartheta_{MC} - \vartheta_{i}$
$\lambda =$	ratio of a deviation from mean to standard deviation
S =	standard deviation
6 <sup>s</sup> =	specific standard deviation of the local uncertainties

$\sigma_i^{s'}=$	relative uncertainty on coolant temperature increase
-	due to the specific standard deviation of the parameter i
G <sup>s,ch</sup> =	the same as for $\sigma_i^{s'}$ , but calculated for a particular channel arrangement
G <sup>s,ch</sup> =	total specific standard deviation on coolant temperature increase for the considered channel arrangement - (relative value)
<sup>cs</sup> i'∆√j-d,max	= specific standard deviation - expressed in $^{\circ}C$ - of the temperature difference $\Delta \mathcal{T}_{j-d}$ , calculated at the axial maximum, due to the uncertainty i
<sup>σ</sup> Δ <sup>η</sup> j-d <sup>,max</sup> =	total specific standard deviation - expressed in $^{\circ}C$ - of the temperature difference $\Delta \widetilde{\mathcal{V}}_{j-d}$ , max
$\sigma^{1}_{\Delta \vartheta_{c}}(z) =$	standard deviation of the coolant temperature increase at height z due to the local uncertainties (relative value)
$\mathcal{C}^{1}_{\Delta \mathcal{J}_{\mathbf{j}-\mathbf{d}}}(\mathbf{z}) =$	standard deviation of the temperature difference $A_{\nu j-d}^{O}$ at height z due to the local uncertainties (°C)
$\mathcal{O}_{\mathcal{J}_{j}}^{1}(z) =$	standard deviation of the temperature $\mathcal{V}_{\mathbf{j}}$ at height $\mathbf{z}$ due to the local uncertainties (°C)
~, <sup>*</sup> , * =	reference standard deviation in equivalent number of spots, channels and zones respectively.
$\chi$ (z) =	specific power of a pin at height z
χ'(z) =	specific power delivered in a channel at height z
χ <sub>max</sub> =	axial maximum of $\chi(z)$ .
(m, °) =	normal distribution with mean m and standard deviation
$(0, \tilde{c}_{h}) =$	distribution of the "channel" uncertainties
(0, 5) =	distribution of the "subassembly" uncertainties
(0, ~) =	distribution of the "zone" uncertainties
(0, ° <sub>c</sub> ) =	distribution of the "core" uncertainties
(m_sp, Sp) =	equivalent "spot" distribution after the reduction procedure

(m<sup>eq</sup><sub>ch</sub>, eq) = equivalent "channel" distribution after the reduction procedure

- (m<sup>eq</sup><sub>s</sub>, e<sup>eq</sup><sub>s</sub>) = equivalent "subassembly" distribution after the reduction procedure
- $(m_z^{eq}, \mathfrak{S}_z^{eq}) =$  equivalent "zone" distribution after the reduction procedure
- (m<sup>eq</sup><sub>c</sub>, eq) = equivalent "core" distribution after the reduction procedure

#### Introduction

This paper extends the previous work / 1,2 / 2 on hot channel factors to the hot spot factors. Many theoretical bases were already given in the referenced papers; therefore only the aspects, which are peculiar to hot spot analysis, are treated extensively in the following. However, some new considerations on hot channel analysis, which actually is only a particular case of the hot spot analysis, are pointed out.

The assumptions of previous methods of analysis are critically reviewed. As in the "Synthesis Method" /3, 4/7, the hot spot factors are evaluated versus the total failure probability of a core, taking into account the probability that every "spot" in the core, deviating from its own nominal temperature, could be "hot". The main improvements, which the proposed method gives to previous analyses, are the following:

1) The correlation among the temperatures of the several cooling channels, due to the global uncertainties (that is,the uncertainties which affect in the same way the temperature of a whole core, or at least of a whole part of a core), is taken into account. As shown in /[1,7], considering the actual correlations, the <u>hot spot factors</u>, at a given confidence level, result to be <u>lower</u> than those evaluated starting from the assumption of independent channels.

Another advantage of this procedure is that it allows <u>a better assess-</u> <u>ment of the relative importance of the individual uncertainties</u>; this enables the designer to identify which are the most limiting uncertainties for a higher power rating, and consequently, where smaller tolerances should be required. Previous methods sometimes drastically over-or underestimated the importance of certain tolerances because of their inadequate theoretical treatment, even if they might occasionally agree with the present method in the numerical value of the overall hot spot factor.

2) A definition of "hot spot" is proposed, which allows to correlate the probability of exceeding critical temperatures to the size of the zone in which they occur. In contrast to previous methods  $/3,4,5_7$ , the <u>hot channel factors</u> are demonstrated to be <u>independent</u>

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of the assumed spot size, if the uncertainties are correctly defined. Therefore, a new criterion is proposed for the specification of the uncertainties: namely, for the uncertainties which are random functions along the fuel pin axis, the term "<u>specific standard deviation</u>" is introduced.

3) The total probability to have <u>at least one hot spot</u> is subdivided into the sum of its components, evaluating the probability that <u>hot</u> <u>spots occur in exactly one, or in exactly two</u>, .... <u>or in exactly</u> <u>n subassemblies</u>. In this way, a better quantitative assessment of the reactor safety against over-temperatures is offered to the designer, which enables him to weigh the cost of the plant versus the financial risk of fuel failure.

The work was performed within the context of sodium cooled fast reactor development, but the proposed method is quite general and immediately applicable to any type of reactor constituted of bundled fuel rods with single phase coolant. The paper is subdivided into three parts. The first part presents the theoretical analysis; the second part describes the Fortran-IV code SH $\phi$ SPA (Statistical HOt SPot Analysis) which operates according to the proposed method and does not require large computational time; the third part shows an application of the code to the sodium cooled fast reactor Na-2. In this numerical example, the influence of the spot size on hot spot factors and the relative importance of the several uncertainties affecting the thermal design are examined.

The author wishes to thank Mr. E.G. Schlechtendahl for his interest and valuable suggestions: particularly, the concept of "specific standard deviation" was introduced after a common discussion on the subject.

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# First Part

#### Theoretical Analysis

#### I. GENERAL REQUIREMENTS AND DEFINITIONS

#### I.1 Scope of a hot spot statistical analysis

The operating temperature of a nuclear reactor is generally limited by the requirement that the coolant at channel outlet and some core components, such as fuel and cladding for instance, do not reach certain critical temperatures, which could provoke failures or an unsafe core operation. Because of the several uncertainties which affect the thermal design, the designer is then faced with the problem of estimating the maximum temperatures, which can occur in the core. The uncertainties are generally random variables, with statistical distributions which can be assessed by measurements. Therefore, also the estimation of the maximum possible temperatures is of statistical nature. That is, the maximum temperatures should be associated with a certain level of probability. In the following, we shall indicate this level of probability as "confidence level", meaning with this term a certain probability that a certain maximum temperature will not be exceeded anywhere in the core by the considered critical parameter.

<u>Scope of a hot spot statistical analysis is</u> then to develop a method of calculation, which allows to determine the relationship between <u>maximum temperature and corresponding confidence level</u>, once the nominal temperature and power profile in the core and the distributions of the uncertainties are assigned. In this way the designer is able to verify that, in an actual core design, critical temperatures are not expected to be exceeded at an <u>adequate</u> confidence level.

This adequate confidence level cannot be defined once for ever: it should be assumed in any design on an economical basis, that is, after

a comparison between financial advantages of a higher power rating and financial risks of component failures. In order to allow this comparison, the developed method associates the maximum temperatures with the geometrical size of the zone in which they occur (<u>hot</u> <u>spot size</u>); moreover it permits to assess, at any confidence level, the expected number of failed subassemblies.

# I.2 Hot spot factors and thermal design

The definition of the hot spot factors offers a useful tool to the designer in order to assess the expected maximum temperatures and consequently to choose the operating temperature. The following general definition may be applied to the fuel or the cladding indifferently.

We define as hot spot factor the safety factor  ${\rm F}_{\rm rs}$ 

$$F_{hs} = F_{hs} \quad (conf. level) \tag{1}$$

which takes into account all uncertainties affecting the thermal design in such a way that, in steady-state operation, the value

 $\widehat{\vartheta} = \overline{\widehat{\vartheta}}_{i} + F_{hs} \quad \overline{\Delta \widehat{\vartheta}}_{max}$ (2)

is not expected to be exceeded by the temperature of any "spot" of the considered component in the core, at the confidence level associated with  $F_{hs}$ . In Eq. (2)  $\overline{\vartheta}_i$  is the nominal coolant temperature at core inlet, and  $\Delta \overline{\vartheta}_{max}$  is the temperature difference between the nominally hottest point of the considered component in the core and  $\overline{\vartheta}_i$ . The definition of "spot" will be given in the following items.

The proposed method of analysis allows to determine explicitly the function in Eq. (1), once the set of uncertainties and the nominal temperature profile in the core are known. Then, if Pr is the "adequate" confidence level assumed for the thermal design, and  $\mathcal{T}_{crit}$  is the critical temperature not to be exceeded, the allowable maximum temperature difference  $\overline{\Delta \mathcal{T}_{all}}$  can be evaluated as:

$$\overline{\Delta \overline{\vartheta}}_{all} = \frac{\vartheta_{crit} - \overline{\vartheta}_i}{F_{hs}(Pr)} .$$
(3)

The optimum condition (with reference to the hot spot problem only)

$$\overline{\Delta \vartheta}_{\max} = \overline{\Delta \vartheta}_{all}$$
(4)

cannot be reached in general for all components at once: there are always one or two components which are the most limiting for higher power ratings. Thermal design optimization should be, therefore, performed with reference to the overall non-failure probability. Neglecting eventual correlations, a conservative figure of the overall non-failure probability is given by

 $P_{S} = \prod_{j} P_{S-j}$ (5)

where  $P_{S-j}$  is the probability that no failure occurs due to the component "j" (j = coolant, fuel, cladding for instance).

A typical optimization problem, which can be solved by means of iterative application of hot spot analysis to successive core thermal designs, is the following: assumed a constant non-failure probability for the core, once the power profile, the mechanical design and the inlet coolant temperature are fixed, in which way should the flow rates be distributed among the several subassemblies, in order to obtain the maximum efficiency, that is the maximum average coolant temperature at core outlet?

#### I.3 Hot spot definition in previous methods of analysis

In order to complete the definition of the hot spot factors, it is necessary to define what is a "hot spot"; that is, which is the geometrical size of the zone in which over-temperatures are expected to occur. Previous methods of statistical analysis have presented very unsatisfactory assumptions: in the following they are critically reviewed, then the assumptions of the proposed advanced method will be discussed.

#### I.3.1 Deterministic method

In the paper of Letourneau and Grimble /67 there is the first systematic exposition of the deterministic method and a clear difference between hot channel and hot spot factors is pointed out: namely, the hot channel factor "is based on average deviations from nominal over the length of the channel, while the spot factors are based on local deviations from nominal at one worst point on the surface (the hot spot)". Consequently, the deterministic method defines the nominally hottest point in the core as the hot spot: then, in this point all "maximum deviations" are assumed to occur simultaneously. This method indeed gives an absolute safety for the whole core, once the "worst" or maximum possible deviations are known. (The concept of worst deviation however must be subject to criticism: infact, the fabrication tolerances, for instance, are results of measurements, which, moreover, are generally performed only on samples drawn from the complete production: they are, therefore, of statistical nature. The "worst deviations" should be consequently associated always with the confidence level of their estimation.) The main limitation of the deterministic method is recognized by the authors themselves: "The philosophy of the assumption that all worst deviations occur at one location is open to question, since the statistical probability of such an occurrence is extremely low ... The advantage gained (with a statistical method) would have to be weighed against the consequences which might occur if the design maximum temperatures were exceeded."

### I.3.2 First statistical methods

The first statistical methods (see for instance / 7, 8, 9 / 7) have erroneously maintained the definition of hot spot as the "worst point" in the axial temperature profile along the fuel, even if some of these have considered that not just the most critical fuel element in the core, but also the other ones contribute to the total hot spot probability. We must in fact distinguish between global and local uncertainties. A global uncertainty, for instance, an error in the power measurement or a deviation from nominal of the flow rate in a subassembly provokes that the whole temperature profile along the fuel length is shifted at once: therefore, with respect to the global uncertainties, if the thermal design guarantees that the <u>nominal maximum</u> in the temperature profile does not exceed a certain value, it guarantees also that this value is not exceeded in <u>any other point</u> of the fuel. So far these methods were justified. This is no longer true for the local uncertainties, such as those of fuel density, heat transfer coefficients, etc., which are random functions along the fuel length. These uncertainties provoke an irregular temperature profile, the maximum of which does not necessarily occur in the point of the nominal maximum: moreover more points could reach the same maximum temperature. The concept of hot spot as the nominally "worst point" is therefore very unsatisfactory in a statistical analysis.

#### I.3.3 The "Spot Model" and the "Synthesis Method"

The multiplicity of possible occurrences of hot spots along the fuel length was taken into account first by Businaro and Pozzi in their "spot model" /57.

These authors divided the fuel length into an arbitrary number of segments, along which parameters such as fuel density, enrichment, diameter, that is the local uncertainties, were assumed as constant. Then, they developed a Monte Carlo routine, drawing sample values of these parameters from their statistical distribution, in order to evaluate the channel and spot correction factors. The conclusions they drew are that, increasing the number of the segments, that is of the spots, the hot spot factors increase, while the channel factors decrease: namely, the decreasing of the channel factors is proportional to  $\frac{1}{\sqrt{n}}$ , where n is the number of spots, or else, in accordance with the statistical theorem of the mean of n values assumed by a random variable  $\sqrt{10}$ , the channel factors take into account an average value of the uncertainties along the fuel length.

Now, there can be no doubt that increasing the subdivision of the pin into more segments must result in larger hot spot factors, since there is a decrease of the <u>physical</u> size of the considered spot; and it is obvious that the <u>probability of having small hot zones</u> is larger that that of having large hot zones: but, the temperature of the coolant at channel outlet is affected by a physical average

of the uncertainties along the fuel length, which should by no means vary with the mathematical model, namely the number of the segments in the assumed subdivision. This consideration shows that the method was in error: the contradiction can be easily removed if we consider that decreasing the size of the segmentation would result in a corresponding increase in the actual deviations, while the authors maintained the standard deviations of the uncertainties constant in this application of the Monte Carlo routine to different number of spots in a fuel element. We shall explain this point with an example: let 1 be the chosen length of the spot, n the number of spots and or the standard deviation of the average value of a certain uncertainty along the length 1 : the standard deviation to consider for channel correction factors is then  $\frac{G}{\sqrt{n}}$ , according to Ref. <u>7</u>. If now the spot length were assumed to be 2 l, the total number of spots would be n/2, and the standard deviation would be that of the average value over two spots each one of length  $l_s$ ; that is the actual standard deviation to consider must be  $\frac{\sigma}{\sqrt{2}}$  and no longer  $\sigma$  as assumed in Ref. / 5 7. Considering the appropriate values, the standard deviation for channel correction factor results to be  $\frac{\sigma/\sqrt{2}}{\sqrt{n/2}} = \frac{\sigma}{\sqrt{n}}$ : that is, it does not depend upon the assumed spot length. Moreover, exact assumptions on the uncertainties would have given spot factors even larger than those calculated in Ref.  $\sqrt{5}$  for small spot lengths, because, together with the increase in the number of possible occurrences of hot spots, larger standard deviations for the uncertainties should have been assumed. The "synthesis method"  $\overline{3}, 4\overline{7}$  extends the previous "spot model" to the whole core: in his work, Guéron / 3\_7 assumes the length of a pellet as length of a circumferential spot and states coherently "it is over a pellet that diameter, density and isotopic composition are defined". This assumption allows him to overcome the recognized (but not clearly removed) ambiguity of the results in Ref.  $/5_7$ ; while this might be a reasonable assumption for pelleted fuels, it is not applicable to other types of fuel. It is hence necessary to give a new general and unambiguous definition of "hot spot".

# I.4 Hot spot definition

From a mathematical point of view a "spot" might be defined as a geometrical point. The number of such "spots" in a core being infinite,

we would be faced with the <u>certainty</u> of the presence of hot spots in a core, if for the uncertainties unlimited distributions were assumed, such as, for instance, normal distributions. Assuming, however, truncated distributions for the uncertainties, the only possible method of analysis would be the evaluation of a statistical factor for the "global" uncertainties and of a deterministic factor including the "worst possible deviations" of the "local" uncertainties.

This assumption, which might appear to be very satisfactory for the reactor safety, has, however, no physical relevance: in fact, according to this definition, the maximum possible deviations of the local uncertainties should be measured on samples of infinitesimal size, which is physically and technically meaningless. It is always necessary to assume a certain finite size for the sample to be measured: consequently the minimum size which allows to perform measurements might be assumed as size of a "hot spot". A finite size of the spot allows a statistical analysis and, at the same time, is of technical relevance.

The necessity of performing the measurements on samples of the same size of the assumed spot was already indicated, criticizing the spot model: some example will clear better this concept. If we want to assess the effects of the enrichment tolerances on the power delivered by a pellet, we should measure the average contents of Plutonium in the pellet, but, if we want to assess the effect of the same parameter on the temperature of every zone of a pellet, we should measure the exact Plutonium concentration in every pellet zone. It is clear that the smaller the sample is, the larger the deviations will be: for instance, the enrichment which at nominal value 20 % can vary in a pellet of mixed Plutonium and Uranium oxides within 19 % and 21 %, varies in microscopical scale within 0 % and 100 %. Moreover, the heat transfer between fuel and cladding differs largely between a point of perfect contact and one with a large gap; whereas over a certain surface the resulting value is an average of several intermediate and extreme conditions, therefore, also in this case, the smaller the examined surface is, the larger the deviations will be.

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Consequently, if the size of the spot were fixed only on the basis of physical possibility of measurement, the hot spot factor could result prohibitively large.

But the occurrence of a hot spot in the fuel or in the cladding can affect the performance of the pin only if a finite zone is concerned, the radius of which can be estimated theoretically or experimentally. For instance, the melting of 1 mgr. of fuel in a pellet does not deteriorate the performance of a pin, but the melting of fuel along 10 cm of a pin may well do so. These considerations bring us to the following conclusions:

- a) The size of the hot spot, against which safeguard must be taken in reactor thermal design, cannot be defined once for ever, but it should be assumed by the designer on the basis of safety and physical requirements.
- b) Once the size of the spot has been assumed, the distribution of the local uncertainties should be determined on samples of corresponding size.

That is, it is not possible to assume a size for the "hot spot", without taking into account the size on which the deviations were assessed. On this basis only, a statistical hot spot analysis can correlate unambiguously the probability of exceeding certain critical temperatures to the size of the zone in which they occur.

Before discussing our particular assumptions for fuel and cladding hot spot, we shall indicate how the local uncertainties should be specified, in order to avoid any possible ambiguity, such as that criticized in the "spot model".

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# 1.5 Specifications of the uncertainties

The basic information required for a statistical analysis, is the complete set of the distributions of the uncertainties, both global and local \*. A single parameter may be affected by different types of uncertainty; therefore, care should be taken to separate the global effects from the local one. The thermal conductivity of Uranium-Plutonium oxides, for instance, is known within certain limits, which depend upon measurement error and fuel chemical composition. The measurement error plays a very large role on the corresponding uncertainty: this is an uncertainty which is <u>common</u> to all pellets in the core and should be treated as a "global uncertainty". On the other hand, the chemical composition affects every pellet individually, its effect should be taken into account as an average over a certain length of the pellet. Other parameters must be treated similarly.

The local uncertainties, that is the uncertainties which are random functions along a fuel element, must always be specified together with the size of the sample, over which they were measured. In order to refer always to a same size, we shall introduce the term of specific standard deviation ( $\sigma^{S}$ ). Namely, we define as "specific standard deviation" the standard deviation of the average value which the con-

\*<u>Note</u>. As shown at /[1]/, the knowledge of the mean (m) and of the standard deviation ( $\sigma$ ) is sufficient in most practical cases: this is because we deal with a sum of several distributions, and this sum tends to a normal distribution if the number of the terms of the sum is large /[10]/, whichever the type of the individual distributions is. The estimation of m and  $\sigma$  should be performed on an adequate number of representative samples, according to the well known statistical experimental technics, since the final confidence level of the analysis will also depend upon the confidence level at which the uncertainties are assessed.

We shall indicate a distribution with the symbol  $(m, \sigma)$  and assume it to be normal: correspondingly in the diagrams we shall indicate it with the block  $-\underline{m,\sigma}$ . Moreover we assume that in the core design the nominal value of each parameter coincides with its mean; that is, each parameter has the distribution  $(0, \sigma)$  about its nominal value.

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sidered parameter may assume over the unit of length, surface or volume according to the nature of the parameter.

For instance, the tolerances of the cladding outer diameter should be specified as follows (the numerical quantities have only an explicative function):

#### Outer diameter:

<u>nominal value</u> = mean = 6 mm, <u>standard deviation of average diameter among</u> pins belonging to <u>different production batches</u>:  $\vec{e}_{do} = 0.013$  mm, <u>specific standard deviation of irregularities along a pin</u>:  $\vec{e}_{do} = 0.012$  mm  $\sqrt{cm}$ .

In this example  $G'_{do}$  is an uncertainty which acts on a whole group of pins, namely those produced in a same batch;  $G'_{do}$  acts locally along a pin. If we have assumed, for instance, for the cladding a circumferential hot spot of length  $l_s$  cm , the corresponding standard deviation to consider is

$$\sigma^{ls} = \frac{\sigma^{s}}{\sqrt{l_{s}}}$$
(6)

since now, it is the standard deviation of the average local diameter - averaged over the length  $l_s$  - which should be taken into account.

For the heat transfer coefficients and for the enrichment, for instance,  $\sigma^{S}$  should be referred to the unit of surface or of volume, respectively. However, for a fixed geometry of the fuel element, we can continue to define it for unit of length: naturally, in this case, expressions different from Eq. (6) should be introduced in the calculations, if in the assumptions of hot spot size only a partial section of a pin is taken into account. This point will be explained extensively in the following.

#### II. EVALUATION OF THE HOT SPOT FACTORS

We shall now concentrate on the analysis of a reactor constituted of bundled fuel rods: particularly, reference will be made to a sodium cooled fast reactor, such as Na-2 / 11 / 7. The method of analysis is, however, quite general and it can be easily extended to other fuel types.

#### II.1 Hot spot size

The spot size, we shall now assume for fuel and cladding respectively, are based on physical and safety considerations: they remain, however, in a certain way arbitrary. Other assumptions might be equally satisfactory. Actually the main point is to get a procedure which allows to evaluate the hot spot factors coherently with the assumptions performed on the spot, since a certain arbitrariness, as already discussed, cannot be avoided in the spot definition.

#### II.1.1 Fuel hot spot

The cooling channel configuration taken into account for a sodium



Fuel hot spot channel arrangement

cooled reactor such as Na-2 is given in Fig. 1. We assume the hot spot size in the fuel as <u>a certain length of the pin</u> center line.

This definition takes into account the fuel geometry at beginning of reactor operation,

that is, when the central channel is not yet formed in the fuel . Moreover, it is not taken into account, that, due to local irregularities, as e.g. large fissile material concentration in a small zone of the fuel, the maximum temperature location might not be at the center of the pin: that is, <u>only average deviations</u> from nominal <u>in the</u> <u>axial section of a pin are considered</u>.

But there are still a few more effects which could displace the point of maximum temperature out from the pin centerline. At the beginning of exposure this can be caused principally by the following effects:

### a) Eccentricity of the fuel in respects to the cladding:

In this case a radial displacement of the maximum fuel temperature occurs versus the maximum thermal resistance point; but this displaced maximum is lower than the nominal maximum at fuel center / 12\_7. Hence, neglecting the effects of pellet eccentricity results in a conservative assessment of the hot spot probability for the fuel. This effect should naturally not be neglected in channel and cladding analysis.

# b) Asymmetrical power generation in the fuel:

This can be provoked by average irregularities of density or enrichment in the fuel and neutron flux gradients: we do not, in fact, consider local irregularities, as previously said. These effects appear however to be not very important. (For instance, it has been verified that, assuming constant average power in the central pellet of a Na-2 fuel pin, a 20 % higher power generation within a 60  $^{\circ}$  angle provokes the maximum temperature to be displaced radially versus the high power region: this maximum is only 2 % higher than the nominal one.) Therefore, it is possible to take into account these irregularities as further sources of uncertainties on the maximum temperature, which will be assumed to occur always at pin center.

If hot spot analysis should be performed for a fuel at a given burnup, no variation in the analysis method would be necessary. It would just require a new spot definition , namely the inner surface of the fuel defined by the boundary of the inner channel over a certain length. The nominal temperature should correspondingly be recalculated for a given inner channel radius, taking into account new nominal values and uncertainties of parameters such as density, thermal conductivity, power density etc. In this case, however, another reasonable assumption might be to consider a triangular channel arrangement, such as that which will be presented for the cladding and correspondingly to define a hot spot as the inner surface of the fuel defined by a  $60^{\circ}$  arc on the boundary of the inner channel over a certain length.

# II.1.2 Cladding hot spot

Since there is no power generation, in the cladding the maximum temperatures occur only at the inner surface of the cladding.



<u>Fig. 2</u>

Cladding hot spot channel arrangement

examined in the following.

the considered configuration of the cooling channel for cladding temperature analysis, as well as for hot channel analysis, is shown in Fig. 2. Hot spot size is assumed to be the <u>inner surface of the</u> <u>cladding defined by an arc of  $60^{\circ}$ </u> <u>as basis over a certain length.</u> The effect of the spot length on both fuel and cladding factors will be

(7)

For a reactor such as Na-2,

#### II.2 Nominal temperature profile

#### II.2.1 Axial temperature profile

At a given radius r, the inner temperature  $\hat{v}_f$  of the fuel at height z is given by the sum of the following terms:

$$\widetilde{\mathcal{Y}}_{f}(z) = \widetilde{\mathcal{Y}}_{i} + \Delta \widetilde{\mathcal{Y}}_{c}(z) + \Delta \widetilde{\mathcal{Y}}_{c-cl}(z) + \Delta \widetilde{\mathcal{Y}}_{cl-t}(z) + \Delta \widetilde{\mathcal{Y}}_{cl-f}(z) + \Delta \widetilde{\mathcal{Y}}_{f-r}(z),$$

that is, by the sum of the inlet coolant temperature  $(\mathscr{V}_i)$ , the coolant temperature increase between inlet and height z  $(\Delta \mathscr{V}_c)$ , and the temperature drops across the interface cladding-coolant  $(\Delta \mathscr{V}_{c-cl})$ , the cladding thickness  $(\Delta \mathscr{V}_{cl-t})$ , the gap fuel-cladding  $(\Delta \mathscr{V}_{cl-f})$  and fuel radius  $(\Delta \mathscr{V}_{f-r})$ .

Assuming the origin of the axis z at core center, and a cosine distribution for the specific power  $\chi$  delivered by a fuel pin:

$$\chi(z) = \chi_{\max} \cos \left(\frac{\hat{\gamma}}{\mathbf{l}_{ex}} z\right),$$
 (8)

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we get for  $\Delta \mathcal{T}_{c}(z)$  the following expression:

$$\Delta \mathcal{J}_{c}(z) = \int_{-\frac{1}{2}}^{z} \frac{x'z}{c_{p} \cdot G} dz = \frac{\chi' \max^{1} ex}{\pi c_{p} G} \sin\left(\frac{\pi}{2}\frac{1}{l_{ex}}\right) + \sin\left(\frac{\pi}{l_{ex}}z\right)$$
(9)

where

1 = active length of a pin

1 = extrapolation length

- c = coolant specific heat, assumed independent of temperature

$$(\chi' = 3\chi, Fig. 1)$$

Indicating with  $\Delta \vartheta_c$  the total temperature span along a channel, the Eq. (9) can be written as:

$$\Delta \mathcal{Y}_{c}(z) = \Delta \mathcal{Y}_{c} \quad \frac{\sin(\frac{\hat{\pi}}{2}\frac{1}{l_{ex}}) + \sin(\frac{\hat{\pi}}{l_{ex}}z)}{2\sin(\frac{\hat{\pi}}{2}\frac{1}{l_{ex}})} \quad (10)$$

Explicite notation of the other terms in Eq. (7) results in:

$$\vartheta_{f}(z) = \vartheta_{i} + \Delta \vartheta_{c} \frac{\sin(\frac{\pi}{2}\frac{1}{l_{ex}}) + \sin(\frac{\pi}{l_{ex}}z)}{2\sin(\frac{\pi}{2}\frac{1}{l_{ex}})} + \chi_{\max}(\frac{1}{\pi h_{cl-c}} + \chi_{max})$$

$$+ \frac{2 t}{\widetilde{\pi} k_{cl}(d_i + d_o)} + \frac{1}{\widetilde{\pi} h_{f-cl} d_i} + \frac{1}{4 \pi k_f} \cos\left(\frac{\widetilde{\pi} z}{l_{ex}}\right) \quad (11)$$

where  $d_{i}, d_{o}$  = inner and outer cladding diameter respectively t = cladding thickness (t <  $(d_{i}+d_{o})/2$ )

h cl-c, h f-cl = heat transfer coefficient cladding-coolant, and fuel-cladding respectively

$$(k_{f} = \frac{1}{\Delta \tilde{v}_{f-r}} \int_{\tilde{v}_{f-\Delta} \tilde{v}_{f-r}}^{\tilde{v}_{f}} k_{f}(\tilde{v}) d\tilde{v} ) .$$

Indicating with  $\Delta \vartheta_{f-d,max}$  the total temperature drop between fuel and coolant at height z = 0, where it is maximum, and introducing the functions  $f_c(z) \leq 1$  and  $f(z) \leq 1$  in order not to restrict the analysis to the considered cosine power distribution, the Eq. (11) can be written as:

$$\mathfrak{I}_{f}(z) = \mathfrak{I}_{i} + \Delta \mathfrak{I}_{c} f_{c}(z) + \Delta \mathfrak{I}_{f-d,\max} f_{p}(z), \qquad (12)$$

where

$$f_{c}(z) = \int_{-1/2}^{z} f_{p}(z) dz / \int_{-1/2}^{+1/2} f_{p}(z) dz$$
(12b)

Eliminating the terms  $\Delta \hat{\mathcal{I}}_{cl-f}$  and  $\Delta \hat{\mathcal{I}}_{f-r}$  in Eq. (7), the inner cladding temperature will be given analogously by:

$$\mathcal{J}_{cl}(z) = \mathcal{J}_{i} + \Delta \mathcal{J}_{c} f_{c}(z) + \Delta \mathcal{J}_{cl-d, \max} f_{p}(z) . \qquad (13)$$

In the case of a sodium cooled reactor such as Na-2 / 11\_7,  $\mathcal{T}_{f}(z)$  assumes its maximum value at a point only slightly above the core central plane,  $\mathcal{T}_{cl}(z)$  at core top.

#### II.2.2 Radial temperature profile

The radial temperature profile in the core is determined by the radial flux profile, and the radial distribution of fissile materials and flow rate. In the Na-2 reactor, for instance, there are two radial zones of different fuel enrichment in order to flatten the power profile, and the flow rate is distributed among the several subassemblies in such a way that the coolant temperature span in the nominally hottest channels within a subassembly is the same for all subassemblies [11\_7 in order to obtain a better efficiency. This provokes a discontinuous radial temperature profile. Other causes of discontinuity in radial, as well as in azimuthal direction, arise generally from different burn-up zones.

Instead of giving an explicit function for the radial - or azimuthal profile, we shall take it into account, in the following analysis, by defining, for each subassembly individually, the maximum fuel and cladding temperatures and the power profile. In order to simplify the analysis it will be assumed as in Ref.  $/1_7$  that the power profile within a subassembly is approximately linear, and the pins are uniformly distributed along the radial direction.

According to these assumptions, the power radial profile in a subassembly is completely characterized by the parameter H:

$$H = \frac{\max. power - average power}{\max. power},$$
(14)

where the terms "maximum" and "average" are to refer to the pins in the considered subassembly.

#### II.3 Analysis of the uncertainties

The uncertainties affecting the parameters, by which the temperature profile is determined, should be divided into the following groups, according to their nature, as discussed in Ref. /1/2:

- Local uncertainties, which are random functions along the channel axis: these uncertainties should be defined by their "specific standard deviations";
- <u>Channel (or pin) uncertainties</u>, which do not vary along a channel (or pin), but among the several channels (or pins) within a subassembly.
- <u>Subassembly uncertainties</u>, which act on all channels within a subassembly in the same way, but vary in an independent way among different subassemblies;

Zone uncertainties, which act on a whole zone, that is on a whole group of subassemblies;

Core uncertainties, which act on the whole core.

The main causes of uncertainties to be considered at beginning of reactor operation for a sodium cooled reactor are reported in Table 1. It ought to be noted that in this table the uncertainties on the critical temperatures, which must not be exceeded, appear as uncertainties on core temperatures.

This procedure is allowable in our analysis, because we want to assure that a certain temperature  $\mathcal T$  does not exceed the value  $\mathcal T_{\rm crit},$  that is

$$\Delta \hat{\mathcal{T}} = \hat{\mathcal{T}}_{\text{crit}} - \hat{\mathcal{T}} \ge 0.$$
 (15)

Now the Eq. (15) is statistically sati\_sfied, at the confidence level associated with the parameter  $\lambda$  (see Table 2), if the nominal value  $\overline{\Delta \vartheta}$  satisfies the equation:

$$\overline{\Delta\vartheta} = \overline{\vartheta}_{\text{crit}} - \overline{\vartheta} > \lambda \widetilde{\vartheta}$$
(16)

(17)

with

$$\mathcal{C}_{\Delta \mathcal{T}}^2 = \mathcal{C}_{\mathcal{T}_{crit}}^2 + \mathcal{C}_{\mathcal{T}}^2$$
.

From Eqs. (16) and (17), we have:

$$\overline{\widehat{\mathcal{I}}}_{\text{crit}} \geq \overline{\widehat{\mathcal{I}}} + \lambda \sqrt{\widehat{\mathcal{I}}_{\widehat{\mathcal{V}}}^{2} + \widehat{\mathcal{I}}_{\widehat{\mathcal{V}}}^{2}},$$

that is, the same equation, we should get, if the uncertainty on  $\vartheta_{\rm crit}$  were considered as a further uncertainty on  $\mathcal T$ .

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Tab. 1 Causes of Uncertainties in Core Thermal Design

Uncertainties	Fuel Hot Spot	Cladding Hot Spot	Hot Channel	Local	Channel	Subassembly	Zone	Core
density	¥	¥	×	65 65			o <sub>5</sub> 1)	
enrichment	×	¥	¥	o <sup>s</sup> a			°a	
cladding inner diam.	×	¥		σ <sup>s</sup> di			di	
cladding outer diam.	ж	¥	¥	o s do			Gdo I	
cladding thickness	×	¥		σt			σ <sub>t</sub>	
density asymmetry	¥			es a		4		
enrichment asymmetry	¥			o aa				
fuel-clad eccentricity	ж	ж	×	6 e				
pin pitch	¥	¥	×	6 p	°p			
pin active length	×	×	×		σ <sub>l</sub>			
orifice calibration	×	×	¥			€¥ €F		
neutron flux 2)	ж	¥	×	5 Øa	Fr	<sup>6</sup> ≠cr	€¢c	
power measurement	ж	×	×					0-pu
inlet temperature	X	¥	ж					0ī
clad critical temp.		¥						cl
fuel melting point	¥			s 3)				4) ۳
fuel thermal conduct.	¥			e <sup>s</sup> ∣ €kf				°kf !
clad thermal conduct.	¥	×		° kcl			·	<sup>6</sup> kcl'
heat transfer clad-fuel	¥			on f-cl		_		Ghr-cl 1
heat transfer clad-cool.	¥	×		o hcl-c			-	G hel-c I
coolant specific heat	<b>X</b> .	×	¥				-	€ <sub>cp</sub> 」

1) Different production batches

2)  $s_{\phi_a}, s_{\phi_r}, s_{\phi_c}, s_{\phi_c}$  = axial, radial, control rod position and flux calculation respectively.

3) Local effects due to chemical composition ,gap etc.

4) Uncertainties in physical knowledge

Confidence level	Exceeding probability	
$P(x)=P(x \le m + \lambda \sigma)$	$1-P(x)=P(x \ge m+\lambda \sigma)$	
(%)	(%)	

# Table 2 Normal Distribution

84.13

93.32

97.72

99.38

99.86

99.977

99.997

As already noted, for the local uncertainties it is necessary to take into account all the spots of a pin, while for the global uncertainties only the spots at maximum nominal temperature are relevant. In opposition to previous methods of analysis, (see for instance Ref. 3) it is necessary then to consider their effects separately, as it will be shown in the following.

# II.4 Effects of the local uncertainties

#### II.4.1 Axial subdivision of a channel

Let us divide the channel axis into a number of segments, each one of a length equal to the assumed spot length  $l_{c}$  (Fig. 3) and consider



for the moment only the local uncertainties. Along a spot length we assume the fuel, cladding and coolant temperatures and the uncertainties as constant. The values, which the local uncertainties assume in spot i, are independent from the values assumed in the other spots. The uncertainties on the fuel(cladding) temperature in the spot i depend upon the uncertainties on the coolant

λ

1.0

1.5

2.0

2.5

3.0

3.5

4.0

Arrial cubdivision of a channel.

15.87

6.68

2.28

0.62

1.4.10<sup>-1</sup>

2.3.10-2

3.2.10-3

temperature rise  $\Delta \vartheta_1$  up to the spot i-1, the uncertainties on the coolant temperature rise  $\Delta \vartheta_2$  in the spot i, and on the uncertainties on the temperature difference fuel (cladding)-coolant  $\Delta \vartheta_3$  in the spot i. Now the uncertainties on  $\Delta \vartheta_1$  are independent of the uncertainties on  $\Delta \vartheta_2$  and  $\Delta \vartheta_3$ , but the uncertainties on  $\Delta \vartheta_2$  are partially correlated to those on  $\Delta \vartheta_3$ : they are both partially due to the same uncertainties on the power delivered from the spot i into the coolant. But if  $l_s \ll 1$ , the temperature rise along a spot length is small so that the corresponding uncertainties give a negligible contribution to the total uncertainty. Therefore we can neglect this correlation and consider the total local uncertainty on  $\vartheta_f(z_1)$  ( $\vartheta_{c1}(z_1)$ ) as due to two independent uncertainties, namely to  $\Phi_{d\vartheta_{f-d}}(z_1)$  ( $z_1$ ) ( $z_1$ )), standard deviation of the coolant temperature rise at height  $z_1$ , and standard deviation of the temperature difference fuel (cladding)-coolant at height  $z_i$  respectively, while the upper index "I" indicates that they take into account the local uncertainty only.

# II.4.2 Standard deviation of the coolant temperature

In the evaluation of  $\mathcal{G}_{\Delta \mathscr{G}C}^{1}(z_{i})$  the actual channel arrangement should be taken into account. Therefore we separate the analysis for the fuel and the cladding, since different channel arrangements were considered. Extension to other channel arrangements can be easily derived , from the very general criteria upon which the following analysis is based.

The temperature of the coolant depends upon the power delivered into the considered channel by all adjacent pins. Every uncertainty might assume different values in each pin; therefore, it is necessary to calculate the "specific standard deviation of the average value of the uncertainty"i" in the considered channel arrangement; this uncertainty will be indicated as  $\sigma_i^{s,ch}$ , and is the basis of the further calculations.

We shall now explain the general criteria of calculation of  $\sigma_i^{s,ch}$ , and show explicitly on example. Further details will be given in the third part of this report, presenting the numerical calculations for the reactor Na-2.
The main local uncertainties to consider are, from Table 1, those on density  $\mathfrak{S}_5^S$ , enrichment  $\mathfrak{S}_a^S$ , eccentricity  $\mathfrak{S}_e^S$  and axial flux  $\mathfrak{S}_a^S$ . Among these,  $\mathfrak{S}_5, \mathfrak{S}_a^S, \mathfrak{S}_a^S$  are to consider both for the central considered pin as for the adjacent ones (Fig. 1); while  $\mathfrak{S}_e^S$  only for the adjacent pins, because it does not affect the total power delivered from the central pin into the channel.  $\mathfrak{S}_e^S$  could provoke that a fraction of power larger than the nominal one (1/3) is delivered from the adjacent pins into the considered channel.

Let us calculate  $\sigma_{\delta}^{s,ch}$ , that is the average uncertainty on the density in a channel section.

From the original uncertainty  $\sigma_{\delta}^{S}$  we can calculate the corresponding <u>relative</u> uncertainty on coolant temperature increase  $\sigma_{\delta}^{S'}$ . (For instance if  $\sigma_{\delta}^{S} = 2 \%$  is  $\sigma_{\delta}^{S'} = 0.02$ , because the relation between density and power is linear. Otherwise the actual relation must be taken into account).

Now the central pin contributes to the power delivered into the channel with its whole section and for 1/3 of the total power, while the 6 adjacent pins each one with 1/3 of their sections and therefore for 1/9 of the total power: the average effect is then:

$$\mathcal{G}_{\delta}^{s,ch} = \sqrt{\left(\frac{1}{3}\right)^{2} \left(\mathcal{G}_{\delta}^{s'}\right)^{2} + 6\left(\frac{1}{9}\right)^{2} \left(\mathcal{G}_{\delta}^{s'} \cdot \sqrt{3}\right)^{2}} = \frac{\mathcal{G}_{\delta}^{s'}}{\sqrt{3}}$$
 (18)

where  $\sqrt{3} \cdot \sigma_{\delta}^{s}$  takes into account the smaller section of the adjacent pins which contributes to the total power, and therefore a larger uncertainty, since  $\sigma_{\delta}^{s}$  was defined for the whole section / see item I.5\_7. It is to note, that in this way also the "density asymmetry" is taken into account, therefore it is not considered any further in the coolant temperature analysis (see Table 1).

The same results in Eq. (18) could be directly obtained considering that the overall volume of fuel to be considered is 3 times the volume of a pin, for which the specific standard deviation was defined.

## b) Channel arrangement (Fig. 2) for cladding hot spot

The parameters to consider are the same as at the point a), but now each one with equal weight (1/3): for instance, for the density we get:

$$\mathfrak{S}_{\delta}^{s,ch} = \sqrt{3} (\mathfrak{S}_{\delta}^{s'} \sqrt{6})^2 (\frac{1}{3})^2 = \sqrt{2} \mathfrak{S}_{\delta}^{s'},$$
 (18b)

because now for each pin only 1/6 of its volume is considered. Having calculated the individual  $\sigma_i^{s,ch}$ , the total relative specific standard deviation is then:

$$(\sigma^{s,ch})^{2} = \sum_{i} (\tilde{\sigma}^{s,ch}_{i})^{2}$$
(19)

at height  $z_j$  the nominal coolant temperature rise is by Eq. (12) or (13):

$$\widetilde{\Delta \vartheta}_{c}(z_{i}) = \widetilde{\Delta \vartheta}_{c} \cdot f_{c}(z_{i})$$
(20)

and the corresponding standard deviation:

$$\mathcal{O}_{\Delta \mathcal{Y}_{c}}^{1}(z_{i}) = \frac{\sigma^{s,ch}}{\sqrt{1/2 + z_{i}}} \quad \widetilde{\Delta \mathcal{V}_{c}} \quad f_{c}(z_{i})$$
(21)

since the local uncertainties should be "averaged" over a length  $1/2 + z_i$  (see Eq. 6).

In Eq. (21) a correction factor slightly larger than 1 was neglected: this factor, which was calculated by Guéron, takes into account that, performing the average along the considered length, the effects of the uncertainties should be weighed according to the axial power profile /3, Appendix C\_7.

#### II.4.3 Standard deviation of the local temperature drops

The nominal value of the temperature drops  $\Delta \mathcal{J}_{j-d}$  (where j indicates fuel or cladding) at height z; is (Eqs. (12) or (13)  $\mathcal{J}$ :

$$\overline{\Delta \vartheta}_{j-d}(z_i) = \overline{\Delta \vartheta}_{j-d,\max} \cdot f_p(z_i) .$$
(22)

Correspondingly, we obtain for  $\Theta^{l}_{\Delta \widehat{\mathcal{T}}_{j-d}}$ :

$$\mathcal{S}_{\Delta \mathcal{V}_{j-d}}^{1}(z_{i}) = \mathcal{S}_{\Delta \mathcal{V}_{j-d}, \max}^{1} f_{p}(z_{i})$$
(23)

The local uncertainties to be considered are shown in Table 1 for fuel and cladding respectively. Contrary to coolant analysis, only the local value acting on the spot at height  $z_i$  must be taken into account, while the other spots have no influence upon  $\Delta \hat{\mathcal{V}}_{i-d}$ .

From the original uncertainty  $\sigma_i^s$ , for every parameter i, the corresponding effect on the total maximum temperature drop  $(\sigma_{i,\Delta}^s \tilde{\gamma}_{j-d,max})$  should be calculated: in this case also, the different pin sections, considered for fuel and cladding analysis respectively, should be taken into account. (Details on the evaluation of this standard deviation for every parameter are given in the numerical example for the Na-2 reactor, in the third part of the report.) It is useful to express  $\sigma_{i,\Delta}^s \tilde{\gamma}_{j-d,max}$ in the dimension of a temperature, rather than as a relative value, since only the uncertainties on the specific power act on the whole  $\Delta \tilde{\gamma}_{j-d}$ , whereas the other ones act only on individual terms of the sum in Eq. (7).

Then the overall uncertainty of the maximum temperature drop for a spot of unit length is:

$$(\sigma_{\Delta \hat{\mathcal{V}}_{j-d, \max}}^{s})^{2} = \sum_{i} (\sigma_{i, \Delta \hat{\mathcal{V}}_{j-d, \max}}^{s})^{2};$$
 (24)

and for a spot of length  $l_s$  we obtain by Eqs. (6), (13) and (24) at height  $z_i$ :

$$\mathcal{G}_{\Delta \mathcal{V}_{j-d}}^{1}(z_{i}) = \frac{\mathcal{G}_{\Delta \mathcal{V}_{j-d}, \max}^{S} f_{p}(z_{i})}{\sqrt{1}s} \qquad (25)$$

## II.4.4 Equivalent number of spots

Having calculated the standard deviation of the coolant temperature and of the temperature drops / Eqs. (21) and (25) /, we can give

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now the general expression for the nominal value and the standard deviation of the fuel (or cladding) temperature in the spots at height  $z_i$ :

$$\widetilde{\widetilde{\mathcal{V}}_{j}}(z_{i}) = \widetilde{\widetilde{\mathcal{V}}_{i}} + \widetilde{\Delta\widetilde{\mathcal{V}}_{c}} f_{c}(z_{i}) + \widetilde{\Delta\widetilde{\mathcal{V}}}_{j-d,\max} f_{p}(z_{i})$$
(26)

$$\begin{bmatrix} c_{\vartheta_{j}} \\ c_{\vartheta_{j}} \\ + \frac{(c_{\Delta\vartheta_{j}-d,\max}^{s})^{2}}{1/2 + z_{i}} & \overline{\Delta\vartheta_{c}}^{2} f_{c}^{2}(z_{i}) + \frac{(c_{\Delta\vartheta_{j}-d,\max}^{s})^{2}}{1/2 + z_{i}} & f_{p}^{2}(z_{i}) + (c_{\vartheta_{crit}}^{s})^{2}/1_{s}$$

$$(27)$$

where j indicates fuel or cladding, the upper index "l" that this standard deviation takes into account the local uncertainties only, and the last term takes into account the local uncertainty on critical temperature, which does not depend upon the axial power.

In Eq. (27) the dependence of the standard deviation of the local temperature upon the assumed length of the spot becomes evident. On the other hand, excluding the term corresponding to the temperature drop  $\Delta \hat{v}_{j-d}$ , and substituting  $z_i = +\frac{1}{2}$ , Eq. (27) gives the standard deviation of the coolant outlet temperature as well,which clearly does not depend upon the assumed spot length. This demonstrates unambigously that the <u>hot channel factors are independent of the spot size</u>; moreover the hot channel analysis results to be only a particular case of the hot spot analysis.

The probability that no spot along the channel axis exceeds a certain temperature  $\mathcal{Y}_{\mathbf{y}}$  is given by<sup>**x**</sup>:

<sup>\*)</sup> Note: the temperature uncertainties are assumed to be independent among the several spots of a pin. This is exact only for the local temperature drops. Since the uncertainty on the temperature rise is generally much smaller than those on  $\Delta \vartheta$ , we can - conservatively /1/ - assume the Eq. (28) to be valid.

$$P(\vartheta_{\mathbf{x}}) = \prod_{i=1}^{1/1} \left[ P\left(\frac{\vartheta_{\mathbf{x}} - \overline{\vartheta}_{j}(\mathbf{z}_{i})}{\widetilde{\vartheta}_{j}(\mathbf{z}_{i})}\right) \right]^{n} \mathbf{sj},$$

where:

 $P(\frac{\hat{\mathcal{V}}_{x} - \bar{\hat{\mathcal{V}}}_{j}(z_{i})}{\tilde{\mathcal{V}}_{j}(z_{i})}) = \text{ probability that a spot at height } z_{i}$ does not exceed the temperature  $\hat{\mathcal{V}}_{x}$ 

 $n_{sj}$  = number of spots at height  $z_i$  (according to our assumptions (Fig. 1 and 2),  $n_s$  = 1 in fuel analysis,  $n_s$  = 3 in cladding analysis respectively)

(28)

 $1/1_s$  = number of segments in the assumed axial subdivision / Fig. 3\_7.

The probability distribution (28) should be added with those of the "global" uncertainties in order to obtain the overall maximum temperature distribution. In order to simplify this procedure, we shall substitute the actual number of spots in a channel with an "equivalent number of spots", /13, 14 / all at the same reference temperature  $\overline{\vartheta}_{rj}$  and with the same standard deviation  $\mathscr{G}_{rj}$ .

According to  $/ 14_7$  an equivalent number of spots ( $N_{sp}^{eq}$ ) has the same probability of exceeding the temperature  $\mathcal{Y}_x$  as the actual distribution (28) if the following equation holds:

$$N_{sp,j}^{eq}(\tilde{\gamma}_{x},\tilde{\tilde{\gamma}}_{rj},\tilde{\varepsilon}_{rj}) = \frac{\sum_{i=1}^{n} \log \left[ P(\frac{\tilde{\gamma}_{x}-\tilde{\tilde{\gamma}}_{j}(z_{i})}{\log \left[ P(\frac{\tilde{\tilde{\gamma}}_{x}-\tilde{\tilde{\gamma}}_{rj}}{\tilde{\varepsilon}_{rj}}) \right]} \right]$$
(29)

As indicated in Eq. (29),  $N_{sp,j}^{eq}$  is a function of the chosen <u>reference</u> <u>temperature</u> ( $\hat{\mathcal{V}}_{rj}$ ) and standard deviation ( $\tilde{\mathcal{C}}_{rj}$ ) besides the temperature  $\hat{\mathcal{V}}_{x}$ , at which the equivalence is evaluated. Since for the global uncertainties it is necessary to take into account only the spots at the nominal maximum temperature  $\tilde{\hat{\mathcal{V}}}_{jM}$  along the channel axis, it is physically appropriate to assume  $\tilde{\hat{\mathcal{V}}}_{rj} = \tilde{\hat{\mathcal{V}}}_{jM} = \tilde{\hat{\mathcal{V}}}_{j}(z_{M})$ , indicating with  $z_{M}$  the abscissa of the maximum. Assuming  $\sigma_{rj} = \sigma_{\tilde{\mathcal{V}}j}^{1}(z_{M})$ , the function  $N_{sp-j}(\tilde{\mathcal{V}}x)$ , calculated in the case of the reactor Na-2, has the qualitative behaviour indicated in Fig. 4: both for cladding as for fuel,  $N_{sp}^{eq}$  is not constant with  $\tilde{\mathcal{V}}x$ ,



as it would be necessary, in order to apply a simple procedure in the further calculations. Therefore we perform the following

pessimization:

We assume for  $\mathfrak{S}_{rj}$  that value,  $\overline{\mathfrak{S}_{ri}}$  evaluated for successive approximations  $\overline{\mathfrak{S}_{jM}}^{+4.5\sigma_{rj}}$  (see Part II), which provokes that  $\overline{\mathfrak{S}_{jM}}^{eq}$  ( $\overline{\mathfrak{S}_{x}}$ ) is always less than  $N_{sp-j}^{eq}$  ( $\overline{\mathfrak{S}_{jM}}$ ) Equivalent number of spots within an interval ( $\mathfrak{T}_{x} - \overline{\mathfrak{F}_{jM}}$ ) of practical relevance (4.5 $\sigma$ ) - dotted

line in Fig. 4 -; then we assume  $N_{sp-j}^{eq}(\mathcal{F}_x)$  as constant and equal to  $N_{sp-j}^{eq}(\overline{\mathcal{F}}_{jM})$ .

Summarizing the previous considerations, we consider the effects of the local uncertainties, as due to a system of  $N_{sp-j}^{eq}$  spots, each one at the same temperature  $\overline{\widetilde{v}_{jM}}$  and the same standard deviation  $\widetilde{v}_{rj}$ ; these parameters being defined as following:

$$N_{sp-j}^{eq} = const. = \frac{\sum_{i=1}^{1/1} \log \left[ P\left(\frac{\overline{\mathcal{Y}_{jM}} - \overline{\mathcal{Y}_{j}}(z_{i})}{\Im_{j}(z_{i})}\right) \right]}{\log (0.5)}$$
(30)

 $\overline{\vartheta}_{jM} = \overline{\vartheta}_{j}(z_{M}) = maximum nominal temperature along a pin$   $\Im_{rj} = reference standard deviation for which the function$ given by (29) results to be:

$$N_{sp-j}^{eq}(\widehat{\vartheta}_{x},\overline{\vartheta}_{jM},\widetilde{\sigma}_{rj}) \begin{cases} < N_{sp-j}^{eq}(\overline{\vartheta}_{jM},\overline{\vartheta}_{jM},\widetilde{\sigma}_{rj}) \text{ if } \overline{\vartheta}_{jM}'\widehat{\vartheta}_{x} < \widehat{\vartheta}_{jM} + 4.5 \widetilde{\sigma}_{rj} \\ \\ = N_{sp-j}^{eq}(\overline{\vartheta}_{jM},\overline{\vartheta}_{jM},\widetilde{\sigma}_{rj}) \text{ if } \widehat{\vartheta}_{x} = \overline{\vartheta}_{jM}' + 4.5 \widetilde{\sigma}_{rj} \\ \end{cases}$$

(31)

From Eq. (30) it can be noted that  $(\overline{\vartheta}_{jM} - \overline{\vartheta}_{j}(z_i))$  and  $\varepsilon_{\vartheta i}^{1}(z_i)$ neglecting in Eq. 27  $\varepsilon_{\vartheta crit}^{s}$  - being proportional to the specific power at constant flow rate,  $N_{sp-j}^{eq}$  is the same for all pins within a subassembly.

In the case of Na-2,  $\mathfrak{S}_{r-cl}$  results to be a little larger than  $\mathfrak{S}_{\mathfrak{V}cl}^{1}(z_{M})$  and  $\mathfrak{S}_{rf}$  a little smaller than  $\mathfrak{S}_{\mathfrak{V}f}^{1}(z_{M})$ : this can be explained, observing that in the case of the cladding the nominal temperature is maximum in the point at which the standard deviation is minimum (namely at the top of the core and because  $\mathfrak{S}$  is proportional to  $\cos(\frac{\mathfrak{N}}{1-z})$  neglecting the coolant uncertainty) whereas for the fuel the nominal temperature is maximum almost at the same point at which the standard deviation is maximum. Moreover the small difference between  $\mathfrak{S}_{\mathfrak{V}j}^{1}(z_{M})$  and  $\mathfrak{S}_{rj}$  - resulted from the calculation - indicates that the pessimization introduced by Eq. (31) is very small.

# II.5 Reduction of the uncertainties distributions to an overall normal distribution

Now we are able to apply a procedure similar to that indicated in Ref. / 1 / 7, in order to reduce the actual distribution of the uncertainties to a simple normal distribution. The procedure is identical for cladding and fuel hot spot (therefore we shall omit for simplicity the previously introduced index "j"); it can be applied to the hot channel as well considering simply that in this case  $N_{sp}^{eq} = 1$ .

A core must be divided into a number  $N_z$  of zones: a zone (which will be indicated with the index "i") is a group of subassemblies, for which the power, the flow rate and the uncertainties are constant. Therefore for a cylindrical geometry a zone is a certain set of subassemblies, not necessarily contiguous, at the same radius, for which the burn-up and the construction data are the same.

For each zone, the maximum nominal temperature  $\hat{\mathcal{T}}_{MS}^{j}$  in a subassembly and the power gradient H<sup>i</sup> (Eq. 14) must be assigned.

Let us now consider one of the channel arrangements in a subassembly of the zone "i", in which the maximum nominal temperature  $\overline{\hat{v}}_{MS}^{i}$  occurs.



### Fig. 5 Reduction scheme: equivalent channel distribution

This scheme indicates that the same uncertainty  $\mathfrak{F}_{ch}$  acts on all the spots of a channel (Fig. 5a and 5b).  $\mathfrak{F}_{ch}$  must be evaluated as the statistical sum of all "channel"uncertainties (see Table 1), calculated from the original standard deviation in Table 1 in the dimension of a temperature in the point where the maximum temperature  $\overline{\mathfrak{V}}_{MS}^i$  occurs (for simplicity of notation, we shall omit the superscript "i" to all uncertainties distributions in Fig. 5 and successive).

According to Ref. 1, the reduction procedure substitutes the distribution of the probability that at least one out of the  $N_{sp}^{eq}$  spots exceeds a certain deviation - that is the distribution of the maximum value in  $N_{sp}^{eq}$  samples - with a more pessimistic normal distribution, determined by the coefficients  $h^{m}(N)$  and  $h^{\circ}(N)$  (Fig. 5b). These coefficients, which were represented graphically in Ref. 1, are explicitly given, with a least square fit, by the following functions:

$$h^{m}(N) = 1.70694 + 0.54372 \log N - 1.70169 e$$
 (32)

and, pessimizing up to 40° confidence level,

$$h^{\circ}(N) = 0.62589 - 0.03584 \log_{10}N + 0.37230 e^{-0.82554 \log_{10}N}$$
(33)

As in Fig. 5 c, a normal distribution is then evaluated, equivalent to the "local" and "channel" uncertainties.

Let us now consider a subassembly in the zone "i". Similar to the channel we can give a graphical representation of a subassembly and its uncertainties in Fig. 6. In this figure,  $N_c$  indicates the number of channels, and  $G_s$  the "subassembly" uncertainty evaluated from the original subassembly uncertainties in Table 1. The channels have been assumed uniformly distributed along the subassembly radius (see item II.2.2).









The nominal flow rate being constant among the several channels in a subassembly, the temperature gradient will be equal to the power gradient  $H_i$ .

The nominal maximum of the axial temperature profile can be consequently expressed as:

$$\overline{\vartheta}_{M}^{i}(h) = \overline{\vartheta}_{i} + (\overline{\vartheta}_{Ms}^{i} - \overline{\vartheta}_{i})(1 - H^{i} + h)$$
(34)

with  $-H^{i} \leq h \leq +H^{i}$  and  $\overline{\mathcal{T}}_{i}$  = inlet coolant temperature.

Taking into account that between h and h +  $\Delta h$  there is a number of channels equal to  $\frac{N_c}{2H^1} \Delta h$ , and that now the maximum temperature along a pin is distributed with standard deviation  $\mathfrak{C}_{h}^{eq-i}$  about  $(\overline{\mathfrak{T}}_{Ms}^{i} + \mathfrak{m}_{ch}^{eq-i})$  (Figs. 5 and 6), we can define an equivalent number of channels - analogously to item II.4.4 - as:

In Eq. 35 an integration has been substituted to the summation in Eq. 30; and as reference temperature it has been assumed  $\tilde{\vartheta}_{Ms}^{i} + m_{ch}^{eq-i}$ . Furthermore, a reference standard deviation  $\sigma_{ch}^{*}$  is evaluated in such a way, that conditions corresponding to Eq. (31) are satisfied.

As indicated in Fig. 6, it is then possible to evaluate the equivalent subassembly distribution  $(m_s^{eq}, \sigma_s^{eq})$ .

Taking into account the number  $(N_s^i)$  of subassemblies in the zone "i" and that all subassemblies in a zone have the same nominal maximum temperature, with the reduction procedure illustrated in Fig. 7, we can evaluate the equivalent zone distribution  $(m_z^{eq}, \sigma_z^{eq})$ . In Fig. 7  $\sigma_z$  is the total standard deviation of the zone



 $\frac{-i}{2^{M_s}} \qquad m_z^{eq} = m_s^{eq} + h^m (N_s^i) \sigma_s^{eq}$   $\sigma_z^{eq} = \sqrt{\sigma_z^2 + \left[h^{\sigma} (N_s^i) \sigma_s^{eq}\right]^2}$ 

Fig. 7 Reduction scheme: equivalent zone distribution

uncertainties evaluated from the original uncertainties in Table 1.

Considering now the  $N_z$  zones in a core, with a total standard deviation  $G_c$  of the core uncertainties evaluated from the original uncertainties in Table 1 for the maximum temperature in the core:

$$\overline{\widehat{\mathcal{T}}}_{MC} = Max. \ (\overline{\widehat{\mathcal{T}}}_{MS}^{i}) = \overline{\widehat{\mathcal{T}}}_{MS}^{i}, \qquad (36)$$

we can (as indicated in Fig. 8) evaluate the core distribution  $\binom{eq}{c} \binom{eq}{c}$ , which is equivalent to all uncertainty distributions and takes into account the axial and radial temperature profile as well as the actual correlations existing among the temperatures of the individual spots and channels of the core.

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Fig. 8 Reduction scheme: equivalent core distribution

In Fig. 8, an equivalent number of zones is defined according to item IV.4.4 as:

$$N_{z}^{eq} = \frac{i=1}{\log \left[ P(\frac{\overline{\mathcal{Y}}_{Mc} + m_{z}^{eq-i}M(\overline{\mathcal{Y}}_{Ms} + m_{z}^{eq-i})}{\sigma_{z}^{eq-i}} \right]}{\log 0.5}$$
(37)

## II.6 Hot spot factors

Having calculated the equivalent core distribution  $(m_c^{eq}, G_c^{eq})$ , we can now evaluate the probability that no spot - of the assumed size - exceeds a certain temperature

$$\hat{\mathcal{V}} = \hat{\mathcal{V}}_{Mc} + m_{c}^{eq} + \lambda \mathfrak{S}_{c}^{eq} ; \qquad (38)$$

this probability, indicated as confidence level, is given by  $P(\lambda)$  (see Table 2).

The corresponding hot spot factors by Eqs. (1), (2) and (38) are given then by:

$$F_{hs}(conf.level) = 1 + \frac{m_c^{eq} + \lambda \sigma_c^{eq}}{\overline{v}_{Mc} - \overline{v}_i}$$
(39)

Even if it is not explicitly expressed in Eq. (39),  $F_{\rm hs}$  is strictly correlated to the size assumed for the spot. In the third part of the report, application of the method to the reactor Na-2 will show the influence of the spot size on the factors.

The complement of the confidence level is the occurrence probability of <u>at least one</u> hot spot - of the assumed size - in the core. It expresses an overall failure probability, but gives no indication on the way of failure, other than that of the size of the spot: for instance, no indication is given on the number of expected failed pins. A better quantitative figure of safety can be obtained as indicated in the next item.

## II.7 Probability of n "hot subassemblies"

As already indicated in hot channel analysis <u>/</u>2<u>7</u>, rather than to evaluate the expected number of failed pins because of hot spot occurrences, the analysis can be simplified, assessing the individual probabilities that in exactly one or in exactly two or <u>in exactly n subassemblies</u> hot spots occur. This corresponds also to a practical necessity: in fact, if one or more pins in a subassembly are damaged by excessive temperature, the affected subassembly must be removed as a whole. In order to simplify the notation we shall indicate as "hot subassembly", a subassembly in which at least one hot spot occurs. An exact solution of the problem will be given for the case of a core constituted of all equal subassemblies (that is  $N_{z} = 1$ ). In this case the schematic representation of the problem is presented in Fig. 9. There is



# Simplified model for assessing the number of hot subassemblies Fig. 9

only one global uncertainty  $6_{c}$ , this time including also the zone uncertainties, which acts in the same way on all N  $_{\rm S}$  subassemblies in a core, each one with the equivalent distribution  $(m_s^{eq}, \sigma_s^{eq})$ calculated as in Fig. 6.

The probability function  $Q_{nhs}(\xi)$  that in exactly n subassemblies a deviation E is exceeded by the temperature of at least one spot is then given by:

$$Q_{nhs}(\xi) = \int_{-\infty}^{+\infty} f(y) {N \choose s} \left[ P(\xi - y) \right]^{N} s^{-n} \left[ Q(\xi - y) \right]^{n} dy$$
(40)

where

is the frequency function of the global distribution f(y) (0, ج);

 $\int Q(\xi - y) n$ 

is the probability that the deviation  $\xi$ -y will be exceeded in n subassemblies, if the global uncertainty is equal y: therefore  $x + y \ge \xi$ .  $Q(\xi-y) = 1 - P(\xi-y) = \int_{\xi-y}^{+\infty} p(x) dx$ , with p(x) the frequency function of the subassembly distribution  $(m_s^{eq}, e_s^{eq});$ 

 $\left[P(\xi-y)\right]^{N}s^{-n}$ 

is the probability that the deviation  $\xi$ -y in the other N<sub>s</sub>-n subassemblies will not be exceeded;

and  $\binom{N}{s}$ 

the number of the corresponding possible combinations (Binomial coefficient).

In the case of a core with more zones, it is possible to reduce the actual scheme to a more pessimistic one, assuming all the subassemblies equal to the most limiting ones and evaluating a pessimizing global distribution for the core and subassembly uncertainties, as it will be indicated in the code description. Then assuming conservatively that the ratios of the probabilities of n hot subassemblies to the total hot spot probability are the same as in the actual model, a conservative estimation of these probabilities can be derived.

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#### Second Part

SHØSPA: a code for Statistical HOt SPot Analysis

#### III. Program description

#### III.l Scope

SHØSPA is a Fortran-IV code for IBM 360 system, which, operating on the presented analysis method, evaluates the hot spot factors for fuel and cladding as well as the hot channel factor as a function of the confidence level. Moreover, it evaluates the probability of "n hot subassemblies" according to item II.7.

The code has been developed with emphasis on sodium cooled fast reactors, but is applicable to any type of reactor constituted of bundled fuel rods with single phase coolant.

In order not to restrict application of the code to our assumptions on the spot size (item II.1) and to the thannel arrangements assumed in Fig. 1 and 2, the calculations of the individual standard deviations from the original uncertainties are not incorporated in the program, since the relations between original uncertainty and corresponding temperature deviations are not in general the same among different reactor types. Therefore some preliminary calculations must be performed in order to assign the input data into the program (see Part III).

Moreover in order not to restrict the code to cosine axial power distribution, the functions f(z) and  $f_c(z)$  (Eq. 12) are not defined in the program, but they must be supplied by the user according to the actual axial profile as separate function subroutines. In the first part of the report, only statistical uncertainties were taken into account: actually, together with statistical deviations, quite systematic causes might provoke that the temperature of some core components deviates from its nominal value: the program allows to take into account such systematic deviations, as will be shown in the following.

## III.2 Code structure

The code is constituted of a main program and several subroutines, according to the interconnection scheme in Fig. 10.



Fig. 10 Routine interconnections

All routines are listed in the following together with their principal functional characteristics:

SHØSPA = Main program. It is principally a routine controlling input, output and sequence of operations. It performs however some simple calculation, such as grouping of local uncertainties and selecting the reference temperatures.

- GRØUP = Subroutine for reading, printing and grouping the global uncertainties.
- STFACT = Subroutine for evaluation of the statistical factors and $probability of n "hot subassemblies" (<math>Q_{nhs}$ ).
- SPØT = Subroutine for evaluation of the equivalent number of spots.
- CHANEQ = Subroutine for evaluation of the equivalent number of channels.
- $F \not Q R = Subroutine controlling integration in evaluation of <math>Q_{nhs}$
- $PV \emptyset NX =$  Function to be integrated in evaluation of  $Q_{nhs}$ .
- KSTØRE = Subroutine for storing of an alphanumeric text.
- <u>PLØTA</u> = Subroutine controlling plot of curves:standard IBM/360 Assembler subroutine in the program library of Kernforschungszentrum Karlsruhe / 15 7
- $\underline{F}$  Subroutine for function integration: standard Fortran-IVsubroutine in the program library of Kernforschungszentrum Karlsruhe / 16\_7
- <u>FP</u> = Function, to be supplied by user, describing the axial power profile.
- $\underline{FC}$  = Function, to be supplied by user, describing the axial temperature profile of the coolant.

## III.3 Input cards

The input cards can be divided into four blocks, according to Fig. 11.

1.	General Data
2.	Zone Data (to be repeated $N_z$ times)
2.1	Zone Characteristics
2.2	Zone Uncertainties
2.3	Subassembly Uncertainties
2.4	Channel Uncertainties
2.5	Local Uncertainties
3.	Core Uncertainties
4.	Program Control

## Fig. 11 Input card blocks

## III.3.1 Block 1 - General Data

This block is constituted of the three following cards:

The first card - FORMAT (20A4) - is an identification card, containing a free comment, which will be printed by the program as output headline.

The second card - FORMAT (1015) - contains in the order:

NZ = number of zones, into which the core has been divided ( $1 \le NZ \le 150$ )

NS = total number of subassemblies in the core

NP = number of channel arrangements in a subassembly considered in fuel hot spot analysis

NC number of channel arrangements in a subassembly considered in hot channel analysis NCL number of channel arrangements in a subassembly considered in cladding hot spot analysis NSPF number of spots assumed for the fuel at a given abscissa Ŧ NSPC number of spots assumed for the cladding at a given abscissa = critical fuel temperature (°C) ITFCR =critical cladding temperature (°C) ITCLCR =critical coolant temperature (<sup>o</sup>C) ITCCR Ξ The third card - FORMAT (8 F 10.4) - contains in the order: active length (cm) XL extrapolation length (cm) XLEX = XLSPF = assumed fuel spot length (cm) (x) assumed cladding spot length (cm) XLSPC =nominal coolant inlet temperature (°C) ΤI = DTC nominal maximum coolant temperature span in = the core  $(^{\circ}C)$ nominal maximum temperature drop inner cladding-DTCL Ξ coolant in the core  $(^{\circ}C)$ nominal maximum temperature drop central fuel-coolant DTF in the core  $(^{\circ}C)$ 

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<sup>\*</sup> Note: the ratios XL/XLSPF and XL/XLSPC must be integer, since the active length should be divided into an integer number of segments (see item II.4.1).

### III.3.2 Block 2 - Zone Data

This block must be repeated  $N_z$  times and is constituted of 5 subblocks, which must be assigned in the following order (Fig. 11):

## a) <u>Subblock 2.1 - Zone characteristics</u>

This subblock is constituted of one card - FORMAT (2 15,5F10.5) - containing in the order:

- ΚZ
- = subscript identifying the zone.  $1 \leq KZ \leq NZ$ . The zone blocks must be ordered according to increasing KZ.
- NSZ = number of subassemblies in the zone KZ. All subassemblies in a zone have the same temperature and uncertainties.
- HP = ratio of nominal maximum of power profile in a subassembly of the zone KZ to nominal maximum in the core (HP ≤ 1)
- HC = ratio of nominal maximum coolant temperature span in the zone KZ to nominal maximum in the core (HP  $\leq$  1)
- H = power profile in a subassembly of the zone KZ, corresponding to Eq. (14), item II.2.2 (H ≥ 0) (The factors HP, HC, H and the temperatures TI, DTC, DTCL, DTF define completely the radial temperature profile)
- FSYSP = factor by which the power in the zone KZ must be multiplied in order to take into account systematic deviations from nominal.
- FSYSC = factor by which the coolant temperature span in the zone KZ must be multiplied in order to take into account systematic deviations from nominal. (The statistical analysis is performed starting from nominal temperatures, recalculated taking into account FSYSP and FSYSC).

b) Subblock 2.2 - Zone uncertainties

The first card of this block - FORMAT (615) - contains informations on the number of the following cards in the subblock, and namely in the order:

-

KZ = zone subscript.

- Nl = total number of following cards in the subblock; every following card containing informations on only one uncertainty, Nl is also the number of uncertainties belonging to this group. Nl ≥ 0.
- Nll = number of uncertainties affecting the power. The temperature uncertainty is proportional to the whole temperature difference between considered point and coolant inlet: these uncertainties must be assigned as relative value. Nll ≥ 0.
- N12 = number of uncertainties affecting only the coolant temperature increase. The temperature uncertainty is proportional to the temperature span of the coolant up to the considered point: these uncertainties must be assigned as relative value. N12 >0.
- N13 = number of uncertainties affecting only the temperature drops fuel (cladding)-coolant at a given abscissa: these uncertainties must be assigned in the dimension of a temperature ( $^{\circ}$ C). N13 ≥ 0.
- N14 = number of uncertainties on fixed temperatures such as fuel melting point, cladding critical temperature etc.: these uncertainties must be assigned in the dimension of a temperature ( $^{\circ}$ C). N14  $\geq$  O. (The following relation must be satisfied: N11 + N12 + N13 + N14 = N1.)

The successive N11 cards contain the power uncertainties according to FORMAT in Table 3 and to the specifications S.1.

Table 3 Card for global uncertainties

FORMAT	13	I3	13	1X	10A4	F10.5	F10.5	F10.5
VARIABLE	KK	K	Kl.		KOMM	SF	SCL	SC

	KK	=	KΖ	= zone subscript
	К	=	2,	indicating a "zone" uncertainty
	Kl	=	l,	indicating a "power" uncertainty
	KOM	íМ	=	free comment for identification of the original
				uncertainty (printed by the program).
S.l	SF		Ξ	standard deviation of the corresponding power
				uncertainty, expressed as relative value, for fuel
				hot spot
	SCI		= .	standard deviation of the corresponding power un-
				certainty, expressed as relative value, for cladding
				hot spot
	SC		=	standard deviation of the corresponding power un-
				certainty, expressed as relative value, for hot
				channel.

The successive N12 cards contain the coolant temperature uncertainties according to FORMAT in Table 3 and to the specifications S.2.

Kl = 2, indicating a coolant temperature uncertainty. SF,SCL,SC = standard deviations of the corresponding coolant temperature increase expressed as relative value.

The other variables identical to specifications S.l.

S.2

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The successive N13 cards contain the uncertainties on the temperature drops fuel (cladding)-coolant at a given abscissa, according to FORMAT in Table 3 and specifications S.3.

	K1 = 3,	indicating a temperature drop uncertainty
	SF,SCL:	standard deviations of the corresponding uncertainty
	÷	on temperature drops expressed in <sup>O</sup> C, calculated
		at the point of maximum specific power in the
S.3 -		core. (In order to simplify the calculations of
		these standard deviations it is opportune to refer
		always to the same power: the program, then,
		elaborates the actual value of SF and SCL according
		to the zone KZ).
	SC =	O.

The other variables identical to specifications S.l.

The successive N14 cards contain the uncertainties on fixed temperatures, according to FORMAT in Table 3 and specifications S.4.

Kl = 4, indicating a fixed temperature uncertainty S.4 SF,SCL,SC: expressed in  $^{\circ}C$ 

The other variables identical to specifications S.l.

## c) Subblock 2.3 - Subassembly uncertainties

This subblock is identical to the 2.2 one, with exception of K in specifications S.l, which now must be equal 3, indicating a "sub-assembly" uncertainty.

## d) Subblock 2.4 - Channel uncertainties

This subblock is identical to the 2.2 one, with exception of K in specifications S.l, which now must be equal 4, indicating a "channel" uncertainty.

## e) Subblock 2.5 - Local Uncertainties

The first card of this subblock - FORMAT (315) - contains informations on the number of local uncertainties, and namely in the order:

ΚZ	п	zone subscript
N15	=	number of uncertainties on coolant temperature span $(\sigma_i^{s,ch})$
N16	=	number of uncertainties on local temperature drop $(5^{S})$
		$i, \Delta \mathcal{F}_{j-d}, \max$

The successive N15 cards contain the N15 uncertainties on coolant temperature span, according to Table 4.

Table 4 Card for local uncertainties on coolant temperature

FORMAT	VARIABLE	SPECIFICATIONS
15	KZ	zone subscript
15	NTIPO	= 5, indicating a local coolant temperature uncertainty
10A4	КОММ	free comment for identification of the uncertainty
F10.5	SPF	specific standard deviation (relative value) for the channel arrangement con- sidered for fuel hot spot
F10.5	SPCL	specific standard deviation (relative value) for the channel arrangement con- sidered for cladding hot spot
F10.5	SPC	specific standard deviation (relative value) for the channel arrangement con- sidered for hot channel

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The successive N16 cards contain the N16 uncertainties on local temperature drops, according to Table 5.

Table 5 Card for uncertainties on local temperature drops

FORMAT	VARIABLE	SPECIFICATIONS
15	KZ	zone subscript
15	NTIPO	= 6, indicating an uncertainty on local temperature drops
10A4	КОММ	free identification comment
F10.5	SPFD	specific standard deviation for fuel temperature ( <sup>°</sup> C), calculated at the spot of maximum power in the core $(\sigma_i^s, \Delta \mathcal{P}_{f-d}, \max)$
F10.5	SPCD	specific standard deviation for cladding temperature ( <sup>°</sup> C) calculated at the spot of maximum power in the core ( $\mathfrak{S}_{i}^{s}$ , $\Delta \mathfrak{T}_{cl-d}$ , max)

The last card of this subblock contains - FORMAT (15, 2 F10.5) - the local uncertainties on critical temperatures and namely in the order

ΚZ	=	zone subscript
SLFCR	=	specific standard deviation of local fuel critical temperature ( $^{\circ}$ C)
SLCCR	=	specific standard deviation of local cladding

critical temperature (<sup>O</sup>C)

### III.3.3 Block 3 - Core uncertainties

This block contains the "core" uncertainties. The first card contains - FORMAT (515) - N1, N11, N12, N13, N14 as defined at point b). The successive cards are identical to these in the subblock 2.2 with the following exceptions in specifications S. 1: KK = 0 (these uncertainties act on all core zones); K = 1, indicating a core uncertainty.

#### III.3.4 Block 4 - Program control

This block is constituted of <u>only one card</u>, containing - FORMAT (A4) - the variable KONTR: if KONTR = LAST, the final control is transferred to a STOP statement otherwise to reading a successive data set and to starting execution of the program for the new data.

#### III.4 Error check on input data set

An error check is performed on the data. If the checked variables are not correctly assigned, execution is stopped and an error message appears. The error messages are listed and explained in the following:

ERROR IN THE ZONE SUBSCRIPT OR IN THE CARD TYPE: This message indicates one out of the following occurrences:

- a) The zones are not ordered according to increasing KZ.
- b) Among the cards of a zone, there is a card not belonging to this zone.
- c) The variable NTIPO (in Tables 4 and 5) has not been correctly assigned. That is there is an error on the type of local uncertainty.

#### ERROR IN THE NUMBER OF SUBASSEMBLIES

The sum of the number of subassemblies of the NZ zones is not equal to the total number of subassemblies NS.

#### ERROR-H NEGATIVE

A negative value is assigned to the parameter H in subblock 2.1.

#### ERROR IN THE NUMBER OF CARDS

N11 + N12 + N13 + N14  $\neq$  N1 (Subblock 2.2 - first card)

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#### ERROR IN THE TYPE OF CARD

It refers to a global uncertainty card and indicates that one of the variables KK, K<sup>1</sup>, Kl in subblocks 2.2 or 2.3 or 2.4 or in block 3 has been assigned in an illegal way: this control is performed in order to avoid that a certain uncertainty might be treated as belonging to a group or to a zone different from the group or the zone to which the uncertainty belongs actually.

## III.5 Program Operation

The program operation can be divided into two successive steps. In the first step the data are read and elaborated in order to evaluate all quantities necessary to the evaluation of the hot spot factors, which is executed in the second step.

#### III.5.1 Data Elaboration

First the block 1 containing the general data is read and printed (Flow chart in Fig. 12). Successively for each zone the program reads and elaborates the zone data. Namely, from the zone characteristics (Subblock 2.1) the nominal temperature differences DTCZ, DTFZ, DTCLZ (see Table 6) are calculated for each zone, and successively the subroutine TMAXZ (Flow chart in Fig. 13) evaluates the nominal maximum temperatures of cladding and fuel and the abscissa at which they occur. (These nominal temperatures take into account the systematic factors FSYSP and FSYSC also.) Successively, the global uncertainties (subblocks 2.2, 2.3, and 2.4) are read and elaborated by the subroutine GRØUP (Flow chart in Fig. 14): namely, from the original data the program evaluates the standard deviations ( $^{\circ}$ C) of each uncertainty in the point of maximum temperature of the axial profile in the considered zone, and the total standard deviation for each group (zone, subassembly and channel uncertainties).

Successively, the local uncertainties are read and the specific standard deviations  $\sigma^{\text{s,ch}}$  and  $\sigma^{1}_{\Delta \widehat{\mathcal{Y}}_{j-d}}$  (Eqs. 19 and 23) are evaluated. Then the reference temperatures and zones are chosen: that is, the program assumes as reference temperature for fuel hot spot the maximum value - among the zones - of the fuel nominal temperature. The same is performed

for cladding hot spot and hot channel. At last, the core uncertainties are read and elaborated in order to determinate the total standard deviation ( $^{\circ}$ C) of the core uncertainties for the chosen reference zone.

In output for each uncertainty the program prints the original values and the elaborated ones and the total standard deviations for each group. An example of the output variables will be given in the third part of the report.

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# Table 6 Main variables of the first step

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Name	Unit	Definition
DTCZ	°C	Nominal maximum temperature span of the coolant in a zone
DTFZ	°c	" " difference fuel-coolant in
	~	a zone
DTCLZ	°c	Nominal maximum temperature difference cladding-coolant in a zone
TFZMAX	°C	Nominal maximum temperature of the fuel in a zone
TCZMAX	°C	" " of the cladding in a zone
ZFZMAX	cm	Abscissa at which TFZMAX occurs in a zone
ZCZMAX	cm	II II TCZMAX II II II
STFZ		Total st.deviation of zone uncertainties for fuel temp.
stclz	o <sub>Z</sub>	" " " " " cladding temp.
stcz J	(°C)	" " " " coolant temp.
ر STFS		" " subassembly uncertainties for fuel temp.
STCLS	°s	" " " " " cladding temp.
STCS	(°c)	" " " " coolant temp.
STFC 、		" " channel uncertainties for fuel temp.
STCLC	<sup>o</sup> ch	n n n n n n n cladding temp
STCC J	(°C)	" " coolant temp.
STCFL		" specific st.deviation for channel arrangement for fuel
STCCL	s,ch	n n n n n n for
{	(rel.	cladding
STCL /	value)	u u u u u u u for coolant
STFL 2	G S	" specific st.deviation of temp.drops for fuel
STCLL	(02) (01) (01) (02)	u u u u u u u cladding
TFREF	°C	Maximum temperature of the fuel in the core
TCLREF	°C	" " of the cladding in the core
DTCREF	°C	" " span of the coolant in the core
KFREF		subscript of the zone in which TFREF occurs
KCLREF		" " " " " TCLREF "
KDTREF		" " " DTREF "
STFO		Total st.deviation of fuel core uncertainties in the zone KFREF
STCLO	°c (°c)	" " of cladding uncertainties in the zone KCLREF
STCO		" " of coolant uncertainties in the zone KDTREF

. . . . . . . . . . . . . . . .



Fig.13 - Flowchart of Subroutine TMAXZ

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Fig.15 - Flowchart of Subroutine STFACT

## III.5.2 Evaluation of the statistical factors

In the second step, under control of the main routine (Fig. 12), the program evaluates at first the fuel hot spot factor, successively the cladding hot spot factor then the hot channel factor. The evaluation of the factors is controlled by the subroutine STFACT (Figs. 10 and 15). Namely if the factor in evaluation is a hot spot factor, the subroutine SPØT is called for each zone in order to evaluate the equivalent number of spots and the reference standard deviation  $\mathfrak{F}_r = SSP$ ØT. According to item II.4.4, the subroutine SPOT evaluates the equivalent number of spots at the reference temperature  $\widehat{\mathcal{V}}_j(z_M) = TMAX$  and then at the temperature TT = TMAX + 4.5  $\mathfrak{S}$  (XNSPOT and XSPOT respectively), for a reference standard deviation equal to  $\mathfrak{S}_{\mathfrak{V}_1}^1(z_M)$ .

According if XSPOT is larger (smaller) than XNSPOT the program evaluates a larger (smaller) SSPOT so that the conditions (31) are satisfied (see Flow Chart in Fig. 16). The reduction procedure of the uncertainties to a final normal distribution follows exactly the reduction scheme in Figs.  $5 \rightarrow 8$ : the equivalent numbers of channels (subroutine CHANEQ) and of zones are evaluated with a procedure very similar to that given for the spots in Fig. 16. The main variables are printed in output (see Fig. 15). Then the expected maximum temperatures and hot spot factors are evaluated and printed according to Eqs. (38) and (39).

In order to evaluate the probability of exactly n hot subassemblies a pessimizing model of the core is chosen according to item II.7. All the subassemblies are assumed equal to the most limiting ones: namely equal to those in the zone "IREF", for which the temperature  $\hat{\mathcal{J}} = \tilde{\mathcal{J}}_{MS}^{i} + m_{s}^{eq} + 3 \sqrt{(\sigma_{s}^{eq})^{2} + \sigma_{z}^{2}}$  is maximum. Then the subassembly distribution  $(m_{s}^{eq}(IREF), \sigma_{s}^{eq}(IREF))$  is assumed (Fig. 9). Successively a pessimizing global distribution  $(m_{g}, \sigma_{g})$  for the core and zone uncertainties is evaluated by:

$$m_{g} = XMG = h^{m}(N_{z}) \cdot \tilde{c}_{z}(IREF)$$

$$\tilde{c}_{g} = SG = \sqrt{\tilde{c}_{c}^{2} + (\tilde{h}(N_{z}) \cdot \tilde{c}_{z}(IREF))^{2}}$$


Then by Eq. 40 the probability  $Q_{nhs}$  (PROBN) of n hot subassemblies is evaluated for values of n within 1 and 10, together with this probability, the total hot spot probability (PROBT) for this pessimizing model is evaluated.

PROBN (for N = 1,2,5 and 10), PROBT and PROBE (hot spot probability in the actual model) are then plotted versus the hot spot factor.

An example of the outputs will be given in the numerical example for the reactor Na=2.

#### III.6 The functions FP(z) and FC(z)

As previously stated, these functions must be defined according to the actual axial power profile as function subroutines.

In the application of the code to the reactor Na-2, the assumption of a cosine power distribution has been maintained. The listing of the corresponding FP(z) and FC(z) subroutines is then the following:

> C AXIAL POWER PROFILE FUNCTION FP(Z) COMMON XL,XLEX FP =COS(3.141593\*Z/XLEX) RETURN END

C AXIAL COOLANT TEMPERATURE PROFILE FUNCTION FC(Z) COMMON XL,XLEX A=SIN(1.570796\*XL/XLEX) FC =(A+SIN(3.141593\*Z/XLEX))/(2.\*A) RETURN END

In this case the active and extrapolated lengthes were assigned in a COMMON statement; however their numerical values can be explicitly assigned in the functions. It is useful to observe that when it is not possible to obtain an explicit function for FC(z), FC(z) can be obtained by integration of FP(z) (Eq. 12b).

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#### III.7 Application of the code to gas cooled reactors

In the development of the program it has been assumed that the heat transfer coefficient cladding-coolant is constant along a channel axis.<sup>\*</sup> This statement is not generally valid for some special types of high performance gas cooled reactors, for which the upper part of external cladding surface is artificially roughened in order to increase heat transfer in the zone of higher temperatures.

Although the code does not directly account for such a situation, problems of this kind can still be solved with SHØSPA by applying the code twice in order to point out which part of the pin is the most critical in respect to hot spot occurrences, and namely: a first application of the code will consider the whole core as constituted by uniform roughened cladding, correspondingly overall hot spots factors will be calculated for the roughened part of the pins; a second application of the code will consider the core as constituted by smooth cladding, correspondingly overall hot spots factors will be calculated for the smooth part of the pins in the functions FP(z) and FC(z) it will be assumed

> FP(z) = 0 for  $z > z_r = abscissa at which roughness$ begins $<math>FC(z) = FC(z_r)$  for  $z > z_r$ .

The total non-failure probability will be given then, conservatively, by the product of the partial non-failure probabilities.

\*Note: The radial variation of this coefficient depending upon the coolant flow rate can be taken into account by appropriate values of the systematic factors FSYSP.

#### IV. Fortran IV Listing

## IV.1 Main Program

BSC= \* F10.4

с С SHOSPA-STATISTICAL HOT SPOT AND HOT CHANNEL AMALYSIS

DIMENSION KOMM(20), NSZ(150), H(150), DTCZ(150), DTFZ(150), DTCLZ(150), 1TCZMAX(150),ZCZMAX(150),TEZMAX(150),ZEZMAX(150),STEZ(150),STCL2(1 250), STCZ(150), STFS(150), STCLS(150), STCS(150), STFC(150), STCLC(150) 3,STCC(150),STCFL(150),STCCL(150),STFL(150),STCLL(150),STCL(150) DIMENSION XB(8), YB(8), NDIR(8), NSC(8), AB(9), SLFCR(150), SLCCR(150) COMMON XL, XLEX, TI, NZ, H, NSZ, DTCZ, NS, CC, ZZ, BB, FAT, J, INDEX, XB, YB, NDIR 1,NSC,AB DATA KKKK/'LAST'/ 1111 READ(5,100)KOMM,NZ,NS,NP,NC,NCL,NSPF,NSPC,ITECR,ITCLCR,ITCCR,XI,XL 1EX, XL SPF, XL SPC, TI, DTC, DTCL, DTF 100 FORMAT(20A4/1015/8F10.4) WRITE(6,200)KOMM, NZ, NS, NP, NC, NCL 200 FORMAT (1H1; 20X; + SHOSPA=STATISTICAL HOT SPOT AND HOT CHANNET ANALYS 115'////1X,20A4////1X, NUMBER OF ZONES WITH IDENTICAL SUBASSEMBLIES 2 NZ=', I5, 12X, 'TOTAL NUMBER OF SUBASSEMBLIES NS=', I5/1X, 'NUMBER OF BCHANNELS CONSIDERED FOR FUEL HOT SPOT NP=', 15, 9X; INUMBER OF CHANNELS IN A SUBASSE 3 4MBLY NC= ', 15/1X, 'NUMBER OF CHANNELS CONSIDERED FOR CLADDING HOT SP AOT NCL=', 15///) WRITE(6,300)XL, XLFX, XLSPF, XLSPC, NSPF, NSPC 300 FORMAT(1X, 'ACTIVE LENGTH XL=', F10.2, 1X, '(CM)', 33X, 'EXTRAPOLATED LE INGTH XLEX=',F10.2,1X,'(CM)'/1X,'FUEL HOT SPOT LENGTH XLSPF=',F10.2 2,1X, '(CM) ', 23X, 'CLADDING HOT SPOT LENGTH XLSPC=', F10.2,1X, '(CM) '/1 3X, FUEL HOT SPOTS AT HEIGHT Z NSPF= ', I5, 28X, 'CLADDING HOT SPOTS AT 4 HEIGHT Z NSPCL=1, 15///) WRITE(6,400)TI, DTC, DTCL, DTF, ITFCR, ITCLCR, ITCCR 400 FORMAT(1X, 'CODLANT INLET TEMPERATURE TI=', E10.2, 1X, '(OC)'/1X, 'NOMI INAL MAXIMUM CODLANT TEMPERATURE SPAN IN CORE DTC=', FLO.2.1X. (OC) 21/1X, NOMINAL MAXIMUM TEMPERATURE DIFFERENCE CLADDING-COOLANT IN C DTCL=', F10.2,1X, '(OC) '/1X, 'NOMINAL MAXIMUM TEMPERATURE DIFFF 30RE 4RENCE FUEL-COOLANT IN CORE DTF=',F10.2,1X, '(OC)'///1X,'FUEL CR 5ITICAL TEMPERATURE ITECR=',I5,' (OC)' ///1X,'CLADDING CRITICAL TEM 6PERATURE ITCLCR=', 15, ' (OC) '///1X, 'COOLANT CRITICAL TEMPERATURE IT 7CCR=1, 15, 1 (OC) 1///) NSS=0. TFREF=0. TCLREF=0. DTCREF=0.  $00 \ 1 \ K=1, NZ$ READ(5,800) KZ, NSZ(K), HP, HC, H(K), FSYSP, FSYSC 800 FORMAT(215,5F10.5) IF(KZ.NE.K)GO TO 999 NSS=NSS+NSZ(K) DTCZ(K)=DTC\*HC \*FSYSC DTFZ(K)=DTF\*HP \*FSYSP DTCLZ(K)=DTCL\*HP \*FSYSP FSYS =HP\*FSYSP CALL TMAXZ(XL,XLSPC,TI,DTCZ(K),DTCLZ(K),TCZMAX(K),ZCZMAX(K)) CALL TMAXZ(XL,XLSPF,TI,DTCZ(K),DTFZ(K),TFZMAX(K),ZFZMAX(K)) WRITE(6,900)K,NSZ(K),HP,HC,FSYSP,FSYSC,H(K),DTC7(K),DTCLZ(K),DTFZ( 2K) • TCZMAX(K) • ZCZMAX(K) • TEZMAX(K) • ZEZMAX(K) 900 FORMAT(1H1, 30X, 'ZONE', I5///1X, 'NUMBER OF SUBASSEMBLIES NSZ=', I5 /1 1X, 'RATIO OF ZONE MAX. SPEC. POWER TO CORE MAX. SPEC. POWER HP=', F13.4 2/1X, 'PATID OF ZONE TO CORE MAX. TEMP. SPAN HC=', F10.4 /1X, 'SYSTEMA ATIC POWER FACTOR FSYSP=', F10.4/1X, 'SYSTEMATIC TEMP.SPAN FACTOR FSY

/1X, '(MAX.TE

3MP.SPAN -AVERAGE TEMP.SPAN) /MAX.TEMP.SPAN -IN A SUBASSEMBLY H= ", 4F10.4/1X, \*NOM.MAX.TEMP.SPAN DTCZ=\*, F10.2,\* (OC)\*/1X,\*NOM.MAX.TEMP. 5DIFF. CLADDING-COOLANT DICLZ=', F10.2 ,' (OC) 1/1X, 'NOM.MAX, TEMP.DIF 6F. FUEL-COOLANT DIFZ=', F10.2,' (OC)'//IX,'NOM.MAX.CLADDING TEMP. 7TCZMAX=",F10.2," (OC) AT HEIGHT ZCZMAX=",F10.2," (CM)"//1X, "NOM. MA 8X.FUEL TEMP. TFZMAX=',F10.2,' (OC) AT HEIGHT ZFZMAX=', F10.2,' (CM 9)1//// I=0 WRITE(6,1000) 1000 FORMAT (30X, 'ZONE UNCERTAINTIES'/) NTIPD=2 2 I = I + 1READ(5,150)KZ,N1,N11,N12,N13,N14 150 FORMAT(615) IF(KZ.NE.K)GO TO 999 CALL GROUP(N1,N11,N12,N13,N14,NTIPO,K,DTCZ(K),DTFZ(K),DTCLZ(K),ZFZ 1MAX(K),ZCZMAX(K),STF,STCLA,STC,DTCZ(K),PTCZ(K),FSYS ,FSYS ) GD TD (3,4,5),I 3 NTIPO=3STFZ(K)=STF STCLZ(K)=STCLA STCZ(K)=STC WRITE(6,250)STF, STCLA, STC 250 FORMAT(1X,65(2H--)/1X, 'TOTAL ZONE UNCERTAINTY-ST.DEV.',66X,3(F10.4 1,2X)///30X, \*SUBASSEMBLY UNCERTAINTIES\*/) GO TO 2 4 NTIP0=4 STFS(K)=STF STELS(K)=STELA STCS(K)=STC WRITE(6,350)STF; STCLA; STC 350 FORMAT(1X,65(2H--)/1X, 'TOTAL SUBASSEMBLY UNCERTAINTY-ST.DEV.',59X, 13(F10.4,2X)///30X, 'CHANNEL UNCERTAINTIES'/) GO TO 2 5 STEC(K)=STE STCLC(K)=STCLA STCC(K)=STC WRITE(6,450)STF, STCLA, STC 450 FORMAT(1X,65(2H--)/1X, 'TOTAL CHANNEL UNCERTAINTY-ST.DEV.',63X,3(E1 10.4,2X1///) WRITE(6,550) 550 FORMAT(30X, COOLANT LOCAL UNCERTAINTIES!//3X, UNCERTAINTY (REL.SPE ICIFIC ST.DEV.)',12X,'FOR FUEL HOT SPOT',7X,'FOR CLADDING HOT SPOT' 2, 7X, 'FOR HOT CHANNEL'/1X,65(2H--)) PEAD(5,650)KZ,N15,N16 650 FORMAT(315) IF(KZ.NE.K)GO TO 999 STCFL(K)=0. STCCL(K)=0. STCL(K)=0. IF (N15.E0.0) GO TO 1212 DO 10 I=1,N15 READ(5,750)KZ, NTTPD, (KOMM(L), L=1,10), SPF, SPCL, SPC750 FORMAT(215,10A4,3F10.5) IF (KZ.NE.K.OR.NTIPO.NE.5)GO TO 999 STCFL(K)=STCFL(K)+SPF\*\*2 STCCL(K)=STCCL(K)+SPCL\*\*2 STCL(K)=STCL(K)+SPC\*\*2

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```
10 WRITE(6,850)(KOMM(L),L=1,10),SPF,SPCL,SPC
 850 FORMAT(1X,10A4,10X,F10.4,20X,F10.4,10X,F10.4)
     STCFL(K)=SORT(STCFL(K))
     STCCL(K) = SOPT(STCCL(K))
     STCL(K)=SOPT(STCL(K))
1212 WRITE(6,950)STCFL(K),STCCL(K) ,STCL(K)
 950 FORMAT(1X,65(2H--)/1X, 'TOTAL COOLANT SPECIFIC UNCERTAINTY (PFL.ST.
    1DEV.)', F12.4, 20X, F10.4, 10X, F10.4
                            ///30X, 'TEMP. DROPS LOCAL UNCERTAINTIES 1//3Y,
    Δ
    2'UNCERTAINTY-SPEC.ST.DEV. (OC)', 5X, '-INPUT-', 5X, 'FUFL HOT SPOT',
    35X, 'CLADD. HOT SPOT', 5X, '-OUTPUT-
                                           FUEL HOT SPOT', 5X, CLADD, HO
    4T SPOT 1/1X,65(2H--))
     STFL(K)=0.
     STCLL(K)=0.
     IF(N16.F0.0)GO TO 2121
     DD 20 I=1,N16
     READ(5,750) K7, NTIPO, (KOMM(L), L=1,10), SPED, SPCD
     IF(KZ.NE.K.OR.NTIPO.NE.6)GD TO 999
     SPFDO=SPFD*FSYS
     SPCD0= SPCD*FSYS
     STFL(K)=STFL(K)+SPFDO**2
     STCLL(K)=SPCDO**2+STCLL(K)
  20 WRITE(6,851)(KOMM(L),L=1,10),SPFD,SPCD,SPFDO,SPCDO
 851 FORMAT(1X,10A4,2(7X,F10.4),16X,2(7X,F10.4))
     STFL(K)=SQPT(STFL(K))
     STCLL(K)=SORT(STCLL(K))
2121 WRITE(6,160)STFL(K); STCLL(K)
 160 FORMAT(1X,65(2H--)/1X, 'TOTAL TEMP.DROPS SPECIFIC UNCERTAINTY-ST.DE
    1V. (OC)-*,46X,F10.4,7X,F10.4////)
     PEAD(5,852)K7,SLFCR(K),SLCCR(K)
 852 FORMAT(15,2F10.5)
     IF(KZ.NE.K) GO TO 999
     WRITE(6,853)SIFCR(K), SLCCR(K)
 853 FORMAT(1X, LOCAL UNCERTAINTIES ON CRITICAL TEMPERATURES: FOR FUEL="
    1,F10.4,'(OC)',10X,'FOR CLADDING=',F10.4,'(OC)'//// )
     IF(TFREF.GE.TFZMAX(K)) GD TO 21
     TFREF=TFZMAX(K)
     KFRFF=K
     FS1=FSYS
  21 IF(TCLREF.GE.TCZMAX(K)) GO TO 22
     TCLREF=TCZMAX(K)
     KCLRFF=K
     FS2=FSYS
  22 IF(DTCREF.GE.DTCZ(K)) GO TO 1
     DTCRFF=DTCZ(K)
     KDTCRF=K
   1 CONTINUE
     IF(NSS.NE.NS)GD TO 998
     WRITE(6,260)KFREF, TFREF, KCLREF, TCLREF, KDTCRF, DTCPEF
260 FORMAT(1H1,1X,'FOR FUEL HOT SPOT THE REFERENCE ZONE IS THE 70NF', I
    13/1X, THE REF. FUEL TEMPERATURE IS', F10.2, ' (OC)'///1X, 'FOP CLADDI
    2NG HOT SPOT THE REFERENCE ZONE IS THE ZONE', 13/19,
                                                THE REF. CLADDING TEMPERA
    Δ
    3TURE IS', F10.2, ' (OC)'///1X, 'FOR HOT CHANNEL THE REFERENCE ZONE I
    4S THE ZONE', I3/1X, 'THE REFERENCE TEMP.SPAN IS', F10.2, ' (0C)'////)
     WRITE(6,600)
600 FORMAT(30X, 'CORE UNCERTAINTIES'//)
     READ(5,500)N1,N11,N12,N13,N14
```

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500 FORMAT(515) CALL GROUP(N1,N11,N12,N13,N14,1,0,DICREF,DIF7(KFREF),DICLZ(KCLPFF) 1,ZFZMAX(KFREF),ZCZMAX(KCLREF),STF0,STCL0,STC0,DTCZ(KFREF),DTCZ(KCL 2REF), FS1, FS2) WRITE(6,700)STF0,STCL0,STC0 700 FORMAT(1X,65(2H--)/1X,'TOTAL CORE UNCERTAINTY -ST.DEV.',65X,3(F10. 14,2X)) WRITE(6,360) 360 FORMAT(1H1,1X, 'FUEL HOT SPOT ANALYSIS'////) CALL STEACT (2, DTFZ, STCFL, STFL, XI SPF, TFZMAX, ZFZMAX, STFC, NS PE, NP, STF 1S, STFZ, KFREF, STFO, FMCORE, SFCORE, ITFCR, SLFCR) WRITE(6,460) 460 FORMAT(1H1,1X, CLADDING HOT SPOT ANALYSIS'////) CALL STFACT(1,DTCLZ,STCCL,STCLL,XLSPC,TCZMAX,7CZMAX,STCLC,NSPC,NCL 1, STCLS, STCLZ, KCLREF, STCLO, CLMCOR, CLSCOR, ITCLCR, SLCCR) WRITE(6,560) 560 FORMAT(1H1, 1X, 'HOT CHANNEL ANALYSIS'////) CALL STFACT (0, DT1, STC1, SDT, X1 S, DTCZ, ZZ, STCC, 1, NC, STCS, STCZ, KDTCPF, 1STCO, CMCORÉ, CSCORE, ITCCR, SLTCR) CALL KSTORE(KOMM, 60H NOMINAL, EXPECTED AND CRITICAL TEMPERATURES 1 ,15) 1=7 DO 23 I=1,9 23 H(I) = TITEMPO=TFREF+FMCORE+4.\*SFCORE TIM = 100. XB(1)=0.3TAM=AMAX1(TEMPO,FLOAT(ITFCR))+100. YB(1) = TAM - 400. XB(8)=0.3 YB(8)=TINDIR(8)=2NSC(8)=1DO 60 I=2,7 60 YB(1)=100. CALL PLOTA(AB, H, 9, 2, 3, 1, 1, 1, 1, 7, , 0., , 0125, TAM, TIM, 0, KOMM, L, 1, 1., 1. ,1,-1,1,1,0,8,XB,YB,NDIR,NSC,11HTEMP.(0C)...,6H. 90..., 1,7.,5HI1 26H. 99..,8H. 99.9.,9H. 99.99..,10H. 99.999..,14H( PEP CENT ).., 314HINLET TEMP. ...) DO 30 J=1,9 DO 31 I=1,9 GN TO (32,33,34,35,36,37,38,39,40),J 32 H(I)=DTCREF+TI GO TO 31 33 H(I)=TCLREF GO TO 31 34 H(I) = TFPEFGO TO 31 35 H(I)=FLOAT(ITECR) GO TO 31 36 H(I)=FLOAT(ITCLCR) GO TO 31 37 H(I)=FLCAT(ITCCR) GO TO 31 38 H(I)=TFREF+FMCORE+AB(I)\*SECORE GO TO 31 39 H(I)=TCLRFF+CLMCOR +AB(I)\*CLSCOR GN TO 31

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	· · ·			
40	H(I)=DTCREF+TI+CMCORE+AB(I)*CSCORE			
31	CONTINUE			
	GC TO (42,43,44,45,46,47,48,49,50),J			
42	X=2.			
	Y=H(1)-80.			
	CALL KSTORE(KOMM , 20HNOM.CONL.TEMP.SPAN,5)			
	GD TO 30			
43	Y=H(1)			
	X=3.			
	CALL KSTORE(KOMM , 20HNOM.CL.T ,5)			
	60 10 30			
44	Y=H(I)			
	X=1.			
	CALL KSTURE (KUMM , ZUHNUM . FUEL TEMP , 5)			
	GU 1U 30			
40				
	COLTO DO			
1. 4				
40			· · · ·	
	CALL KSTORE (KOMM 20HCPIT.CL.T			
	CALL REPORT AND AN AZONCKI I CLAITA			
47	V=H(1)			
• •	X=1.5			
	CALL KSTORE (KOMM . 20HCRIT. COOL.T			
	GO TO 30			
48	X=4.			
	Y=H(9)			
	CALL KSTORE(KOMM , 20HEXP.FUEL TEMP ,5)			
	GO TO 30			
49	Y=H(9)+30.			
	CALL KSTORE(KOMM , 20HEXP.CLADD.TEMP ,5)			
	GO TO 30			
50	Y=H(9)			
	CALL KSTORE(KOMM , 20HEXP.COOL.TEMP ,5)		· · · · · · · ·	
30	CALL PLOTA(AB,H,9,2,3,1,1,1,0,7.,0.,.0125,TA	M,TIM,O,	KUWW, L,	1,1.,1.
]	1,7.,5HI1 ,1,-1,1,1,0,1,X,Y,2,1,KDMM)			
0070	READ(5,9970)KUNIR			
9910	FURMATIAA)			
	IFIKUNIRANEAKKKIGU IV 1111			
000	VALL EXII			
0080	CODMATING TEDDOD IN THE NUMBED OF CURACCEMPT	IECIN	•	
720V	TALL FALL	11 <b>3</b> 1	4	
990	WRITE (A.9990)			
9996	FORMATINA FERROR IN THE TONE SUBSCRIPT OF IN	THE CAR	n TVPE	• •
,,, <b>,</b> ,	CALL FXIT	THE CAN	5 8 8 7 9 <b>1</b> 2 	,
	STOP			
	END	•	•	

7

IV.2 Subroutine GRØUP GROUPING OF THE GLOBAL UNCERTAINTIES SUBROUTINE GROUP(N1,N11,N12,N13,N14,N,NN,DT1,DT2,DT3,7MF,ZMC,STF,S 1TCL,STC,DT4,DT5,FS1,FS2) DIMENSION KOMM(10) STE=0. STCL=0. STC = 0. IF(N1)1.1.2 **1 RETURN** 2 IF(N1-N11-N12-N13-N14)3,4,3 3 WRITE(6,100) 100 FORMAT(1X. 'ERROR IN THE NUMBER OF CARDS') 1000 CALL EXIT 4 WRITE(6,200) 200 FORMAT(3X, 'UNCERTAINTY', 20X, '-INPUT-', 4X, 'FUEL', 5X, 'CLADDING', 5X, 1'COOLANT', 3X, 'INPUT UNIT ', 2X, '-OUTPUT- FUEL(OC) CLADD. (OC) COOL 2.(0C) 1/1X,65(2H--)) IF(N11)5,5,6 6 DO 10 I=1,N11 RFAD(5,300)KK,K,K1,KOMM,SF,SCL,SC 300 FORMAT(313,1X,10A4,3F10.5) IF(K.NE.N.OR.KI.NE.1.OP.KK.NE.NN)GD TO 999 SFC = SF\*(DT4\*FC(ZMF)+DT2\*FP(ZMF))SCLC=SCL\*(DT5\*FC(ZMC)+DT3\*FP(ZMC)) SCC=SC\*DT1 STF=STF+SFC\*\*2 STCL=STCL+SCLC\*\*2 STC=STC+SCC\*\*2 10 WRITE(6,400)KOMM, SF, SCL, SC, SFC, SCLC, SCC 400 FORMAT(1X, 10A4, 3(F10, 4, 2X), 'REL. TO (T-TI)', 7X, 3(F10, 4, 2X)) 5 IF(N12)15,15,16 16 DO 20 I=1,N12 READ(5,300)KK,K,K2,KOMM;SF;SCI,SC IF(K.NE.N.OR.K2.NE.2.OR.KK.NE.NN)GO TO 999 SFC=SF\*DT4\*FC(ZMF) SCL C=SCL\*DT5\*FC(ZMC) SCC=SC\*DT1 STF=STF+SFC\*\*2 STCL=STCL+SCLC\*\*2 STC=STC+SCC\*\*2 20 WRITE(6,500)KOMM, SF, SCL, SC, SFC, SCLC, SCC 500 FORMAT(1X, 10A4, 3(F10.4, 2X), 'REL. TO DTC ',8X,3(F10.4,2X)) 15 IF(N13)25,25,26 26 DO 30 I=1,N13 READ(5,300)KK,K,K3,KOMM,SF,SCL,SC IF(K.NE.N. OP.K3.NF.3)GD TO 999 IF(K.NE.N.OR.K3.NE.3.OR.KK.NE.NN )GO TO 999 SEC=SE\*EP(ZME) \*ES1 SCLC=SCL\*FP(ZMC)\*FS2 STF=STF+SFC\*\*2 STCL=STCL+SCLC\*\*2 30 WRITE(6,600 )KOMM, SF, SCL, SC, SFC, SCLC, SC 600 FORMAT(1X,10A4,3(F10.4,2X), (00)1,7X,3(F10.4,2X)) 25 IF(N14)35,35,36 36 DO 40 I=1,N14 READ(5,300)KK,K,K4,KOMM,SF,SCL,SC IF(K.NF.N.OR.K4.NF.4.OR.KK.NF.NN)GO TO 999 STF=STF+SF\*\*2

С

STCL=STCL+SCL\*\*2 STC=STC+SC\*\*2 40 WRITE(6,600)KDMM,SF,SCL,SC,SF,SCL,SC

- 35 STF=SORT(STE) STCL=SORT(STCL) STC=SORT(STC) RETURN
- 999 WRITE(6,2000) 2000 FORMAT(1X,'ERROR IN THE TYPE OF CARD') GO TO 1000 END

IV.3 Subroutine TMAXZ

C EVALUATION OF THE MAXIMUM AXIAL TEMPERATURES SUBROUTINF TMAXZ(XL,XLSP,TI,DT1,DT2,TMAX,ZMAX) N=XL/XLSP TMAX=0. DO 1 I=1,N Z=-XL/2.+(FLOAT(I)-0.5)\*XLSP T=TI+DT1\*FC(Z)+DT2\*FP(Z) IF(T-TMAX)1,1,2

2 TMAX=T ZMAX=Z 1 CONTINUE RETURN END IV.4 Subroutine STFACT

```
EVALUATION OF THE STATISTICAL FACTORS
    SUBROUTINE STEACT(IND, DT1, SSC, SDT, XLS, TMAX, ZMAX, SCH, NSP, NCH, SSA, SZ
   10, KREF, SCO, MCORE, SCORE, ITCR, SLTCR)
    DIMENSION DTC(150), DT1(150), SSC(150), SDT(150), TMAX(150), ZMAX(150),
   1SCH(150),H(150),NSZ(150),DTCZ(150),SSA(150),SZO(150),MZDEQ(150),
   2SZOEQ(150) , SLTCR(150)
    DIMENSION XB(8), YB(8), NDIR(8), NSC(8), KDMM(15), AB(9), HEC(9)
    PIMENSION HET(21), PROBT(21), PROBN(21,10), PROBE(21), PROBNN(21)
    COMMON XL,XLEX,TI,NZ,H,NSZ,DTCZ,NS,CC,ZZ,BB,FAT,J,INDEX,XB,YB,NDIR
   1,NSC,AB
    PEAL MCHEO, NCHEO, MSAEO, MZDEO, MS, MCORE, NZDEO
    HM(X)=1.70694+0.54372*ALOG10(X)-1.70169*EXP(-0.81888*ALOG10(X))
    HS(X)=0.62589-0.03584*ALDG10(X)+0.37230*EXP(-0.82554*ALDG10(X))
    WRITE(6,100)
100 FORMAT(1X, 'ALL TEMPERATURES, MEANS AND ST. DEVIATIONS ARE EXPRESSED
   1IN OC'//6X,'ZONE',5X,'NS(Z)',5X,'TNOM',6X,
                                                            'NSPEO', 5X, 'SS
   2PEQ', 5X, 'SCHEQ', 5X, 'MCHEQ', 5X, 'NCHEQ', 5X, 'SCHEQ*', 4X,
                                                'SSAE0', 5X, 'MSAE0', 5X, 'S7
   3PEQ', 5X, 'MZDEQ'/1X, 65(2H--))
    TC=0.
    DO 1 I=1,NZ
    JF('IND .EQ.0)TMAX(I)=TMAX(I)+TI
    DTMAX = TMAX(I) - TI
    IF(IND.EQ.0)GO TO 2
    CALL SPOT(DTCZ(I), DT1(I), SSC(I), SDT(I), XLS, TMAX(I), ZMAX(I), SSPF0, X
   INSPEQ, SLTCR(I))
    XNSPEQ=XNSPEO*FLOAT(NSP)
    GO TO 3
  2 SSPE0=SSC(I)/SOPT(XL)
    XNSPEQ=0.
    SCHEQ=SORT(SSPEQ**2+SCH(I)**2)
    MCHEQ=0.
    GO TO 4
  3 MCHEQ=HM(XNSPFQ)* SSPEQ
    SCHED=SORT((HS(XNSPED)*SSPED)**2+SCH(I)**2)
  4 CALL CHANEQ(NCH, H(I), SCHEQ, NCHEQ, DTMAX, SCHE1, MCHEO)
    MSAEQ=MCHEQ+HM(NCHFQ)*SCHE1
    SSAEQ=SORT((HS(NCHEQ)*SCHE1)**2+SSA(I)**2)
    MZOEQ(I)=MSAEQ+HM(FLOAT(NSZ(I)))*SSAEQ
    SZDEQ(I)=SQRT((HS(FLOAT(NSZ(I)))*SSAFQ)**2+SZO(I)**2)
    T=TMAX(I)+MSAEQ+3*SQRT(SSAEQ**2+SZO(I)**2)
    IF(T.LE.TC) GO TO 1
    TC=T
    MS=MSAE0
    SS=SSAE0
    IREF=I
  1 WRITE(6,150)I,NSZ(I),TMAX(I),XNSPF0,SSPE0,SCHE0,MCHE0,NCHE0,SCHF1
   1 ,SSAEQ, MSAEQ, SZOEQ(I), MZOEQ(I)
150 FORMAT(4X, 15, 5X, 15, 2X, 11F10.3)
    AA = ALOG(0.5)
    BB=ALOG(0.5+0.5*ERF(4.5/1.414232))
    SZ=SZOEO(KREE)
    TT = TMAX(KREF) + MZDEO(KREF)
    INDEX=0
    J=0
 20 XZ0E0=0.
    DO 10 I=1,NZ
```

с С

10 XZ0EQ=XZ0E0+AL0G(0.5+0.5\*ERF((TT-TMAX(I)-MZ0E0(I))/(SZ0E0(I)\*1.414 1232111 IF(INDEX.GT.O) GO TO 40 NZDEQ=XZDEQ/AA INDEX=1 50 TT=TMAX(KREF)+MZDEQ(KREE)+4.5\*SZ GO TO 20 40 XZDEQ=XZDEQ/BB J = J + 1IF(INDEX.EQ.3)GO TO 11 IF(INDEX.E0.1.AND.NZDE0.LT.XZDE0)GD TO 12 INDEX=2 IF (NZOEQ.LF. XZOEQ) GO TO 60  $SZ = SZ - 0.01 \times SZ \times FLOAT(J)$ GO TO 50 12 INDEX=3 11 IF(NZOE0.GE.XZOE0)GO TO 60  $SZ = SZ + 0.01 \times FLOAT(J) \times SZ$ GO TO 50 60 MCORE=MZDEO(KREF)+HM(NZDEO)\*SZ SCORE=SORT((HS (NZOE0 )\*SZ)\*\*2+SCO\*\*2) WRITE(6,200)KREF, TMAX(KREF), NZDEQ, MZDEQ(KREF), SZ, MCDRE, SCORE 200 FORMAT(1X,65(2H--)////1X, 'REF.ZONE',5X, 'REF.TEMP.',5X, 'NZOEO',8X, 1'MZOFQ',8X,'SZOEQ\*',7X,'MCORE',8X,'SCORE'/1X,16,5X,3F12.4,3X,3F12. 24///1X, 'PROB.OF EXCEEDING', 15X, 'CONFIDENCE LEVEL', 8X, ' FHS 3 ',10X, 'MAX. TEMP'/1X,45(2H--)) DTMAX=TMAX(KPEF)-TI 00 70 I=1,9 AB(I) = 0.5 \* FLOAT(I-1)PROB=0.5-0.5\*ERF(AB(I)/1.414232) CONF=1.-PROB HFC(I)=1+(MCORE+AB(I)\*SCORE)/DTMAX TEMP=TI+DTMAX\*HFC(I)70 WRITE(6,250)PROB, AB(I), CONF, HFC(I), TEMP 250 FORMAT(1X, E17.6, 10X, '(', F3.1, 'SIGMA)', E14.6, 6X, F13.4, 10X, F8.2) PROB=0.5-0.5\*ERF((FLOAT(ITCR)-TMAX(KREF)-MCORF)/(1.414232\*SCORF)) WRITE(6,300)ITCR, PROB 300 FORMAT(1X,45(2H--)////1X, THE PROBABILITY OF EXCEEDING THE CRITICA 1L TEMPERATURE (', 15, ' OC) IS=', F10.8) 800 XB(1)=0.5 YB(1) = HFC(6)NDIR(1)=1NSC(1)=1DO 17 I=2,7 YB(I) = 1.005NDIR(I)=117 NSC(I) = 1XB(2) = 1.35155XB(3)=2.39635 XB(4) = 3.16023XB(5)=3.78902 XB(6) = 4.33489XB(7) = 6.IF(INDEX.EQ.100) RETURN N=IND+1GC TO (61,62,63), N FUEL HOT SPOT FACTOR VERSUS CONFIDENCE LEV 63 CALL KSTORE (KOMM, 60H 1FL ,15)



L=4 GO TO 74 71 CALL KSTORE (KOMM, 60H HOT CHANNEL ANALYSIS-PROBABILITY OF N HOT SU 1BASSEMBLIES ,15) L=674 XB(1) = HFT(1) + 0.01YB(1)=1. X3(2) = HET(19)YB(2) = 0.05NDIR(2)=2NDIR(3)=2XB(3) = HFT(17)DO 76 I=1,KP PROBT(I)=ALOG10(PROBT(I)) 76 PROBE(I)=ALOG10(PROBE(I)) YB(3) = PROBT(17)CALL PLOTA(HFT, PROBT, KP, 2, 9, 1, 1, 1, 1, HFT(21), HFT(1), 0, 2., -10., 0.012 15,KOMM,L,0,-1,1.E-10,0.,1.E+00,5HE8.1 ,1,1,-1,2,3,XB,YB,NDI0,NSC, 28HPROB. ..., 8HFACT. ..., 16H MIN.1 (PESS.)..) CALL PLOTA(HET, PROBE, KP, 1, 7, 1, 1, 1, 0, XMAX, XMIN, SX, YMAX, YMIN, SY, KOMM 1,L,0,-1,1.E-10,DY,1.E+00,5HE8.10,1,1,-1,2,1,HET(20), PROBE(20),2,1, 28HMIN.1 ...) PO 720 I=1.10 GO TO (721,722,729,720,723,720,720,720,720,724),1 721 NP=0 CALL KSTORE (KOMM, 4H1 ...1) GO TO 725 722 NP=3 CALL KSTORE(KOMM, 4H2 ..., 1) GO TO 725 729 IF(NS-10)709,720,720 723 NP = 4CALL KSTORE(KOMM, 4H5 ...1) GO TO 725 724 NP=9 CALL KSTORE (KOMM, 4H10..,1) 725 DO 727 J=1,KP KS=J-1PROBNN(J) = ALOG10(PROBN(J,I))IF(PROBN(J,I)-1.E-10)750,750,727 727 CONTINUE 750 CALL PLOTA(HFT, PROBNN, KS, 3, NP, 1, 1, 2, 0, X, XM, SX, Y, YM, SY, NT, L, 0, -1, 1. 1E-10,1.,1.E+00,5HE8.1 ,1,1,+1,2,1,HFT(16), PRDBNN(16),2,1,KOMM) 720 CONTINUE 709 INDEX=100 IF(IND.EQ.0) GO TO 800 RETURN END

IV.5 Subroutine SPØT

EVALUATION OF THE EQUIVALENT NUMBER OF SPOTS SUBROUTINE SPOT(DTC,DT1,SSC,SDT,XLS,TMAX,ZMAX,SSPOT,XNSPOT,SFT) COMMON XL, XLEX, TI T(X) = TI + DTC \* FC(X) + DT1 \* FP(X)S(X)=SQRT((SSC\*\*2/(XL/2.+X))\*DTC\*\*2\*FC(X)\*\*2+1./XLS\*SDT\*\*2\*FP(X)\*\* 12+1./XLS\*SFT\*\*2) N=XL/XLS AA = ALOG(0.5)BB=ALOG(0.5+0.5\*ERF(4.5/1.414232)) SSPOT=S(ZMAX)IF(SSPOT.GT.0.0)G0 T0 20 XNSPOT=1. RETURN 20 TT=TMAX SM=0. J=0INDEX=0 2 XSPOT=0.  $D_{1} = 1, N$ Z=-XL/2.+(FLDAT(I)-0.5)\*XLS IF(INDEX.EQ.O.AND.S(Z).GT.SM)SM=S(Z) XSPOT= XSPOT+ALOG(0.5+0.5\*ERF((TT -T(Z))/(S(Z)\*1.414232))) 1 • IF(INDEX)3,3,4 3 XNSPOT=XSPOT/AA INDEX=1 5 TT=TMAX+4.5\*SSPPT GO TO 2 4 XSPOT=XSPOT/BB J = J + 1IF(INDEX.E0.3)GO TO 11 IF(INDEX.EQ.1.AND.XNSPOT.LT.XSPOT) GO TO 10 INDEX=2 IF(XNSPOT.LE. XSPOT ) RETURN SSPOT=SSPOT-0.01\*SSPOT\*FLOAT(J) GO TO 5 10 INDEX=3 11 IF (XNSPOT.GE.XSPOT) RETURN SSPOT=SSPOT+0.02\*(SM-SSPOT)\*FLOAT(J) GO TO 5

END

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# IV.6 Subroutine CHANEQ

C C

EVALUATION OF THE EQUIVALENT NUMBER OF CHANNELS SUBROUTINE CHANEQ(N, H, SIG, XNCHAN, TMAX, SCH, XMM) DIMENSION P(25) FVONX(X,Z)=ALOG(0.5+0.5\*ERF((Z+(H-X)\*(TMAX+XMM))/(1.414232\*SIG\*(1. 1 - H + X))))SCH=SIG P(1) = -HP(2) = HP(5)=1.E-5 IF(H)10,20,25 10 WRITE(6,100)' 100 FORMAT(1X, 'ERROR-H NEGATIVE') CALL EXIT 20 XNCHAN=FLOAT(N) RETURN 25 T=0. IF(SIG.GT.0.0)G0 T0 70 XNCHAN=0.5/H GO TO 60 70 BB=ALOG(0.5+0.5\*ERF(4.5/1.414232)) INDEX=0 J=0 30, K=0 1 CALL FORHAL(K,P) GO TO (2,2,3,3),K 2 P(4) = FVONX(P(3), T)GO TO 1 3 XCHAN=P(4)IF (INDEX.EQ.1)GO TO 40 XNCHAN=XCHAN/ALOG(0.5)\*0.5/H INDEX=1 50 T=4.5\*SCH . GO TO 30 40 XCHAN=XCHAN/BB\*0.5/H IF(XCHAN.GE.XNCHAN)GO TO 60 J = J + 1SCH=SIG-0.01\*SIG\*FLOAT(J) GO TO 50 60 XNCHAN=XNCHAN\*FLOAT(N) RETURN END

# IV.7 Subroutine FØR

C C		INTEGRATION OF PVONX(X)
		SUBROUTINE FOR(A, B, C, D)
		DIMENSION P(25)
		DX = (B - A) * 0.01
		DO 10 I=1,100
		H=A+DX*FLOAT(I)
		F = PVONX(H) * (H - A)
		KK = I - 1
		IF(ABS(E)-1.E-25)10,20,20
	10	CONTINUE
	20	G1=A+DX*FLOAT(KK)
		DO 40 I=1,100
		D=B-DX*FLOAT(I)
		F=PVONX(D)*(B-D)
		KK = I - I
		IF(ABS(E)-1.E-25)40,50,50
	40	CONTINUE
	50	G2=B-DX*FLOAT(KK)
		IF(G2-G1)70,80,80
	70	G2=G1
	80	P(1) = G1
		P(2) = G2
		P(5)=C
		K=0
	1	CALL FORHAL(K, P)
		GO TO (2,2,3,3),K
	. 2	P(4) = PVONX(P(3))
		GO TO 1
	3	D=P(4)
		RETURN
		END

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# IV.8 Subroutine KSTØRE

C PLOT TEXT C SUBROUTINE KSTORE(NT,MT,NN) DIMENSION NT(15),MT(15) DO 1 KM=1,NN 1 NT(KM)=MT(KM) RETURN

END

# IV.9 FUNCTION PVØNX

с С	- 	FUNCTION FOR NUMBER OF HOT SUBASSEMBLIES
		FUNCTION PVONX(X)
		DIMENSION H(150), NS7(150), DTC2(150)
		COMMON XL, XLEX, TI, NZ, H, NSZ, DTCZ, NS, CC, ZZ, RB, FAT, J, INDEX
		GO TO (40,50), INDEX
	40	A=0.5+0.5*ERF((ZZ-X)*CC)
		IF(A-1.E-50**(1./FLOAT(NS)))41,41,42
	41	PVONX=EXP(BB*X**2)
		PETURN
	42	PVONX=EXP(BB*X**2)*(1A**NS)
		RETURN
	50	P=0.5-0.5*ERF((ZZ-X)*CC)
		$\hat{\mathbf{Q}} = \hat{1}_{\bullet} - \mathbf{P}$ is the constant of the function of the constant of the constan
		IF(Q-1.E-50**(1./FLOAT(NS-J+1)))51,51,52
	51	PVONX=0.
		RETURN CONTRACTOR STATES AND A CONTRACTOR OF A CONT
	52	PVONX=EXP(BB*X**2)*FAT*P**J*Q**(NS-J) RETURN
		$END^{\mathrm{res}}$ . The second secon

### Third Part

# Application to the Reactor Na-2

#### V. Data of the Reactor Na-2

#### V.1 General data

The reactor Na-2 / 11 / is a conceptual design of a sodium cooled fast reactor with a nominal thermal power of 730 MWth corresponding to an output of 300 MWe. The fuel is constituted of pellets of mixed  $UO_2 - PuO_2$ , subdivided into two radial zones with different Plutonium enrichment (21.14 % and 31.14 % respectively). The maximum specific power is 420 W/cm. In this numerical application of the described hot spot analysis, it will be considered the reactor at start up. Consequently the reactor has been subdivided into 7 radial zones, corresponding to the 7 concentric rings formed by the 150 subassemblies each one constituted of 169 fuel pins. It has been assumed a cosine axial power profile. Therefore the functions  $f_p(z)$  and  $f_c(z)$  correspond to those listed at item III.6.

The data necessary to define the variables in the first block of input cards are specified in Table 7, in the order given at item II.3.1. The assumptions on the hot spot size are those performed at items I.1.1 and I.1.2; at first it has been assumed a spot length of 1 cm, corresponding to a pellet length: the influence of this length will be successively investigated at item VI.2.

# Table 7 General Data for the Reactor Na-2

Parameter	Value	Unit
Number of zones	7	
Total number of subassemblies	150	
Number of channels for fuel hot spot	169	
Number of channels for hot channel	336	
Number of channels for cladding hot spot	336	
Number of fuel spots at a given z	1	
Number of cladding spots at a given z	3	
Fuel critical temperature	2700	°c
Cladding critical temperature	700	°C.
Coolant critical temperature	880	°C
Active length	95.0	cm
Extrapolation length	132.0	cm
Fuel spot length	1.0	cm
Cladding spot length	1.0	cm
Nominal coolant inlet temp.	380.0	°c
Nominal max. temperature span of the coolant in the core	212.4	°c
Nominal max. temperature drop inner cladding - coolant in the core	58.6	°c
Nominal max. temperature drop central fuel - coolant in the core	1623.6	°c

# V.2 Zone characteristics

The zone characteristics are summarized in Table 8. This table defines the parameters upon which the radial temperature profile depends. The systematic factors take into account the effects of the control rods on the neutron flux distribution. In fact the flux profile varies during the reactor operation according to the position of the control rods. Correspondingly in every point of the core the flux will deviate from nominal within certain minimum and maximum values which can be evaluated theoretically, repeating the calculations of the flux profile for every possible control rod position: we are then faced with the certainty of deviations from nominal during operation and therefore with systematic factors. In every zone there will į. be a certain number of subassemblies for which these deviations are maximum, other subassemblies however will be affected by intermediate and minimum values of flux deviations from nominal; the occurrence of the worst conditions for hot spot generation just at the location at which the control rod influence is maximum is a statistical event. Therefore we have considered this effect by assuming a systematic factor, which takes into account the average flux deviation in the considered zone, and a statistical uncertainty which takes into account the difference between the actual deviations in every subassembly and the average deviation in the zone, and the quite statistical error of the perturbed flux calculations.

Zone	: 1	2	3	4	5	6	7
Number of subassemblies (NSZ)	6	12	12	24	30	24	42
HP= <u>max.power in a zone</u> max.power in the core	0.965	0.934	0,895	0.834	1	0.894	0.715
HC= <u>max.coolant temp.span in a zone</u> max.coolant temp.span in the core	1	l	1	1,	1	1	1
H = power profile in a subassembly (Eq. 14)	0.0146	0.0297	0.0400	0.0505	0.0890	0.1555	0.2370
FSYSP = systematic power factor	1.1	1.07	1.04	1.04	1.07	1.09	1.12
FSYSC = systematic temp.span factor	1.1	1.07	1.04	1.04	1.07	1.09	1.12

Table 8 Radial Temperature Profile 79 1

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#### V.3 Uncertainties

According to Table 1, the original uncertainties for the reactor Na-2 are reported in Table 9. In this analysis the uncertainties have been assumed as constant among the zones, with exception of the statistical effects of control rod position on the flux distribution, which are reported in Table 10.

## Table 10 Flux Uncertainty Due to Control Rods

Zone	1	2	3	4	5	6	7
Øcr <sup>(%)</sup>	2	2	3	3	2	2.5	3

From the original values of the uncertainties the standard deviations to be assigned in the program are evaluated in the following.

#### V.3.1 Global uncertainties

# a) Zone

Density, enrichment, flux calculation: these uncertainties act proportionally on the whole temperature difference fuel (cladding)-inlet coolant (Kl = 1, see Table 3). Therefore they are to be assigned directly as relative value (Table 11).

<u>Outer diameter:</u> this parameter acts principally on the hydraulic diameter, and therefore on the coolant temperature span (K1 = 2 in Table 3). Its effect has been calculated as relative value by /177

$$\sigma'_{do} = \frac{1}{3} \left( \left[ \frac{\frac{1}{2} \sqrt{3} \ \bar{p}^2 - \bar{do}^2 \ \tilde{t}}{\frac{1}{2} \sqrt{3} \ \bar{p}^2 - (\bar{do} + 3 \ \tilde{6}_{do})^2 \ \tilde{t}}{\frac{1}{2} \sqrt{3} \ \bar{p}^2 - (\bar{do} + 3 \ \tilde{6}_{do})^2 \ \tilde{t}} \right]^{\frac{3}{2}} \sqrt{\frac{\bar{do} + 3 \ \tilde{6}_{do}}{\bar{do}} - 1} \right).$$

		Standard deviation						
Variable	Nominal value	Local [σ <sup>s</sup> ] = [σ]·√cm	Channel	Subassembly	Zone	Core		
Density	80% of theor.density	G <sub>δ</sub> <sup>5</sup> = 2 %			<sup>6</sup> δ = 1 %			
Enrichment	I=21.14%;II=31.14%	$\sigma_{\alpha}^{s} = 1 \%$			6 <sub>a</sub> = 1 %			
Cladding inner diameter	5.24 mm	$\sigma_{\rm di}^{\rm s} = 0.006  \rm mm$			<sub>℃di</sub> =0.012mm			
Cladding outer diameter	6 mm	$G_{do}^{s} = 0.006 \text{ mm}$			5 <sub>do</sub> =0.012mm			
Cladding thickness	0.38 mm	$\sigma_t^{s} = 0.008 \text{ mm}$			$\sigma_{\tilde{t}} = 0.008$ mm			
Density asymmetry	0	$6_{\delta a}^{s} = 5 \%$		,				
Enrichment asymmetry	0	5 <sup>s</sup> = 2.5 %						
Fuel-clad eccentricity	0	$\sigma_e^s = 0.6$			-			
Pin pitch	7.9 mm	6 <sup>4</sup> ≱ = 0.05 mm	∝ <sub>p</sub> =0.13 mm					
Pin active length	95 cm		°į =0.5 cm					
Orifice calibration				or = 2.3 %				
Neutron flux		$\sigma_{\phi_{a}}^{s} = 2.5 \%$	§ <sub>r</sub> = 1 %	Ger Table 10	Sø <sub>c</sub> = 2 %			
Power measurement	·				-	<b>5</b> <sub>Pw</sub> = 2.5 %		
Inlet temperature	380 °C					$\sigma_{i} = 1 ^{\circ}C$		
Cladding critical temp.	700 °C					$\sigma_{ii} = 7 °C$		
Fuel melting point	2700 °C	$\sigma_{M}^{s} = 10^{\circ} C$				$\sigma_m = 60 °C$		
Fuel thermal conduct.	0.024 W/cm <sup>0</sup> C	$\sigma_{kf}^{s} = 1 \%$				Skp = 4.5 %		
Cladding thermal conduct	. 0.21 W/cm C	$S_{\mu_{cl}}^{-s} = 0$				° <sub>hel</sub> = 1 %		
Heat transfer clad-fuel	1.5 W/cm <sup>2</sup> °C	$\sigma_{4p-d}^{s} = 0.20 \text{W/cm}^{2\bar{0}} C$				~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~		
Heat transfer clad-sodiu	m 14.5 W/cm <sup>20</sup> C	$\mathcal{G}_{h,l-c}^{5} = 3 \%$				§1. = 3 %		
Sodium specific heat						Cip = 1 %		

Table 9 Uncertainties for the Reactor Na-2

- 81 - (The relationship between  $d_0$  and flow rate being not linear, the deviation has been calculated at 3  $\tilde{\sigma}_{d0}$ , then divided by 3).

Inner diameter: the effect of this parameter results to be negligible.

<u>Cladding thickness</u>: This parameter acts on the temperature drop across the cladding (Kl = 3 in Table 3), therefore it has been determined for the maximum specific power  $(X_{max})$  in the dimension of a temperature by (see Eq. 11)

$$\mathcal{D}_{t}^{\mathcal{Y}} = \overline{\chi}_{\max} \frac{2_{o_{t}}}{k_{cl}(\overline{d}_{i} + \overline{d}_{o})^{\pi}} \cdot$$

The numerical values are given in Table 11.

# b) Subassembly

The subassembly uncertainties (flux perturbation due to control rod and orifice calibration uncertainty) can be directly assigned as relative values (Table 11).

# c) Channel

Flux (local radial): it can be directly assigned as relative value.

<u>Pin pitch:</u> From Ref. (17) we assume the coolant temperature span to be proportional to  $(D_h)^{-2}$ ,  $D_h$  being the hydraulic diameter. Since  $D_h$  is proportional to the flow section (S), we can write

with  $\overline{S}$  and  $\overline{\sigma}_{S}$  the nominal value and standard deviation of S respectively. This holds if we assume that no mixing occurs: in other case, it is possible to decrease  $\sigma_{p}^{*}$  of a quantity corresponding to mixing effects.

# Table 11 Global Uncertainties

Group	Uncertainties	Type (Kl)	Fuel Hot Spot	Cladding Hot Spot	Hot Channel	Unit
	Flux calculation	1	0.02	0.02	0.02	rel.to $\vartheta_j - \vartheta_j$
¢)	Density (Prod.Batch)	1	0.01	0.01	0.01	11 11 11
lon	Enrichment "	1	0.01	0.01	0.01	1.11 11 13
2	Outer Diameter "	2	0.008	0.008	0.008	rel.to ∆9 <sub>c</sub>
	Thickness "	3	0.9	0.9	-	°C
Subass.	Flux (control rod V zone) Orifice calibration	1 2	0.02 0.023	0.02 0.023	0.02 0.023	rel.to ϑ <sub>j</sub> -ϑ <sub>i</sub> rel.to ∆ỹ <sub>c</sub>
	Flux (local radial)	1	0.01	0.01	0.01	rel.to $\vartheta_{\cdot}-\vartheta_{\cdot}$
nel	Pin pitch	2	0.0146	0.0274	0.0274	j i rel.to ልን
Chan	Active length	, 2	0.0023	0.003	0,003	rel.to ∆ϑ
	Measurement error	1	0.025	0.025	0.025	rel.to JJ.
	Sodium heat capacity	2	0.01	0.01	0.01	rel.to $\Delta \vartheta_{c}^{-1}$
	Fuel thermal conductivity	3	62	алар алар алар алар адар алар адар алар — ар	-	°C
	Cladding thermal conductivity	3	0.46	0.46	-	°c
ore	Heat transfer cladding-fuel	3	11.9	-	-	°c
U	Heat transfer cladding-sodium	3	0.5	0.5	-	Do C
	Inlet sodium temp.	4	1	1	1	°c
	Fuel melting point	4	60	-	-	°c
	Cladding critical temperature	4		7		• • • • • • • • • • • • • • • • • • •

the channel arrangement for fuel hot spot and that for cladding and hot channel.

Considering the hexagonal arrangement for the fuel (Fig. 1 and Fig. 17), the section is given by



Fig. 17

Any  $p_i$  has the distribution  $(\bar{p}, \sigma_p)$ ; therefore applying error propagation theory we obtain:  $\bar{S} = \frac{3}{2}\sqrt{3} \bar{p}^2 - 3 \bar{d}_0^2 \frac{n}{4}$  (42)  $\sigma_{\bar{S}} = \frac{3}{\sqrt{2}} \bar{p} \sigma_p$ .

 $s = \frac{\sqrt{3}}{4} (p_1 p_2 + p_2 p_3 + p_3 p_4 + p_4 p_5 + p_5 p_6 \sqrt{1 - 3} \overline{d_0}^2 \frac{\pi}{4}$ 

Then we get from Eqs. (41) and (42) for fuel channel arrangement

$$\mathbf{e}_{\mathbf{p},\mathbf{f}}^{\mathbf{x}} = \frac{1}{3} \left[ \left( \frac{\frac{3}{2}\sqrt{3}}{\frac{2}{2}\sqrt{3}} \frac{\mathbf{p}^{2}}{\mathbf{p}^{2}} - \frac{3}{\sqrt{2}} \frac{\mathbf{d}_{o}^{2} \frac{\pi}{4}}{\frac{3}{\sqrt{2}}} \right)^{\frac{3}{2}} - 1 \right]$$

Analogously for the triangular arrangement we get

$$\sigma_{\mathbf{p},\mathbf{d}}^{\mathbf{x}} = \frac{1}{3} \left[ \left( \frac{\frac{1}{4} \sqrt{3} \ \bar{\mathbf{p}}^2 - \bar{\mathbf{d}}_0^2 \ \hat{\mathbf{g}}}{\frac{1}{4} \sqrt{3} \ \bar{\mathbf{p}}^2 - \frac{3}{4} \sqrt{6} \ \bar{\mathbf{p}} \ \bar{\mathbf{p}}_p^{-} - \bar{\mathbf{d}}_0^2 \ \frac{\pi}{8}} \right)^2 - 1 \right]$$

Moreover, it has been assumed the pitch as constant along the distance between two consecutive spacer grids; therefore the final value  $\sigma'_n$  in Table 11 has been obtained as

$$\sigma'_{p} = \frac{\sigma'_{p}}{\sqrt{10}} \, .$$

10 being the number of spacer grids.

<u>Active length</u> The corresponding uncertainty has been averaged over the number of pins in a channel arrangement with a weight corresponding to the fractional power of each pin.

# d) Core

<u>Measurement error</u>, <u>sodium heat capacity</u> These uncertainties have been directly assigned as relative value.

The effects of <u>fuel and cladding thermal conductivities</u>, and <u>heat</u> <u>transfer cladding-fuel</u> and <u>cladding-sodium</u> have been calculated in the dimension of a temperature at the maximum specific power in the core.

For <u>inlet sodium temperature</u>, <u>fuel melting point</u> and <u>cladding critical</u> <u>temperature</u> no calculation is required, being fixed temperature (K1 = 4)

#### V.3.2 Local Uncertainties

# a) Coolant temperature

The corresponding standard deviations must be assigned as relative specific standard deviations for the considered channel arrangement  $(\sigma_i^{s,ch} - \text{item II.4.2})$ . The numerical values are presented in Table 12. <u>Density and enrichment</u>: for these parameter Eqs. (18) and (18b) hold for the fuel and cladding channel arrangements respectively. <u>Axial flux</u>: it has been assumed that the same cause provoking a deviation from nominal axial profile at a given abscissa of a pin will act in the same manner on all pins of a channel. Therefore in both cases  $\sigma_{\phi a}^{s,ch} = \sigma_{\phi a}^{s'}$ .

<u>Fuel-cladding eccentricity</u>: it has been assumed that every value of eccentricity within 0-and 1 (no eccentricity and eccentricity with contact) are equally probable - (rectangular distribution, therefore  $\sigma_e^s = \frac{1}{\sqrt{3}} \simeq 0.6$ ). From Ref. (12) it follows that the power flux versus the direction of minimum thermal resistance is  $\simeq 5$  % higher than the nominal one. Therefore  $\sigma_e^{s'} = 0.05$ . As precedently stated, (item II.4.2)

for the channel arrangement for fuel hot spot only the pins adjacent to the considered one must be considered: with the procedure indicated in the derivation of Eqs. (18) and (18b) we obtain:

$$\sigma_{e}^{s, ch-f} = \frac{\sqrt{6}}{9} \quad \sigma_{e}^{s'}$$
$$\sigma_{e}^{s, ch-cl} = \frac{\sigma_{e}^{s'}}{\sqrt{3}}$$

#### Table 12 Local Uncertainties for Coolant Temperature

	$\sigma_{i}^{s,ch}$ (relative value)					
Parameter	Fuel hot spot	Cladding and hot channel				
Density	0.015	0.028				
Enrichment	0.008	0.014				
Axial flux	0.025	0.025				
Eccentricity fuel-cladding	0.013	0.028				

# b) Temperature drops

The specific standard deviations for this group of uncertainties have been calculated in the dimension of a temperature for the maximum specific power in the core. The numerical values are reported in Table 13, and have been calculated according to the following considerations.

and the first of the second second

Parameter	$\sigma_{j,\Delta}^{s} \mathcal{T}_{j-d}, \max$ (°C)				
	Fuel	Cladding			
Density	32.0	2.8			
Density asymmetry	8.0	n na santa n 🕶			
Enrichment	16.0	1.4			
Enrichment asymmetry	4.0	_			
Pitch and local diam. on	negligible	negligible			
Cladding thickness	0.9	0.9			
Axial flux	40.0	1.5			
Cladding-fuel eccentricity	-	3.0			
Fuel thermal conductivity	16.7				
Heat transfer cladding-fuel	26.5	-			
Heat transfer cladding-sodium	0.5	1.2			
Fuel melting point	10.0				

## Table 13 Uncertainties on local temperature drops

<u>Density and enrichment</u>: for fuel hot spot the whole section of a pin has been considered, whereas for cladding hot spot only  $\frac{1}{6}$  of the section has been taken into account.

<u>Density and enrichment asymmetry</u> were considered only for fuel hot spot as stated at item I.l.l.

The effect of cladding local diameter and local pitch on  $h_{cl-c}$  results to be negligible using the relations given in Ref. 17.

Fuel cladding eccentricity was considered only for cladding hot spot and calculated as at point a) assuming  $\sigma_e^{s'} = 0.05$ . - 88 -

V.4	Listing	of	input	cards

•

REAK	TOR	NA-2	·* · • •		-				
	7	50 169 33	36 336	1	3 2700	700 880			
	95.	132.	1.		1.	380.	212.4	58.6	1623.6
	1	6 0.965	1.		.01464	1.1	1.1		
	1	5 3	1 1	Q					
1	2	1 FLUX CAL	CULATION		(2 P	ER CENT)	0.02	0.02	0.02
1	2	1 DENSITY	(P.B)		(1 2)	ER CENT)	0.01	0.01	0.01
1	2	1 ENRICHME	NT(P.B)		(1 P	FR (ENT)	0.01	0.01	0.01
1	2	2 DHTER DI	[AM. (P.B.)	HYDR	A111 - 1	0.012 MM	0.008	0.008	0.008
1	2	3 THICKNES	(8. C) 22		10.008	MM )	n. e	0.0	n n
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1	12			V .	12 0		0 02	0.02	0 0 2
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_	1	3 1	2 0	.0					
1	4	I FLUX(LOC	AL RADIAL	) - ·	(1 P	ER CENT)	0.01	C.01	0.01
1	4	2 PIN PITC	CH (10GRID)	51	(0-13 )	4M	0.0146	0.0274	<b>⊕</b> •0274
1	4	2 ACTIVE L	ENGTH		10.5	CM )	0.0023	0.003	0.003
	1	4 10							
	1	5 DENSITY			(2 PER	CENT)	0.015	0.028	0.028
	1	5 ENRICHME	ENT		(1 PER	CENT)	0.008	0.014	0.014
	1	5 FUEL-CL	ADD. ECCENTI	₹.	(0.6)		0.013	0.028	0.028
	1	5 AXIAL FL	. UX	(2	.5 PEP	CENTI	0.025	0.025	C. 025
	1	6 DENSITY			(2 0)	FR CENT)	32.	2.8	
	1	6 DENSITY	ASYMM. (	5 PFR	CENT )		8.0	2.0	
	1	6 ENRICHME	INT		(1 0	FR CENTI	16.	1.4	
	1	6 ENRICHME	NT ASYMM.	12.5	DED CEN	τ)	4.	n.n	
	1	6 CLADD. TH	ITCKNESS	12.00	10.008	1814 3	n 0	0.9	
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	1		,						
	2	12 0.934	1.	-	.0297	1.07	1.07		
	2	5 3	1 1	0					
2	2	1 FLUX CAL	CULATION	12	PER CE	VT)	0.02	0.02	0,02
2	· 2	I DENSITY	(P.B)	(1	PEP CE	(T)	0.01	0.01	0.01
2	2	L ENRICHME	NT(P,B)	(1	PER CE	NT)	0.01	0.01	0,01
2	2	2 OUTER DI	AM. (P.B.)	HYDR	AIIL.	0.012 MM	0.008	0.008	0.008
2	2	3 THICK NES	S (P.8)	•	10.008	мт )	0.9	0.9	0.0
	2	2 1	1 0	0					
2	3	1 FLUX(CON	TRAL RAN)	12.	PER CH	INT)	6.02	0.02	0.02
2	3	2 ORIFICE	CALIBRATIC	NI 2.3	PEP CI	ENT)	0.023	0.023	n. n <u>2</u> 3
	2	3 1	2 0	ŋ					
2	4	1 FLUX (LC	CAL RADIAL	)(1	PEP CI	ENT)	0.01	0.01	0.01
2	4		H	(0.1	3 (MM)		0.0146	0.0274	0.0276
2	4	2 ACTIVE I	ENGTH		10.5 (	°M )	0.0023	0.003	0.003
<u>د</u>	2	4 10					0.00000		···• · ·
	2	5 DENCITY			12 DE	CENTI	0.015	0 028	0 028
	2	5 ENOTOUME	NIT			CENT)	0.000	0.010	0.014
	2			<b>,</b>	10 Z1 503	e e create a ch	0.012	0.020	0.029
	2	S FOELTOLA	NURDE E CALEMER HIV	<b>`</b> •	10.01	TO CENTY	0.025	0 020	0.005
	2	DENETTY	.UX			TE SERVIT	01/27 ·	1 10 10 10 10 10 10 10 10 10 10 10 10 10	19 <b>6</b> 47 2
	2	O DENSITY				TH UTNIF	54.	<. 7 • 7	
	2	O DENSITY	азуям. (:	) PER			ĕ•0	3.5 • 5.2	
	2	6 ENRICHME			(T D)	-K (FW1)	16.	J.4	
	2	6 ENRICHME	NI ASYMM.	(2.5	PER CEN		4.	0.0	
	2	6 CLADD.TH	ICKNESS		10.008	MM )	0.9	0.9	
	2	6 AXIAL FL	UX		(2.5 P	R CENT)	40.	1.5	
			*						
					· · ·				

	2 2	6 6	CLADD-FUEL ECCENTR. (0. FUEL THERMAL CONDUCT.	6) - (1 PER CENT)	0. 16.7	3.0	
	22	6	HEAT TRANSFER CLADD.FUF HCL-SOD. (3 PER CENT)	L 0.20W/CM2-0C	26.5	0.0	
	2	1	0. 0.0				
	3	12		.04 1.04	1.04		
3	2	1	FLUX CALCULATION	(2 PER CENT)	0.02	0.02	0.02
3	2	1	DENSITY(P.B)	(1 PER CENT)	0.01	0.01	0.01
<u>`</u> 3	2	1	ENRICHMENT(P.B) (1	PFR (FNT)	0.01	0.01	0.01
3	2	2	DUTER DIAM. (P.B.) HYDR	AUL. 0.012 MM	0.008	0.008	0.008
3	2	3	THICKNESS (P.B).	(0.008 MM)	0.0	0.9	0.0
	3	2	1 1 0 0				
3	3	1	FLUX(CONTROL ROD)	(3. PER CENT)	0.03	0.03	0.03
3	3	2	DRIFICE CALIBRATION	(2.3 PER CENT)	0.023	0.023	0.123
	3 -	3					
3	4	1	FLUX (LOCAL RADIÁL)	(1 PER CENT)	0.01	0.01	0.01
3	4	2	PIN PITCH	(0.13 MM )	0.0146	0.0274	0.0274
3	4	2	ACTIVE LENGTH	(0.5 CM)	0.0023	0.003	0.003
	3	4	10	1			
	3	5	DENSITY	(2 PER CENT)	0.015	0.028	0.028
	3	5	ENR ICHMENT	(1 PER CENT)	0.008	0.014	0.014
	3	5	FUEL-CLADD. ECCENTR.	(0.6)	0.013	0.028	0.028
	3	5	AXIAL FLUX	(2.5 PER CENT)	0.025	0.025	0.025
	3	6	DENSITY	(2 PER CENT)	32.	2.8	
	3	6	DENSITY ASYMM. (5 PER	CENT )	. 8.0	0.0	
	3	6	ENRICHMENT	(1 PER CENT)	16.	1.4	
	3	6	ENRICHMENT ASYMM. (2.5	PER (ENT)	4.	0.0	
	3	6	CLADD. THICKNESS	(0.008 MM)	0.9	• 0 • <del>0</del>	
	3	6	AXIAL FLUX	(2.5 PER CENT)	40.	1.5	
	3	6	CLADD-FUEL ECCENTR. (0.	6)	0.	3.0	
	3	6	FUEL THERMAL CONDUCT.	(1 PER CENT)	16.7	0.0	
	3	6	HEAT TRANSFER CLADD.FUE	L 0.20W/CM2-0C	26.5	0.0	
	3	6	HCL-SOD. (3 PER CENT)		0.5	1.2	
	3	11		0 0505 1 0/	1 04		
	4	24		0.0505 1.04	1.04	×.	
	4	, ?			0 02	0.00	0.00
4	2	1	PLUX CALCULATION	(2 PER UENT)	0.02	0.02	0.01
4	2	1		(1 PER CENT)	0.01	0:01	()• ()) ()• ()
4	2	1	CHTED DIAM /D D ) HVDD		0.000	0.01	0.00
4	2	2	THICKNESS (D B)	10 000 MM)	n o	0.000 0.0	a a
-4		່ ວ	$\frac{1}{1} \frac{1}{1} \frac{1}$	(U•U08 mm)	° <b>∦ ● 7</b>	57 <b>•</b> 72	· · · · ·
1.	72	َم <sup>د</sup>		13 DER CENTI	0.03	0 03	0 03
4	3	2	ODIETCE CALIBRATION	(3. DEP CENT)	0.023	0.023	0.023
-	. 4	<u>د</u>	The 2 CALIBRATICHY		(3 <b>●</b> (32) J		•
4	4	້		(1 DER CENT)	0.01	0.01	0.01
4	4	2	PIN DITCH	(0.13 MM)	0.0146	0.0274	0.0274
4	4	2	ACTIVE LENGTH	(0.5 CM)	0.0023	0.003	0.003
*	4	- 4	10				-
	4	5	DENSITY	(2 PER CENT)	0.015	0.028	0.028
	4	5	ENRICHMENT	(1 PER CENT)	0.008	0.014	.014
	4	5	FUEL-CLADD. FCCENTR.	(0.6)	0.013	0.028	0.028
	4	5	AXIAL FLUX	(2.5 PEP CENT)	0.025	0.025	0.025
	4	6	DENSITY	(2 PER CENT)	32 -	2.8	
	4	6	DENSITY ASYMM. (5 PFR	CENT )	8.0	0.0	
	4	6	ENRICHMENT	(1 PER CENT)	16.	1.4	
	4	6	ENRICHMENT ASYMM. (2.5	PER CENT)	4.	0.0	

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4	6 CLADD, THICKNESS (0, 008 MM)	0.9	0.9	
÷				
4	$6 \text{ AXIAL FLUX} \qquad (2.5 \text{ PFP (FNT)})$	40.	1.3	
4	6 (1  ADD - EUEL - ECCENTP, (0, 6)	<b>n</b> .	3.0	
			2.0	
4	6 FUEL THERMAL CUNDUCT. (1 PER CENT)	15.7	$0 \bullet 0$	
4	6 HEAT TRANSFER CLADD FUEL 0.20W/CM2-0C	26.5	0.0	
-1	C HEAT TOMPOSED CENTRAL GELGELGELGELGELGELGE	2000		
4	6 HCL-SDD. (3 PEP CENT)	0.5	1.2	
4	10 0.0			
-7	10. 0.0			
5	30 1. 1. 0.089 1.07	1.07		
5	5 7 1 1 <b>0</b>			
2				
5 2	1 FLUX CALCULATION (2 PER CENT)	0.02	0.02	0.02
 ۲	1 DENCITY(D D) /1 DED CENT)	0 01	0.01	0 01
⊃ <u>८</u>	T DEMOTION TT SEA CENTY	9•01	· · · · · · · · · · · · · · · · · · ·	2 8 🖷 🔧 5 🧎
5 2	1 ENRICHMENT(P.B) (1 PER CENT)	0.01	0.01	0.01
Ê Â		0.000	0.000	0 000
5 2	Z UDIER HIAM (P.B.) HYDRAUL. U.UIZ MM	V.UC8	0.008	11 • 1313 M
5 2	3 THICKNESS (P.B) (0.008 MM)	0.9	0.9	0.0
/ _ <u>_</u>		0.		
5	2 1 1 0 0 $($			
5 2	1 FLUY (CANTON POD) (2 DED CENT)		0.02	- 0.02
2 2	$1  \text{JECA (COMPARE MOD)} \qquad \text{CA EVE COMP}$	SZ ● SZ €		17 ● 127 Au
5 3	2 ORIFICE CALIBRATION (2.3 PER CENT)	0.023	0.023	D. 023
5	2 1 2 0 0		1. N.	
ر				
5 4	1 FLUX (LOCAL RADIAL) (1 PER CENT)	0.01	0.01	0.01
۲. ۲	2 DIM DITCH IA 12 MM	0 0146	0 0274	0 0276
2 4	C RIN RITUL (A.TO WW)	U• 9140	11011214	₩.0214
54	2 ACTIVE LENGTH (0.5 CM)	0.0023	0.003	0.003 .
- ·	Δ 1Δ · · · · · · · · · · · · · · · · · ·			
5	4 10 -			
5	5 DENSITY (2 PER CENT)	0.015	0.028	0.028
		0 000	0 017	
5	5 ENRICHMENT (I PER CENT)	17. OCH	₹2 <b>.</b> 11 <u>1</u> 14	0.14
5	5 EUEL-CLADD, ECCENTR, $(0, 6)$	0.013	0.028	0.028
-				····
5	5 AXIAL FLUX (2.5 PER CENT)	0.025	0.025	0.025
5	6 DENSITY (2 DED CENT)	32	2.8	<i>2</i>
		2 C •	2.00	
5	6 DENSITY ASYMM. (5 PER CENT)	8.0	-0.0	
5	6 ENDICHMENT (1 DED CENT)	16	1 1	
)	U LINE I CAPTERNY (I FACE SERVER)	T C e	3.87	
5	6 ENRICHMENT ASYMM. (2.5 PER CENT)	4.	0.0	
5	6 CLADD THICKNESS (0 DOG MM)	. A O	0 0	
<i></i>	O CLADE HICKNESS (0.000 mm)	G ● 3	2 · • · · · ·	
5	6 AXIAL FLUX (2.5 PER CENT)	40.	1.5	
5	4 CLADD-EUEL ECCENTR (0.4)	0	2 0	
2	O CLADD-FUEL FOCENTR. (0.07	×J ●	2.0	
5	6 FUEL THEPMAL CONDUCT. (1 PER CENT)	16.7	0.0	
E	4 HEAT TRANSEER CLARR EVEL & 2014/CM2-00	24 5	0 0	
2	O BEAT TRANSFOR CLEDTFFUEL GEZONACHREQU	20.00	N. ● N	
5	6 HCL-SOD. (3 PER CENT)	0.5	1.2	
Ē				
2				
6	24 0.894 1. 0.1555 1.09	1.09		
,				
6	D D I I I I			
6 2	1 FLUX CALCULATION : (2 PER CENT)	0.02	0.02	0.02
		0.01	0.01	0 01
0 Z	I RENZITIAN OF CLARK CENTL	↓書書書書 1 1 1 1	25.0 43.T	10 <b>.</b> 11.
6 2	I ENPICHMENT (P.B) (1 DER CENT)	0.01	0.01	0.01
~ ~		0 000	0.000	0.000
6 Z	Z UUTER DIAM. (".B.) HYDRAUL. U.UIZ MM	N. 0028	0.008	5 * <b>a</b> \$2*2 **
6 2	3 THICKNESS (P.R) (0.008 MM)	0.0	0.0	0.0
		· · · · ·	· •	
.6	2  1  1  0  0  0  0  0  0  0  0			
6 2	1 FILLY (CONTROL POD) (2.5 PER CENT)	0.025	0.025	0.025
			0	
6 3	2 ORIFICE CALIBRATION (2.3 PER CENT)	9.023	· 0. 023	D. 1773
6	3 1 2 0 0			
5 4	A FUUX (LOUAL RADIAL) (1 PER CENT)	0.01	0.91 ·	0.11.
6 1.	2 DIN DITCH (A 12 MM)	0,0146	0.0274	0.0274
			······································	• · · · · · · · · ·
6 4	2 ACTIVE LENGTH (0.5 CM)	0.0023	0.003	. n. <u>n.n.</u> 2
4	4 10 to the second s	•	2	
0			· · · · · · ·	
-6	5 DENSITY (2 PER CENT)	0.015	0.028	0.728
6	5 ENDICHMENT /1 DED CENTI	0.008	0.014	0.014
0	→ Flow TOHNOTAL		···• (7,1**	20 X X X
6	5 FUEL-CLADD.ECCENTR. (0.6)	0.013	0.028	n.028
4	E AVIAL CLUV 12 E DED CENTI	0 025	0.025	0.025
0	D AATAL DEVIA 12.0 PER UDWEE	2.04.2	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	5. •
6	6 DENSITY (2 PEP CENT)	32.	2.8	
~	C DENCITY ACMIN JE DED CENT N		0.0	
5			1년 🔶 1 문 👘	

-	91	-
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	6 6 6 6 6 6 6 6 6	6 ENRICHMENT (1 PEP CENT) 6 ENRICHMENT ASYMM. (2.5 PER CENT) 6 CLADD.THICKNESS (0.008 MM) 6 AXIAL FLUX (2.5 PER CENT) 6 CLADD-FUFL ECCENTR. (0.6) 6 FUEL THERMAL CONDUCT. (1 PER CENT) 6 HEAT TRANSFER CLADD.FUEL 0.20W/CM2-0C	16. 4. 0.9 40. 0. 16.7 26.5	1.4 0.0 0.9 1.5 3.0 0.0	
	6	6 HCL-SOD. (3 PER CENT)	0.5	1.2	
	6	10. 0.0			
	7	42 0.715 1. 0.237 1.12	1.12		
7	2	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.025	0.025	0 025
7	2	1 DENSITY(P.R) (1. DER CENT)	0.01	0.01	0.01
7	2	1 ENRICHMENT(P.R) (1. DER CENT)	0.01	0.01	0.01
7	2	2 OUTER DIAM. (P.B.) HYDRAUL. 0.012 MM	0.008	0.008	01008
7	2	3 THICKNESS (P.B.) (0.008 MM)			
	7	2 1 1 0 0		•	
7	3	1 FLUX (CONTROL ROD) (3 PER CENT)	0.03	0.03	0.03
7	3	2 ORIFICE CALIBRATION (2.3 PER CENT)	0.023	0.023	0.023
	7	3 1 2 0 0			
7	4	1 FLUX (LOCAL RADIAL) - (1. PER CENT).	0.01	0.01	0.01
7	4	2 PIN PITCH (0.13 MM)	0.0146	0.0274	6.0274
7	4	2 ACTIVE LENGTH (0.5 CM)	0.0023	0.003	0.003
	7	4 10			
	7	5 DENSITY (2 PER CENT)	0.015	0.028	0.028
	7	5 ENRICHMENT (1 PER CENT)	0.008	0.014	0.014
	-	5 FUEL-CLADD. ECCENTR. (0.6)	0.013	0.028	0,028
	/	5 AXIAL FLUX (2.5 PER CENT)	0.025	· 6.025 ·	B. <b>•</b> €2.5
	1	6 DENSITY (2 PER CENIT	32.	2.8	
	ן ד	6 DENSILY ASYMM. (5 PER LENI)	8.0	0.0	
	7	2 ENDICHMENT ACVMM (2 5 DED CENT)	10	1.4···	
	7	6 CINDICHPENS ASTERS (2.5 PER CENT) 6 CINDICHTCKNESS (0.000 MM)	4.	· · · · · · · · · · · · · · · · · · ·	
	7	6  AYTAL FLAY	40.	1 5	
	7	6 CLADD-FUEL ECCENTE. (D.A)	0.	3.0	
	7	6 FUEL THERMAL CONDUCT. (1 PER CENT)	16.7	0.0	
	7	6 HEAT TRANSFER CLADD EVEL 0. 20W/CM2-0C	26.5	0.0	
	7	6 HCL-SOD. (3 PER CENT)	0.5	1.2	
	7	10. 0.0			
	9	1 1 4 3			
0	1 -	1 MEASUREMENT ERROR (2.5 PER CENT)	0.025	0.025	0.025
0	1	2 SODIUM HEAT CAPACITY (1 PER CENT)	0.01	0.01	A.01
0	1	3 FUEL THERMAL COND. (4.5 PER CENT)	62.	Ο.	10 <b>.</b>
0	1	3 CLADD. THEP MAL COND. (1 PER CENT)	0.46	0.46	18 <b>.</b>
0	1	3 HEAT TRANSFER CL-FUEL 0.10 W/CM2-OC	11.9	0.	<b>?</b> •
0	1	3 HEAT TRANSFER CL-SOPILIM (3 PER CENT)	0.50	0.50	
0	1	4 INLET SODIUM TEMP. 1 OC	1.	1.	1.
0	1	4 FUEL MELTING POINT 60 OC	60.	<u>o</u> .	<u>^</u> .
0	. 1	4 CLADD.CRITICAL TEMP. 7. OC	· C •	1.	

LAST

## VI. Analysis of the Results

#### VI.1 Code outputs

The outputs of the program for the reactor Na-2 are presented in the following. First the general data are printed, then the variables and the uncertainties of every zone. For sake of briefness, only the outputs corresponding to the zone of maximum power (zone 5) are reported here. From the original zone data, the program evaluates and prints the maximum nominal temperature of every zone, together with the abscissa of the maximum in the axial profile: it ought to be noted that these nominal maximum temperatures take into account also the systematic factors FSYSP, and FSYSC. The original uncertainties are printed together with the standard deviations calculated at the abscissa of the maximum temperature. Taking into account the actual value of the power and temperature span of the zone-then the program prints the reference temperatures - that is the maximum temperatures of fuel, cladding and coolant in the core - and the zones in which they occur. The standard deviations of the core uncertainties are then calculated corresponding to the reference temperatures: for instance, in the present example, the standard deviation of the fuel temperature is calculated for the temperature of the zone 5, whereas the zones 1 and 7 are considered for the cladding and coolant temperatures respectively.

Then the results of fuel hot spot analysis are printed and successively those of cladding hot spot and hot channel analysis. The notation of the intermediate variables of the reduction procedure follows the given symbols. The hot spot factors are printed together with their corresponding confidence level, probability of occurrence of at least one hot spot, and core maximum temperatures.

Then the variables of the pessimizing scheme for evaluating the probability of n hot subassemblies are printed. At last the probability (PROBT) that at least one subassembly exceeds the maximum temperature corresponding to the indicated safety factor in the pessimistic model is printed together with the probabilities that this temperature is exceeded in exactly <u>n</u> subassemblies (n within 1 and 10). Table 14 summarizes the results for a confidence level of 97.7 %, for the assumed hot spot length of 1 cm. The last column shows

the ratio of the probability of two subassemblies to that of only one subassembly exceeding the maximum expected temperatures in Table 14.

	Nom.Temp. (°c)	F hs (20)	Max.expected temperature	Prob.of ex- ceed. Vcrit	Prob2 Prob1
Fuel	2235	1.29	2777	9.3%	0.46
Cladding	640	1.27	710	10.3 %	0.39
Coolant	618	1.24	675	0	0.33

	Table 1	14	Results	for	Reactor	Na-2
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The outputs of the program are completed by diagrams. Namely Fig. 18 shows the cladding hot spot factor versus confidence level, analogous diagrams are drawn for fuel and hot channel factors. In these diagrams the abscissa scale is divided linear in  $\lambda$ , therefore a normal distribution is represented by a straight line. Figs. 19, 20 and 21 show the exact total probability of exceeding the maximum temperature corresponding to the hot spot factor in the abscissa (dotted curve indicated with  $\Delta$ , together that corresponding to the pessimistic model and the probability of exactly 1,2,3 or 10 hot subassemblies.

At constant total failure probability there is a difference of circa 5 % in the hot spot factor between the two models: however, it is possible to assume conservatively that the ratios among the individual components of the total probability are the same in both models. The probability of exactly 1 hot subassembly converges more rapidly to that of <u>at least one</u> for hot channel and cladding hot spot than for fuel hot spot. This depends upon the relative importance of the core and subassembly uncertainties in the scheme of Fig. 9 as it will be shown in the following. The last diagram in Fig. 22 resumes the whole thermal design of the core: in this diagram the nominal, critical and expected temperatures are reported versus the confidence level.

The computation time for this case resulted to be 1.32 minutes (IBM 360/65).
#### SHOSPA-STATISTICAL HOT SPOT AND HOT CHANNEL ANALYSIS

REAKTOR NA-2

NUMBER OF ZONES WITH IDENTICAL SUBASSEMBLIES NZ= 7 NUMBER OF CHANNELS CONSIDERED FOR FUEL HOT SPOT NP= 169 NUMBER OF CHANNELS CONSIDERED FOR CLADDING HOT SPOT NCL= 336 TOTAL NUMBER OF SUBASSEMBLIES NS= 150 NUMBER DF CHANNELS IN A SUBASSEMBLY NC= 336

ACTIVE LENGTH XL= 95.00 (CM) FUEL HOT SPOT LENGTH XLSPF= 1.00 (CM) FUEL HOT SPOTS AT HEIGHT Z NSPF= 1

CLADDING HOT SPOT LENGTH XLSPC= 1.00 (CM) CLADDING HOT SPOTS AT HEIGHT Z NSPCL= 3

EXTRAPOLATED LENGTH XLEX= 132.00 (CM)

COOLANT INLET TEMPERATURE TI= 380.00 (0C) NOMINAL MAXIMUM COOLANT TEMPERATURE SPAN IN CORE DTC= 212.40 (0C) NOMINAL MAXIMUM TEMPERATURE DIFFERENCE CLADDING-COOLANT IN CORE DTCL= 58.60 (0C) NOMINAL MAXIMUM TEMPERATURE DIFFERENCE FUEL-COOLANT IN CORE DTF= 1623.60 (0C)

FUEL CRITICAL TEMPERATURE ITFCR= 2700 (0C)

CLADDING CRITICAL TEMPERATURE ITCLCR= 700 (0C)

COOLANT CRITICAL TEMPERATURE ITCCR= 880 (OC)

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ZONE 5

NUMBER OF SUBASSEMBLIES NSZ= 30 RATIO OF ZONE MAX.SPEC.POWER TO CORE MAX.SPEC.POWER HP= 1.0000 RATIO OF ZONE TO CORE MAX. TEMP. SPAN HC= 1.0000 SYSTEMATIC POWER FACTOR FSYSP= 1.0700 SYSTEMATIC TEMP.SPAN FACTOR FSYSC= 1.0700 (MAX.TEMP.SPAN -AVERAGE TEMP.SPAN) /MAK.TEMP.SPAN -IN A SUBASSEMBLY H= 0.0890 NOM.MAX.TEMP.SPAN DTCZ= 227.27 (OC) NOM.MAX.TEMP.DIFF. CLADDING-COOLANT DTCLZ= 62.70 (OC) NOM.MAX.TEMP.DIFF. FUEL-COOLANT DTFZ= 1737.25 (OC)

NOM.MAX.CLADDING TEMP. TCZMAX= 634.02 (OC) AT HEIGHT ZCZMAX= 47.00 (CM)

NOM.MAX.FUEL TEMP. TFZMAX= 2235.42 (OC) AT HEIGHT ZFZMAX= 3.00 (CM)

ZONE UNCERTAINTIES

UNCERTAINTY	en Norsen (199	-TNPUT-	FUEL	CLADDING	COOL ANT	INPUT UNIT	-DUTRUT- FHEL(OC)	CLABD. (OC)	CODE . (0C)	
FLUX CALCULATION	(2	PER CENT)	0.0200	0.0200	0.0200	RFL.TO (T-TT)	37.1084	5.0804	4.5454	1
DENSITY(P.B)	(1	PER CENT)	0.0100	0.0100	0.0100	REL.TO (T-TI)	18.5542	2.5432	2.2727	ف
ENR ICHMENT (P.B)	:(1	PER CENT)	0.0100	0.0100	0.0100	REL.TO (T-TI)	18.5542	2.5402	2.2727	6
OUTER DIAM. (P.B.)	HYDRAUL .	0.012 MM	0.0080	0.0080	0.0080	REL.TO DTC	0.9808	1.8130	1.8181	1
THICKNESS (P.B)	(0.0	( MM 800	0.9000	0.9000	0.0	(00)	0.9605	0.4208	3.0	•
TOTAL ZONE UNCERTAI	NTY-ST.DEV	/•			p <sub>man</sub> whe plate days days days and data days age as		45.4690	6.4946	5.8563	

	SUBASSEMBLY U	NCERTAINTI	IES						
UNCERTAINTY	-INPUT-	FUEL	CLADDING	COOLANT	INPUT UNIT	-OUTPUT-	FUEL(OC)	CLADD.(00)	chal.(ac)
FLUX (CONTROL ROD) ORIFICE CALIBRATION	(2. PER CENT) (2.3 PER CENT)	0.0200 0.0230	0.0200 0.0230	0.0200 0.0230	REL.TO (T-TI) REL.TO DTC	, ,	37.1084 2.8197	5.0804 5.2123	4.5454 5.2272
TOTAL SUBASSEMBLY UNCER	TAINTY-ST.DEV.	-188 war 199 199 199 199 199 199 199 199 199 19					37.2153	7.2787	6.9273

ſ	CHANNEL UNCER	TAINTIES			ź				
UNCERTAINTY	-INPUT-	FUEL	CLADDING	COOLANT	INPUT UNIT -OL	JTRUT- FUFL(OC)	CLADD.(0C)	CODL.(CC)	
FLUX (LOCAL RADIAL) PIN PITCH ACTIVE LENGTH	(1 PER CENT) (0.13 MM) (0.5 CM )	0.0100 0.0146 0.0023	0.0100 0.0274 0.0030	0.0100 0.0274 0.0030	RFL.TO (T-TI) RFL.TO DTC REL.TO DTC	18.5542 1.7899 0.2829	2.5402 6.2095 0.6799	2.2727 6.2271 0.6818	
TOTAL CHANNEL UNCERTAIL	NTY-ST.DEV.					18.6424	6.7433	6.6639	

# COOLANT LOCAL UNCERTAINTIES

UNCERTAINTY (REL.SPECIFIC ST.DEV.)	FOR FUEL HOT SPOT	FOR CLADDING HOT SPOT	FOR HOT CHANNEL	
DENSITY (2 PER CENT) ENRICHMENT (1 PER CENT) FUEL-CLADD.ECCENTR. (0.6)	0.0150 0.0080 0.0130	0.0280 0.0140 0.0280	0.0280 0.0140 0.0280	
TOTAL CODIANT SPECIFIC UNCERTAINTY (RELAST-DEV.)	0.0250	0.0250	0.0489	

# TEMP.DROPS LOCAL UNCERTAINTIES

UNCERTAINTY-SPEC.ST.DEV. COCT -INPUT-	FUEL HOT SPOT	CLADD. HOT SPOT	OUTPUT-	FUEL HOT SPOT	CLADE. HOT SPE	r
DENSITY (2 PER CENT)	32.0000	2.8000		34.2400	2,9960	
DENSITY ASYMM. (5 PER CENT )	8.0000	0.0		8,5600	0.9	
ENRICHMENT (1 PER CENT)	16.0000	1.4000	, >	17.1200	1.4980	
ENRICHMENT ASYMM. (2.5 PER CENT)	4.0000	0.0		4.2800	0.0	
CLADD.THICKNESS (0.008 MM)	0.9000	0.9000		0.9630	0.9630	
AXIAL FLUX (2.5 PER CENT)	40.0000	1.5000		42.8000	1.6050	
CLADD-FUEL ECCENTR. (0.6)	0.0	3.0000		0.0	3.2107	
FUEL THERMAL CONDUCT. (1 PER CENT)	16.7000	0.0		17.8690	0.0	
HEAT TRANSFER CLADD.FUEL 0.20W/CM2-0C	26.5000	0.0	1	28.3550	0.0	. ,
HCL-SOD. (3 PER CENT)	0.5000	1.2000		0.5350	1.2843	, i
TOTAL TEMP.DROPS SPECIFIC UNCERTAINTY-ST.DEV.	(OC)-	**************************************		67.1820	5.1649	

LOCAL UNCERTAINTIES ON CRITICAL TEMPERATURES: FOR FUEL= 10.0000(0C)

FOR CLADDING= 0.0

0.0 (00)

FOR FUEL HOT SPOT THE REFERENCE ZONE IS THE ZONE 5 THE REF.FUEL TEMPERATURE IS 2235.42 (OC)

FOR CLADDING HOT SPOT THE REFERENCE ZONE IS THE ZONE 1 THE REF.CLADDING TEMPERATURE IS 640.16 (OC)

FOR HOT CHANNEL THE REFERENCE ZONE IS THE ZONE 7 THE REFERENCE TEMP.SPAN IS 237.89 (OC)

CORE UNCERTAINTIES

UNCERTAINTY		-INPUT-	FUEL	CLADD ING	COOLANT	INPUT UNIT	-OUTPUT- FUFL(DC)	CLADD. (OC)	COOL.(0C):
MEASUREMENT ERROR	(2.5 PER	CENT)	0.0250	0.0250	0.0250	REL.TO (T-TI	46.3855	6.5039	5.9472
SODIUM HEAT CAPACITY	(1 PER	CENT)	0.0100	0.0100	0.0100	REL.TO DTC	1.2260	2.3298	2.3789
FUEL THERMAL COND.	(4.5 PER	CENT)	62.0000	0.0	0.0	(00)	66.1709	0.0	0.1
CLADD.THERMAL COND.	(1 PER	CENT)	0.4600	0.4600	. 0.0	(00)	0.4909	0.2134	0.0
HEAT TRANSFER CL-FUEL	0.10 W/C	M2-0C	11.9000	0.0	0.0	(00)	12.7006	0.0	0.0
HEAT TRANSFER CL-SODIUM	(3 PER	CENT)	0.5000 -	0.5000	0.0	(00)	0.5336	0.2319	0.0
INLET SODIUM TEMP.	1 0C		1.0000	1.0000	1.0000	(00)	1.0009	1.0000	1.0000
FUEL MELTING POINT	60 OC		60.0000	0.0	0.0	(00)	60.0000	0.0	0.0
CLADD.CRITICAL TEMP.	7		0.0	7.0000	0.0	(00)	/ <b>0.0</b>	7.0000	0.0
TOTAL CORE UNCERTAINTY -	ST.DEV.		a salih sing nakih 400 kinak waka tana man dana mina mina		in ann 2014 ann ado ann ann ann aite ann ann		101.4619	9.8908	6.4829

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# ALL TEMPERATURES, MEANS AND ST. DEVIATIONS ARE EXPRESSED IN OC

ZONE NS(Z)	TNOM NSPEQ SSPEQ	SCHEQ	MCHEQ	NCHEQ SCHEO*	SSAFO	MSAFQ	SZOFO	MZREO
1 6 2 2 12 2 3 12 2 4 24 1 5 30 2	2225.091 17.410 63.269   2121.053 17.427 59.651   2006.554 17.454 55.663   1903.922 17.463 51.882   2235.420 17.408 63.764	48.896 46.104 43.027 40.139 49.261	111.704 105.342 98.340 91.673 112.575	85.606 45.962   47.409 42.877   35.284 40.015   27.842 37.330   15.914 45.813	47.057 44.936 55.825 52.496 49.756	222.382 199.102 181.139 165.255 191.572	58.651 54.094 57.405 52.232 56.763	280.273 270.573 269.928 265.407 291.204
6 24 2 7 42 1	2082.987 17.443 58.201 1805.733 17.454 48.038	45.000 37.236	102.807 84.869	9.069 41.850 5.887 34.630	53.232 50.965	163.519 127.106	55.792 53.178	265.076 236.126
		nna 1994 una alta des filo dei dei uno esse ess	e ezyn wegn-acam fillen 4025 ezan mann fillef azar vegn gapa g	129 129 129 129 129 129 129 129 129 129		1600 1666 1660 988 989 1697 1699 1699 979 989 989 49		4 Min (Co (ar an an Cin An an an
REF.ZONE REF.TEMP. 5 2235.4199	NZOEQ MZDEQ 1.6506 291.2041	SZOEQ* 56.1953	MCORE 313.7588	SCOPF 114.1091		, ta		
an an an Arthur an Arthur an Arthur An Arthur an Arthur an Arthur an Arthur An Arthur an Arthur an Arthur an Arthur			*		4.			
PROB.OF EXCEEDING	CONFIDENCE LEVEL		FHS	MAX. TEMP				
0.500000E 00 0.308540E 00 0.158658E 00 0.668097E-01 0.227515E-01 0.621021E-02 0.135005E-02 0.232697E-03 0.316501E-04	(0.0SIGMA) 0.500000E 00 (0.5SIGMA) 0.691460E 00 (1.0SIGMA) 0.841342E 00 (1.5SIGMA) 0.933190E 00 (2.0SIGMA) 0.977248E 00 (2.5SIGMA) 0.993790E 00 (3.0SIGMA) 0.998650E 00 (3.5SIGMA) 0.999767E 00 (4.0SIGMA) 0.999968E 00	х. <sup>1</sup>	1.1691 1.1999 1.2306 1.2614 1.2921 1.3229 1.3536 1.3844 1.4151	2549.18 2606.23 2663.29 2720.34 2777.40 2834.45 2891.50 2948.56 3005.61				

THE PROBABILITY OF EXCEEDING THE CRITICAL TEMPERATURE ( 2700 OC) IS=0.09313202

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# EVALUATION OF THE NUMBER OF HOT SUBASSEMBLIES

# ALL SUBASSEMBLIES ARE ASSUMED IDENTICAL TO THOSE IN THE ZONE 1

SUBASSEMBLY DISTRIBUTION -MEAN= 222.3823 ST.DEV.= 47.0565 GLOBAL DISTRIBUTION -MEAN= 59.4436 ST.DEV.= 107.4306

FACTOR	PROBT	PROB1	PROB 2	PR OB 3	PRO84	PR OB 5	PROB6	PROR7	PROB8	PROB9	PPOBLO
1.1691	0.802E 00	0.419E-01	0.260E-01	0.195E-01	0.159E-01	0.136E-01	0.119E-01	0.107E-01	0.975E-02	0.8995-32	0.836F-02
1.1814	0.739E 00	0.488E-01	0.294E-01	0.217E-01	0.175E-01	0.148E-01	0.129E-01	0.115E-01	0.104E-01	0.9555-02	0.883E-02
1.1937	0.667E 00	0.543E-01	0.318E-01	0.231E-01	0.184E-01	0.154E-01	0.144E-01	0.1185-01	0.1065-01	0.969F-02	0.892F-02
1.2060	0.588E 00	0.579E-01	0.330E-01	0.236E-01	0.185E-01	0.154E-01	0.132E-01	0.116E-01	0.104E-01	0.940E-02	0.861E-02
1.2183	0.505E 00	0.590E-01	0.327E-01	0.229E-01	0.178E-01	0.147E-01	0.125E-01	0.109E-01	0.968E-02	0.8725-02	0.7955-02
1.2306	0.422E 00	0.576E-01	0.310E-01	0.214E-01	0.164F-01	0.134E-01	0.113F-01	0.977F-02	0.864E-02	0.7745-02	0.7025-02
1.2429	0.343E 00	01539E-01	0.281E-01	0.191E-01	0.144E-01	0.116F-01	0.975E-02	0.839E-02	0.737E-02	0.656E-02	0.592E-02
1.2552	0.270E 00	0.482E-01	0.244E-01	0.162E-01	0.122E-01	0.970E-02	0.806E-02	0.689E-02	0.601E-02	0.532E-72	0.478E-02
1.2675	0.205E 00	0.412E-01	0.202E-01	0.132E-01	0.979E-02	0.773E-02	0.6375-02	0.540E-02	0.468F-02	0.413E-02	0.369E-02
1.2798	0.151E 00	0.338E-01	0.160E-01	0.103E-01	0.7535-02	0.5895-02	0.481E-02	0.405E-02	0.349F-02	0.306E-02	0.272E-02
1.2921	0.107E 00	0.265E-01	0.122E-01	0.770E-02	0.554E-02	0.429E-02	0.348F-02	0.291E-02	0.2495-02	0.217E-02	0.192F-02
1.3044	0.741E-01	0.199E-01	0.885E-02	0.549E-02	0.390E-02	0.299E-02	0.2406-02	0.199E-02	0.170F-02	0.147E-02	0.129F-02
1.3167	0.486E-01	0.143E-01	0.615E-02	0.374E-02	0.263E-02	0.1998-02	0.159E-02	0.131E-02	0.1115-02*	* 0.952E-03	0.833E-03
1.3290	0.310E-01	0.987E-02	0.408E-02	0.244E-02	0.169E-02	0.127E-02	0.100E-02	0.820E-03	0.689E-03	0.5.90F-03	0.513F-03
1.3413	0.190E-01	0.650E-02	0.260E-02	0.152E-02	0.104E-02	0.774E-03	0.606E-03	0.4925-03	0.410E-03	0.349F-03	0.302E-03
1.3536	0.112E-01	0.411E-02	0.158E-02	0.9096-03	0.613E-03	0.451E-03	0.350E-03	0.282E-03	0.234E-03	0.198E-03	0.170F-03.
1.3659	0.636E-02	0.248E-02	0.919E-03	0.518E-03	0.345E-03	0.251E-03	0.1935-03	0.1556-03	0.127E-03	0.107E-D3	0.917F-04
1.3782	0.347E-02	0.144E-02	0.512E-03	0.283E-03	0.186E-03	0.145E-03	0.102E-03	0.810E-04	0.663E-04	0.555F-04	0.4728-04
1.3905	0.182E-02	0.796E-03	0.2735-03	0.148E-03	0.955E-04	0.680E-04	0.515E-04	0.406E-04	0.3305-04	0.2745-04	D.232E-04
1.4028	0.917E-03	0.423E-03	0.139E-03	0.737E-04	0.471E-04	0.332E-04	0.2496-04	0.1956-04	0.1576-04	0.1375-04	0.109E-04
1.4151	0.443E-03	0.215E-03	0.677E-04	0.352E-04	0.221E-04	0.154E-04	0.1156-04	0.892F-05	0.715E-05	0.5885-05	0.492E-05
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# ALL TEMPERATURES, MEANS AND ST. DEVIATIONS ARE EXPRESSED IN OC

ZONE	NS(Z)	TNOM	NSPEQ	SSPEO	SCHEO	MCHEQ	NCHEO	SCHFQ*	SSAEO	MSAEQ	SZDEO	MZOEÓ
1	6	640.156	21.482	2.835	7.209	5.272	179.277	6.776	8.514	23.320	9.483	33.794
2	12	632.211	19.627	2.685	6.999	4.885	102.194	6.509	8.316	20.971	8.915	34.197
3	12	624.102	17.527	2.391	6.763	4.228	76.379	6.290	9.765	19.114	9.550	34.644
4	24	622.477	14.725	2.264	6.741	3.822	60.810	6.269	9.749	18,129	9.192	36.729
5	30	634.020	23.795	2.840	7.027	5.408	34.160	6.535	8.514	18.845	8.717	35.894
6	24	635.809	17.477	2.503	7.088	4.424	19.655	6.592	9.531	16.422	9.315	34.605
7	42	637.717	10.647	2.047	7.193	3.135	13.052	6.690	10.661	14.046	10.430	36.852
REF.ZONE 1	REF.TEMP. 640.156	N ZO EQ 0 3.7	M) 394 3	ZDEQ 3•7939	SZDEQ* 10+0630	MCORE 43.39	SCN 921 12•	R E 9936				
PROB.OF EXC	EEDING		CONFIDENCI	E LEVEL		FHS	MAX	• TEMP				
0.5000			TGMA) 0.	500000E 00		1.1668		683.55				
0.3085	540F 00	(0.55	IGMA) O.	691460F 00		1,1918		690.04				
0,1586	58F 00	(1.05	IGMA) O.	841342F 00		1.2167		696.54				
0.6680	)97E-01	(1.55	IGMA) 0.	933190E 00		1.2417		703.04				
0.227	51 5E-01	(2.05	IGMA) O.	977248E 00		1.2667		709.54				
0.6210	21E-02	(2.55	IGMA) O.	993790E 00		1.2917		716.03				
0.1350	05E-02	(3.05	IGMAL O.	998650E 00		1.3166		722.53				
0.2326	97E-03	(3.55	IGMA) 0.	999767E 00		1.3416		729.03				
0.3165	501E-04	(4.05	IGMA) 0.	999968E 00		1.3666		735.52				

THE PROBABILITY OF EXCEEDING THE CRITICAL TEMPERATURE ( 700 OC) IS=0.10273242

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### EVALUATION OF THE NUMBER OF HOT SUBASSEMBLIES

#### ALL SUBASSEMBLIES ARE ASSUMED IDENTICAL TO THOSE IN THE ZONE 1

## SUBASSEMBLY DISTRIBUTION -MEAN= 23.3198 ST.DEV.= 8.5138 GLOBAL DISTRIBUTION -MEAN= 8.7458 ST.DEV.= 11.1721

FACTOR	PROBT	PROB1	PROB 2	PROB3	PRO84	PROR5	PROB 6	PPOB7	PP 788	PRORO	PROPIO
1.1668	0.831E 00	0.653E-01	0.442E-01	0.344E-01	0.286E-01	0.247E-01	0.218E-01	0.1975-01	0.179F-01	0.165E-01	9.1535-01
1.1768	0.769E 00	0.783E-01	0.505F-01	0.3825-01	0.311F-01	0.264E-01	0.230F-01	0.204F-01	0.184F-01	0.168F-01	0.154F-01
1.1868	0.696E 00	0.892E-01	0.549E-01	0.402E-01	0.3205-01	0.267E-01	0.229E-01	0.2015-01	0.179F-01	0.1615-01	2.147E-01
1.1968	0.614E 00	0.967E-01	0.566E-01	0.403E-01	0.313E-01	0.256E-01	0.216E-01	0.187F-01	0.165F-01	0.1475-01	0.133F-01
1.2067	0.527E 00	0.998E-01	0.554E-01	0.382E-01	0.290E-01	0.233E-01	0.194=-01	0.166F-01	0.144E-01	0.127F-01	0.114F-01
1.2167	0.439E 00	0.979E-01	0.515E-01	0.344E-01	0.255E-01	0.201E-01	0.165E-01	0.1395-01	0.119E-01	0.104E-01	0.923E-02
1.2267	0.354E 00	0.915E-01	0.455E-01	0.294E-01	0.213E-01	0.165F-01	0.1335-01	0.111E-01	0.939E-02	0.811E-02	0.710F-02
1.2367	0.275E 00	0.813E-01	0.382F-01	0.239E-01	0.168E-01	0.128E-01	0.102E-01	0.834F-02	n.707F-02	0.598E-02	0.510F-02
1.2467	0.207E 00	0.690F-01	0.304F-01	0.184E-01	0.127E-01	0.944E-02	0.738E-02	0.597F-02	0.495F-02	0.418E-02	0.3595-02
1.2567	0.149E 00	0.555E-01	0.230E-01	0.134E-01	0.904E-02	0.660E-02	0.508E-02	0.405E-02	0.331E-02	0.277E-02	0.235F-02
1.2667	0.104E 00	0.426E-01	0.166E-01	0.932E-02	0.612E-02	0.438F-02	0.331E-02	0.2605-02	n.211F-02	0.174-02	n.146F-02
1.2767	0.696E-01	0.311E-01	0.113E-01	0.614E-02	0.393E-02	0.276E-02	0.205E-02	0.1595-02	0.127F-02	0.104F-02	7.864E-03
1.2867	0.447E-01	0.217E-01	0.735E-02	0.384E-02	0.239E-02	0.164E-02	0.120E-02	0.919E-03	0.725F-03	0.586F-03	0.483F-03
1.2967	0.276E-01	0.144F-01	0.454F-02	0.228E-02	0.138F-02	0.930E-03	0.670F-03	0.504F-03	0.393F-03	0.314F-03	1.256F-03
1.3066	0.163E-01	0.914E-02	0.266E-02	0.128E-02	0.758E-03	0.500E-03	0.354E-03	0.262E-03	0.202E-03	0.159E-03	0.129F-03
1.3166	0.927E-02	0.552E-02	0.148E-02	0.686E-03	0.394E-03	0.255E-03	0.1775-03	0.129E-03	0.9835-04	0.768F-34	0.613F-04
1.3266	0.505E-02	0.318E-02	0.785E-03	0.348E-03	0.195F-03	0.123E-03	0.841E-04	0.606F-04	0.4545-04	0.3505-04	0.277E-04
1.3366	0.264E-02	0.175E-02	0.395E-03	0.168E-03	0.912E-04	0.564E-04	0.379F-04	0.269F-04	0.1905-04	0.152F-04	0.1195-04
1.3466	0.132E-02	0.921E-03	0.189F-03	0.769E-04	0.406E-04	0.245E-04	0.162F-04	0.1135-04	0.826F-05	P.623F-05	0.482F-05
1.3566	0.637E-03	0.463E-03	0.859E-04	0.334E-04	0.171E-04	0.101E-04	0.656E-05	0.451F-05	0.3255-05	0.242E-05	1.186F-05
1.3666	0.294E-03	0.222E-03	0.372E-04	0.138E-04	0.684E-05	0.396E-05	0.252F-05	0.171F-05	0.121E-05	0.895E-06	3.678F-06

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# ALL TEMPERATURES, MEANS AND ST. DEVIATIONS ARE EXPRESSED IN OC

ZONE	NS(Z)	TNOM	NSPEQ	SSPEQ	SCHEO	MCHEQ	NCHEQ	SCHF0*	SSAFO	MSAFO	S70F0	MZDEO
1	6	613.640	0.0	0.005	6.851	0.0	187.464	6.440	8.106	17.247	8.812	27.219
2	12	607.268	0.0	0.005	6.664	0.0	109.370	6.197	7.927	15.463	8.289	28.071
3	12	600.896	00	0.005	6.477	0.0	82.007	6.024	9.182	34.412	8.864	29.014
4	24	600,896	0.0	0.005	6.477	0.0	65.005	6.024	9.208	13.897	8.569	31.464
5	30	607.268	0.0	0.005	6.664	0.0	36.886	6.197	8.082	12.935	8.047	29.110
6	24	611.516	0.0	0.005	6.788	0.0	21.111	6.313	9.031	11.691	8.663	28.921
7	42	617.888	0.0	0.005	6.975	0.0	13.855	6.487	10.163	10.765	9.790	32.505
			د من هم که که هم منت که چه د									
REF.ZONE	REF. TEMP.	NZOFO	M	10F0	S70F0*	MCDRE	scr	RE	i.			
7	617.887	9 1.50	519 32	2.5060	9.5068	35.9	294 11.	0113				
									i			
PROB.OF EXC	EEDING	·	ONFIDENCE	LEVEL		FHS	MAX	TEMP		- -		
0.5000	00E 00	(0.05)	GMA) 0.	500000E 00		1.1510		653.82	i			
0.3085	40E 00	(0.55)	IGMAI) 0.0	591460E 00		1.1742		659.32				
0.1586	58E 00	(1.05)	IGMA) 0.8	341342E 00		1.1973		664.83				
0.6680	97E-01	(1.55)	IGMA) 0.9	33190E 00		1.2205		670.33				
0.2275	15E-01	(2.05)	IGMA) 0.	977248E 00		1.2436		675.84				
0.6210	21E-02	(2.55)	IGMA) 0.	993790E 00	·	1.2668		681.35				
0.1350	05E-02	(3.05)	IGMA) 0.9	998650F 00		1.2899		686.85				
0.2326	97E-03	(3.55)	IGMA) 0.9	999767E 00		1.3130		692.36				
0.3165	01E-04	(4.05	IGMA) 0.9	999968E 00		1.3362		697.86				

THE PROBABILITY OF EXCEEDING THE CRITICAL TEMPERATURE ( 880 OC) IS=0.0

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#### EVALUATION OF THE NUMBER OF HOT SUBASSEMBLIES

ALL SUBASSEMBLIES ARE ASSUMED IDENTICAL TO THOSE IN THE ZONE 7

SUBASSEMBLY DISTRIBUTION -MEAN= 10.7650 ST.DEV.= 10.1635 GLOBAL DISTRIBUTION -MEAN= 9.3246 ST.DEV.= 8.5269

FACTOR	PROBT	PROB 1	PROB2	PROB3	PROB4	PR OB5	PROB6	PROB7	PROB8	PROBA	PROBIO
1.1510	0.879E 00	0.797E-01	0.6205-01	0.5146-01	0.441E-01	0.387E-01	0.345E-01	0.311E-01	0.283F-01	0.260F-01	0.239E-01
1.1603	0.825E 00	0.101E 00	0.733E-01	0.580E-01	0.4805-01	0.409E-01	0.356E-01	0.3146-01	0.283E-01	0.252F-01	0.228E-01
1.1695	0.758E 00	0.121E 00	0.816E-01	0.615E-01	0.491E-01	0.406E-01	0.344F-01	0.297E-01	0.260F-01	0.229F-01	0.205E-01
1.1788	0.679E 00	0.136E 00	0.8555-01	0.614E-01	0.4725-01	0.378E-01	0.312E-01	0.263E-01	0.226E-01	0.196E-01	0.172F-01
1.1881	0.592E 00	0.146E 00	0.845E-01	0.575E-01	0.426E-01	0.331E-01	0.266E-01	0.219F-01	0.184E-01	0.1575-01	0.135E-01
1.1973	0.499E 00	0.147E 00	0.786E-01	0.508E-01	0.361E-01	0.272E-01	0.2135-01	0.1715-01	0.141E-01	0.118E-01	0.998E-02
1.2066	0.408E 00	0.141E 00	0.689E-01	0.421E-01	0.287E-01	0.210E-01	0.160F-01	0.126E-01	0.101E-01	0.830E-02	0.691E-02
1.2158	0.321E 00	0.128E 00	0.569E-01	0.329E-01	0.215E-01	0.152E-01	0.1125-01	0.863E-02	0.681E-02	0.548E-02	3.449F-02
1.2251	0.244E 00	0.110E 00	0.443E-01	0.2415-01	0.1515-01	0.103E-01	0.744F-02	0.5575-02	0.430F-02	0.3395-02	0.273E-02
1.2344	0.178E 00	0.895E-01	0.326E-01	0.167E-01	0.100E-01	0.659E-02	0.461E-02	0.3375-02	0.255E-02	0.1975-02	0.156E-02
1.2436	0.125E 00	0.693E-01	0.226E-01	0.108E-01	0.622E-02	0.396E-02	0.269E-02	0.1925-02	0.142E-02	0.1078-02	0.835E-03
1.2529	0.843E-01	0.509E-01	0.148E-01	0.664E-02	0.363E-02	0.223E-02	0.147E-02	0.102E-02	0.738E-03	0.550F-03	0.419E-03
1.2621	0.547E-01	0.356E-01	0.912E-02	0.3835-02	0.200E-02	0.118E-02	0.7558-03	0.5125-03	0.361E-03	0.263F-73	0.197E-03
1.2714	0.341E-01	0.237E-01	0.532E-02	0.208E-02	0.103E-02	0.587E-03	0.364E-03	0.240E-03	0.166E-03	0.1195-03	0.871E-04
1.2806	0.204E-01	0.151E-01	0.293E-02	0.106E-02	0.501E-03	0.274E-03	0.165E-03	0.1065-03	0.714E-04	0.500F-04	0.361E-04
1.2899	0.117E-01	0.914E-02	0.153E-02	0.511E-03	0.229E-03	0.120E-03	0.700E-04	0.438F-04	0.289E-04	0.198E-04	0.14DE-04
1.2992	0.651E-02	0.529E-02	0.751E-03	0.2326-03	0.983E-04	0.496E-04	0.280E-04	0.170E-04	0.109F-04	0.734E-75	0.510F-05
1.3084	0.347E-02	0.294E-02	0.350E-03	0.989E-04	0.397F-04	0.192E-04	0.105E-04	0.6208-05	0.389F-05	0.256 -05	0.1746-05
1.3177	0.179E-02	0.1568-02	0.154E-03	0.398E-04	0.151E-04	0.700E-05	0.369E-05	0.2125-05	0.1305-05	0.835E-06	2.558F-06
1.3269	0.888E-03	0.797E-03	0.644E-04	0.151E-04	0.539E-05	0.240E-05	0.1226-05	0.682F-06	0.407E-06	0.256E-26	0.168E-06
1.3362	0.425E-03	0.391E-03	0.255E-04	0.542F-05	0.1825-05	0.772E-06	0.379F-06	0.206F-06	0.1205-06	0.736E-07	1.473F-07
1.3177 1.3269 1.3362	0.179E-02 0.888E-03 0.425E-03	0.156E-02 0.797E-03 0.391E-03	0.154E-03 0.644E-04 0.255E-04	0.398E-04 0.151E-04 0.542E-05	0.151E-04 0.539E-05 0.182E-05	0.700E-05 0.240E-05 0.772E-06	0.369E-05 0.122E-05 0.379E-06	0.212E-05 0.682E-06 0.206E-06	0.133E-05 0.497E-06 0.120E-06	0.835E-06 0.256E-06 0.736E-07	0.558F-06 0.168E-06 0.473E-07

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# VI.2 Influence of spot length

Fig. 23 shows the hot channel and hot spot factors at a confidence level of 25 (97.7 %) as a function of the assumed spot length  $(1_s)$ . As stated at item II.4.4, the hot channel factor does not depend upon  $1_s$ . The dependence of the fuel hot spot factor on  $1_s$  is larger than that of the cladding hot spot factor: this indicates that the local uncertainties are more important for the fuel than for the cladding (see next item). The very high values of the hot spot factors for little  $1_s$  can be explained by taking into account that for  $1_s \rightarrow 0$  both the number of spots as the local standard deviation  $(\frac{6^s}{\sqrt{1}})$  tend to infinite.Such large factors are however not to be expected in reality: in fact for very small values of  $1_s$  axial conductivity plays a not negligible role in the determination of fuel and cladding temperature, moreover the local standard deviation in any practical cases will tend to a certain finite maximum value rather than to the theoretical infinite value as assumed.

Although some doubts might still exist on the assumed value of the uncertainties, which should be subject to further research for a better assessment, some conclusions can be drawn for the safety of the thermal design of a reactor such as Na-2. As it results from Fig. 23, the critical temperature of the fuel is expected to be reached - at 97.7% confidence level, which, referred to the whole reactor, might be a quite acceptable one - by a length of circa 2 cm: taking into account the actual behaviour of the fuel with the formation of the central channel and the consequent decreasing of the nominal temperature, the safety of such a | fuel thermal design can be considered satisfactory. For the cladding, however, it must be observed that the critical temperature 700 °C is expected to be exceeded also by very large cladding lengths; moreover the cladding safety is decreased by the occurrence of hot spots of little size; therefore it is advisable to modify the thermal design in such a way that the nominal maximum temperature of the cladding in the core will be decreased of circa 10 - 15 °C.



Fig.23 Hot Spot Factors vs Spot Length

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As already stated, one of the main improvements that the present method gives to hot spot analysis is the possibility of a precise assessment of the influence of any uncertainty on the total uncertainty in the maximum temperatures. This assessment allows the designer to choose the parameters for which better tolerances must be required in order to obtain a better performance of the reactor.

Since the most limiting uncertainties within every group of uncertainties are of immediate determination, it was examined the influence of the different groups of uncertainties on the hot spot factors. Namely, the standard deviation of the core uncertainties  $(\mathcal{G}_{C})$  was put equal zero; correspondingly the new value of the hot spot factors were calculated. The consequent decreasing of the hot spot factors indicates which is the maximum possible advantage, which can be expected if the core uncertainties were actually zero. The same procedure was followed for the other types of uncertainties.

The results are reported in Table 14 for fuel temperature and in Table 15 for cladding temperature. In this tables a spot length of 1 cm was assumed and the factors are given at 97.7 % confidence level.

From Table 14, it results that the groups of uncertainties which are the most limiting for high power rating in respects to fuel temperature are the core and the local uncertainties: in fact if  $\mathfrak{S}_{c}$  were zero the hot spot factor would be reduced by about 7 %, and if  $\mathfrak{S}_{r}$  were zero, it would be reduced by about 10 %, whereas negligible reductions would be provoked, if the other uncertainties were zero. It is also to note that the local uncertainties are more limiting than the core uncertainties, although the value of local standard deviation  $(3.4 \% \text{ of the maximum temperature difference } \overline{\vartheta}_{f} - \overline{\vartheta}_{j})$  is 2 % lower than the core standard deviation (5.5 %).

This results, as already stated in  $/ 1_7$ , confirms that the more important uncertainties are those which have a larger number of independent occurrences; consequently the usually adopted semistatistical methods, which assume the uncertainties acting on a whole core as deterministic ones, are clearly wrong.

Reactor Na-2 l <sub>s</sub> = 1 cm	Core	Zone	Subassembly	Channel	Local
total st.dev. (°C)	6 <sub>c</sub> = 101.5	6 = 45.5 z = 45.5	$\tilde{s} = 37.2$	<sub>ch</sub> = 18.6	r = 63.3
total st.dev. (% of $\overline{\vartheta}_{f}^{-\overline{\vartheta}}_{i}$ )	5.5	2.4	2.0	1.0	3.4
$F_{hs}$ for $G = 0$ (97.7%) (Actual $F_{hs} = 1.29$ )	1.225	1.277	1.271	1.286	1.186
Expected max.temp. for $\sigma = 0$ (°C) (Actual max.temp. 2777 (97.7%) ) Nominal temp. 2235	2 <b>6</b> 54	2750	2738	2766	2580
Main uncertainties (st.dev. <sup>O</sup> C)	Fuel therm.conduct. 66 Fuel melting point 60 Power measurement 46	Flux Calcul. 37 Density <sup>*</sup> 18 Enrichment <sup>*</sup> 18 *(Production batch)	Flux(Control Rod) 37	Flux(local rad.) 18	<u>Axial flux</u> 39 <u>Density</u> 32 Heat transfer Claddfuel 26

Table 14 Influence of the different groups of uncertainties on fuel hot spot factor

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Reactor Na-2 l = l cm s	Core	Zone	Subassembly	Channel	Local
total st.dev. ( <sup>0</sup> C)	6 = 9.9	<sup>6</sup> z = 6.6	$\frac{6}{s} = 7.5$	$\frac{G}{ch} = 6.9$	6 = 2.8 r
total st.dev. (% of $\tilde{\vartheta}_{cl} - \tilde{\vartheta}_{i}$ )	3.8	2.5	2.9	2.7	1.1
$F_{hs}$ for $G=0$ (97.7%) (Actual $F_{hs}=1.27$ )	1.232	1.242	1.215	1.215	1.248
Expected max.temp. for $\mathfrak{S} = 0$ (°C) Nom. temp. 640 (Actual max.temp. 710 (97.7%)	700	703	696	696	705
Main Uncertainties (St.dev. <sup>O</sup> C)	Cladd.crit.temp. 7 Power measurement 6.5	Flux Calcul. 5 Density <sup>*</sup> 2.6 Enrichment <sup>*</sup>	Flux (Control Rod) 5 Orifice calibratio 5	<u>Pin Pitch</u> 6 n	Eccentricity 1.6 Density 1.6

2.6

\*(Production batch)

#### Table 15 Influence of the different groups of uncertainties on cladding hot spot factor

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Moreover the previous statistical methods give an errated assessment of the importance of the singular uncertainties, because they treat all uncertainties in the same way and therefore the influence of the singular uncertainties depends only on the value of this standard deviation and not on the number of their independent occurrences.

From Table 15, it results that the most important uncertainties for cladding hot spot are the subassembly and the channel uncertainties. From Table 14 and 15 it is possible to point clearly out for which uncertainties better tolerances should be required in order to have lower hot spot factors.

The local uncertainties (Table 15) have a very low influence on cladding hot spot, at least for  $l_s = 1$  cm. These results explain the stronger dependence of the fuel hot spot factor on the spot length in comparison to cladding hot spot factor (Fig. 23).

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